





85 temperature. Parasite-induced mortality rate  $(\alpha)$  was set to the product of infection load 86 and  $5.12 \times 10^{-6}$ , the per-parasite added mortality previously estimated (Kirk et al. 2018). We note that because we did not track individuals through time, per-parasite added mortality is constant for an infected individual and cannot change through time, though it 89 can change with the shape parameter in Kirk et al. (2018). For  $\lambda$  and  $\omega$ , which both relate to parasite spores rather than parasite clusters, we assume that a spore cluster has twenty- four spores, which is generally the average that we observe in the lab (between 16 and 32 92 spores per clusters). For  $\lambda$ , we assumed that when a parasite cluster bursts, half of the twenty-four spores are released into the environment while the other twelve remain in the host (to either re-infect or die), and that this bursting process occurs every seven days. We note that Refardt and Ebert (2006) estimate that the parasite may burst approximately every three days at room temperature, but we assume here that this takes significantly 97 longer in our  $10 - 13.5$ °C range since within-host parasite growth rate is depressed (Kirk et al. 2018). We refer readers to the main text for implications of modeling these rates as functions of temperature.



 Fig. S1. Daphnia magna abundances in experimental populations. Blue and red points and lines represent 102 populations in constant 10.0°C and warming conditions respectively. Points represent the mean of three<br>103 counts for each population, and error bars represent the maximum and minimum value from these three 103 counts for each population, and error bars represent the maximum and minimum value from these three 104 counts.

## **Model sensitivity to assumptions**

We explored how sensitive our model results are to five different assumptions: 1)

infection load proportion of equilibrium abundance, 2) *Daphnia* size, 3) spores per

cluster, 4) cluster burst time, and 5) the number of spores released out of the host per

cluster. Assumptions were tested by simulating our model 250 times (without

demographic stochasticity) in which we allowed parameters to take a value from along

- uniform distributions (with replacement) in which the median is the value used for the
- main analysis, and upper and lower range limits are our best estimates at realistic ranges
- for the parameter values (see Table S1). While changing the assumptions incorporated
- 115 more variation, our results were generally robust across the entire parameter space (Fig.
- S2).
- 117 We also investigated the effects of sampling noise on our results, as we sampled a
- 118 subset of the population (twelve individuals) on each sampling day. We simulated the
- 119 model in a deterministic framework 250 times, and then used a binominal sampling
- 120 process to randomly select twelve individuals every three days. This process captures the
- 121 sampling noise observed in the warming samples well, but somewhat overestimates the
- 122 sampled prevalence in the constant 10<sup>o</sup>C populations (Fig. S3).
- 123
- $124$ <br> $125$

125 **Table S1. Range of parameter values used in simulations to test model sensitivity to assumptions.**

|                                | Lower range limit | Main text value | Upper range limit |
|--------------------------------|-------------------|-----------------|-------------------|
| Parasite load as proportion of | 0.132             | 0.182           | 0.232             |
| equilibrium parasite abundance |                   |                 |                   |
| Parasite cluster burst time    | 4.5 $d^{-1}$      | $7 d^{-1}$      | $9.5 d^{-1}$      |
| Spores per cluster             | 16                | 24              | 32                |
| Proportion of spores released  | 0.25              | 0.50            | 0.75              |
| into environment per cluster   |                   |                 |                   |
| Daphnia length                 | $2500 \mu m$      | $2700 \mu m$    | $2900 \mu m$      |

