Supplementary Information for

Tipping point and noise-induced transients in ecological networks

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Appendix A: Steady state solutions and stability analysis based on the reduced model

The steady state solutions of the reduced model [Eqs. (7) and (8) in the main text] are determined by

$$\frac{dx}{dt} = \alpha x - \beta x^2 + \frac{\langle\gamma_x\rangle y}{1 + h\langle\gamma_x\rangle y} x + \mu = 0, \quad \text{(S1.1)}$$

$$\frac{dy}{dt} = \alpha y - \beta y^2 + \frac{\langle\gamma_y\rangle x}{1 + h\langle\gamma_y\rangle x} y_\mu = 0. \quad \text{(S1.2)}$$

The stability of the steady solutions is determined by the Jacobian matrix:

$$\mathbf{J} = \begin{pmatrix} \frac{\langle\gamma_x\rangle y}{1+\langle\gamma_x\rangle hy} + \alpha - \kappa - 2\beta x & -\frac{\langle\gamma_x\rangle^2 hxy}{(1+\langle\gamma_x]\rangle hy)^2} + \frac{\langle\gamma_x\rangle x}{1+\langle\gamma_x\rangle hy} \\ -\frac{\langle\gamma_y\rangle^2 hxy}{(1+\langle\gamma_y\rangle hx)^2} + \frac{\langle\gamma_y\rangle y}{1+\langle\gamma_y\rangle hx} & \frac{\langle\gamma_y\rangle x}{1+\langle\gamma_y\rangle hx} + \alpha - \kappa - 2\beta y \end{pmatrix}$$

To obtain all possible solutions of Eqs. (S1.1) and (S1.2) is an algebraically lengthy process. We store the details into a file that is available to those interested upon request.

