

S1 Appendix. Endemic Equilibrium Expressions

Expressions for $g_{00}, g_{01}, \dots, g_{11}, q_{01}, q_{02}, \dots, q_{20}, h_{01}, h_{02}, \dots, h_{20}, C_1, C_2, \dots, C_5$ and $C_{11}, C_{21}, \dots, C_{51}$ in section 2.3.3.

$$\left\{ \begin{aligned}
 g_{00} &= \frac{\rho_{ct}^{f1} \mu_{f5} + \rho_{ct}^{f1} \rho_{ct}^f}{\mu_{f5} \mu_{f6} - \rho_{ct}^{f1} \rho_{ct}^f}, & g_{01} &= \frac{\rho_{ct}^f (1 + g_{00})}{\mu_{f5}}, & g_{02} &= 1 + \rho_{ht}^m, & g_{03} &= \frac{\Lambda_{fa} \rho_{ht}^{m1}}{\mu_{f4}}, & g_{04} &= \frac{\Lambda_{fu} \rho_{ht}^{m1} \rho_{ht}^f}{\mu_{f4}}, \\
 g_{05} &= \frac{\Lambda_{fu} \rho_{ht}^m}{\mu_{f4}} + \frac{\Lambda_{fu} \rho_{ht}^f}{\mu_{f3} \mu_{f4}}, \\
 g_{06} &= \frac{\rho_{ct}^{m1} \mu_{m5} + \rho_{ct}^{m1} \rho_{ct}^m}{\mu_{m5} \mu_{m6} - \rho_{ct}^{m1} \rho_{ct}^m}, & g_{07} &= \frac{\rho_{ct}^m (1 + g_{06})}{\mu_{m5}}, & g_{08} &= 1 + \rho_{ht}^f, & g_{09} &= \frac{\Lambda_{ma} \rho_{ht}^{f1}}{\mu_{m4}}, & g_{10} &= \frac{\Lambda_{mu} \rho_{ht}^{f1} \rho_{ht}^m}{\mu_{m4}}, \\
 g_{11} &= \frac{\Lambda_{mu} \rho_{ht}^f}{\mu_{m4}} + \frac{\Lambda_{mu} \rho_{ht}^m}{\mu_{m3} \mu_{m4}}, \\
 q_{01} &= N_f^* (\bar{\mu}_f + \delta_f) - (\Lambda_{fu} + \Lambda_{fa}), & q_{02} &= q_{01} g_{02} \rho_{ht}^f, \\
 q_{03} &= (g_{02} q_{01} \mu_{f2} + q_{01} \mu_{f1} \rho_{ht}^f) - (\Lambda_{fu} \rho_{ht}^{f1} \delta_f + \Lambda_{fa} g_{02} \delta_f), \\
 q_{04} &= q_{01} \mu_{f1} \mu_{f2} - (\Lambda_{fu} \mu_{f2} \delta_f + \Lambda_{fa} \mu_{f1} \delta_f + \rho_{ht}^f \Lambda_{fu} \delta_f), & q_{05} &= \delta_f \rho_{ht}^f g_{00} g_{02}, \\
 q_{06} &= g_{00} g_{02} \delta_f \mu_{f2} + g_{00} \delta_f \mu_{f1} \rho_{ht}^f, & q_{07} &= g_{00} \delta_f \mu_{f1} \mu_{f2}, \\
 q_{08} &= g_{02} g_{03} \rho_{ht}^{m1} + g_{05} \rho_{ht}^m \rho_{ht}^{m1}, \\
 q_{09} &= g_{02} g_{03} \mu_{f2} + g_{03} \mu_{f1} \rho_{ht}^{m1} + g_{05} \mu_{f1} \rho_{ht}^{m1} + g_{05} \mu_{f2} \rho_{ht}^m + g_{04} \rho_{ht}^m, \\
 q_{10} &= g_{03} \mu_{f1} \mu_{f2} + g_{05} \mu_{f1} \mu_{f2} + g_{04} \mu_{f2}, & q_{11} &= g_{02} \rho_{ht}^m \rho_{ht}^{m1}, \\
 q_{12} &= g_{02} \mu_{f2} \rho_{ht}^m + g_{02} \mu_{f2} \rho_{ht}^{m1} + \mu_{f1} \rho_{ht}^m \rho_{ht}^{m1}, \\
 q_{13} &= g_{02} \mu_{f2}^2 + \mu_{f1} \mu_{f2} \rho_{ht}^m + \mu_{f1} \mu_{f2} \rho_{ht}^{m1}, & q_{14} &= \mu_{f1} \mu_{f2}^2, \\
