**Supplemental Materials**

*Appendix 1: Bayesian model parameters and specification*

<u>Observed data consisted of:</u>

| | |
|---|---|
| { i } | = Set of Philadelphia ZIP codes that have reported ≥1 case of COVID-19 |
| $Y_i$* | = Number of observed positive SARS-CoV-2 tests in ZIP code i |
| $N_i$ | = Total population in ZIP code i |
| $A_i$ | = Area deprivation index in ZIP code i |

<u>Models consisted of:</u>

*ZIP code prevalence models:*

*Model #1: Unstructured random effect:* $\text{Logit}(r_i) = \eta_0 + \eta_1 {*} A_i + u_i$

True prevalence of COVID-19 in ZIP code i, conditioned on $A_i$ and unstructured random effect $u_i$.

*Model #2: Prior directly on r:* $r_i \sim \text{Unif}(a=0, b=0.1)$

True prevalence of COVID-19 in ZIP code i defined by uniform prior.

*Count of infected*: $Y.t_i \sim \text{Binom}(r_i, N_i)$

Number of positive SARS-CoV-2 tests in ZIP code i based on true prevalence, r.

*Observed infected if infected*: $Y.tp_i \sim \text{Binom}(SN_i, Y.t_i)$

Number of true positive SARS-CoV-2 tests in ZIP code i based on ZIP code surveillance SN.

*Observed infected if not infected*: $Y.fp_i \sim \text{Binom}(1\text{-}SP, N_i - Y.t_i)$

Number of false positive SARS-CoV-2 tests in ZIP code i based on ZIP code surveillance SP.

*Observed cases*: $Y_i{*} \sim (Y.tp_i + Y.fp_i)$

Observed cases are a function of observed infected if infected model and observed infected if not infected model. We let a variable $E_i = Y.t_i - Y_i{*}$ to capture the difference between the true and observed cases.

<u>Priors consisted of:</u>

| | | |
|---|---|---|
| $SN_i$ | ~ Beta(14.022, 9.681) | Sensitivity of inclusion in surveillance data corresponding to a median SN of 59% with 5- and 95-percentiles of 42% and 75%, respectively. |
| $SP$ | ~ Beta (100, 3.02) | Specificity of inclusion in surveillance data corresponding to a median SP of 95% with 5- and 95-percentiles of 94% and 99%, respectively. |
| $\eta_0$ | ~ Norm(-3.5, 0.5) | True prevalence of COVID-19 (logit scale) when $A_i$=0 corresponding to a mean prevalence of 3% with a standard deviation of 2%. |
| $\eta_1$ | ~ Norm(0.04, 0.04) | Effect of $A_i$ on true prevalence of COVID-19 (logit scale) |

| | | |
|---|---|---|
| | | where each standard deviation increase in ADI is associated with a change in prevalence of 6% (95% CI: -1%, 14%). |
| $u_i$ | $\sim$ Norm(0, $\tau_u$) | Random error (unstructured) in in ZIP code i. |
| $\tau_u$ | $\sim$ Gamma(1, 0.1) | Precision of the random parameter $u_i$ with a weak prior. |
| $r_i$ | $\sim$ Unif($a$=0, $b$=0.1) | True prevalence of COVID-19 with a minimum of 0% and maximum of 10% for each ZIP code i. |

*Appendix 2: Simulation study and sensitivity analysis to inform parameter specification in the Bayesian model*

Generating simulated data

To evaluate the performance of the Bayesian models with different priors, we undertook a simulation study where the true sensitivity and specificity of the surveillance program, as well as the true prevalence in each ZIP code were known. In the simulated data, the number of residents ($N_i$) and spatial representation of the 47 ZIP codes ($i$) were identical to the observed data. However, the observed (simulated) counts ($Y_i$*) were generated as follows.

First, we specified a logistic model (Eq. A) of true ZIP code prevalence ($r_i$) conditional on $ADI_i$ based on three hypothetical city-wide prevalence values: 1%, 3%, and 5%. For each ZIP code and each of the three hypothetical prevalences, $\lambda_0$ was drawn from a normal distribution with the respective means=-4.5, -3.5, and -3, and standard deviation=0.1; $\lambda_1$ was sampled from a normal distribution with mean=0.04 and standard deviation=0.04, where each standard deviation increase in ADI (when $\lambda_0$ = -3.5) was associated with a change in prevalence of 0.1% (95% confidence interval: -5.6%, 5.2%). This was informed by effect estimates obtained from regressing ZIP code aggregated cases of COVID-19 by ADI (Bilal et al., 2020). We calculated the true prevalence ($r_i$) under the three hypothetical scenarios for $\lambda_0$ using Eq A.

Eq A. $$logit(r_i) = \lambda_0 + \lambda_1 * ADI_i$$

Second, we simulated three hypothetical surveillance programs with average sensitivities (*SN*) of 30%, 50%, and 70%. To operationalize, for each $i^{th}$ ZIP code we simulated $SN_i$ from a beta distribution with the respective shape=3, 5, and 7 and scale=7, 5, and 3 parameters, which provides approximately the same standard deviation (15%) to each level. Specificity (*SP*) was fixed at a constant value of 99%. The observed prevalence ($p_i$) due to misclassification was generated as shown in Eq. B:

Eq B. $$p_i = SN_i * r_i + (1-SP)*(1-r_i)$$

Finally, we used $p_i$ to sample from a binominal distribution for each ZIP code, where the number of trials equaled the population of the ZIP code, $N_i$. The number of successes thus corresponded to the simulated observed counts ($Y_i^*$) of cases of COVID-19 for each ZIP code. Each of the three proportion scenarios (determined by $\lambda_0$) were combined with three distributions of $SN_i$, thereby producing 9 total simulated datasets: observed counts when average $r_i$ (when ADI=0) is about 1%, 3%, or 5% and average $SN_i$ of 30%, 50%, or 70%. These datasets were generated stochastically to better reflect real world expectations of data generation.

Evaluated Bayesian models

The model of true and observed counts is shown in the main text, Eqs. 1 – 4. Two Bayesian models were simultaneously evaluated to assess the robustness of different approaches in recovering the posteriors of true ZIP code prevalence values ($r_i$) when only the observed counts ($Y_i^*$) and population sizes ($N_i$) are known. In the first model (Eq. C), the true prevalence was conditioned on ADI and incorporated unstructured random effects $u_i$, where $u_i \sim$ Norm($\mu$=0, $\sigma^2=1/\tau_u$) and $\tau_u$ is the precision of the random effect. We considered two priors on $\eta_0$ (1) an informative prior Norm($\mu$=-3.5, $\sigma^2$=*0.25*), which corresponds to a 3% prevalence with an approximate standard deviation of 2%, and (2) a vague prior Norm($\mu$=0, $\sigma^2$=*2.71*), which gives a close approximation to a standard logistic distribution (Lunn et al., 2012). We derived the informative prior from data in a neighboring jurisdiction of the State of Delaware that conducted sewage-based PCR testing of the most populous and urban county in the state and reported an estimate of prevalence of infection of 3% (Wilson, 2020). The effect of ADI on the logit of true prevalence received an informative prior of $\eta_1$ ~ Norm($\mu$=0.04, $\sigma^2$=0.0016), which was informed by effect estimates obtained from regressing ZIP code aggregated cases of COVID-19 by ADI (Bilal et al., 2020). We used a weakly informative prior $\tau_u$ ~ Gamma($\kappa$=1, $\theta$=0.1) for the precision of the exchangeable random effects.

Eq C. $$logit(r_i) = \eta_0 + \eta_1 * ADI_i + u_i$$

In the second model (Eq. D), true prevalence was not conditioned on any covariates, and therefore the prior was set directly using a uniform distribution, with a minimum and maximum prevalence of 0% and 10%, respectively. These bounds represent our reasonable extremes for the true prevalence of COVID-19 in any ZIP code in Philadelphia during our study period based on a seroprevalence study across multiple cities (Havers et al., 2020).

Eq D. $$r_i \sim Unif(a=0, b=0.1)$$

We used two informative priors on sensitivity for both models just described, $SN_i$ ~ Beta($\alpha$=14.022, $\beta$=9.681) and $SN_i$ ~ Unif($a$=0.25, $b$=0.75), and one informative prior on specificity, $SP$ ~ Beta($\alpha$=100, $\beta$=3.02). Because the accuracy of the surveillance program was unknown in the study area, we assumed that surveillance performed no better than the accuracy of the diagnostic test alone which, early on in the pandemic, had an estimated SN between the 70-90% range and near perfect SP (Yap et al., 2020). $SN_i$ ~ Beta(14.022, 9.681) corresponded to a median sensitivity of 59% with 5- and 95-percentiles of 42% and 75%, respectively; $SN_i$ ~ *Unif*($a$=0.25, $b$=0.75) allowed for greater uncertainty in where sensitivity was most likely distributed.

In total, six Bayesian models were fitted for each of the 9 simulated datasets: for both specifications of the prior on $SN_i$, we estimated models with two specifications of the priors on $\eta_0$, plus analysis with the prior directly on $r_i$.

<u>Results</u>

Modeling prevalence conditioned on ADI under the prior specifications for $\eta_0$ ~ Norm($\mu$=-3.5, $\sigma^2$=0.5) and a Beta prior on $SN_i$ resulted in posterior distributions all centered around 3% prevalence regardless of the true simulated prevalence, with varying degrees of accuracy (Supplemental Figure 1). The model tended not to include the true prevalence values in the 95% credible interval when prevalence was around 1%, but did far better, with most of the 95% credible intervals including true prevalence values as prevalence increased to 3% and 5% (Supplemental Figure 2). The Bayesian models demonstrated little learning about surveillance sensitivity, with all posteriors spanning the prior, however we improved our understanding of surveillance specificity which was nearly perfect (Supplemental Figures 3 & 4).

Modeling prevalence conditioned on ADI under the prior specifications for $\eta_0$ ~ Norm($\mu$=0, $\sigma^2$=2.71) resulted in posterior distributions spanning unrealistic values of hypothesized prevalence, in some cases >50% (Supplemental Figures 5 & 6). The bimodal distribution of sensitivity demonstrated the lack of identifiability of the model (and hence the need to have informative prior on prevalence); posterior specificity settled on approximately 1.00 (Supplemental Figures 7 & 8).

Placing the prior directly on prevalence, $r_i \sim$ Unif($a$=0, $b$=0.1), resulted in posterior distributions that were markedly narrower than the prior and that largely captured the true prevalence in their credible intervals (Supplemental Figures 9 & 10), even when true prevalence was around 1%. The Bayesian models did not demonstrate meaningful learning about surveillance sensitivity, with all posteriors spanning the prior, however we improved our understanding of surveillance specificity which was nearly perfect (Supplemental Figures 11 & 12).

Placing a uniform prior on the $SN_i \sim$ Unif($a$=0.25, $b$=0.75), showed our results to be robust to this prior specification (Supplemental Figures 13 – 24) relative to the Beta prior on $SN_i$. The posterior distributions of prevalence and specificity were visually indistinguishable compared to the prior of $SN_i \sim$ Beta(14.022, 9.681), meanwhile the posterior distributions of sensitivity demonstrated the expected wider variability and lower median values inherent in the $SN_i \sim$ Unif($a$=0.25, $b$=0.75) prior specification.

We examined the minimum mean squared error, estimated as the average of the squared difference between the posterior mean of prevalence and its true value, for the three models with a Beta prior on sensitivity. The two approaches that used informative priors, that is, $\eta_0$= Norm($\mu$=-3.5, $\sigma^2$=$0.25$) and $r_i \sim$ Unif(a=0, b=0.1), appeared to have comparable performance when true prevalence was 3% or higher and true sensitivity was at least 50% on average.

## Conclusions

We concluded three main findings from this sensitivity analysis. First, the Bayesian model demonstrated learning about true prevalence in the posterior distribution as compared to the prior distribution, so long as the prior was not nearly flat. Second, when true prevalence was at the lower value of our hypothetical scenarios (i.e., 1%) the posterior distributions tended to overestimate true prevalence; when true prevalence was at the center of our hypothetical scenarios (i.e., 3%) the posterior distributions tended to most accurately reflect true prevalence; when true prevalence was at the upper value of our hypothetical scenarios (i.e., 5%) the posterior distributions tended to underestimate true prevalence. Third, as accuracy of the surveillance program improved (i.e., higher sensitivity values corresponding to less bias), the credible intervals captured the true value more consistently (except, at 1% prevalence).

Based on these results, in the main text we decided to compare results of our best model conditioned on ADI to that of simpler one that does not account for ADI; both require informative priors on prevalence. We believe that the model with ADI may be more scientifically justified given that there is some evidence that ADI is associated with prevalence of COVID-19 (Bilal et al., 2020), and that under most realistic expectations of prevalence in many locales being ≥3% (Havers et al., 2020), both approaches showed similar performance.

## References

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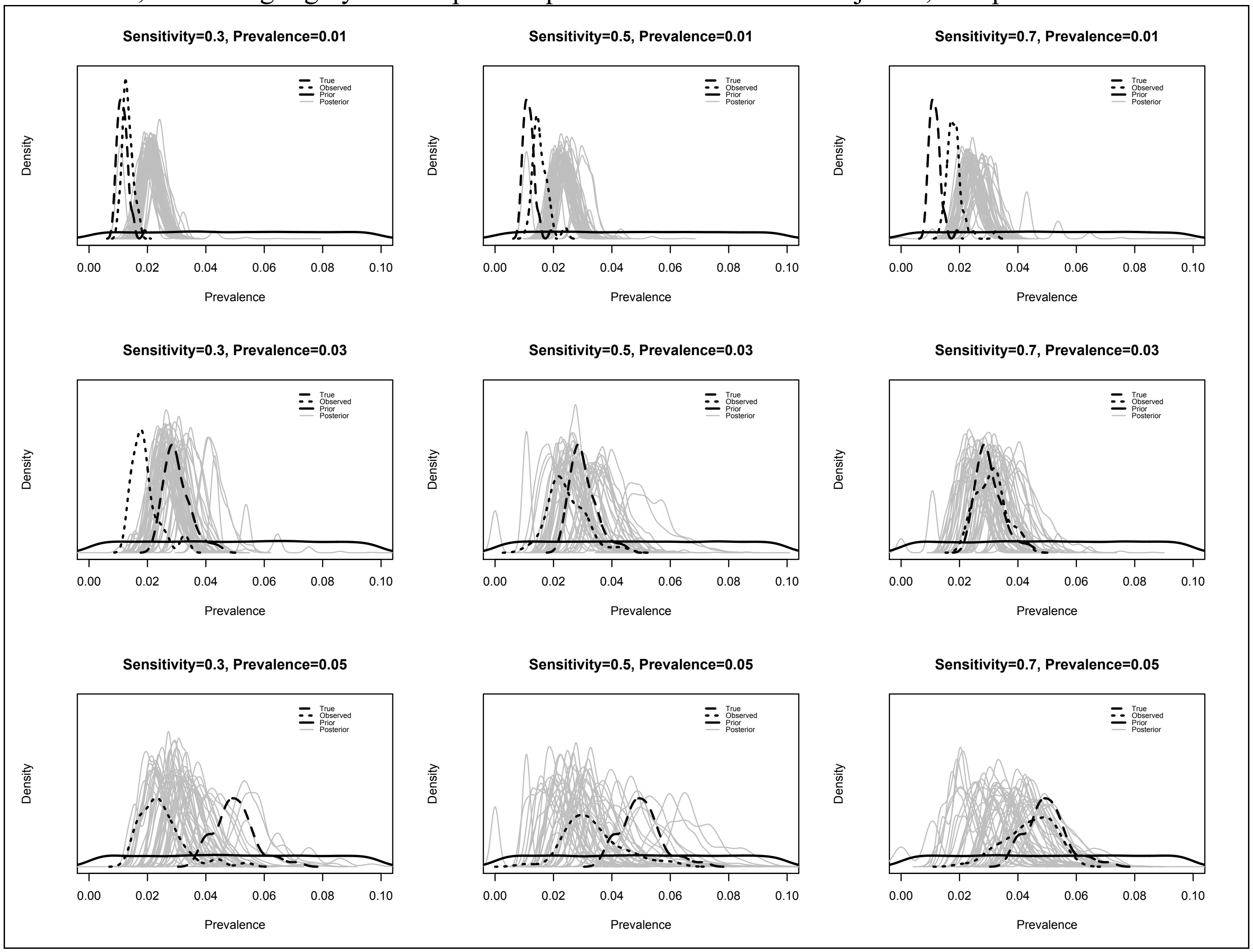
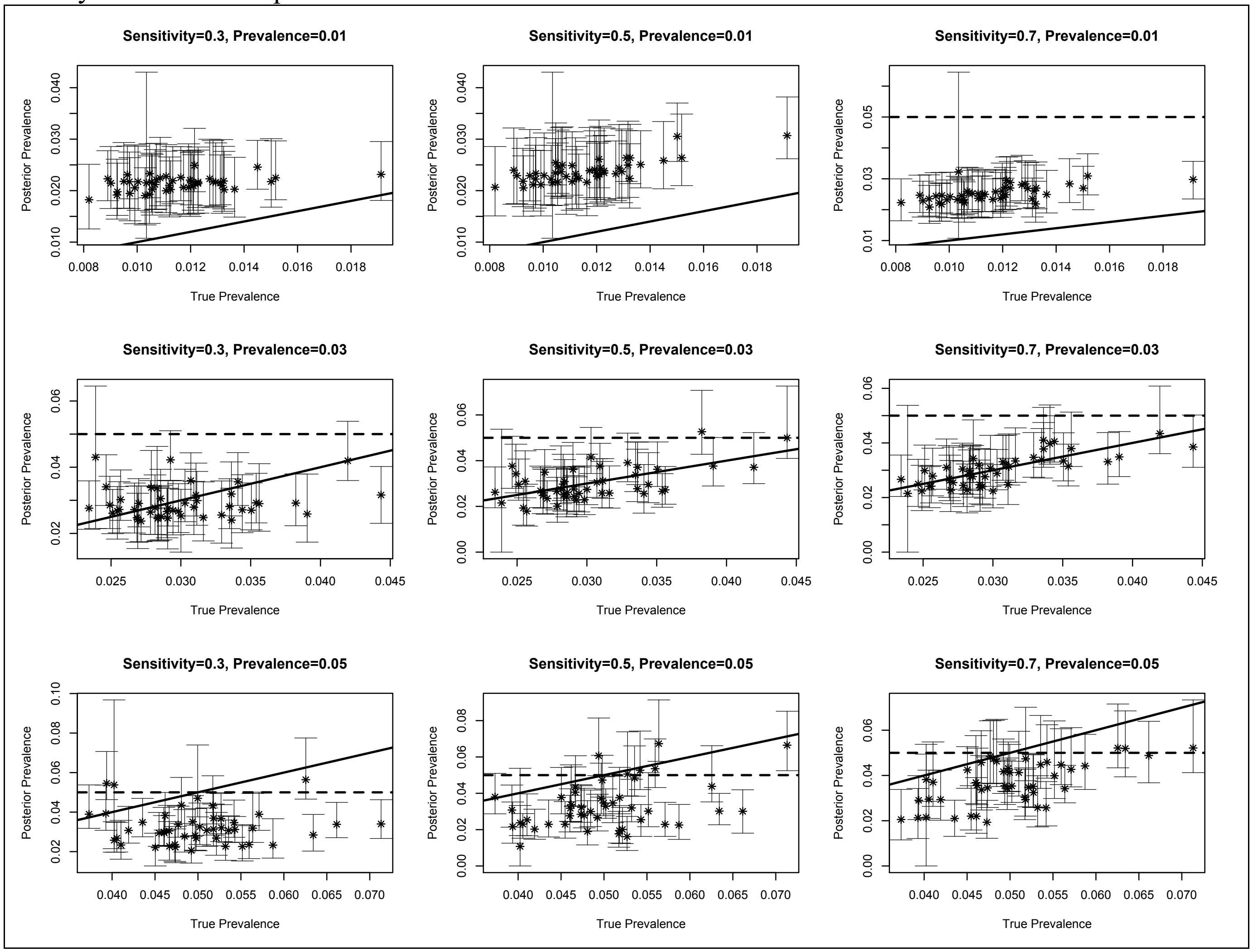
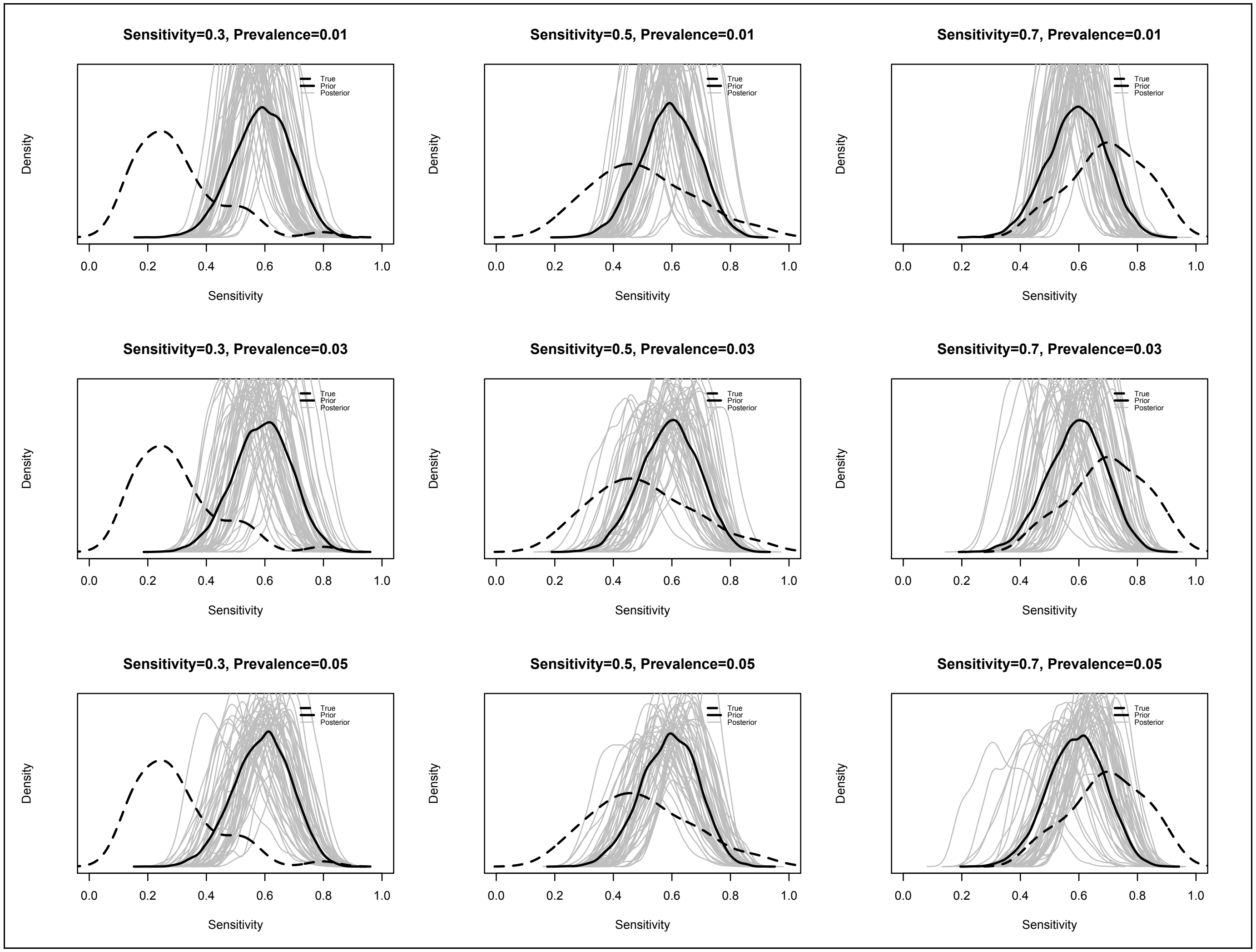
**Supplemental Figure 1**. Posterior distribution of true prevalence when priors on $\eta_0$ ~ Norm(μ=-3.5, $\sigma^2$=*0.25*) and $SN_i$ ~ Beta(α=14.022, β=9.681) for each of the nine simulated datasets. Dashed black line depicts the averaged true prevalence of COVID-19 in the simulated data, the dotted line depicts the averaged observed, misclassified prevalence, the solid line depicts the prior distribution, and the light gray lines depict the posterior distribution of adjusted, true prevalence for each ZIP code.