Appendix B: Noise-induced collapse in empirical mutualistic networks B-D

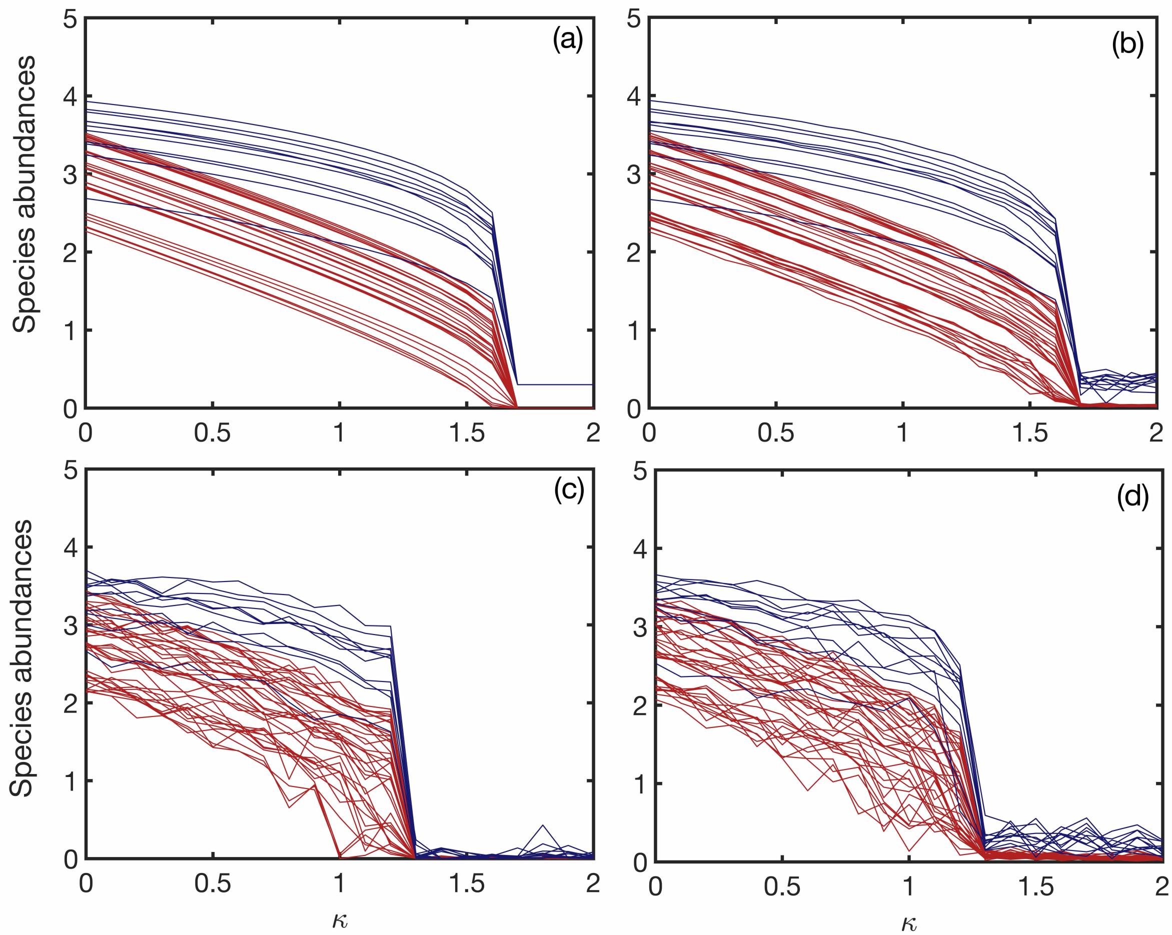

FIG. S1. Noise induced collapse through a tipping point transition in network B. (a-d) Species abundances versus the normalized decay rate κ in the absence of noise, with EN, DN and EDN, respectively. The red and blue curves represent the pollinator and plant abundances. In (b,d), the environmental noise amplitude is $\sigma = 0.1$. In (c,d), the demographic noise amplitude is $\zeta = 0.25$. Other parameters and the simulation setting are the same as those in Fig. 1 in the main text.