 q_{15} &= q_{08} q_{05} - q_{02} q_{11}, & q_{16} &= q_{06} q_{08} + q_{05} q_{09} - (q_{02} q_{12} + q_{03} q_{11}), \\
 q_{17} &= q_{05} q_{10} + q_{06} q_{09} + q_{07} q_{08} - (q_{02} q_{13} + q_{03} q_{12} + q_{04} q_{11}), \\
 q_{18} &= q_{06} q_{10} + q_{07} q_{09} - (q_{02} q_{14} + q_{03} q_{13} + q_{04} q_{12}), \\
 q_{19} &= q_{07} q_{10} - (q_{03} q_{14} + q_{04} q_{13}), & q_{20} &= q_{04} q_{14}, \\
 h_{01} &= N_m^* (\bar{\mu}_m + \delta_m) - (\Lambda_{mu} + \Lambda_{ma}), & h_{02} &= h_{01} g_{08} \rho_{ht}^m, \\
 h_{03} &= (g_{08} h_{01} \mu_{m2} + h_{01} \mu_{m1} \rho_{ht}^m) - (\Lambda_{mu} \rho_{ht}^{m1} \delta_m + \Lambda_{ma} g_{08} \delta_m), \\
 h_{04} &= h_{01} \mu_{m1} \mu_{m2} - (\Lambda_{mu} \mu_{m2} \delta_m + \Lambda_{ma} \mu_{m1} \delta_m + \rho_{ht}^m \Lambda_{mu} \delta_m), & h_{05} &= \delta_m \rho_{ht}^m g_{06} g_{08}, \\
 h_{06} &= g_{06} g_{08} \delta_m \mu_{m2} + g_{06} \delta_m \mu_{m1} \rho_{ht}^m, & h_{07} &= g_{06} \delta_m \mu_{m1} \mu_{m2}, \\
 h_{08} &= g_{08} g_{09} \rho_{ht}^{f1} + g_{11} \rho_{ht}^f \rho_{ht}^{f1}, \\
 h_{09} &= g_{08} g_{09} \mu_{m2} + g_{09} \mu_{m1} \rho_{ht}^{f1} + g_{11} \mu_{m1} \rho_{ht}^{f1} + g_{11} \mu_{m2} \rho_{ht}^f + g_{10} \rho_{ht}^f, \\
 h_{10} &= g_{09} \mu_{m1} \mu_{m2} + g_{11} \mu_{m1} \mu_{m2} + g_{10} \mu_{m2}, & h_{11} &= g_{08} \rho_{ht}^f \rho_{ht}^{f1}, \\
 h_{12} &= g_{08} \mu_{m2} \rho_{ht}^f + g_{08} \mu_{m2} \rho_{ht}^{f1} + \mu_{m1} \rho_{ht}^f \rho_{ht}^{f1}, \\
 h_{13} &= g_{08} \mu_{m2}^2 + \mu_{m1} \mu_{m2} \rho_{ht}^f + \mu_{m1} \mu_{m2} \rho_{ht}^{f1}, & h_{14} &= \mu_{m1} \mu_{m2}^2, \\
 h_{15} &= h_{08} h_{05} - h_{02} h_{11}, & h_{16} &= h_{06} h_{08} + h_{05} h_{09} - (h_{02} h_{12} + h_{03} h_{11}), \\
 h_{17} &= h_{05} h_{10} + h_{06} h_{09} + h_{07} h_{08} - (h_{02} h_{13} + h_{03} h_{12} + h_{04} h_{11}), \\
 h_{18} &= h_{06} h_{10} + h_{07} h_{09} - (h_{02} h_{14} + h_{03} h_{13} + h_{04} h_{12}), \\
 h_{19} &= h_{07} h_{10} - (h_{03} h_{14} + h_{04} h_{13}), & h_{20} &= h_{04} h_{14}, \\
 C_1 &= \frac{q_{16}}{q_{15}}, & C_2 &= \frac{q_{17}}{q_{15}}, & C_3 &= \frac{q_{18}}{q_{15}}, & C_4 &= \frac{q_{19}}{q_{15}}, & C_5 &= \frac{q_{20}}{q_{15}}, \\
 C_{11} &= \frac{h_{16}}{h_{15}}, & C_{21} &= \frac{h_{17}}{h_{15}}, & C_{31} &= \frac{h_{18}}{h_{15}}, & C_{41} &= \frac{h_{19}}{h_{15}}, & C_{51} &= \frac{h_{20}}{h_{15}}.
 \end{aligned} \right. \tag{15}$$