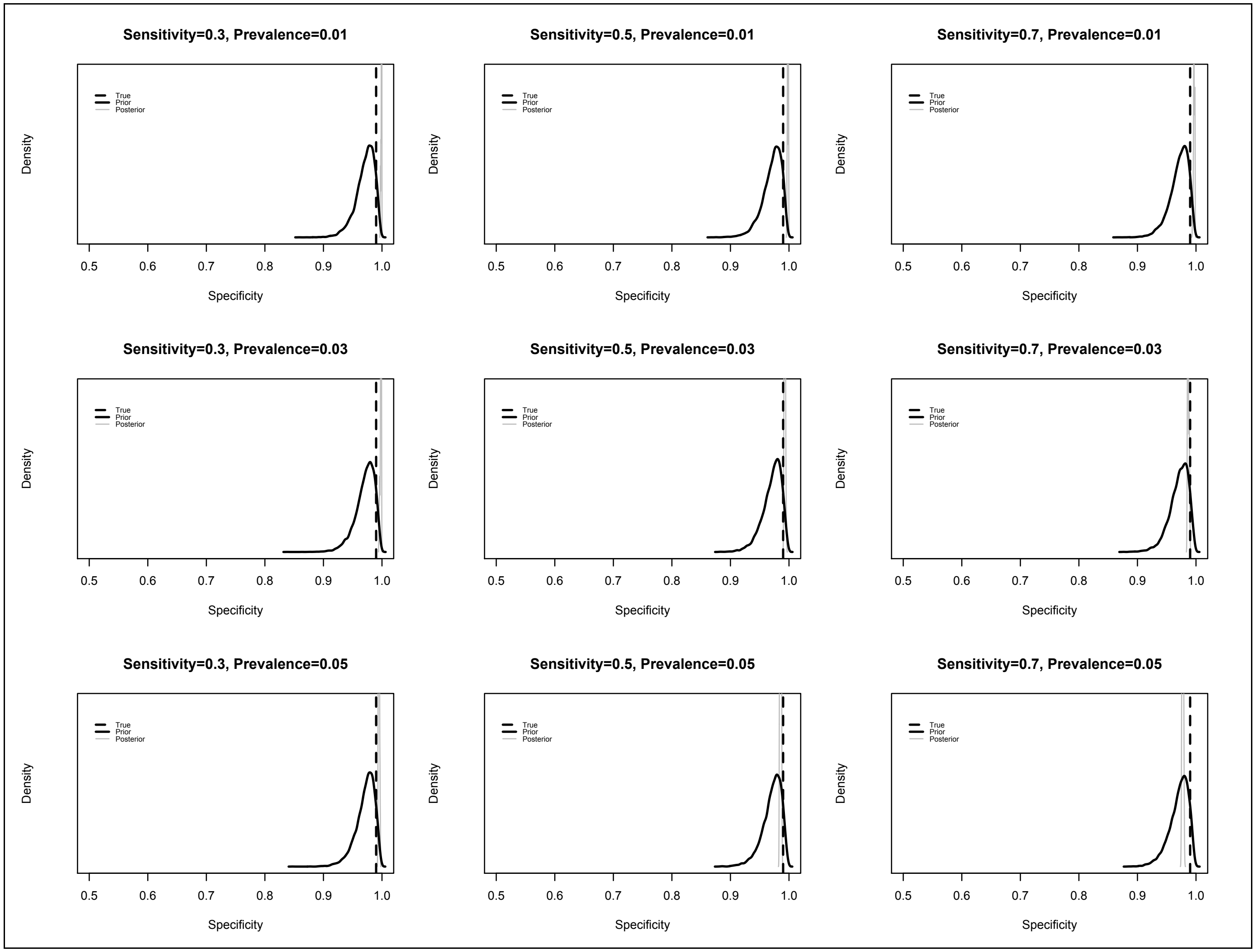
**Supplemental Figure 2**. Relationship between true prevalence in the simulated dataset and the posterior distribution of adjusted, true prevalence when priors on $\eta_0$ ~ Norm(μ=-3.5, $\sigma^2$=*0.25*) and $SN_i$ ~ Beta(α=14.022, β=9.681) for each of the nine simulated datasets. The asterisk represents the median of the posterior distribution with accompany 95% credible intervals. The solid line represents identity between the depicted values.



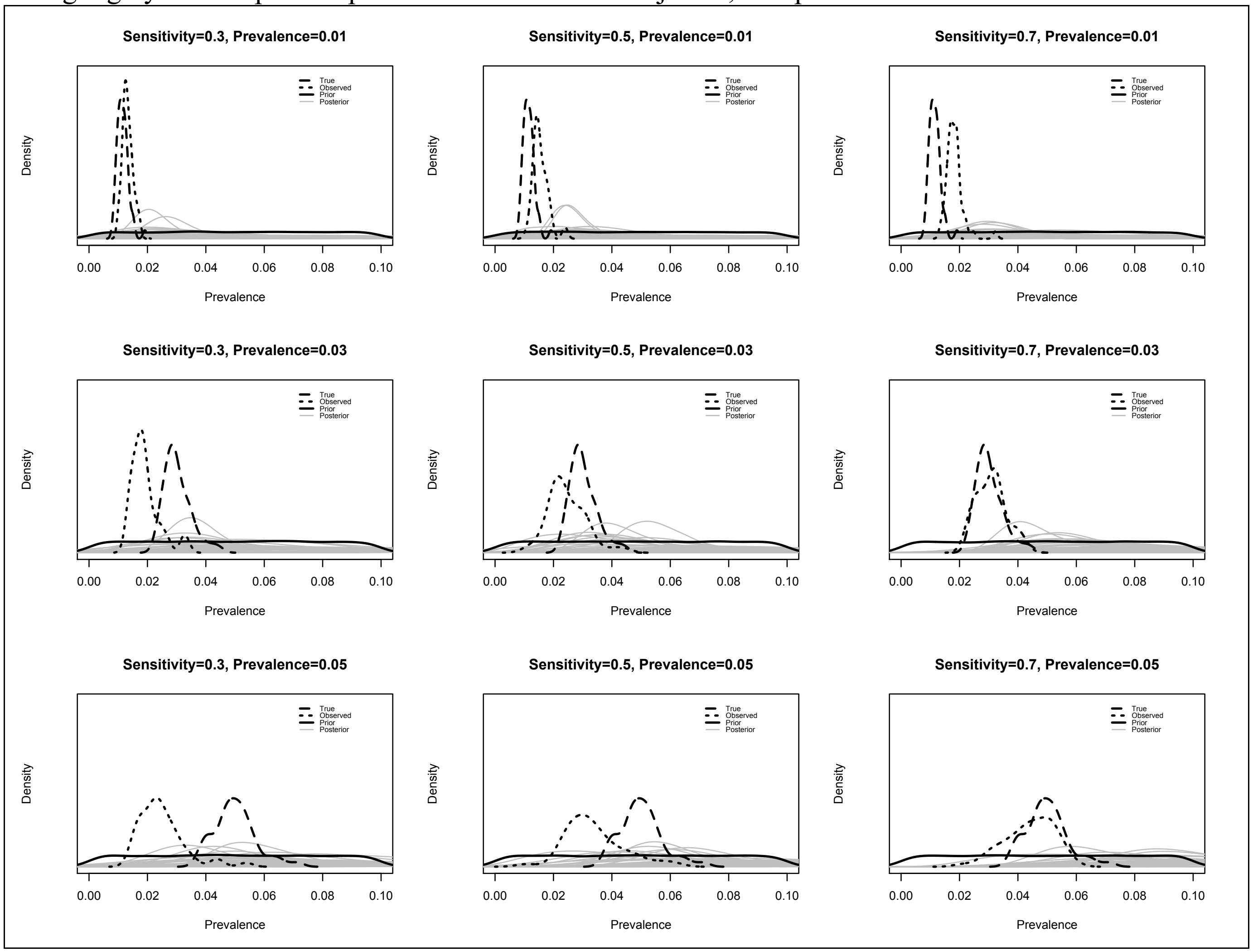
**Supplemental Figure 3**. Posterior distribution of surveillance sensitivity when priors on $\eta_0$ ~ Norm(μ=-3.5, $\sigma^2$=*0.25*) and $SN_i$ ~ Beta(α=14.022, β=9.681) for each of the nine simulated datasets. Dashed black line depicts the averaged true sensitivity in the simulated data, the solid line depicts the prior distribution, and the light gray lines depict the posterior distribution of sensitivity for each ZIP code.



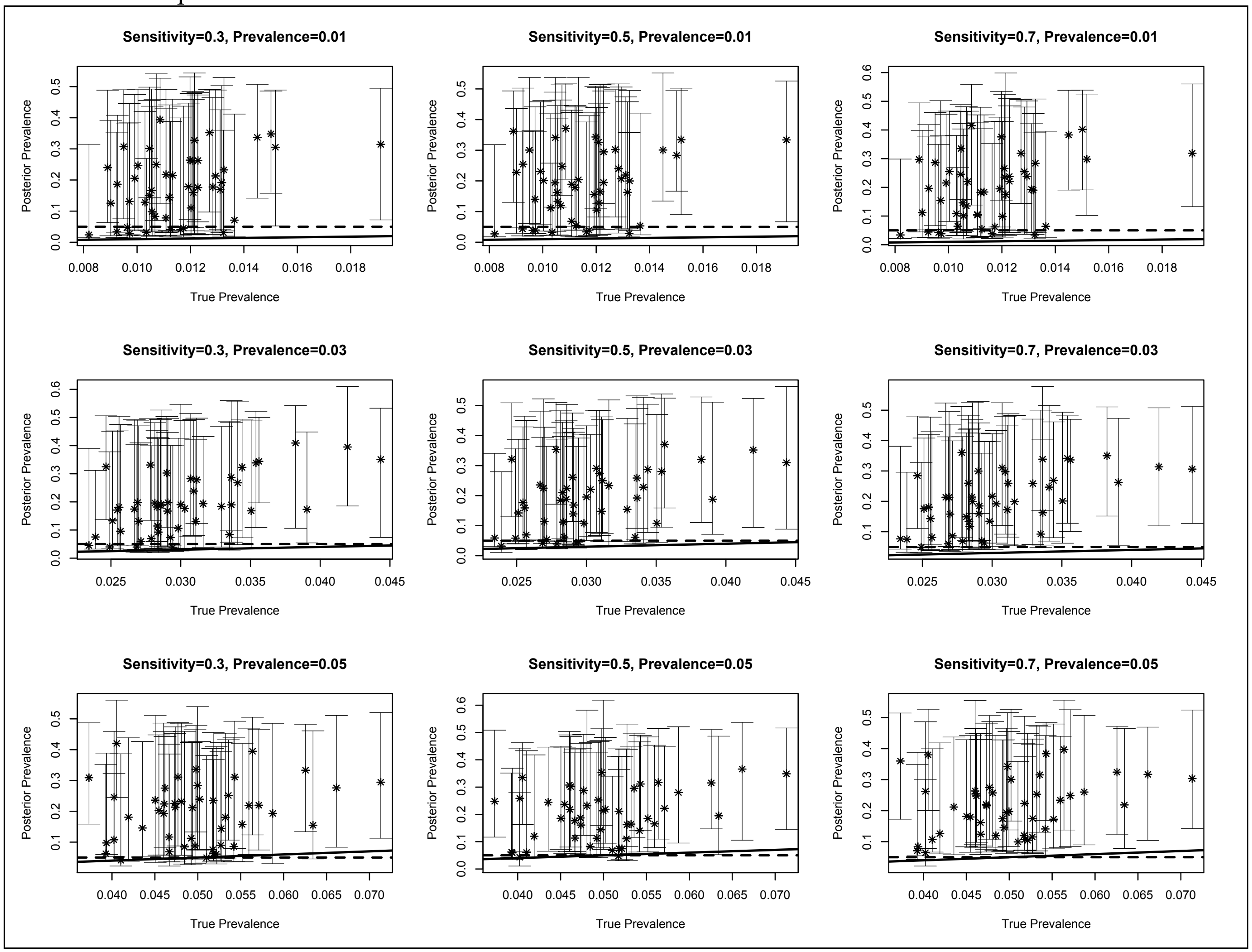
**Supplemental Figure 4**. Posterior distribution of surveillance specificity when priors on $\eta_0$ ~ Norm(μ=-3.5, $\sigma^2$=*0.25*) and $SN_i$ ~ Beta(α=14.022, β=9.681) for each of the nine simulated datasets. Dashed black line depicts the averaged true specificity in the simulated data, the solid line depicts the prior distribution, and the light gray lines depict the posterior distribution of specificity for each ZIP code.



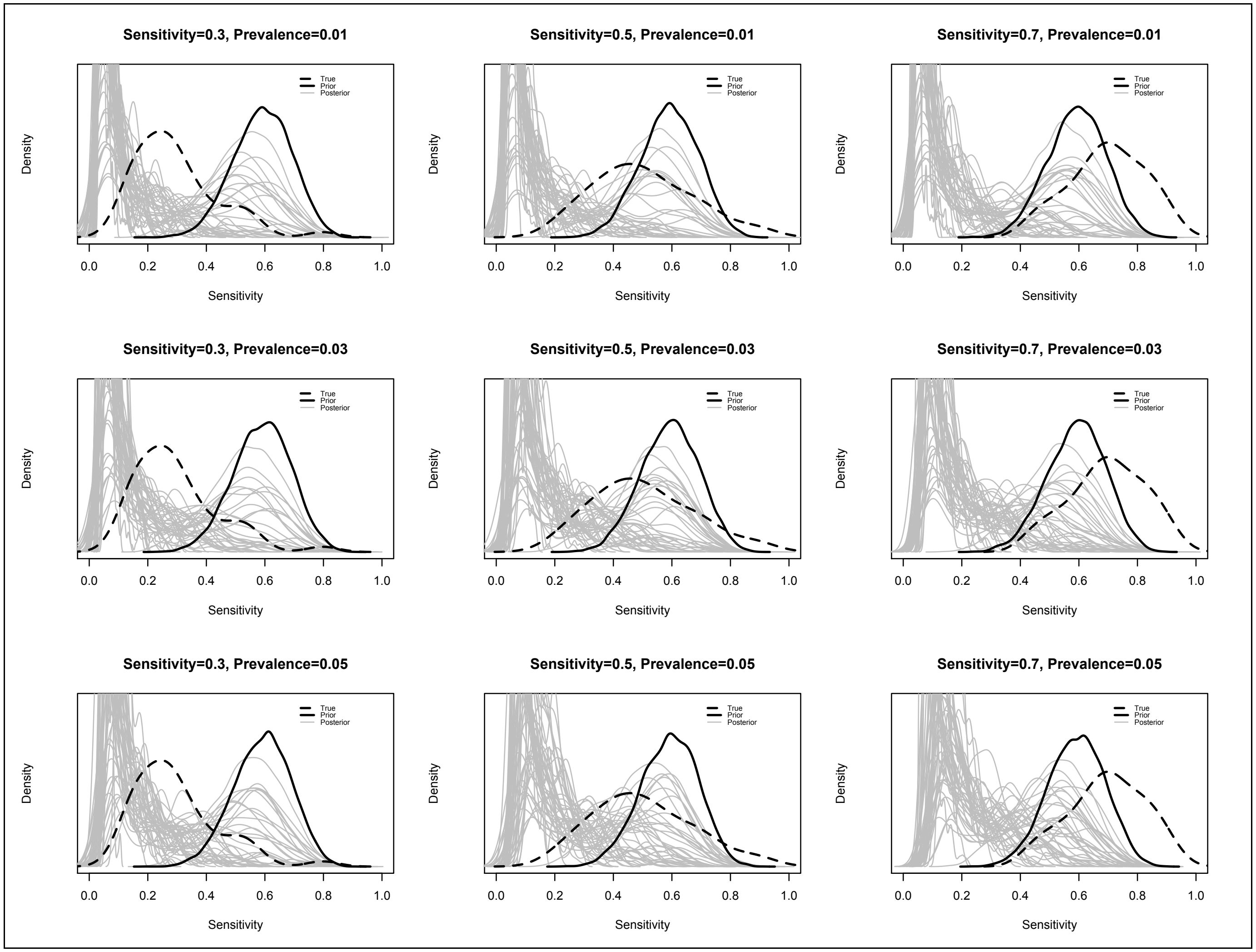
**Supplemental Figure 5**. Posterior distribution of true prevalence when priors on $\eta_0$ ~ Norm($\mu$=0, $\sigma^2$=*2.71*) and $SN_i$ ~ Beta($\alpha$=14.022, $\beta$=9.681) for each of the nine simulated datasets. Dashed black line depicts the averaged true prevalence of COVID-19 in the simulated data, the dotted line depicts the averaged observed, misclassified prevalence, the solid line depicts the prior distribution, and the light gray lines depict the posterior distribution of adjusted, true prevalence for each ZIP code.