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130 Fig. S2. Sensitivity to model assumptions. Blue (top panel) and red lines (bottom panel) represent mean 131 disease prevalence for constant 10.0°C and warming conditions, respectively, across 250 simulations that 131 disease prevalence for constant 10.0°C and warming conditions, respectively, across 250 simulations that sample from the parameter space. The shaded red region in the bottom panel represents the 95% confidence 132 sample from the parameter space. The shaded red region in the bottom panel represents the 95% confidence interval under warming conditions. The small shaded blue region in the top panel represents the 95% 133 interval under warming conditions. The small shaded blue region in the top panel represents the 95% confidence interval, but is not visible due to the parameter assumptions having negligible effects on di 134 confidence interval, but is not visible due to the parameter assumptions having negligible effects on disease<br>135 prevalence at  $10^{\circ}$ C. The yellow, dashed vertical line represents the temperature/time point at whic 135 prevalence at 10°C. The yellow, dashed vertical line represents the temperature/time point at which the 136 MTE model predicts  $R_0 > 1$  for warming conditions. MTE model predicts  $R_0 > 1$  for warming conditions.

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139 Figure S3. Effects of sampling noise. Blue and red lines represent 250 random samples from deterministic simulation of the model. Grey lines represent experimental data for constant 10.0°C (top panel) and 140 simulation of the model. Grey lines represent experimental data for constant  $10.0^{\circ}$ C (top panel) and warming populations (bottom panel) respectively. warming populations (bottom panel) respectively. **Observed versus predicted prevalence**  The goal of this work was not to specifically quantify model performance, but rather to

- leverage an experimental system to provide a proof of principle that the MTE approach
- can be used to predict warming-induced disease emergence. Nevertheless, below we
- provide observed versus model predicted values of disease prevalence for our four
- warming populations. We note that observed prevalence values are discretized at only six

153 different levels due to sampling (i.e. 0/12 infected, 1/12 infected... up to 5/12 infected), while the model predicts prevalence continuously as it is averaged across our 250 155 stochastic simulations.  $R^2$  values ranged from 0.297 – 0.533 across the four warming populations.



158 Figure S4. Experimentally observed versus model predicted values of prevalence for the four warming populations.  $R^2$  values are provided for each population. populations.  $\mathbb{R}^2$  values are provided for each population.

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### 166 **R0 formulation**

167 In our  $R_0$  formulation (equation 5 of the main text, reproduced as Eq. S1 here), the 168 first term in parentheses represents the total number of spores produced per infected 169 individual, on average.

$$
R_0(T) = \left(\frac{\lambda(T)}{\mu(T) + \alpha(T) + h} + \omega(T) * \frac{(\mu(T) + \alpha(T))}{(\mu(T) + \alpha(T) + h)}\right) \left(\frac{\chi(T) * \sigma(T) * S_{eq}}{\gamma}\right)
$$
 Eq. S1

170

171 In this first term,  $\frac{\lambda(T)}{\mu(T)+\alpha(T)+h}$  represents the number of spores shed by an infected 172 individual during their lifetime, and  $\omega(T) * \frac{(\mu(T) + \alpha(T))}{(\mu(T) + \alpha(T) + h)}$  represents the number of spores released by an infected individual after they die, weighted by the fraction of infected hosts that remain in the system until death and are not harvested prior. The second term in parentheses represents the probability a spore infects a new host as opposed to being lost via medium removal, and has two key assumptions: 1) Spores that are ingested but do not infect the host are expelled, re-enter the water 178 column, and remain viable; 2) The rate of spore loss from the water column due to ingestion and subsequent 180 infection is very small compared to spore loss via media removal; i.e.,  $\sigma \chi S \ll \gamma$ . Regarding assumption 1, to our knowledge, there have been no studies in this host – parasite system that investigate the proportion of spores that remain viable after passing through the host gut. However, evidence from a similar system with this same host and a bacterial parasite, *Daphnia magna* – *Pasteuria ramosa*, shows that the parasite is not killed if it fails to infect the host (King et al. 2013). Moreover, based on our observations working with this system, we believe that at least a large proportion of spores must

 remain viable after passing through the host gut. This is because an average sized *Daphnia* filters ~1ml of medium per hour (Kirk et al. 2019), meaning that the dense populations we maintain under lab conditions (~200 hosts/2L at 20C) should filter through all of their medium in their mesocosm every 10 hours. If spores not causing infection were destroyed upon ingestion, this scenario would lead to very low levels of spores in the medium resulting in little or no infection in the population, which is not concordant with the high levels of infection prevalence we regularly observe in our stock populations (47% prevalence, ref: this study). Moreover, we know that new viable spores are released after host cell lysis within the anterior of the *Daphnia* gut. These spores then must pass through the remainder of the gut before entering the environment, implying that passage through the gut does not kill spores. Finally, microsporidian spores are generally durable, and have been shown to survive months of winter in other *Daphnia* – microsporidian systems (Ebert 2005).

