Supplemental Document

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Subdiffuse scattering and absorption model for single fiber reflectance spectroscopy: supplement

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Supplemental materials

Here we will demonstrate that the diffuse contribution to the SFR signal can be rewritten as a function of μ_s'd and μ_a/μ_s'.The diffuse reflectance per unit area as a function of radial coordinate ρ is given by Farrell *et al* as [1]:

$$R(\rho) = \frac{a\prime}{4\pi}\left(z_0\left[\mu_{eff} + \frac{1}{r_1}\right]\frac{e^{-\mu_{eff}r_1}}{r_1^2} + (z_0 + 2z_b)\left[\mu_{eff} + \frac{1}{r_2}\right]\frac{e^{-\mu_{eff}r_2}}{r_2^2}\right) \quad (S1)$$

where $a' = \mu_s'/(\mu_s'+\mu_a)$ and μ_{eff} is the effective attenuation coefficient, $\mu_{eff} = \sqrt{\mu_a/D} = \sqrt{3\mu_a\mu_s'}$. The position of the point source in the medium is set at one reduced mean free path given by $z_0 = 1/\mu_s'$. The location of the 'virtual boundary', where the fluence rate has to be zero, is given by $z_b = 2AD$ where A is a parameter that depends on the refractive index mismatch between the fiber and the tissue [2]. We can express equation S1 in terms of dimensionless variables $x = \mu_s'd$, $y = \mu_a/\mu_s'$ and $\beta = \rho/d$. We also use the constant $c_A = 1+ 4A/3$ to compact the expression. Using these substitutions, the distances r_1 and r_2 are:

$$r_1^2 = z_0^2 + \rho^2 = \left(\frac{d}{x}\right)^2 + d^2\cdot\beta^2 = \frac{d^2}{x^2}(1 + x^2\beta^2) \quad (S2)$$

$$r_2^2 = (z_0 + 2z_b)^2 + \rho^2 = \left(\frac{d}{x} + \frac{4A\cdot d}{3x}\right)^2 + d^2\cdot\beta^2 = \frac{d^2}{x^2}\left(\left(1 + \frac{4A}{3}\right)^2 + x^2\beta^2\right) \quad (S3)$$

and

$$\frac{d^2}{x^2}\cdot R(y, x\beta) = \frac{(1+y)^{-1}}{4\pi}\left[\left(\frac{\sqrt{3y}}{1+(x\beta)^2} + \frac{1}{(1+(x\beta)^2)^{\frac{3}{2}}}\right)e^{-\sqrt{3y(1+(x\beta)^2)}} + c_A\left(\frac{\sqrt{3y}}{c_A^2+(x\beta)^2} + \frac{1}{(c_A^2+(x\beta)^2)^{\frac{3}{2}}}\right)e^{-\sqrt{3y(c_A^2+(x\beta)^2)}}\right] \quad (S4)$$

Likewise, the probability distribution for distances on the fiber surface:

$$p(\rho) = \frac{16\rho}{\pi d^2}cos^{-1}\left(\frac{\rho}{d}\right) - \frac{16}{\pi d}\left(\frac{\rho}{d}\right)^2\sqrt{1-\left(\frac{\rho}{d}\right)^2} \quad (S5)$$

can be rewritten as:

$$d\cdot p(\beta) = \left(\frac{16}{\pi}\beta cos^{-1}(\beta) - \frac{16}{\pi}\beta^2\sqrt{1-\beta^2}\right) \quad (S6)$$

Such that the integrated reflectance

$$R_{dif} = \frac{\pi}{4}\cdot d^2\cdot\int_0^d R(\rho)\cdot p(\rho, d)d\rho \quad (S7)$$

Is rewritten as:

$$R_{dif}(x, y) = \frac{\pi}{4}\cdot x^2\cdot\int_0^1 R(y, \beta x)\cdot p(\beta)d\beta \quad (S8)$$

For the integration in this form with integration variable β; the factors x and y are simply constants that are carried over. Consequently, in the resulting expression the absorption coefficient will appear in the ratio μ_a/μ_s', and the fiber diameter will appear in the product μ_s'd.

References

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