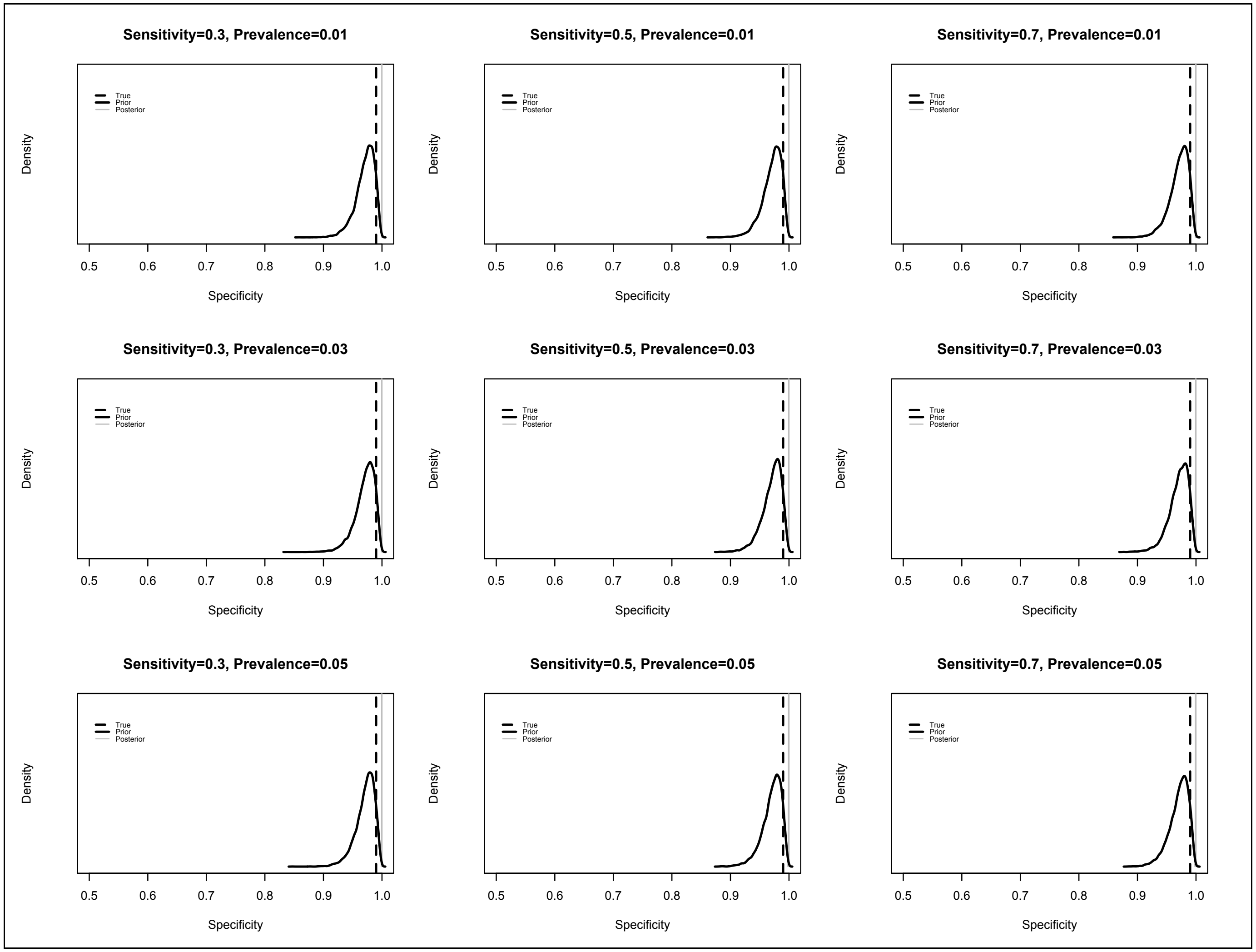
**Supplemental Figure 6**. Relationship between true prevalence in the simulated dataset and the posterior distribution of adjusted, true prevalence when priors on $\eta_0 \sim \text{Norm}(\mu=0, \sigma^2=2.71)$ and $SN_i \sim \text{Beta}(\alpha=14.022, \beta=9.681)$ for each of the nine simulated datasets. The asterisk represents the median of the posterior distribution with accompany 95% credible intervals. The solid line represents identity between the depicted values.



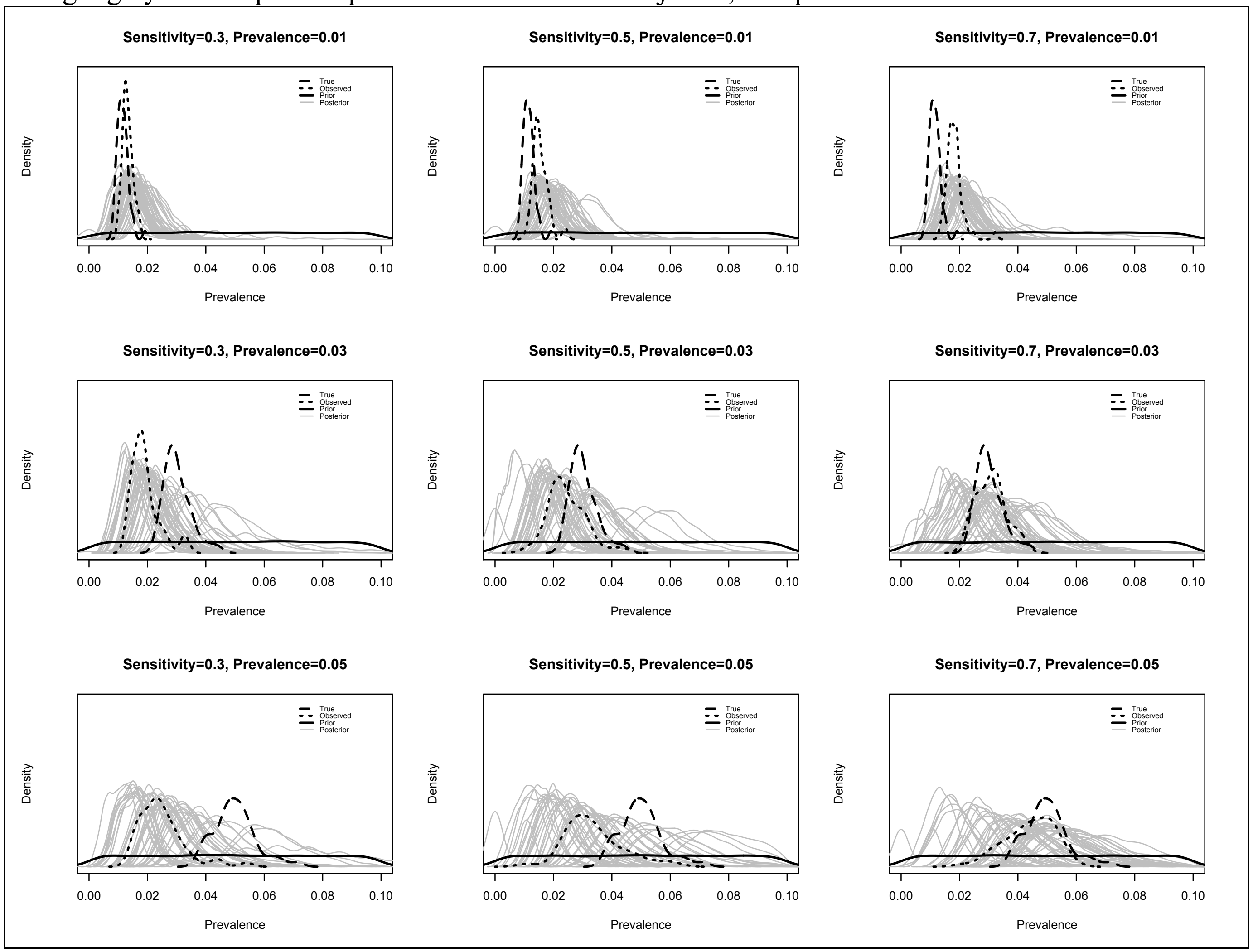
**Supplemental Figure 7**. Posterior distribution of surveillance sensitivity when priors on $\eta_0$ ~ Norm(μ=0, $\sigma^2$=*2.71*) and $SN_i$ ~ Beta(α=14.022, β=9.681) for each of the nine simulated datasets. Dashed black line depicts the averaged true sensitivity in the simulated data, the solid line depicts the prior distribution, and the light gray lines depict the posterior distribution of sensitivity for each ZIP code.



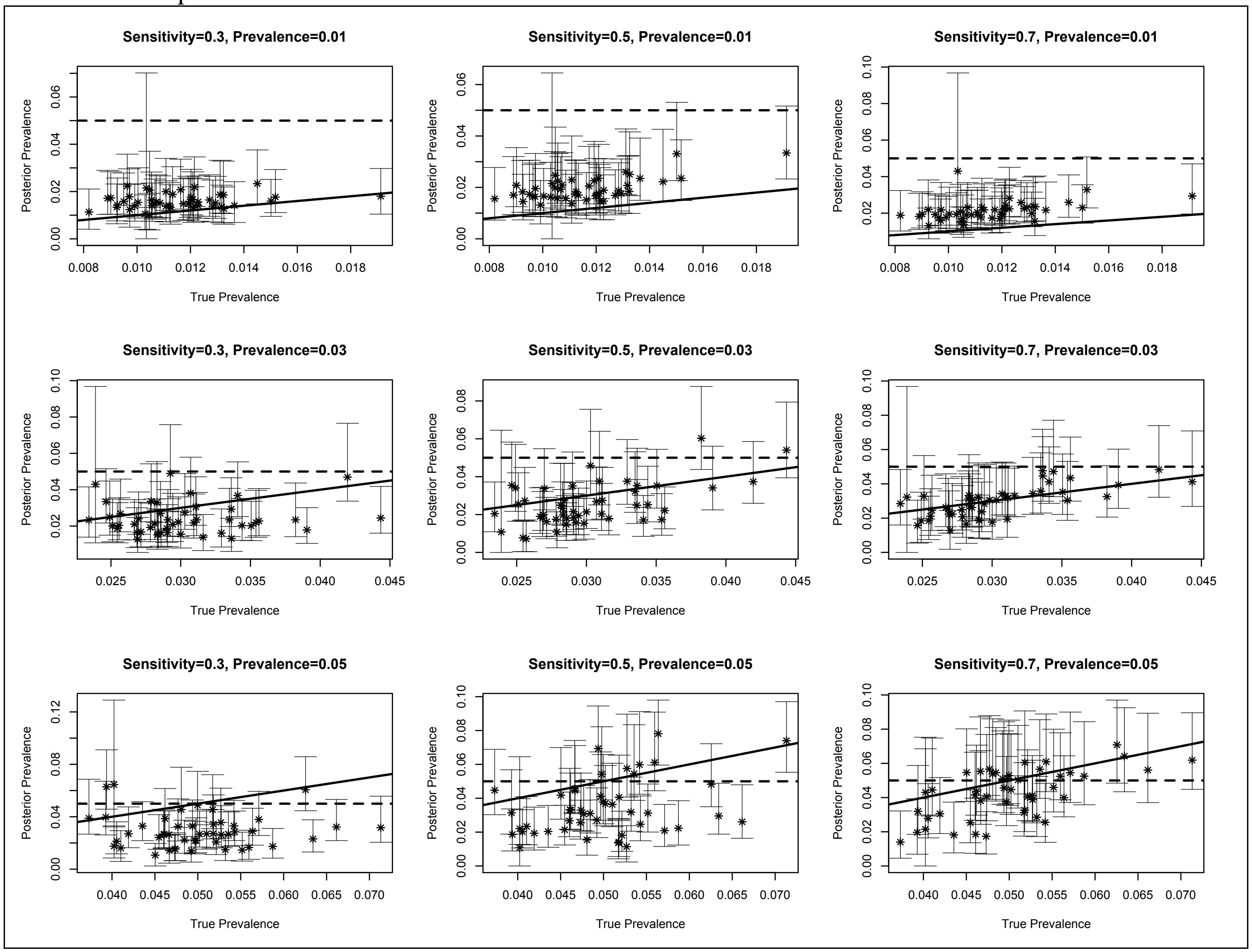
**Supplemental Figure 8**. Posterior distribution of surveillance specificity when priors on $\eta_0$ ~ Norm($\mu$=0, $\sigma^2$=*2.71*) and $SN_i$ ~ Beta($\alpha$=14.022, $\beta$=9.681) for each of the nine simulated datasets. Dashed black line depicts the averaged true specificity in the simulated data, the solid line depicts the prior distribution, and the light gray lines depict the posterior distribution of specificity for each ZIP code.



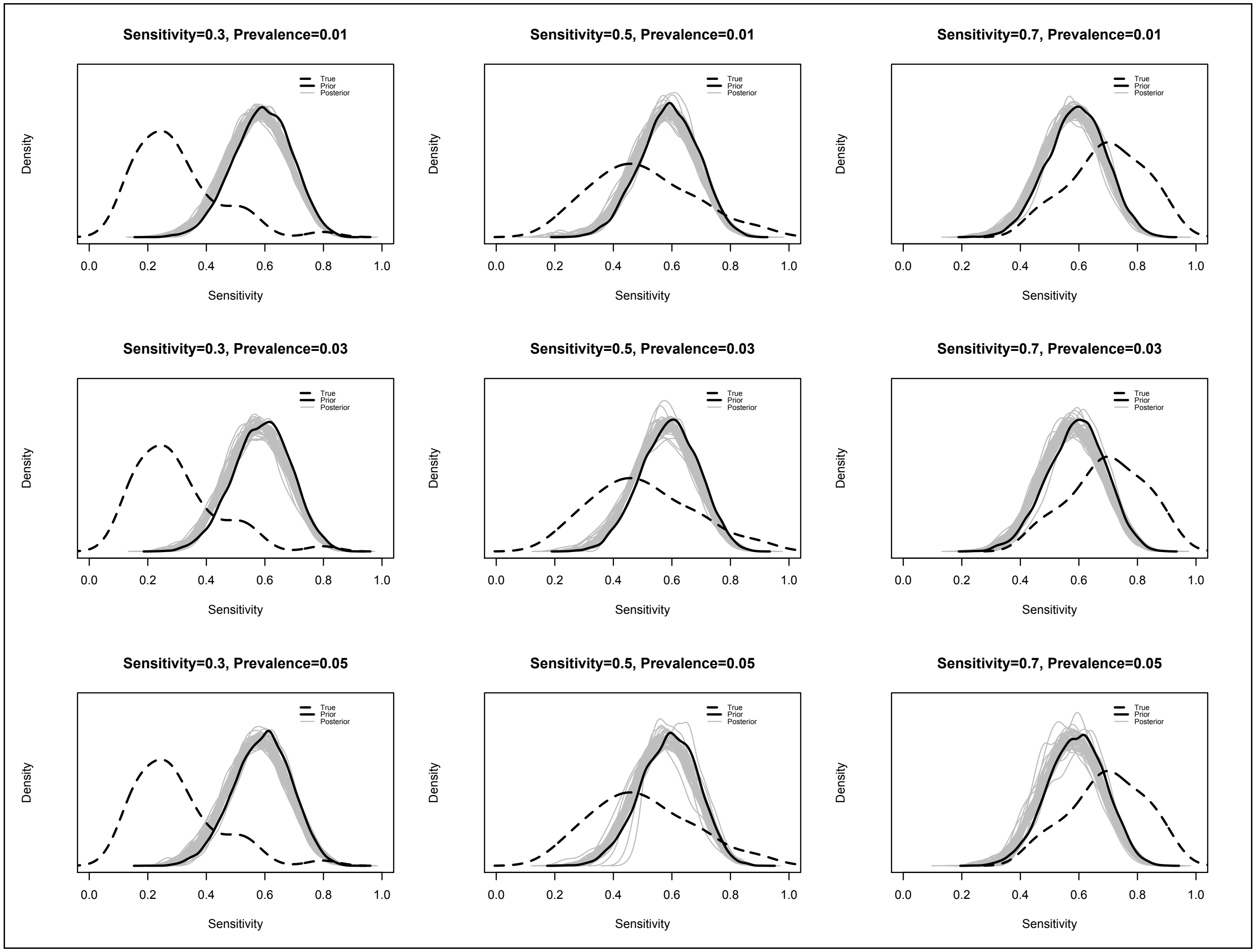
**Supplemental Figure 9**. Posterior distribution of true prevalence when priors on $r_i$ ~ Unif($a$=0, $b$=0.1) and $SN_i$ ~ Beta($\alpha$=14.022, $\beta$=9.681) for each of the nine simulated datasets. Dashed black line depicts the averaged true prevalence of COVID-19 in the simulated data, the dotted line depicts the averaged observed, misclassified prevalence, the solid line depicts the prior distribution, and the light gray lines depict the posterior distribution of adjusted, true prevalence for each ZIP code.



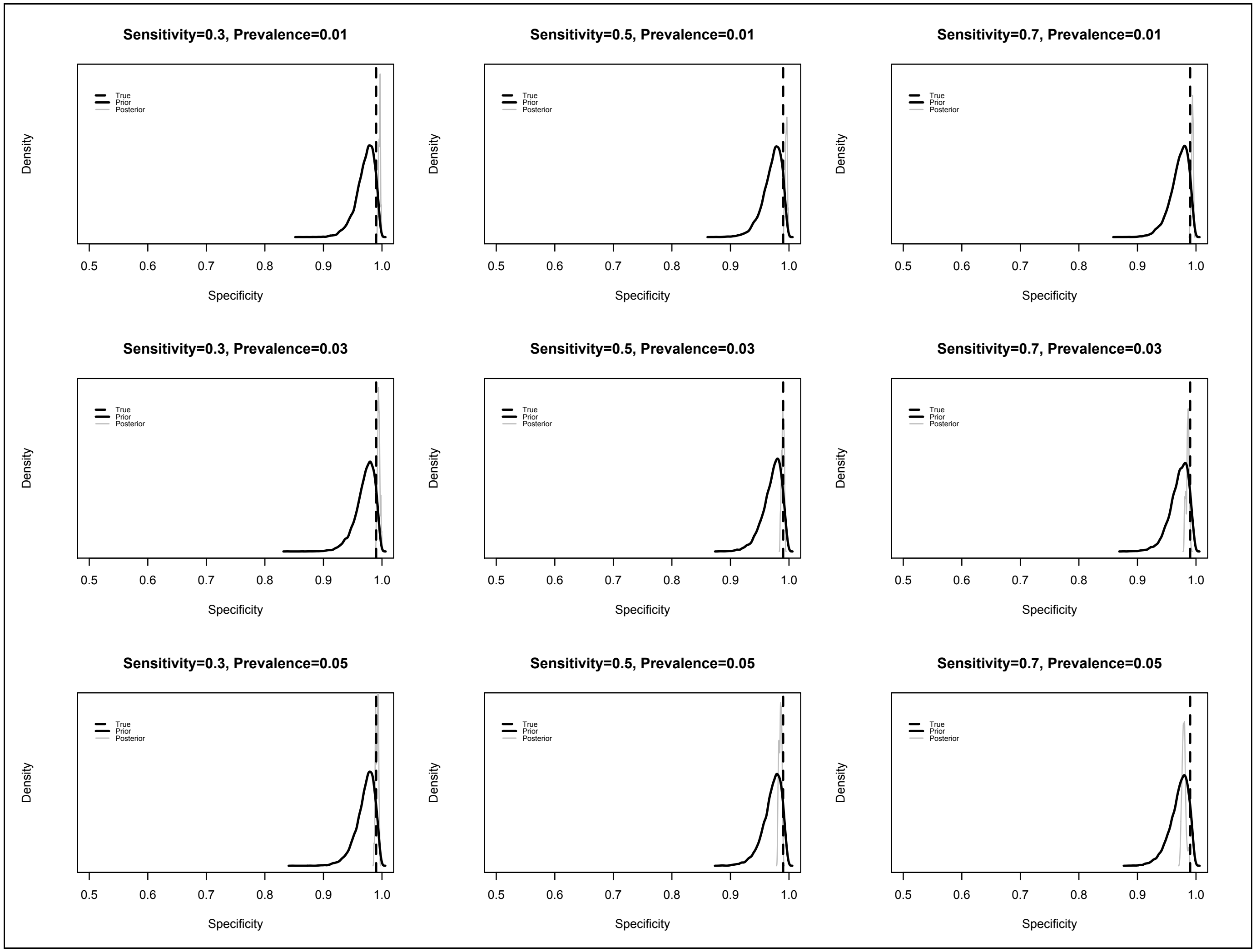
**Supplemental Figure 10**. Relationship between true prevalence in the simulated dataset and the posterior distribution of adjusted, true prevalence when priors on $r_i \sim \mathrm{Unif}(a=0, b=0.1)$ and $SN_i \sim \mathrm{Beta}(\alpha=14.022, \beta=9.681)$ for each of the nine simulated datasets. The asterisk represents the median of the posterior distribution with accompany 95% credible intervals. The solid line represents identity between the depicted values.