 With the assumption that ingested spores that do not cause infection remain viable after passing through the gut, the probability that a spore causes an infection is

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$$
[\sigma \chi S/(\chi S + \gamma)] * \sum_{i=0}^{\infty} [(1 - \sigma) \chi S/(\chi S + \gamma)]^{i}
$$
 Eq. S2

203 where  $\chi$  is the filtrate rate,  $\sigma$  is the probability that an ingested spore causes infection, *S* is 204 the abundance of susceptible hosts, and  $\gamma$  is the rate of medium exchange. This equation represents the sum of the probabilities that a spore infects a host upon its first ingestion 206  $(i = 1)$ , or that it passes through the gut on its first ingestion and infects a host the 207 second time it is ingested  $(i = 2)$ , or that it passes through the host gut for the first two 208 ingestions and infects a host on its third ingestion  $(i = 3)$ , and so on. Via the formula for the sum for a geometric power series the equation becomes

211 
$$
[\sigma \chi S / (\chi S + \gamma)] * [1 - (1 - \sigma) \chi S / (\chi S + \gamma)]^{-1}
$$
 Eq. S3

212 which simplifies to

$$
\sigma \chi S / (\sigma \chi S + \gamma) \qquad \qquad Eq. S4
$$

 We therefore assume there are only two ultimate fates for a spore: either it is ingested 215 and infects (at rate  $\sigma \gamma S$ ) or it is removed from the system via medium exchange (at rate  $\gamma$ ). Additionally, spores could of course also die in the environment, but microsporidian spores are highly durable and we assume their death rate is negligible over the timescale of the experiment as we have noted in the main text.

219 Under the approximation that  $\sigma \chi S \ll \gamma$ , we arrive at the expression for the second 220 parenthesis for  $R_0$ :  $\sigma \gamma S/\gamma$ , but as an approximation rather than exactly correct.

221 This assumption that  $\sigma \gamma S \ll \gamma$  is strongly supported for our system, as we know that  $\sigma$  is

222 very small (Kirk et al. 2019). For example, at  $12^{\circ}\text{C}$ ,  $\sigma\gamma S=0.000018 \text{ d}^{-1}$ , while  $\gamma$  is

223 constant across temperature and equals  $0.0286$  d<sup>-1</sup>. This means that spore loss from

224 medium exchange ( $\gamma$ ) is nearly 1600x larger than spore loss from infection ( $\sigma \chi S$ ) at this

225 temperature, and the assumption that  $\sigma \chi S \ll \gamma$  is valid. Because of this, if we look at the

226 temperature range of the experiment  $(10^{\circ}C - 13.5^{\circ}C)$ , there is no discernible difference

227 between predictions from the simpler  $R_0$  expression (Eq. 5) that assumes spore loss only

- 228 from medium removal (black line; Fig. S2) compared to a more complicated expression
- 229 that explicitly accounts for removal of spores from infection (dashed blue line; Fig. S5).



231 Fig. S5. Comparing  $R_0$  expressions in relation to temperature, with (black line) and without 232 (dashed blue line) the assumption that spore loss from infection is negligible compared to sp 232 (dashed blue line) the assumption that spore loss from infection is negligible compared to spore<br>233 loss from medium removal. The temperature range of our experiment was  $10.0^{\circ}\text{C} - 13.5$ , loss from medium removal. The temperature range of our experiment was  $10.0$ °C – 13.5,

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## **REFERENCES**



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