nanomaterials

MDPI

**Supplementary material**

The electric fields in propagation through the samples can be described by considering the wave equation [29]:

$$\nabla^2 E_{\pm} = -\frac{n_{\pm}^2 \omega^2}{c^2} E_{\pm} \quad \text{(S1)}$$

In this mathematical equation E+ and E- represent the circular components of the left and right electric fields in the TWM interaction. The optical frequency is described by ω, c corresponds to the speed of the light and n is the refractive index dependent on irradiance which can be also described by [29]:

$$n_{\pm}^2 = n_0^2 + 4\pi\left(\chi_{1122}^{(3)}\left|E_{\pm}\right|^2 + (\chi_{1122}^{(3)} + \chi_{1212}^{(3)})\left|E_{\mp}\right|^2\right) \quad \text{(S2)}$$

where $n_0$ is the weak-field refractive index. $\chi_{1122}^{(3)}$ and $\chi_{1212}^{(3)}$ are the independent components of the third-order susceptibility tensor $\chi^{(3)}$.

In the interaction of a beam of light with nonlinear optical media can be involved an intensity-dependent refractive index n:

$$n = n_0 + n_2 I \quad \text{(S3)}$$

where, $n_0$ is the weak-field refractive index, $n_2$ in the nonlinear refractive index, and *I* refers to the intensity of the optical field.

The Lorentz–Lorenz equation relates the refractive index of a material to its polarizability $\alpha$, and it can be written in terms of mass density $\rho_m$ [29]:

$$\frac{n^2 - 1}{n^2 + 2} = \frac{4\pi}{3}\frac{N_A \rho m}{M}\alpha \quad \text{(S4)}$$

where $N_A$ is the Avogadro's number and *M* is the molecular weight of the chemical element.