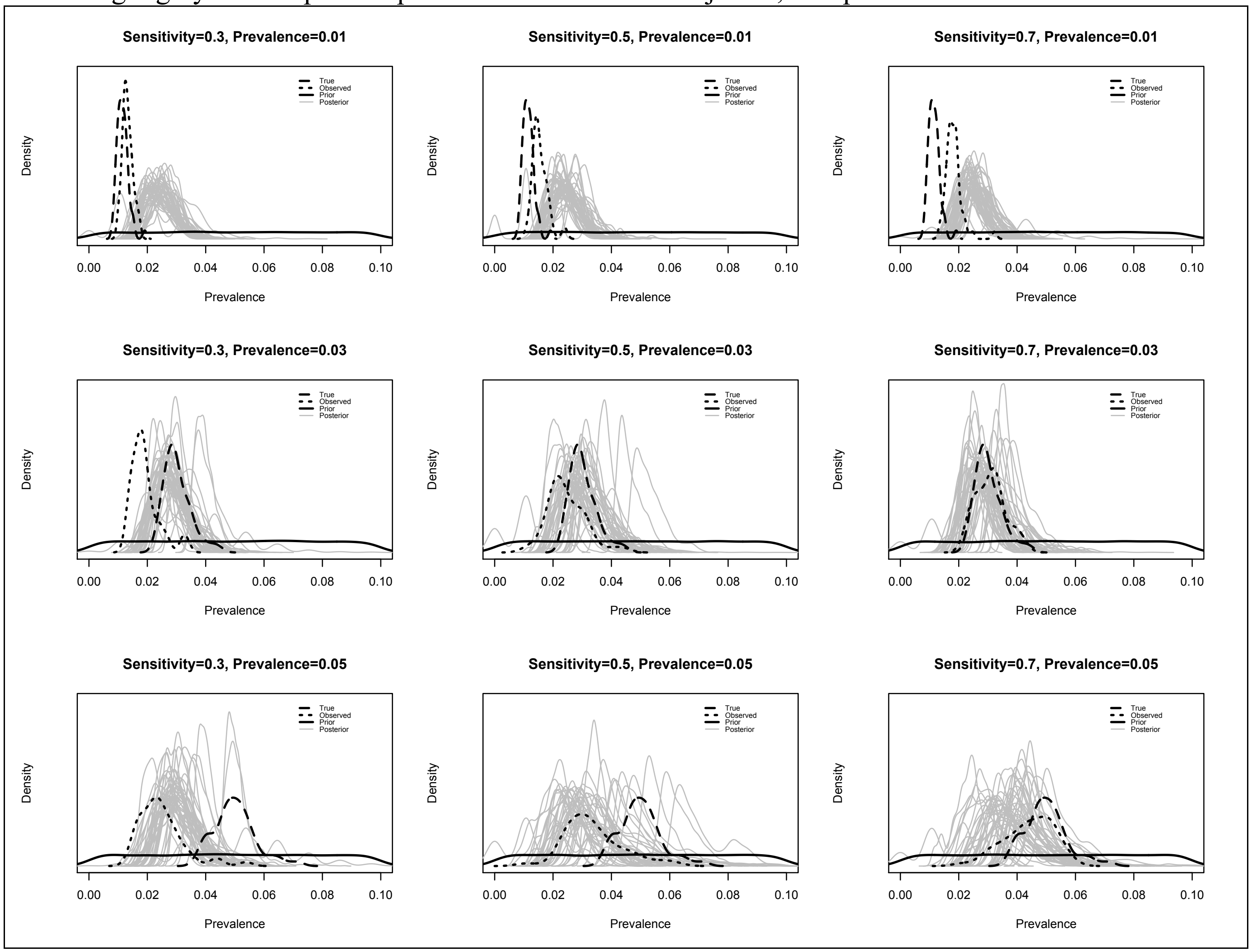
**Supplemental Figure 11**. Posterior distribution of surveillance sensitivity when priors on $r_i$ ~ Unif($a$=0, $b$=0.1) and $SN_i$ ~ Beta($\alpha$=14.022, $\beta$=9.681) for each of the nine simulated datasets. Dashed black line depicts the averaged true sensitivity in the simulated data, the solid line depicts the prior distribution, and the light gray lines depict the posterior distribution of sensitivity for each ZIP code.



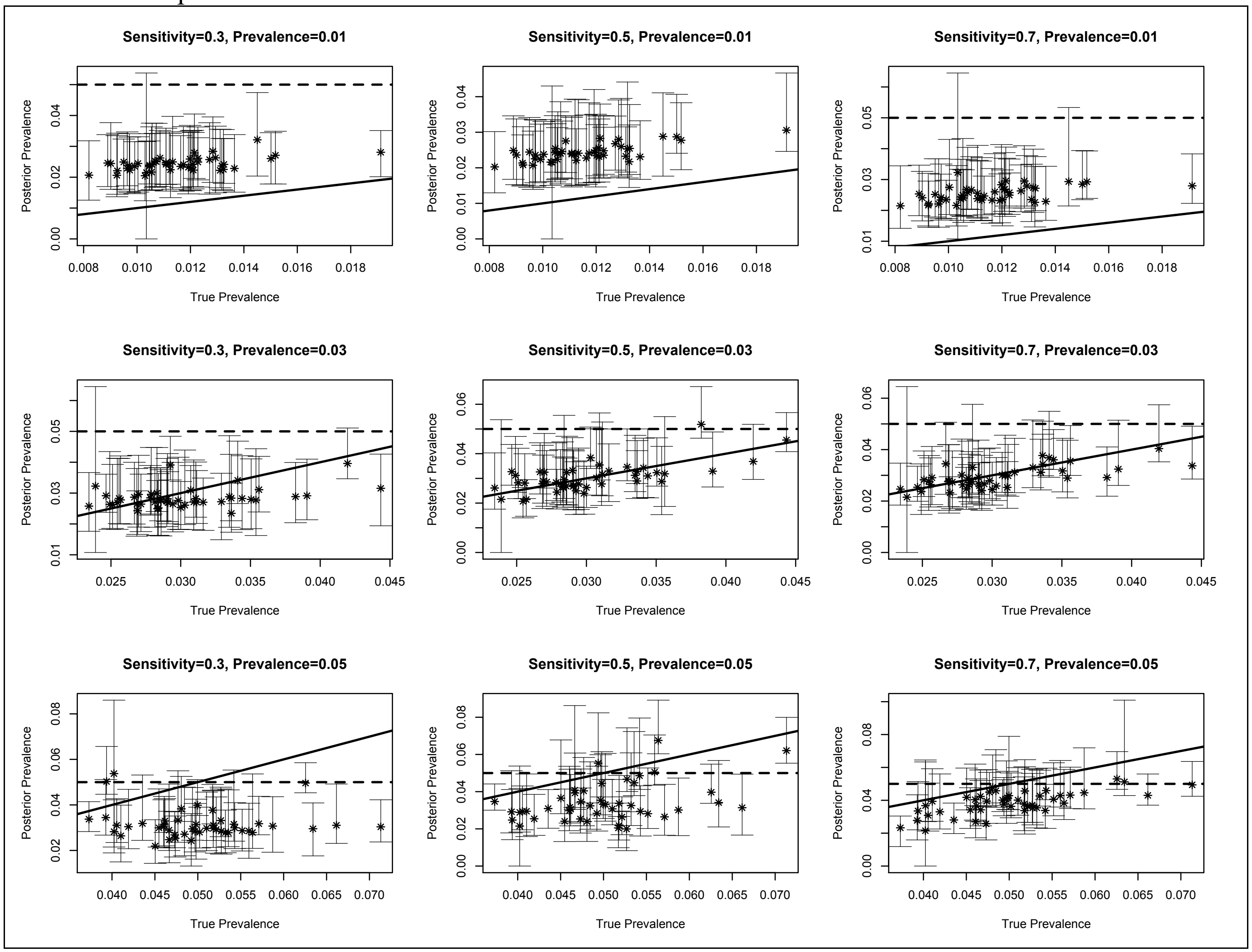
**Supplemental Figure 12**. Posterior distribution of surveillance specificity when priors on $r_i$ ~ Unif($a$=0, $b$=0.1) and $SN_i$ ~ Beta(α=14.022, β=9.681) for each of the nine simulated datasets. Dashed black line depicts the averaged true specificity in the simulated data, the solid line depicts the prior distribution, and the light gray lines depict the posterior distribution of specificity for each ZIP code.



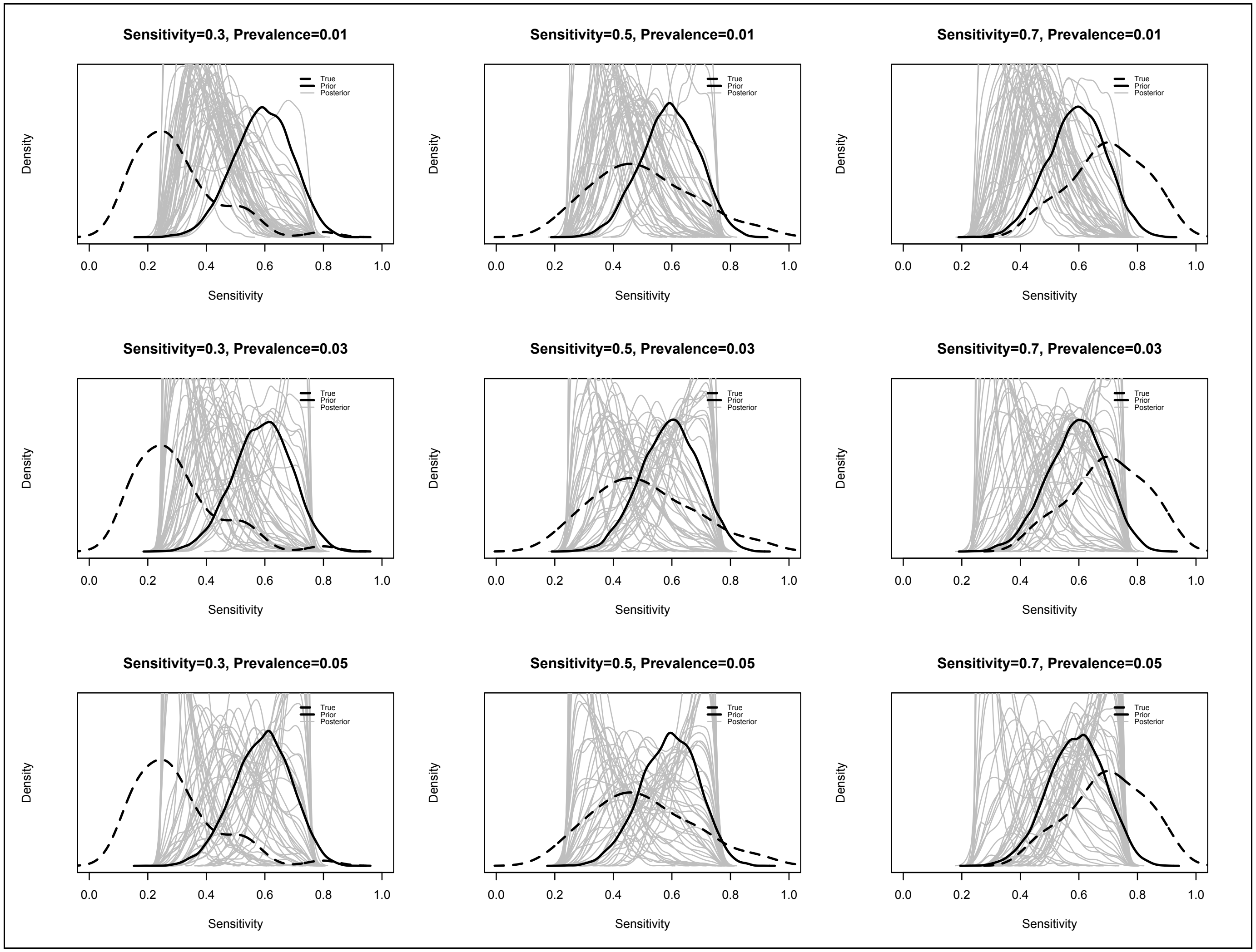
**Supplemental Figure 13**. Posterior distribution of true prevalence when priors on $\eta_0$ ~ Norm(μ=-3.5, $\sigma^2$=*0.25*) and $SN_i$ ~ Unif($a$=0.25, $b$=0.75) for each of the nine simulated datasets. Dashed black line depicts the averaged true prevalence of COVID-19 in the simulated data, the dotted line depicts the averaged observed, misclassified prevalence, the solid line depicts the prior distribution, and the light gray lines depict the posterior distribution of adjusted, true prevalence for each ZIP code.



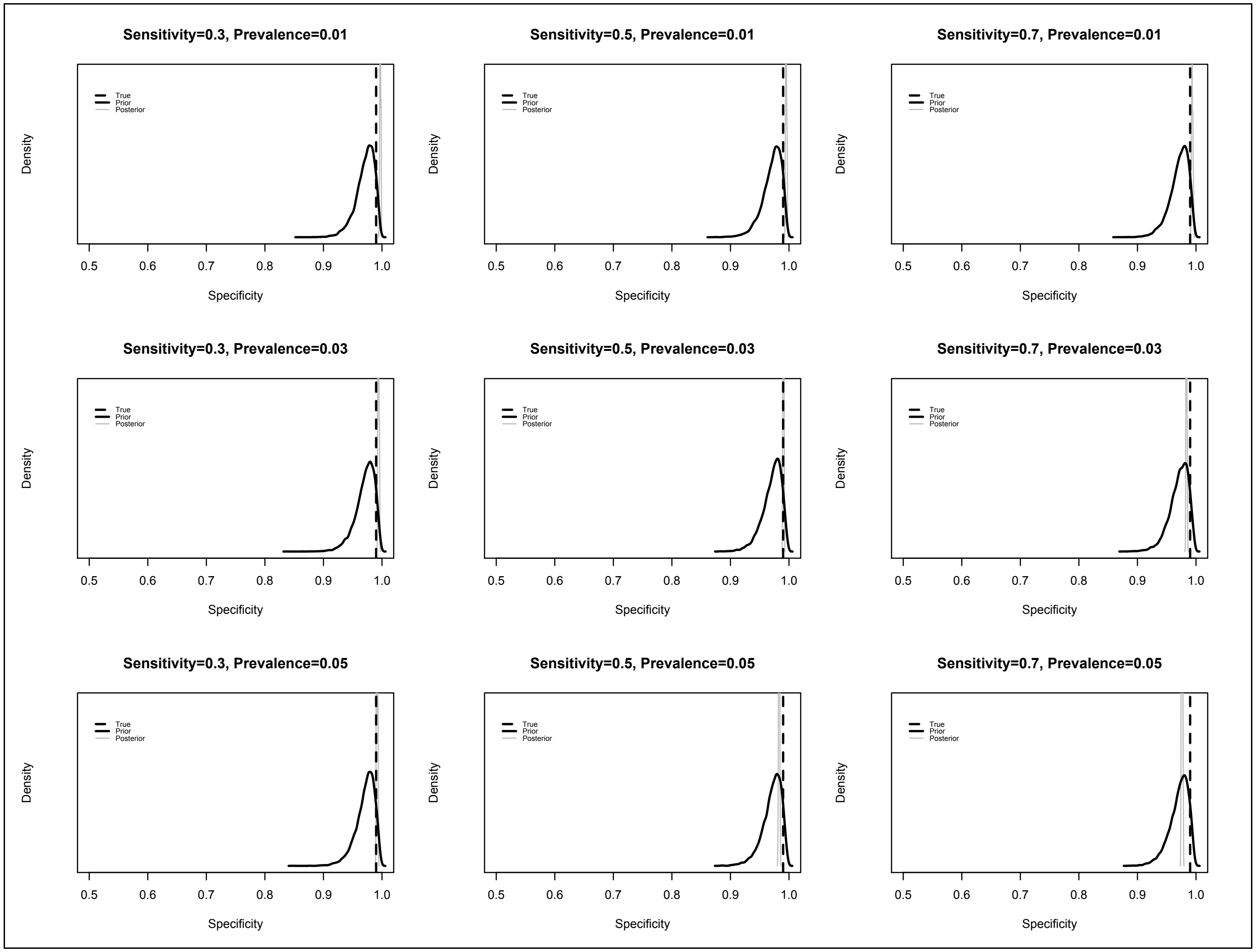
**Supplemental Figure 14**. Relationship between true prevalence in the simulated dataset and the posterior distribution of adjusted, true prevalence when priors on $\eta_0$ ~ Norm(μ=-3.5, $\sigma^2$=*0.25*) and $SN_i$ ~ Unif($a$=0.25, $b$=0.75) for each of the nine simulated datasets. The asterisk represents the median of the posterior distribution with accompany 95% credible intervals. The solid line represents identity between the depicted values.



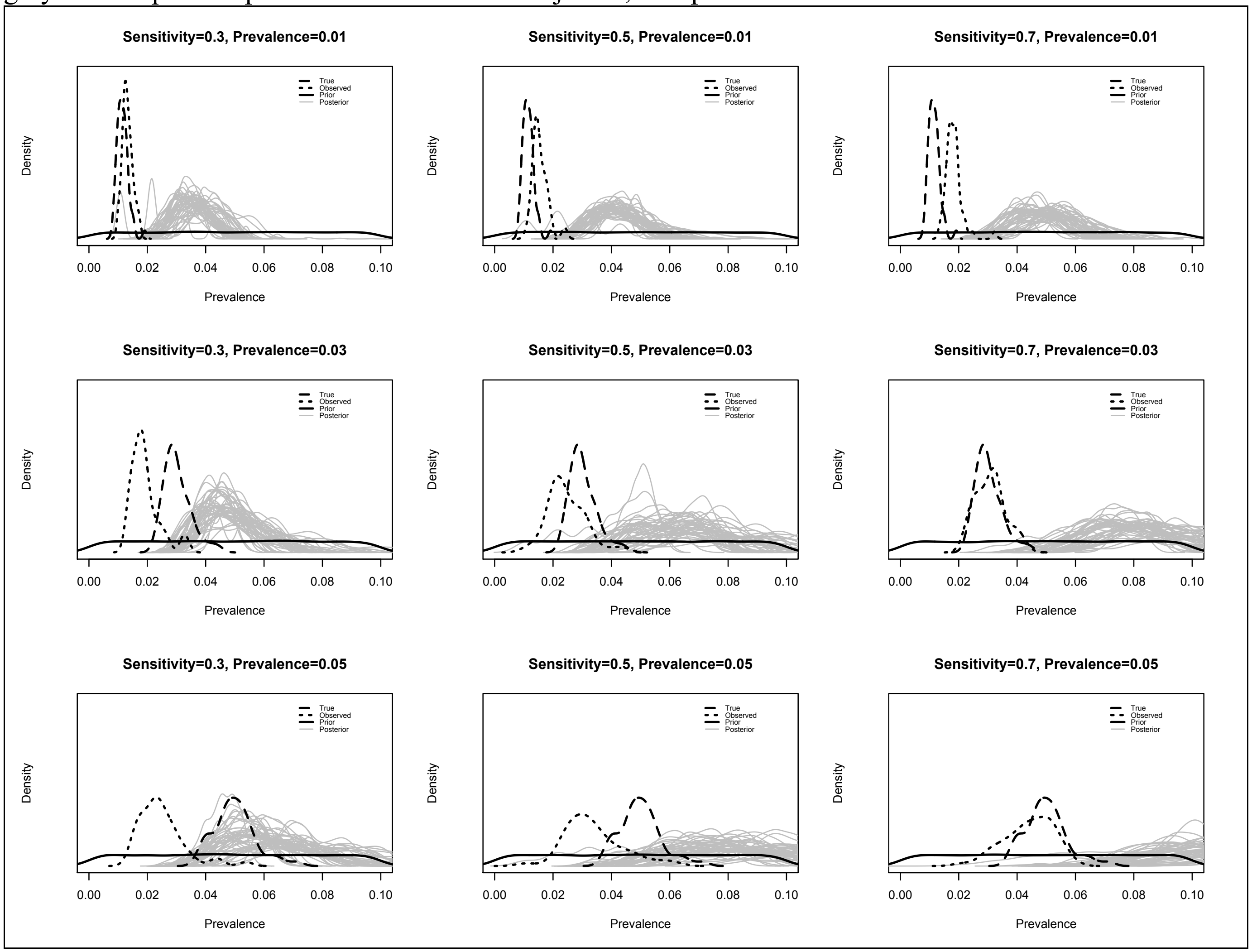
**Supplemental Figure 15**. Posterior distribution of surveillance sensitivity when priors on $\eta_0 \sim$ Norm($\mu$=-3.5, $\sigma^2$=*0.25*) and $SN_i \sim$ Unif($a$=0.25, $b$=0.75) for each of the nine simulated datasets. Dashed black line depicts the averaged true sensitivity in the simulated data, the solid line depicts the prior distribution, and the light gray lines depict the posterior distribution of sensitivity for each ZIP code.



