Online Supplement – Prior Distributions

For the priors in Bayesian estimation, the following four distributions were used in the current study: a standard normal distribution (*N*), an inverse gamma distribution (*IG*), an inverse Wishart (*IW*) distribution, and a uniform distribution (*U*). Briefly, the normal distribution is specified as $N(\mu, \sigma^2)$ with density function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}. \tag{1}$$

The inverse gamma distribution is specified as $IG(\alpha, \beta)$ with density function

$$f(x) = x^{-\alpha-1} e^{-\frac{\beta}{x}}. \tag{2}$$

The inverse Wishart distribution is specified as $IW(\Omega, d)$, where Ω is a positive definite matrix of size p, and d is an integer. The density function is

$$f(\Sigma) = |\Sigma|^{\frac{-(d+p+1)}{2}} e^{-\frac{TR(\Omega\Sigma^{-1})}{2}}. \tag{3}$$

The uniform distribution is specified as $U(\alpha, \beta)$, where α and β are two boundaries of the distribution. The density function is

$$f(x) = \frac{1}{\beta-\alpha} \quad for \;\; \alpha \leq x \leq \beta, \qquad 0 \quad otherwise. \tag{4}$$

The noninformative (Mplus default) prior distributions of all residual variances for indicator variables were $IG(-1, 0)$. The prior distributions of the variances and covariance of latent intercept and slope variables were $IG(-1, 0)$ in the presence of a binary outcome variable in a model, and $IW(0, -3)$ without a binary outcome. The priors of coefficients for logistic regressions formed by a binary distal outcome variable followed $N(0, 5)$, which is considered a weakly informative prior in the context of logistic and probit models (Gelman, Jakulin, Pittau, & Su, 2008). The priors of all other parameters (i.e., intercept and slope means, regression coefficients of intercepts and slopes on covariates, and threshold values of the binary distal outcome variable) followed $N(0, \infty)$.

Choosing the distributions for the informative priors of the model parameters followed Gelman (2006) and Liu, Zhang, and Grimm (2016), while specifying the priors followed Depaoli (2013). The mean hyperparameter for each prior was fixed at the corresponding parameter value, and the variance hyperparameter was fixed at 5% of the parameter value. The residual variances for indicators were $IG(11, 5)$. The residual variances for intercept and slope growth factors were $IG(22, 21)$ and $IG(6, 1)$, respectively and the covariance between the growth factors were $U(-0.45, 0.45)$, following Lunn, Jackson, Best, Thomas, and Spiegelhalter's (2012) separation strategy. The means for the intercept and slope were $N(0, 0.05)$ and $N(1, 0.05)$, respectively. The regression coefficients for the intercept and slope were N(0.5, 0.025) and $N(0.3, 0.015)$, respectively, and the logistic regression coefficients of the intercept and slope were $N(0.5, 0.025)$ and $N(0.6, 0.03)$, respectively. Finally, the weakly informative prior of $N(0, 10)$ was used for the threshold parameter because the Depaoli's (2014) strategy did not work very well for this parameter.

For the wrong priors, the mean hyperparameter was increased by 3 and the variance hyperparameter was fixed at 50% of the parameter value. The residual variances for indicators were $IG(9, 28)$. The residual variances for intercept and slope growth factors were $IG(10, 36)$ and $IG(8.4, 23.7)$, respectively, and the covariance between the growth factors were $U(-0.45, 0.45)$, again following Lunn et al.'s (2012) separation strategy. The means for the intercept and slope were $N(3, 1.5)$ and $N(4, 2)$, respectively. The regression coefficients for the intercept and slope were $N(3.5, 1.75)$ and $N(3.3, 1.65)$, respectively, and the logistic regression coefficients of the intercept and slope were $N(3.5, 1.75)$ and $N(3.6, 1.8)$, respectively. Finally, the noninformative prior of $N(0, 100)$ was used for the threshold parameter.

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