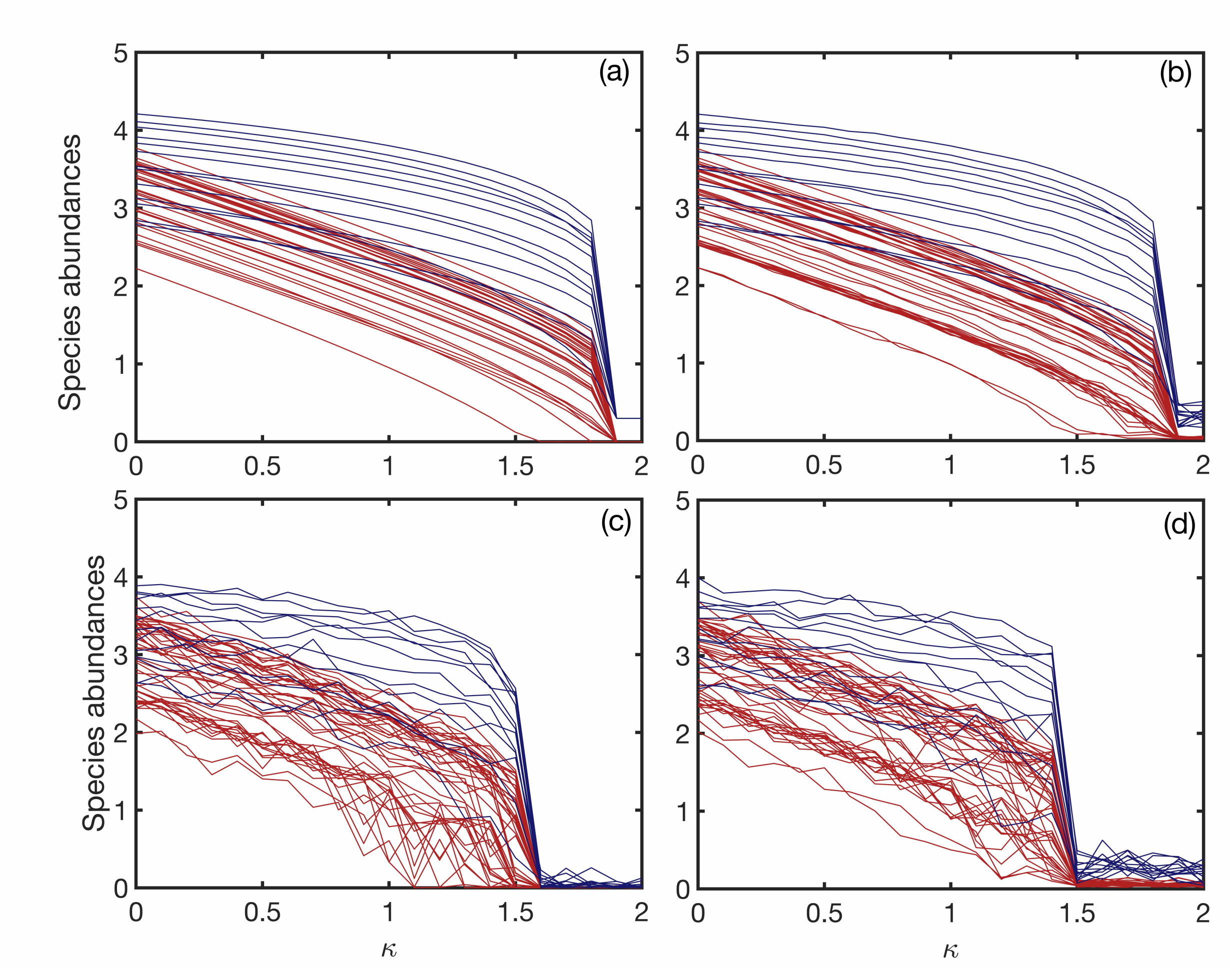

FIG. S2. Noise induced collapse through a tipping point transition in network C. Legends are the same as in Fig. S1.

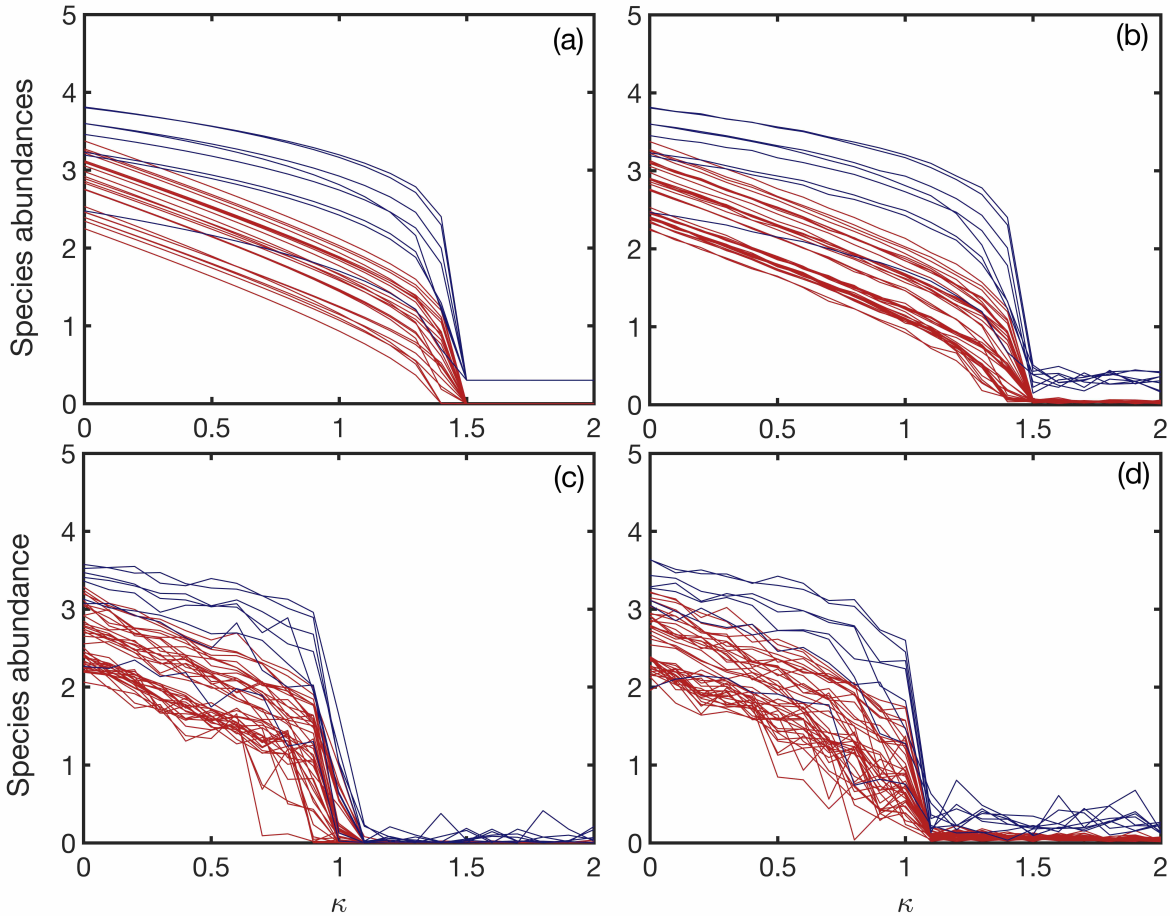

FIG. S3. Noise induced collapse through a tipping point transition in network D. Legends are the same as in Fig. S1.