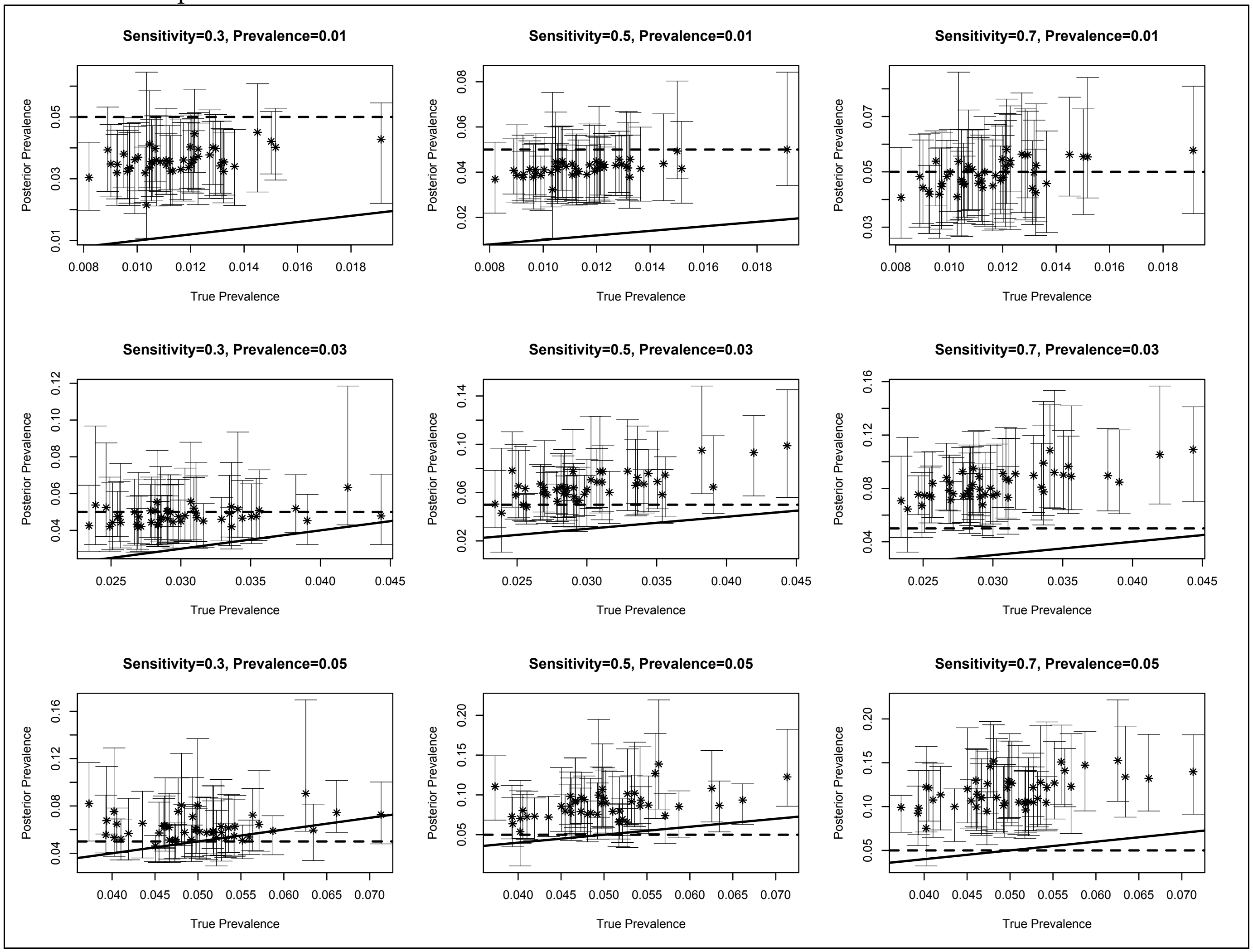
**Supplemental Figure 16**. Posterior distribution of surveillance specificity when priors on $\eta_0$ ~ Norm(μ=-3.5, $\sigma^2$=*0.25*) and $SN_i$ ~ Unif($a$=0.25, $b$=0.75) for each of the nine simulated datasets. Dashed black line depicts the averaged true specificity in the simulated data, the solid line depicts the prior distribution, and the light gray lines depict the posterior distribution of specificity for each ZIP code.



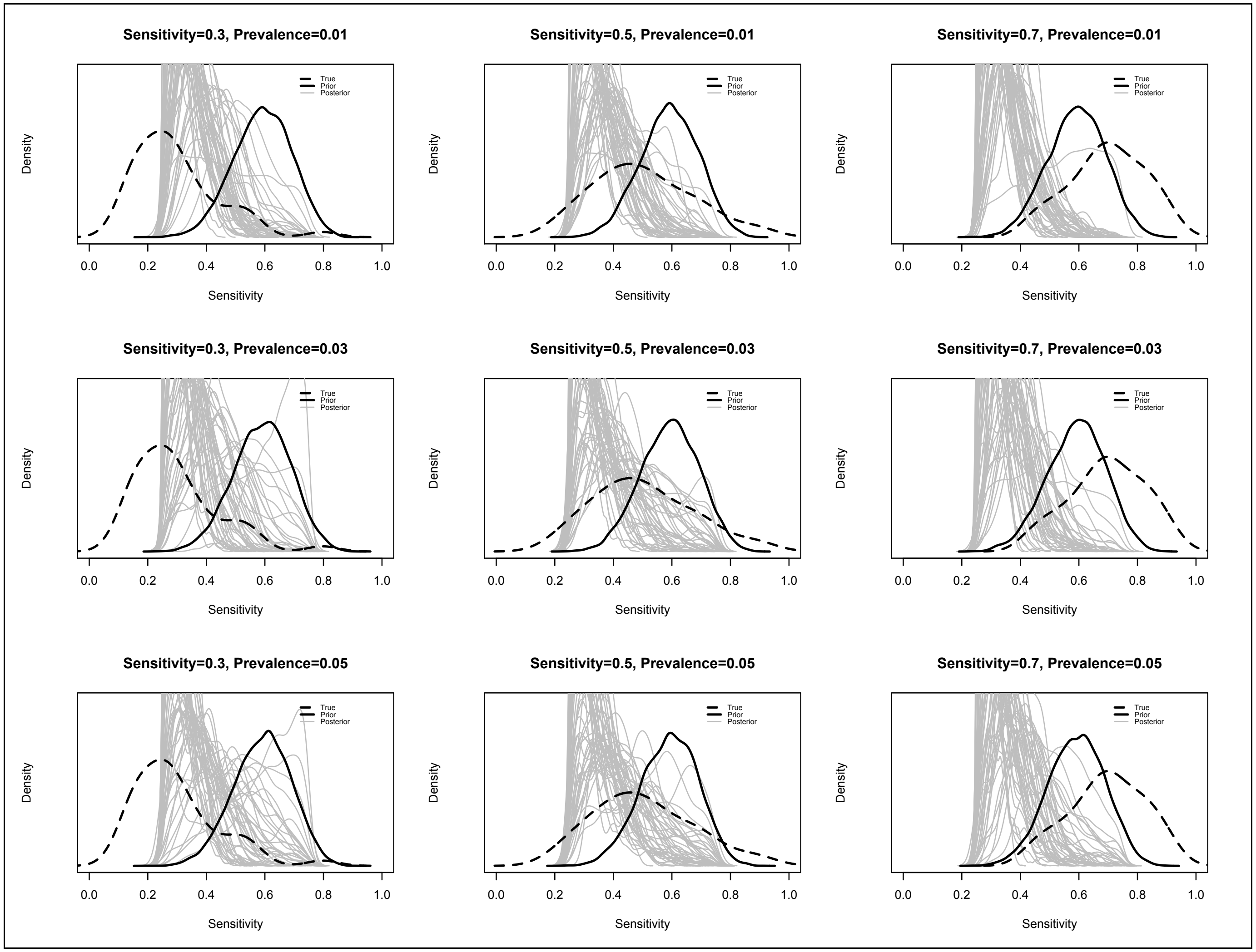
**Supplemental Figure 17**. Posterior distribution of true prevalence when priors on $\eta_0$ ~ Norm($\mu$=0, $\sigma^2$=*2.71*) and $SN_i$ ~ Unif($a$=0.25, $b$=0.75) for each of the nine simulated datasets. Dashed black line depicts the averaged true prevalence of COVID-19 in the simulated data, the dotted line depicts the averaged observed, misclassified prevalence, the solid line depicts the prior distribution, and the light gray lines depict the posterior distribution of adjusted, true prevalence for each ZIP code.



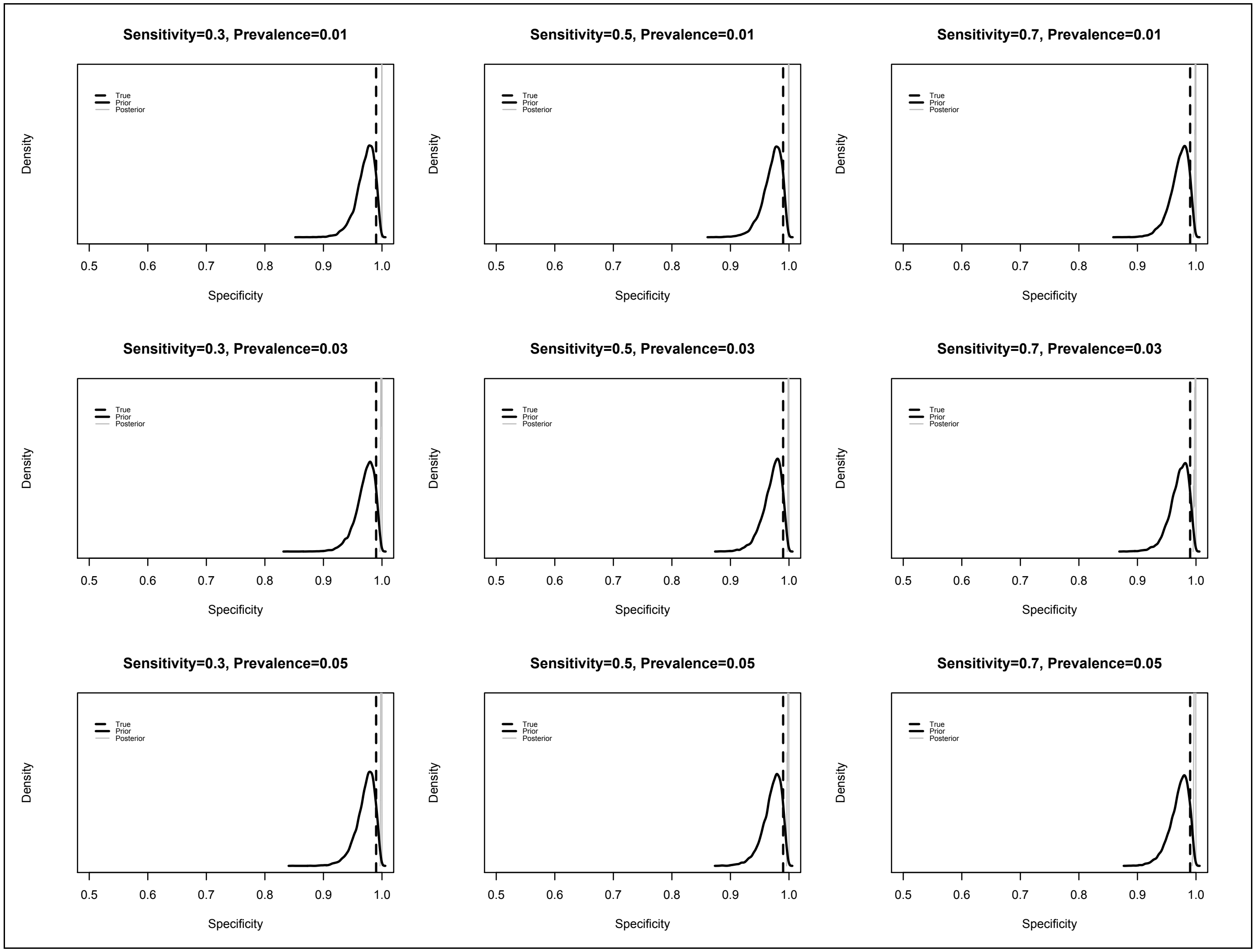
**Supplemental Figure 18**. Relationship between true prevalence in the simulated dataset and the posterior distribution of adjusted, true prevalence when priors on $\eta_0$ ~ Norm(μ=0, $\sigma^2$=*2.71*) and $SN_i$ ~ Unif($a$=0.25, $b$=0.75) for each of the nine simulated datasets. The asterisk represents the median of the posterior distribution with accompany 95% credible intervals. The solid line represents identity between the depicted values.



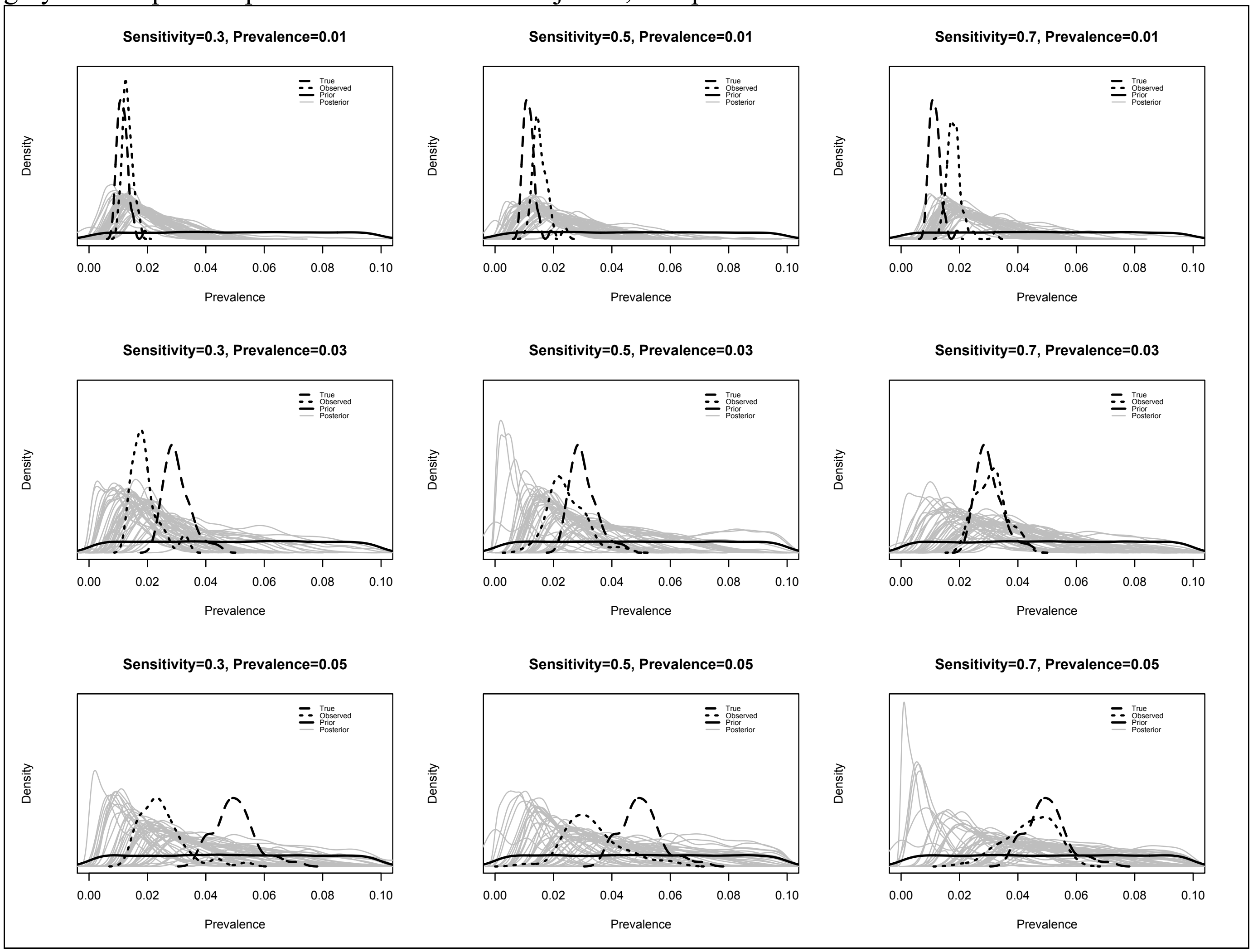
**Supplemental Figure 19**. Posterior distribution of surveillance sensitivity when priors on $\eta_0$ ~ Norm(μ=0, $\sigma^2$=*2.71*) and $SN_i$ ~ Unif($a$=0.25, $b$=0.75) for each of the nine simulated datasets. Dashed black line depicts the averaged true sensitivity in the simulated data, the solid line depicts the prior distribution, and the light gray lines depict the posterior distribution of sensitivity for each ZIP code.



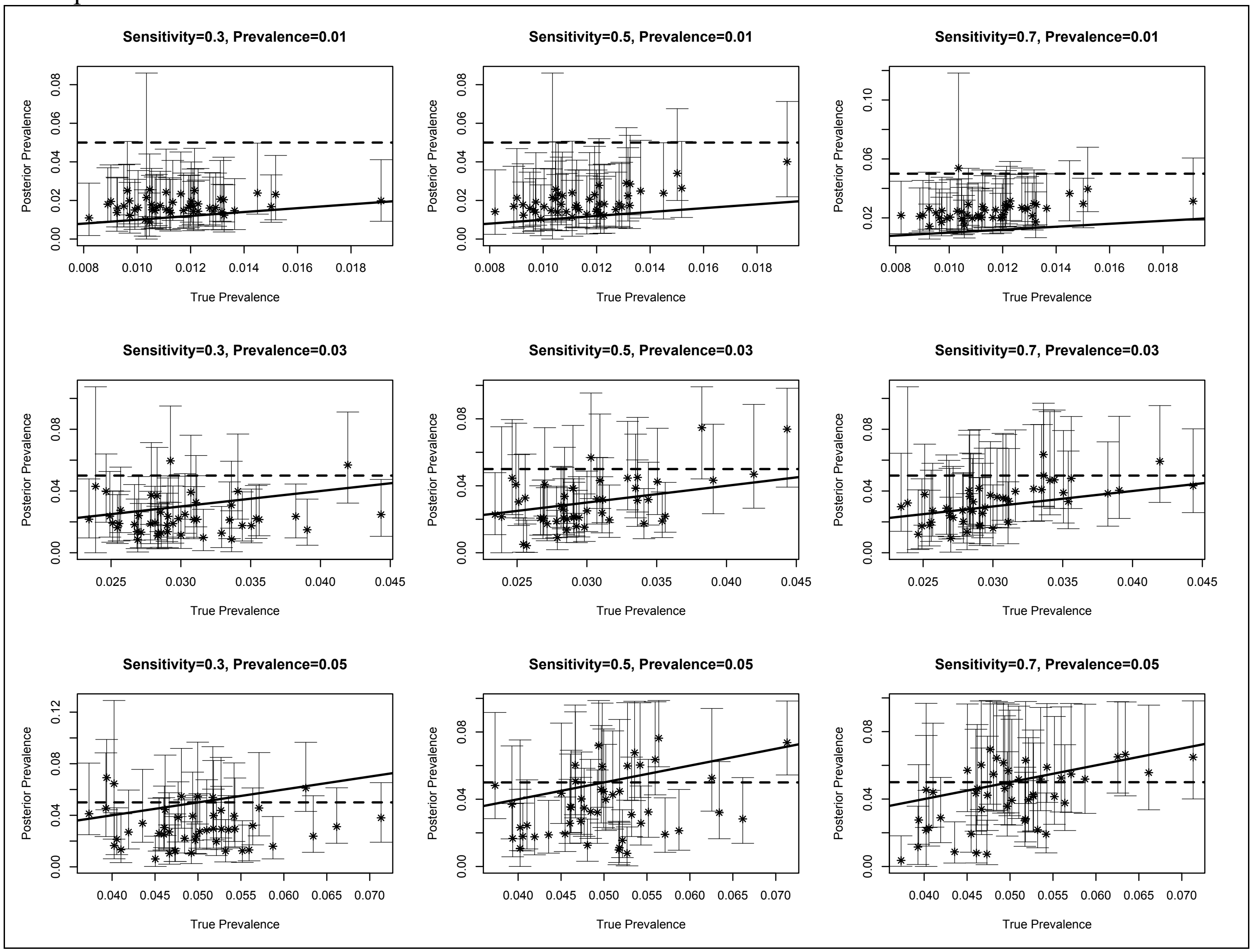
**Supplemental Figure 20**. Posterior distribution of surveillance specificity when priors on $\eta_0$ ~ Norm($\mu$=0, $\sigma^2$=*2.71*) and $SN_i$ ~ Unif($a$=0.25, $b$=0.75) for each of the nine simulated datasets. Dashed black line depicts the averaged true specificity in the simulated data, the solid line depicts the prior distribution, and the light gray lines depict the posterior distribution of specificity for each ZIP code.