Appendix C: Noise-induced recovery in empirical mutualistic networks B-D

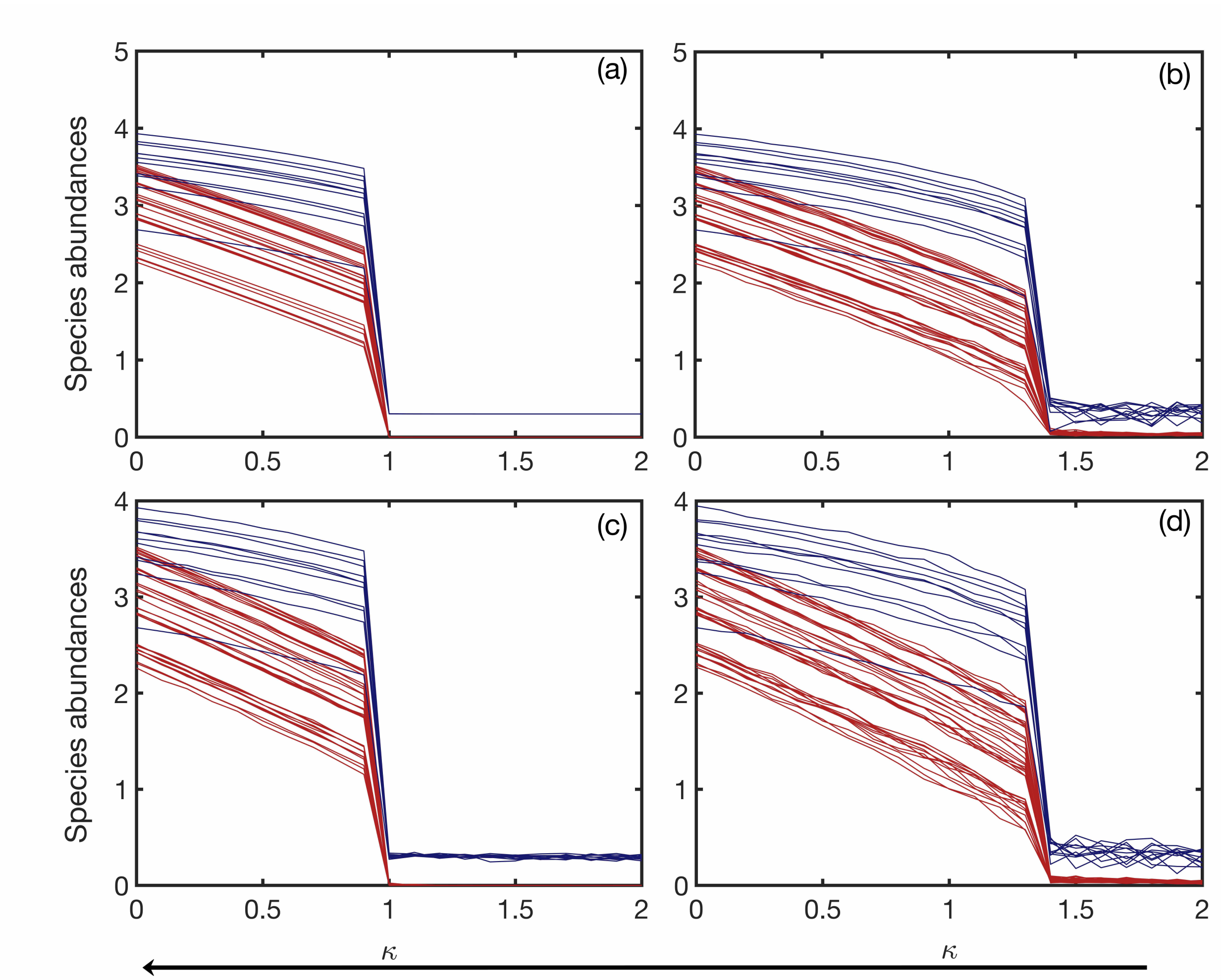

FIG. S4. Noise induced recovery process in network B. (a-d) Species abundances versus the bifurcation parameter as its value continuously decreases from that associated with the extinction state for the four cases of absence of noise, EN, DN and EDN, respectively. The red and blue