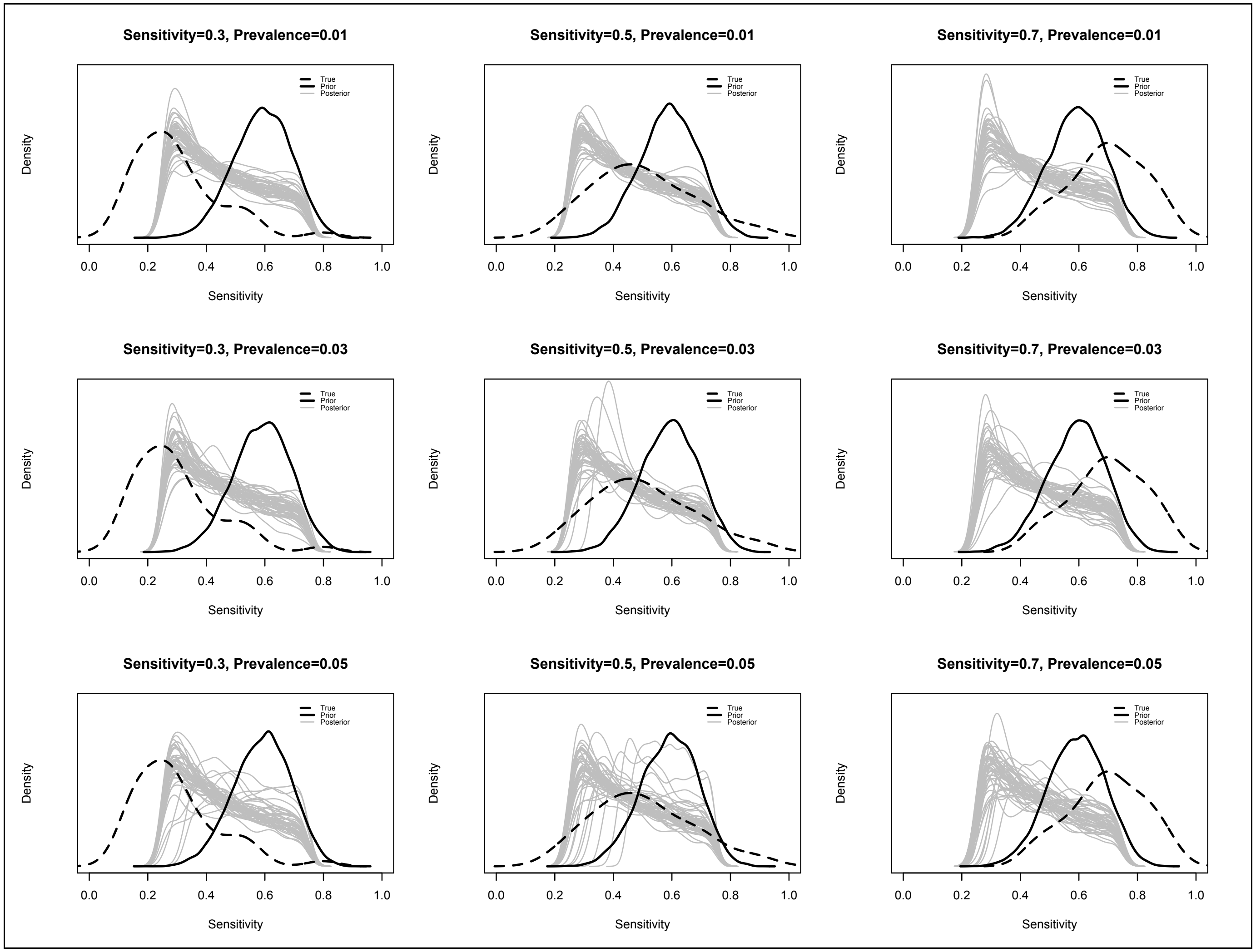
**Supplemental Figure 21**. Posterior distribution of true prevalence when priors on $r_i$ ~ Unif($a$=0, $b$=0.1) and $SN_i$ ~ Unif($a$=0.25, $b$=0.75) for each of the nine simulated datasets. Dashed black line depicts the averaged true prevalence of COVID-19 in the simulated data, the dotted line depicts the averaged observed, misclassified prevalence, the solid line depicts the prior distribution, and the light gray lines depict the posterior distribution of adjusted, true prevalence for each ZIP code.



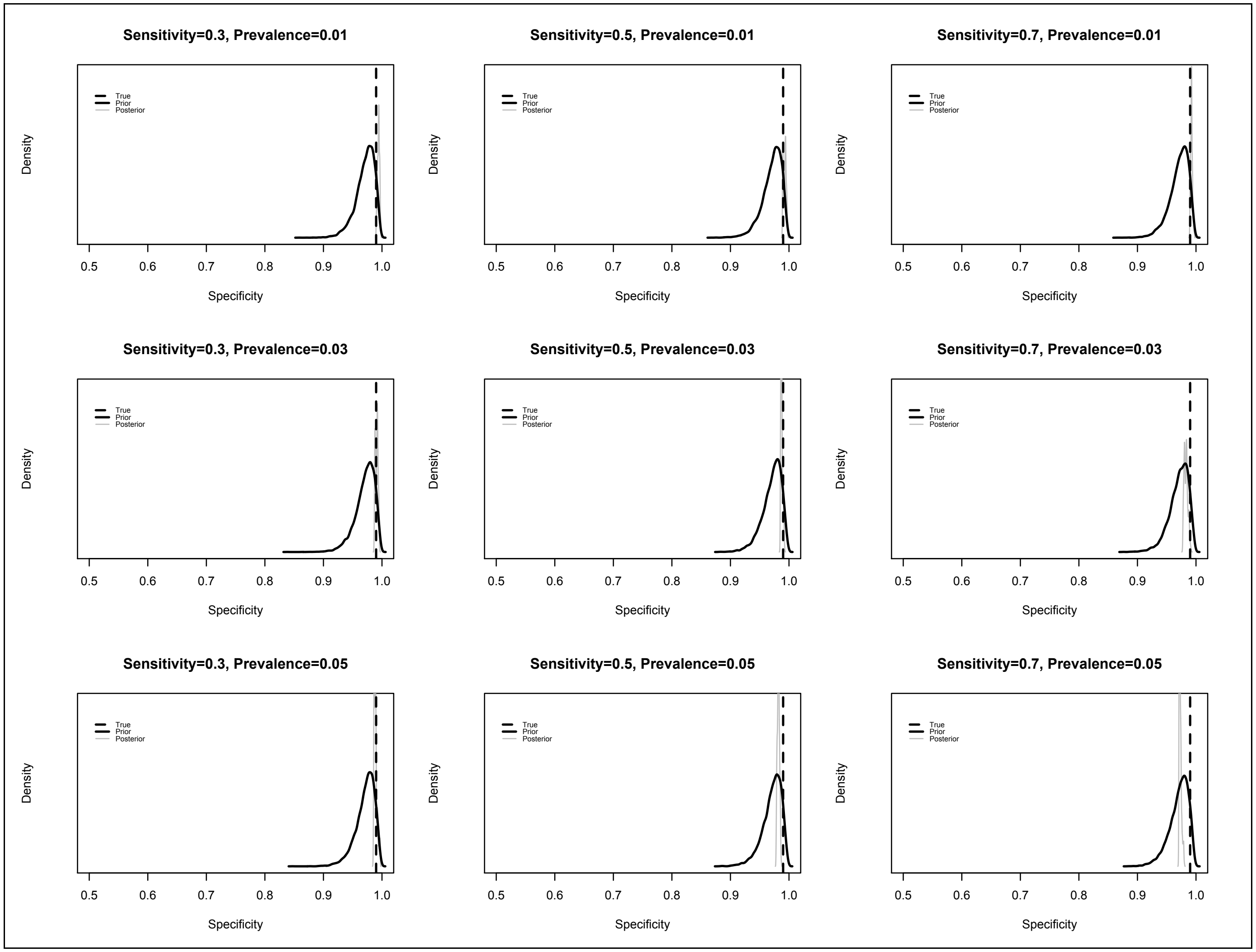
**Supplemental Figure 22**. Relationship between true prevalence in the simulated dataset and the posterior distribution of adjusted, true prevalence when priors on $r_i \sim$ Unif($a$=0, $b$=0.1) and $SN_i \sim$ Unif($a$=0.25, $b$=0.75) for each of the nine simulated datasets. The asterisk represents the median of the posterior distribution with accompany 95% credible intervals. The solid line represents identity between the depicted values.



**Supplemental Figure 23**. Posterior distribution of surveillance sensitivity when priors on $r_i \sim$ Unif($a$=0, $b$=0.1) and $SN_i \sim$ Unif($a$=0.25, $b$=0.75) for each of the nine simulated datasets. Dashed black line depicts the averaged true sensitivity in the simulated data, the solid line depicts the prior distribution, and the light gray lines depict the posterior distribution of sensitivity for each ZIP code.



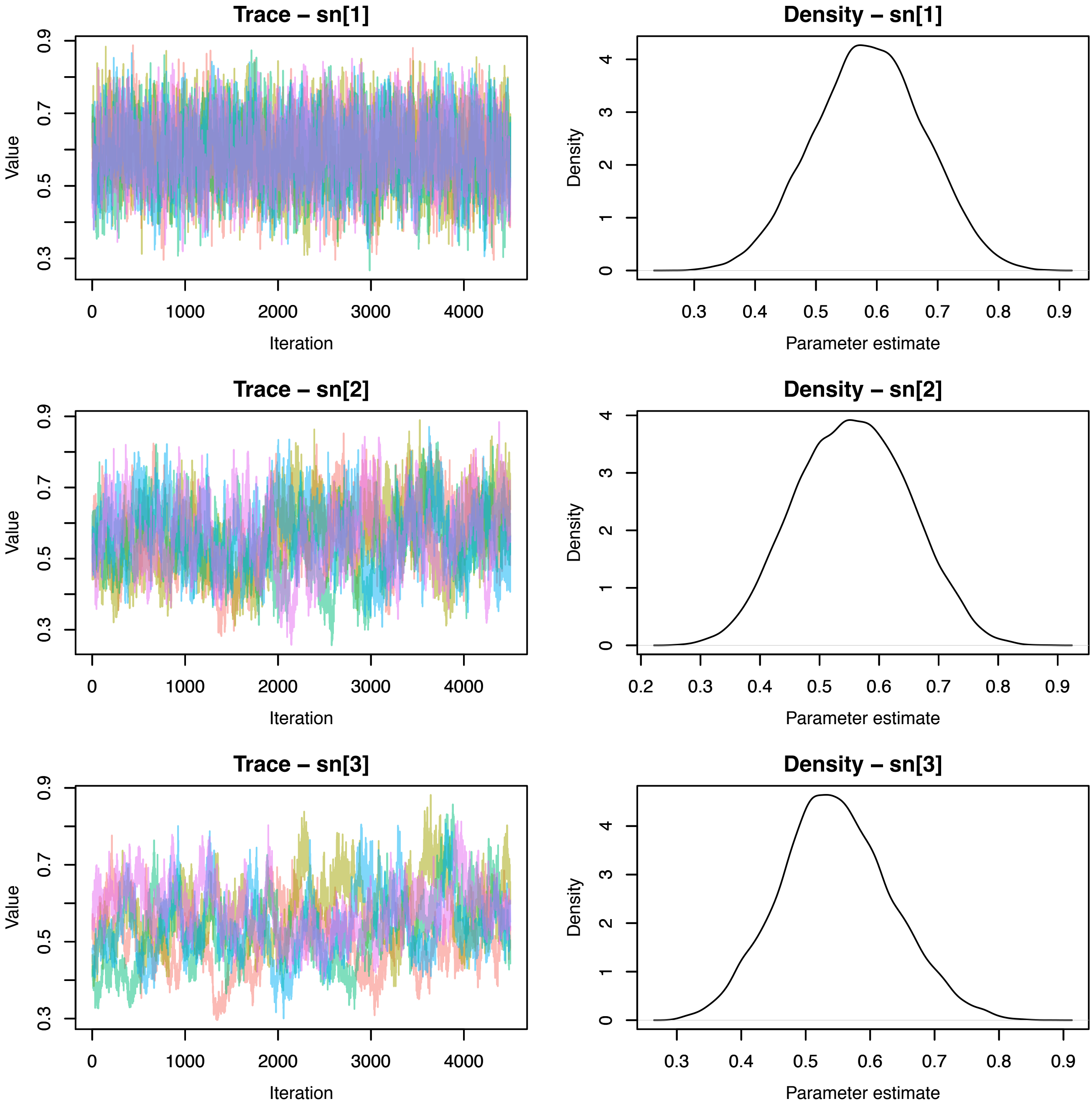
**Supplemental Figure 24**. Posterior distribution of surveillance specificity when priors on $r_i$ ~ Unif($a$=0, $b$=0.1) and $SN_i$ ~ Unif($a$=0.25, $b$=0.75) for each of the nine simulated datasets. Dashed black line depicts the averaged true specificity in the simulated data, the solid line depicts the prior distribution, and the light gray lines depict the posterior distribution of specificity for each ZIP code.

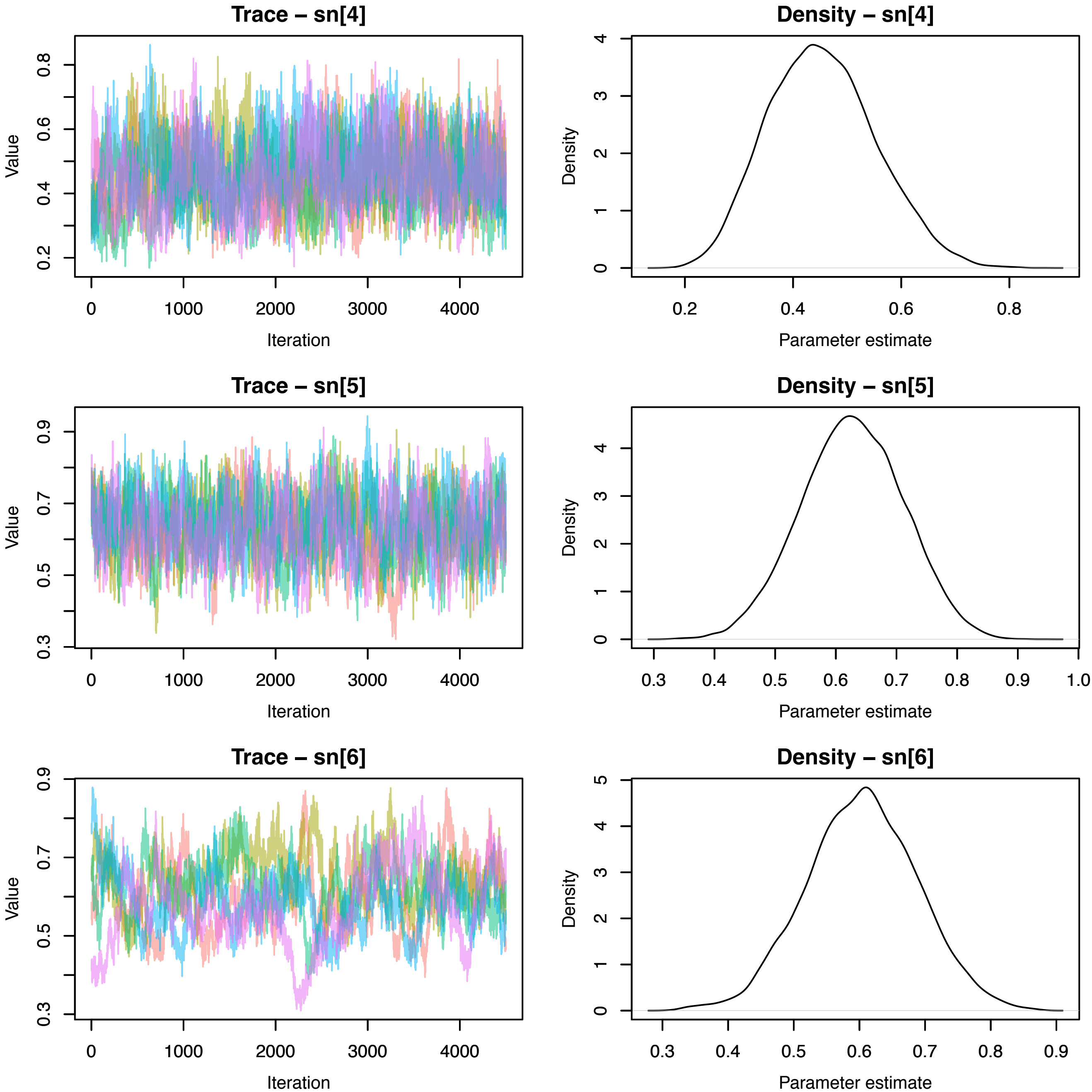


**Supplemental Table 1**. Mean squared error of prevalence comparing models for each simulated dataset (see text for details); **bolded** value denotes the lowest mean square error for a given simulation.

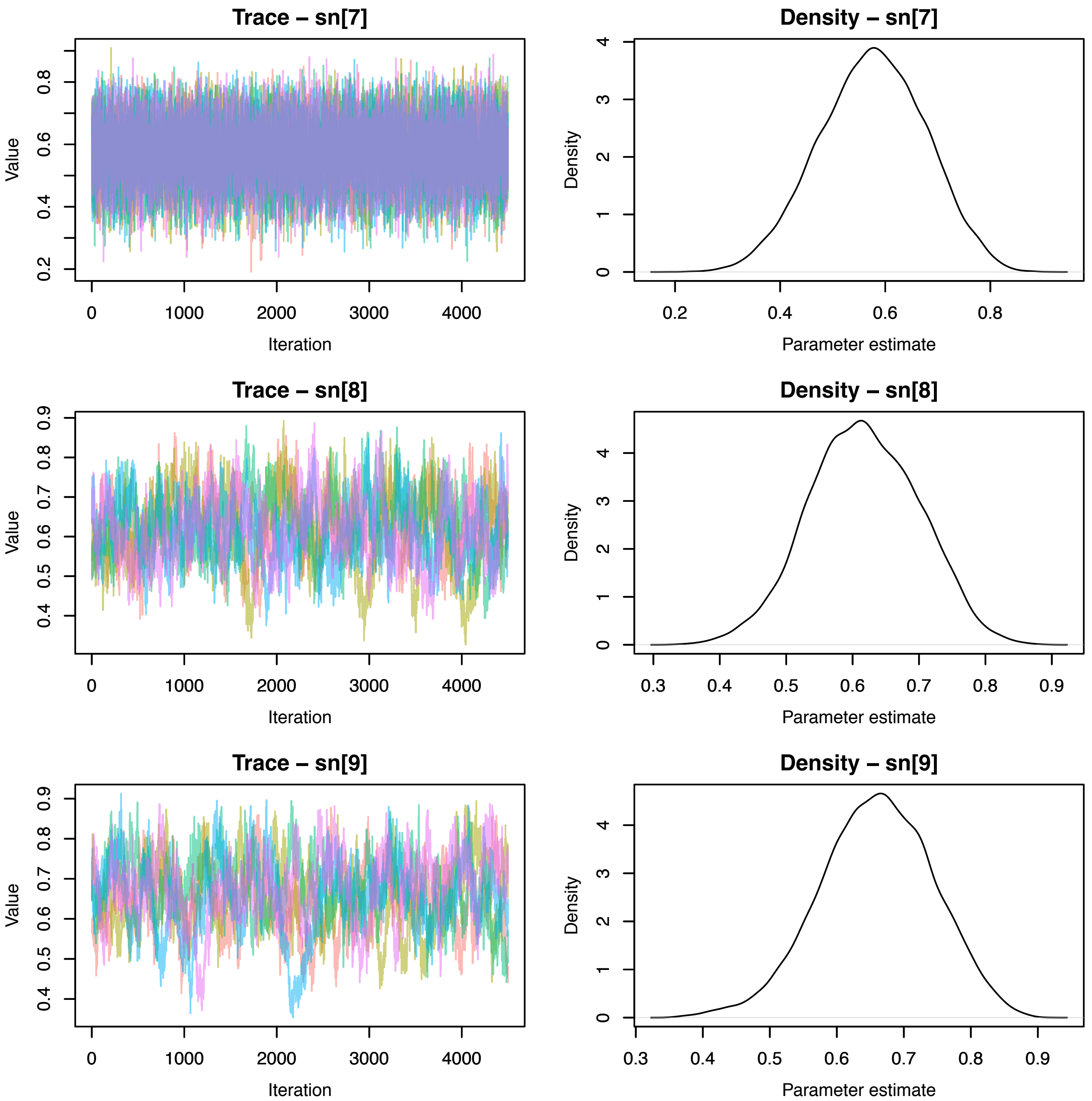
| **Simulation**<br>Means of prevalence and sensitivity, % | **Model #1**<br>Eq. C with priors:<br>$\eta_0$ ~ Norm($\mu$=-3.5, $\sigma^2$=*0.25*)<br>$SN_i$ ~ Beta($\alpha$=14.022, $\beta$=9.681) | **Model #2**<br>Eq. C with priors:<br>$\eta_0$ ~ Norm($\mu$=0, $\sigma^2$=*2.71*)<br>$SN_i$ ~ Beta($\alpha$=14.022, $\beta$=9.681) | **Model #3**<br>Eq. D with priors:<br>$r_i$ ~ Unif($a$=0, $b$=0.1)<br>$SN_i$ ~ Beta($\alpha$=14.022, $\beta$=9.681) |
|---|---|---|---|
| 1, 30 | 0.0032 | 1.79 | **0.0005** |
| 1, 50 | 0.0032 | 1.86 | **0.0009** |
| 1, 70 | 0.0024 | 1.94 | **0.0006** |
| 3, 30 | 0.0046 | 1.83 | **0.0016** |
| 3, 50 | 0.0014 | 1.58 | **0.0010** |
| 3, 70 | **0.0002** | 1.69 | 0.0008 |
| 5, 30 | 0.0028 | 1.75 | **0.0020** |
| 5, 50 | **0.0005** | 1.63 | 0.0022 |
| 5, 70 | 0.0029 | 1.70 | **0.0019** |