curves represent the pollinator and plant abundances. In (b,d), the environmental noise amplitude is $\sigma = 0.1$. In (c,d), the demographic noise amplitude is $\zeta = 0.025$. Other parameter values and the simulation setting are the same as Fig. 2 in the main text.

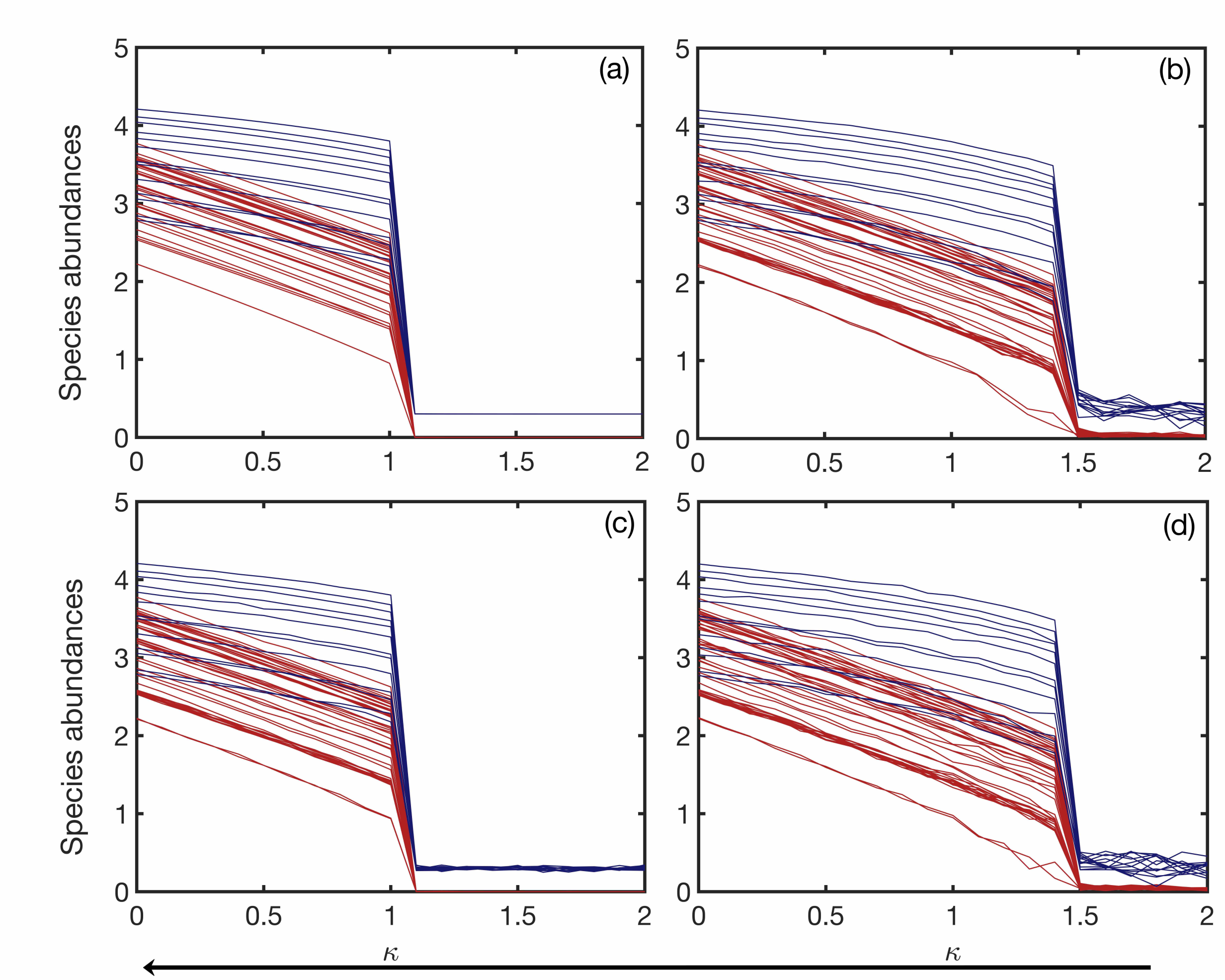

FIG. S5. Noise induced recovery in network C. Legends are the same as in Fig. S4.