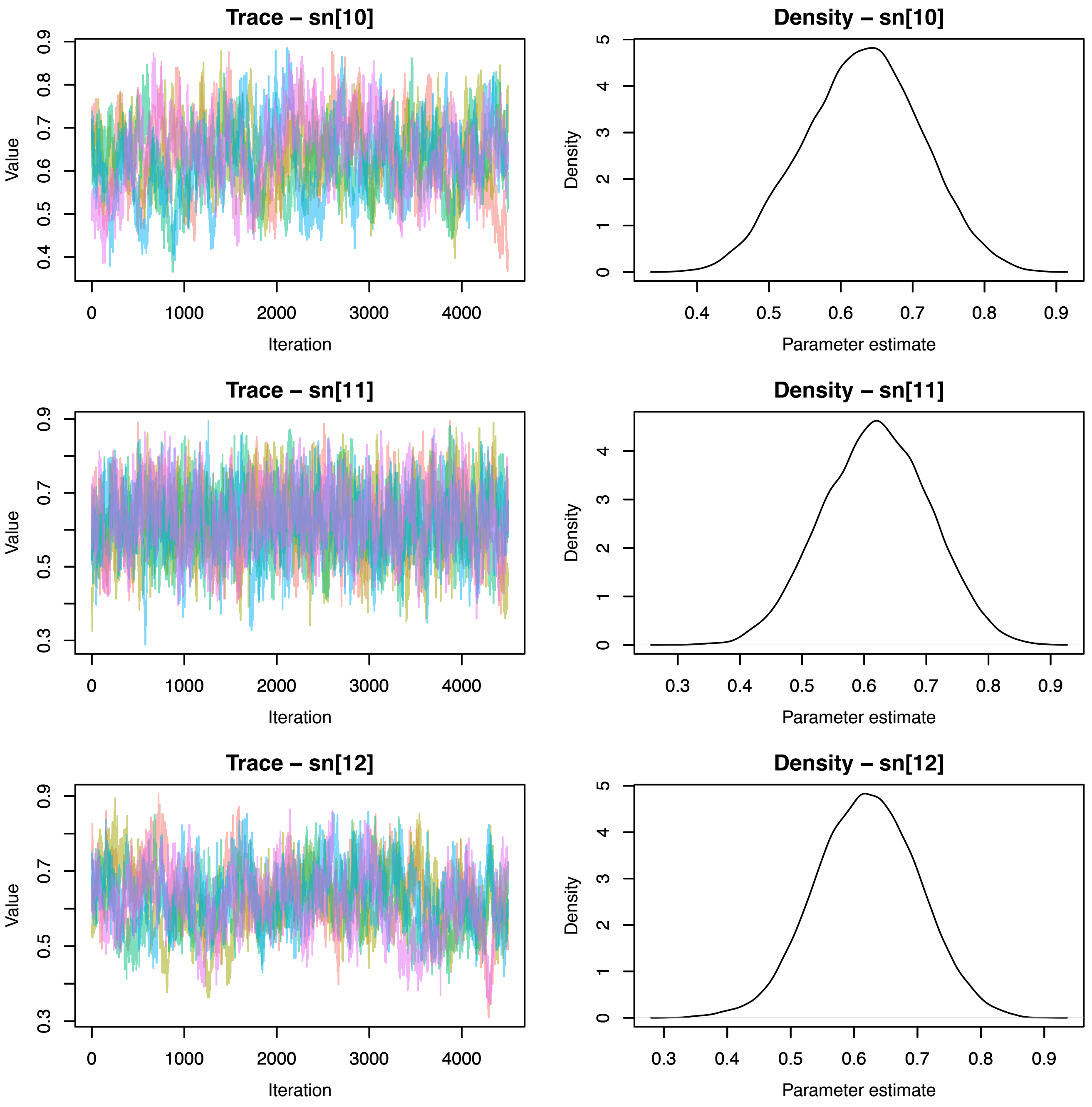
*Appendix 3: Diagnostic plots of select parameters from Model #1 (Eq. 5)*

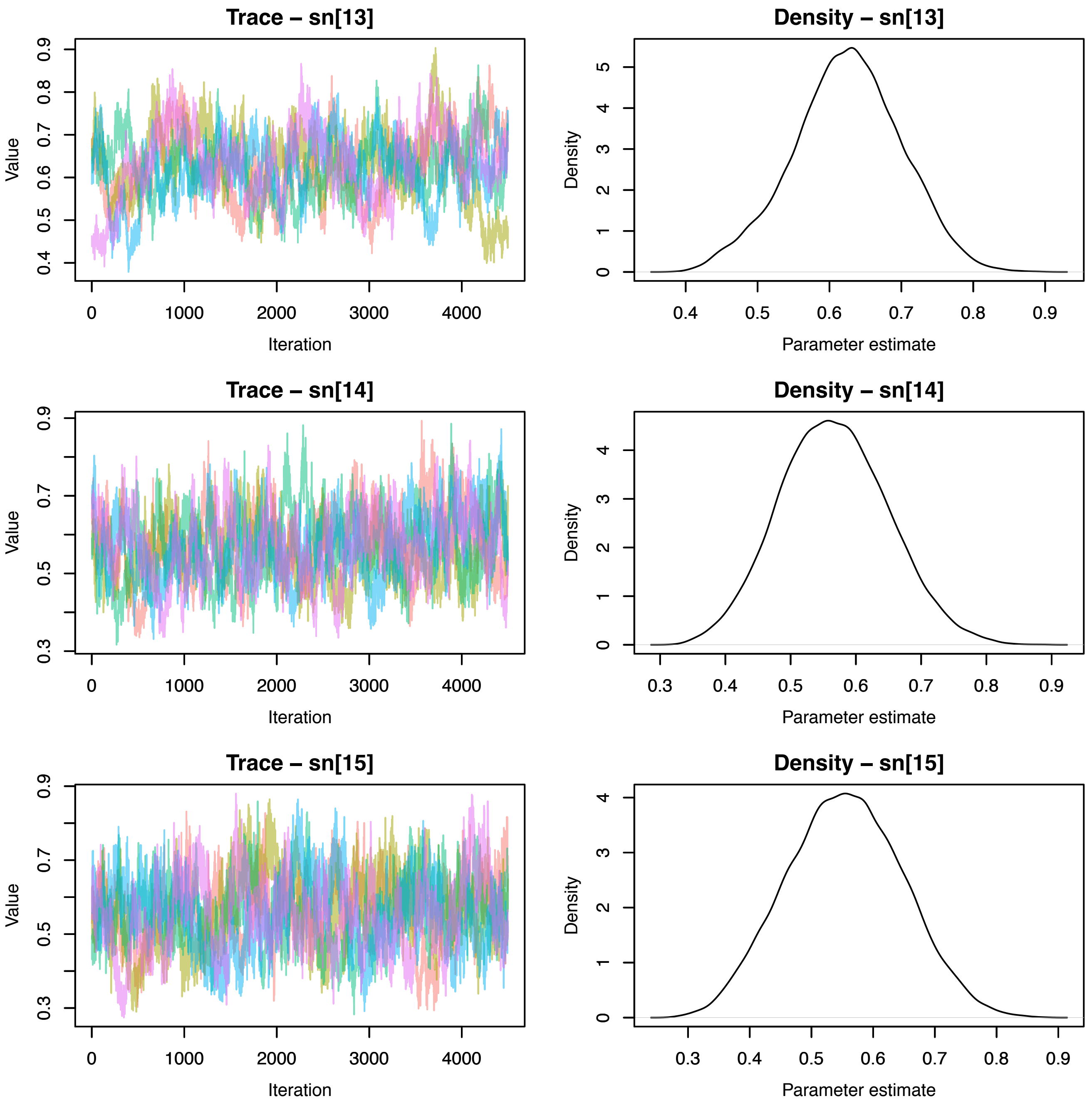

Trace – sn[1]
Density – sn[1]
Trace – sn[2]
Density – sn[2]
Trace – sn[3]
Density – sn[3]
Value
Iteration
Density
Parameter estimate
0
1000
2000
3000
4000
0.3
0.5
0.7
0.9
0.2
0.4
0.6
0.8
1
2
3
4

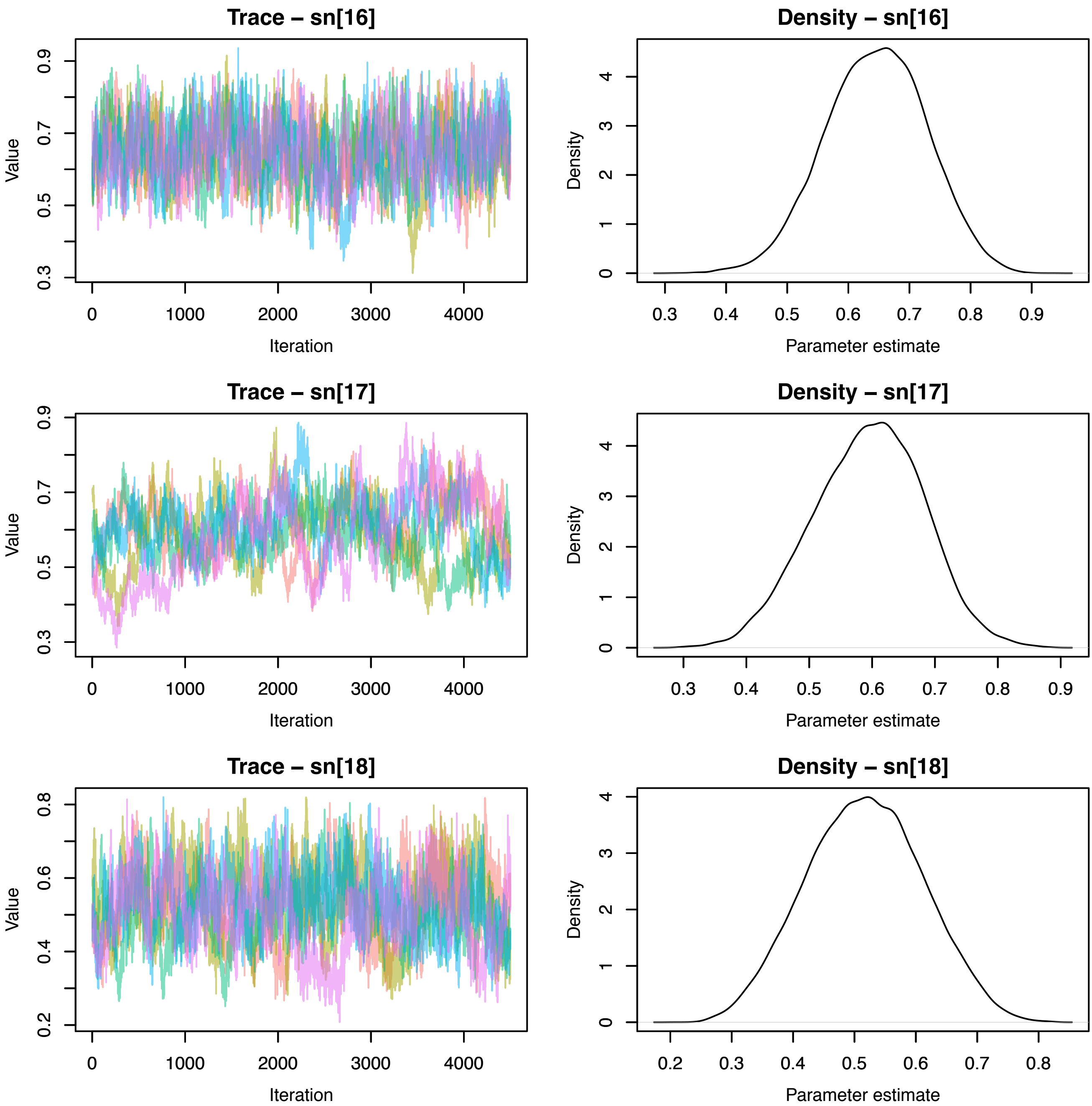


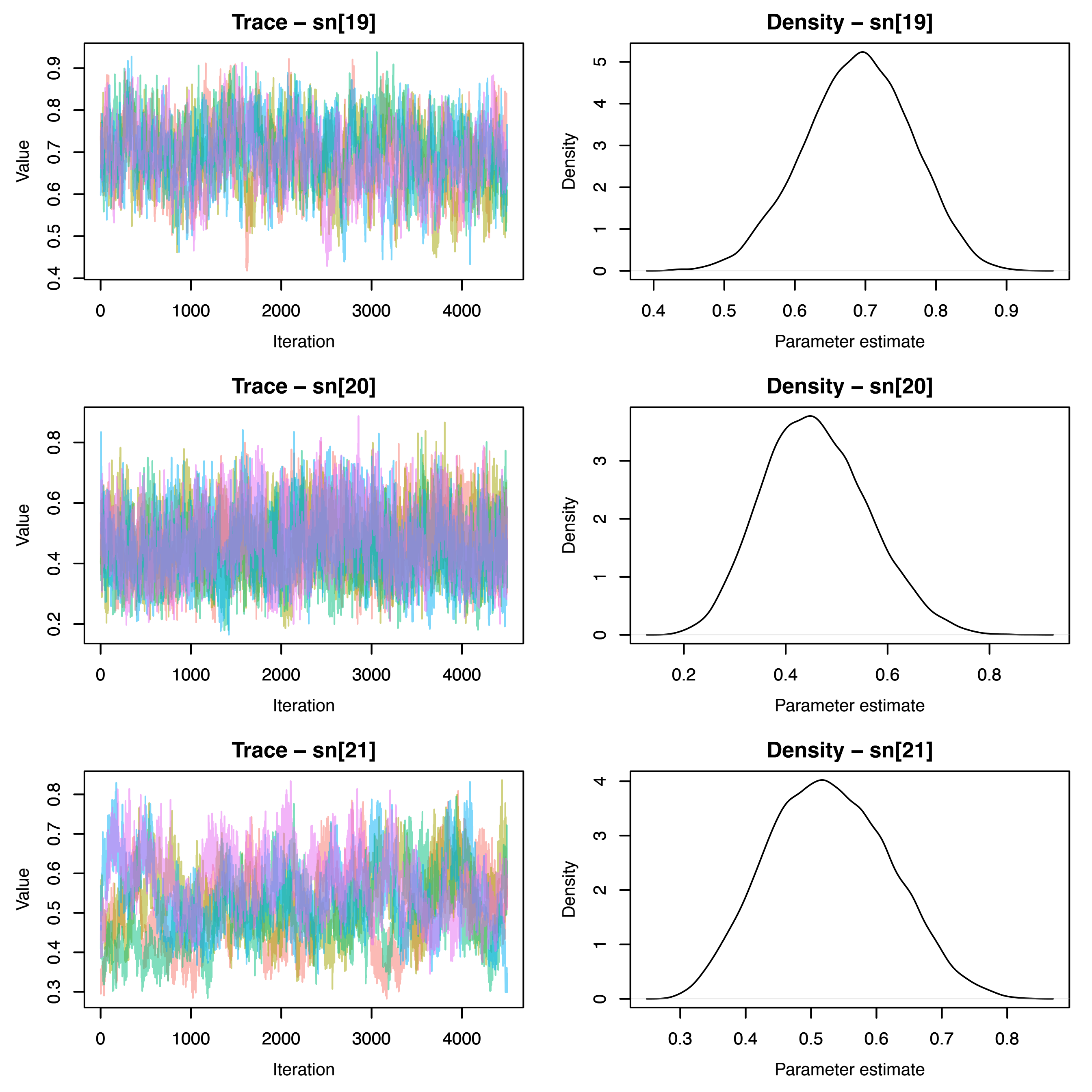
Trace – sn[4]
Value
Iteration
Density – sn[4]
Density
Parameter estimate
Trace – sn[5]
Value
Iteration
Density – sn[5]
Density
Parameter estimate
Trace – sn[6]
Value
Iteration
Density – sn[6]
Density
Parameter estimate

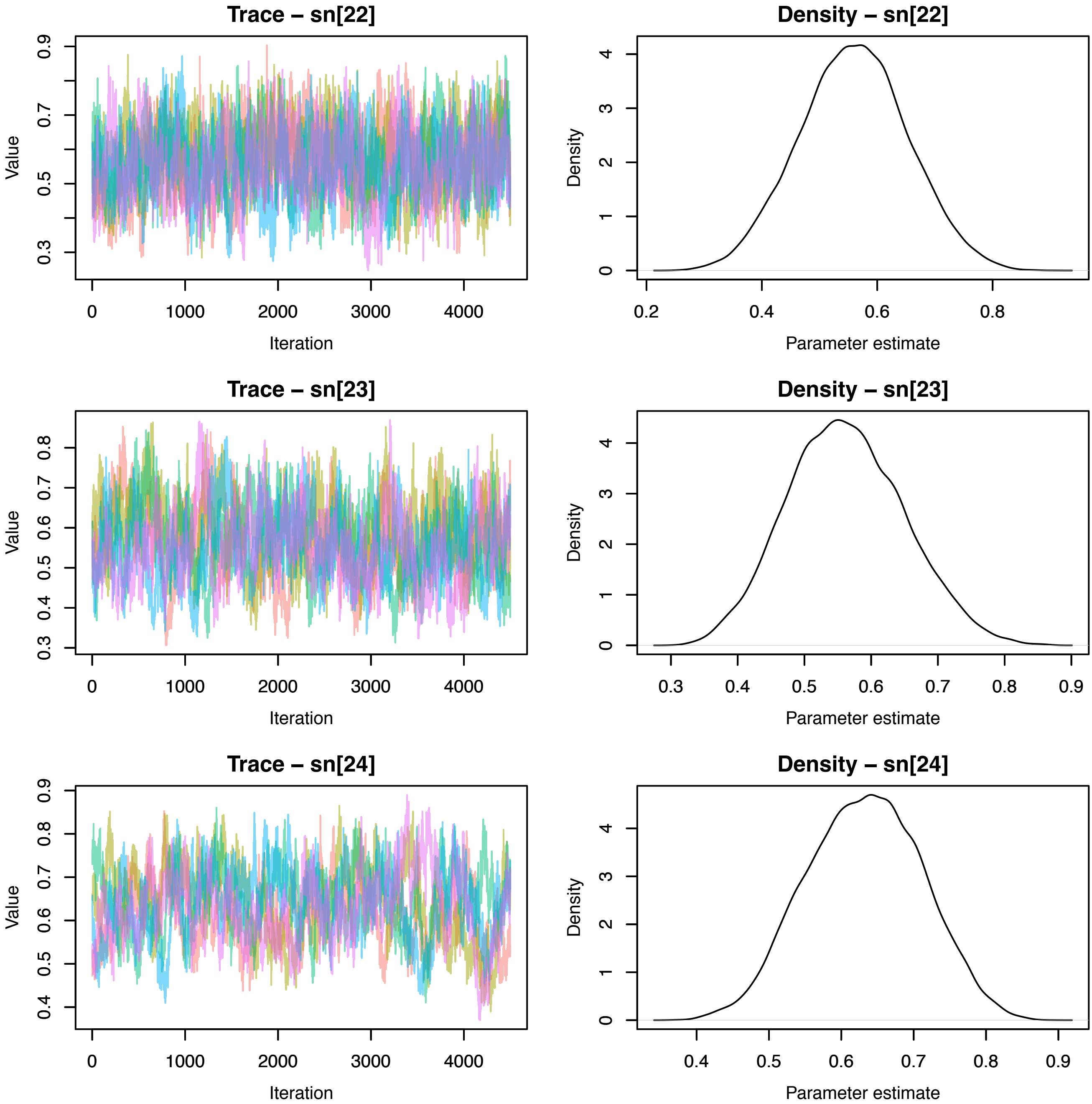

Trace – sn[7]
Value
Iteration
Density – sn[7]
Density
Parameter estimate
Trace – sn[8]
Value
Iteration
Density – sn[8]
Density
Parameter estimate
Trace – sn[9]
Value
Iteration
Density – sn[9]
Density
Parameter estimate