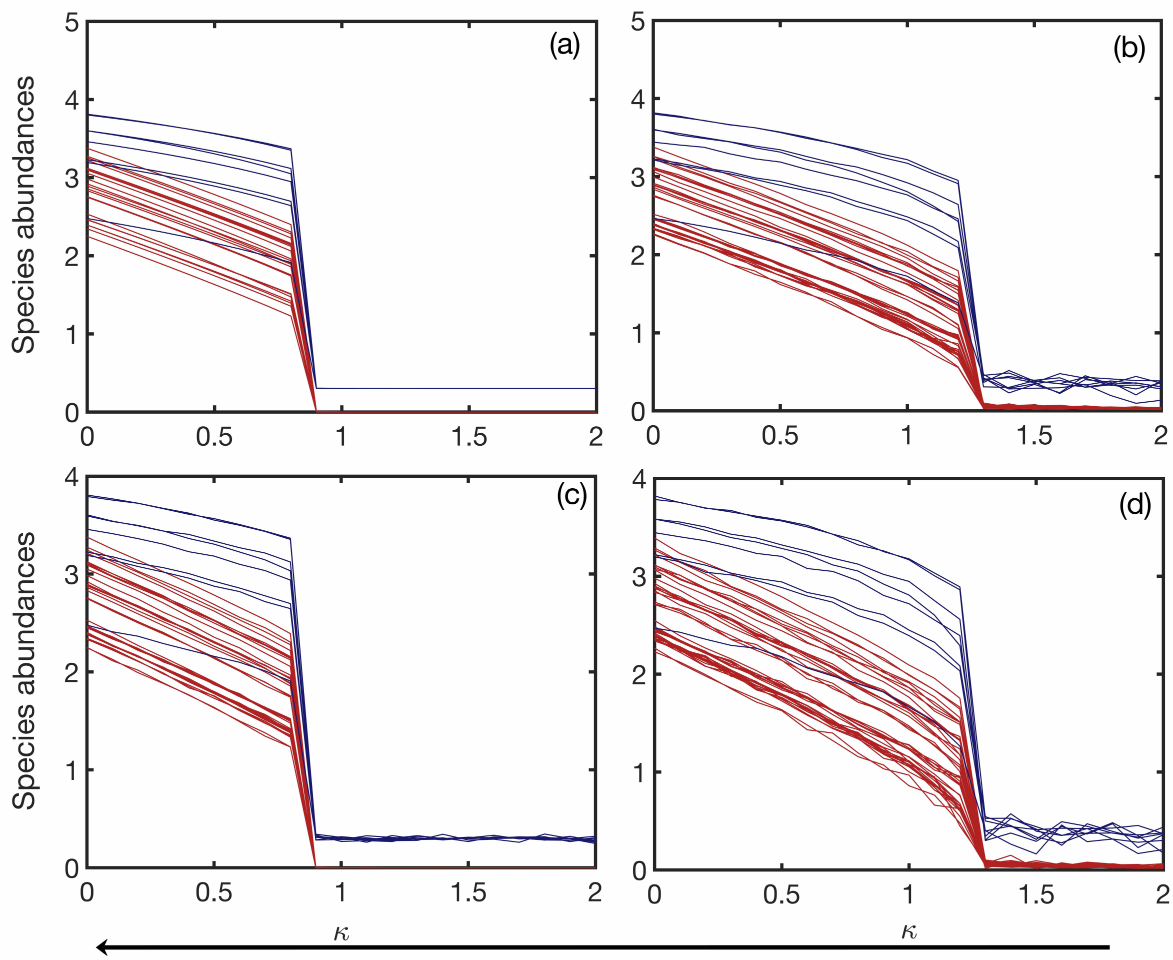

FIG. S6. Noise induced recovery in network C. Legends are the same as in Fig. S4.