Trace – sn[10]
Value
0.4 0.5 0.6 0.7 0.8 0.9
0 1000 2000 3000 4000
Iteration
Density – sn[10]
Density
0 1 2 3 4 5
0.4 0.5 0.6 0.7 0.8 0.9
Parameter estimate
Trace – sn[11]
Value
0.3 0.5 0.7 0.9
0 1000 2000 3000 4000
Iteration
Density – sn[11]
Density
0 1 2 3 4
0.3 0.4 0.5 0.6 0.7 0.8 0.9
Parameter estimate
Trace – sn[12]
Value
0.3 0.5 0.7 0.9
0 1000 2000 3000 4000
Iteration
Density – sn[12]
Density
0 1 2 3 4 5
0.3 0.4 0.5 0.6 0.7 0.8 0.9
Parameter estimate

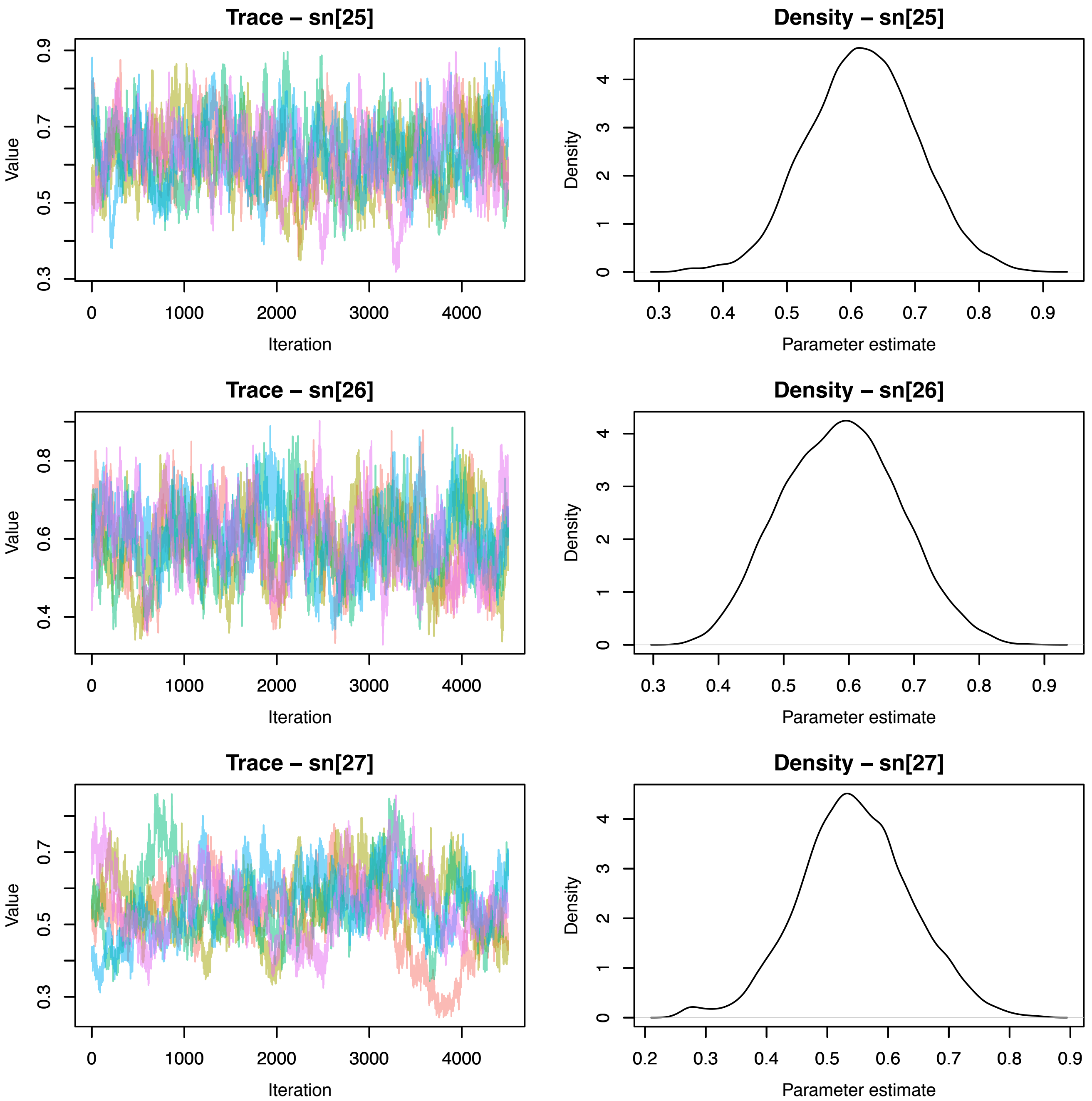

Trace – sn[13]
Value
Iteration
Density – sn[13]
Density
Parameter estimate
Trace – sn[14]
Value
Iteration
Density – sn[14]
Density
Parameter estimate
Trace – sn[15]
Value
Iteration
Density – sn[15]
Density
Parameter estimate


Trace – sn[16]
Value
Iteration
Density – sn[16]
Density
Parameter estimate
Trace – sn[17]
Value
Iteration
Density – sn[17]
Density
Parameter estimate
Trace – sn[18]
Value
Iteration
Density – sn[18]
Density
Parameter estimate

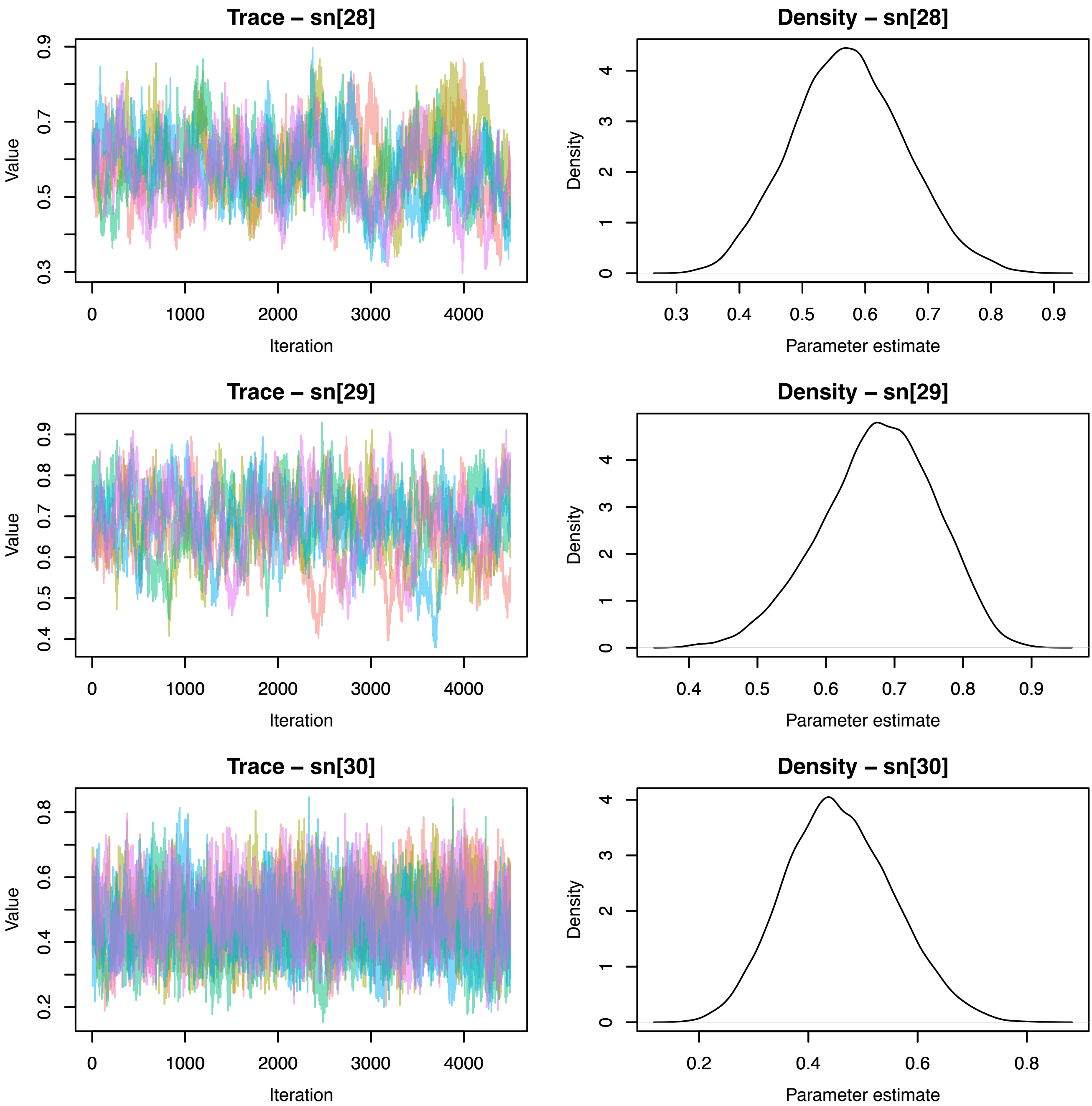

Trace – sn[19]
Value
Iteration
Density – sn[19]
Density
Parameter estimate
Trace – sn[20]
Value
Iteration
Density – sn[20]
Density
Parameter estimate
Trace – sn[21]
Value
Iteration
Density – sn[21]
Density
Parameter estimate

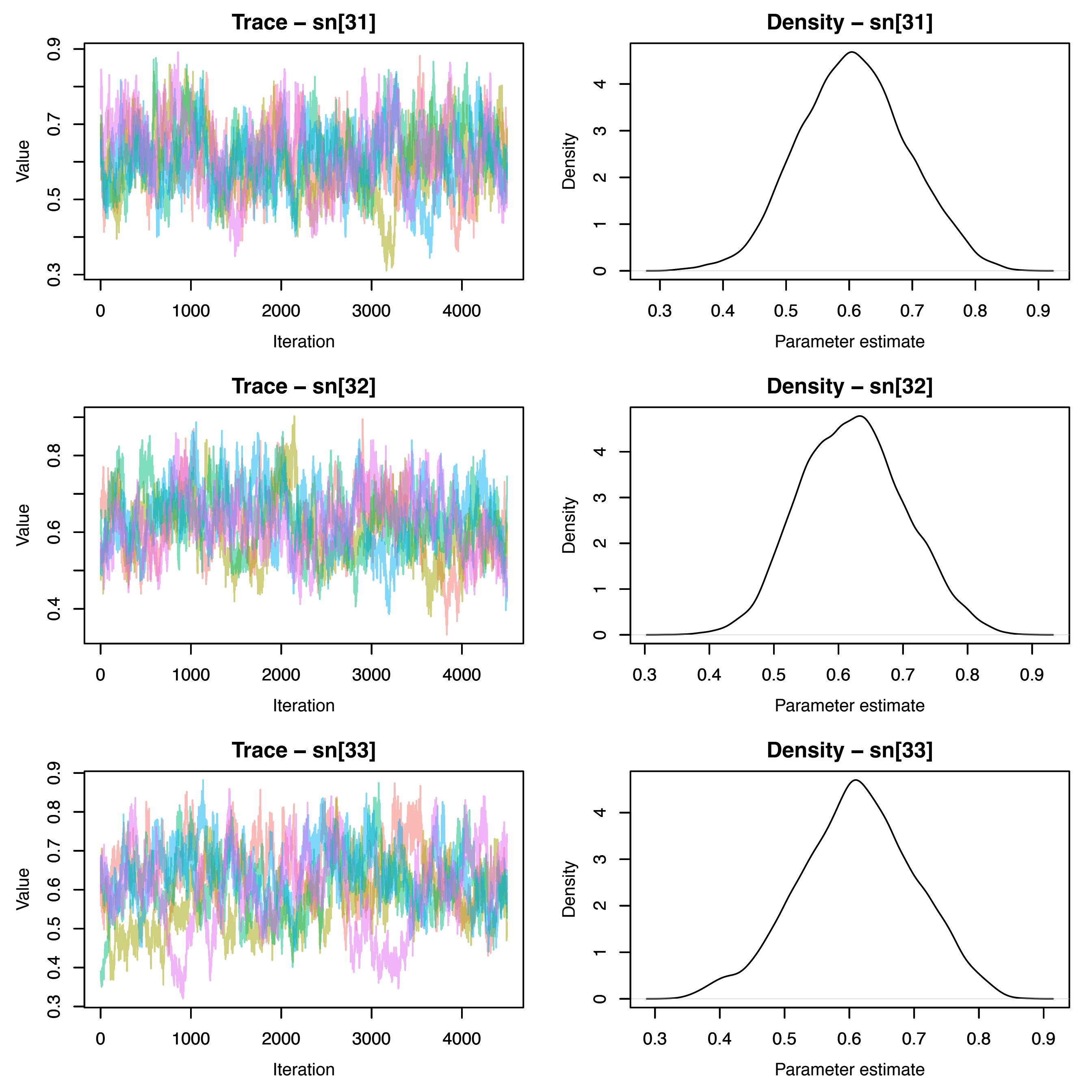


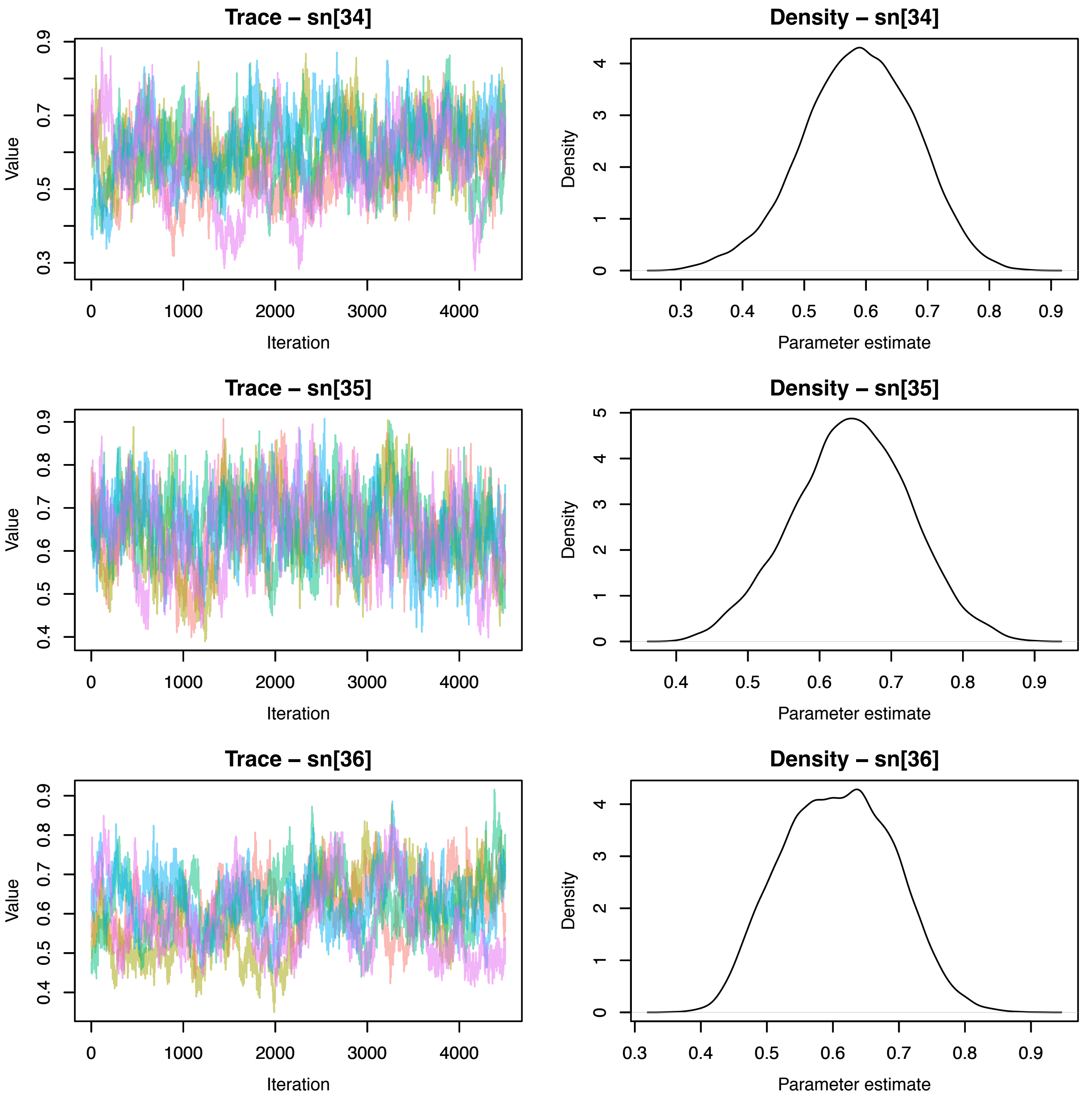
Trace – sn[22]
Density – sn[22]
Trace – sn[23]
Density – sn[23]
Trace – sn[24]
Density – sn[24]
Value
Iteration
Density
Parameter estimate
0
1000
2000
3000
4000
0.3
0.5
0.7
0.9
0.2
0.4
0.6
0.8
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
2
3
4

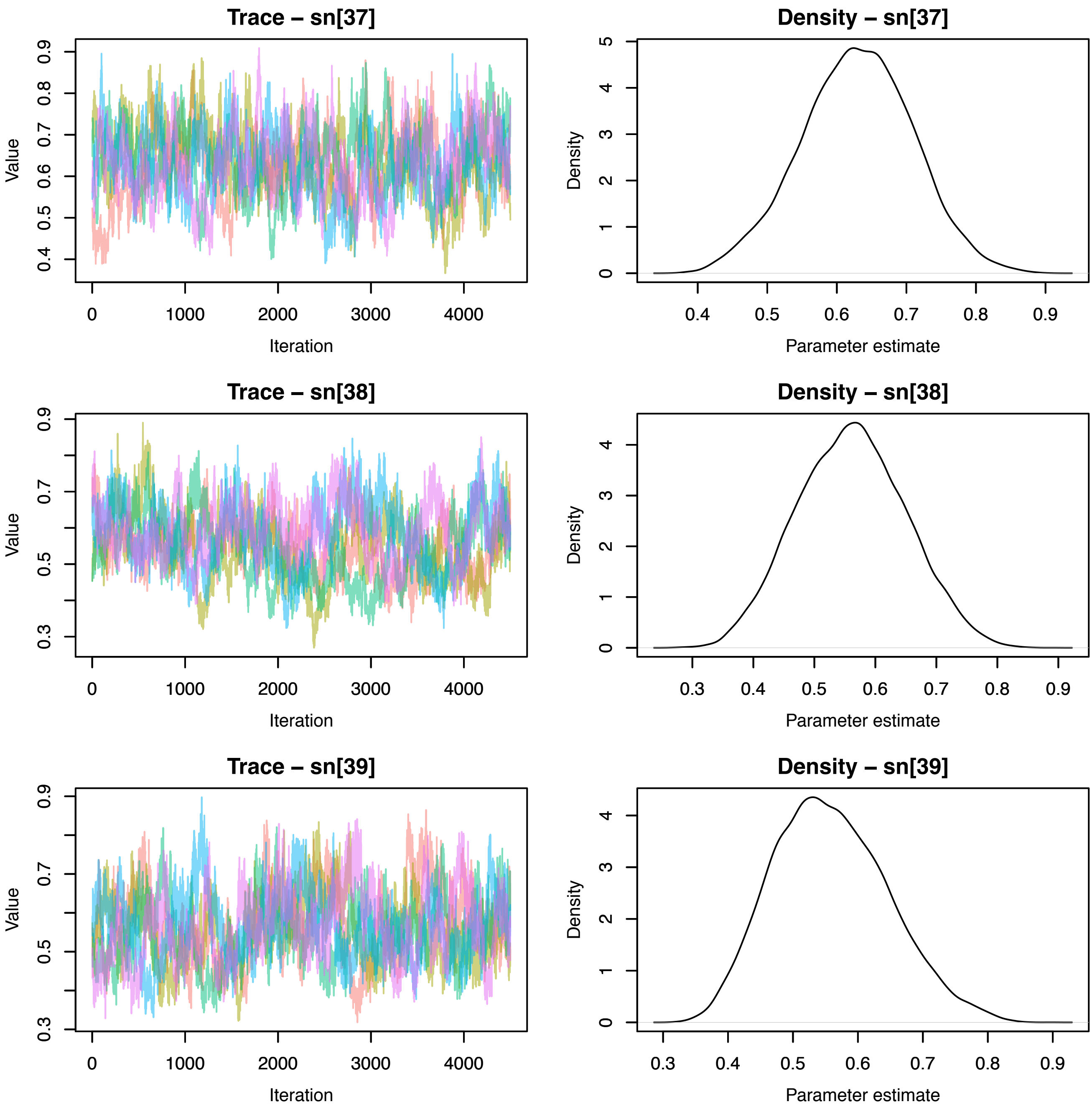


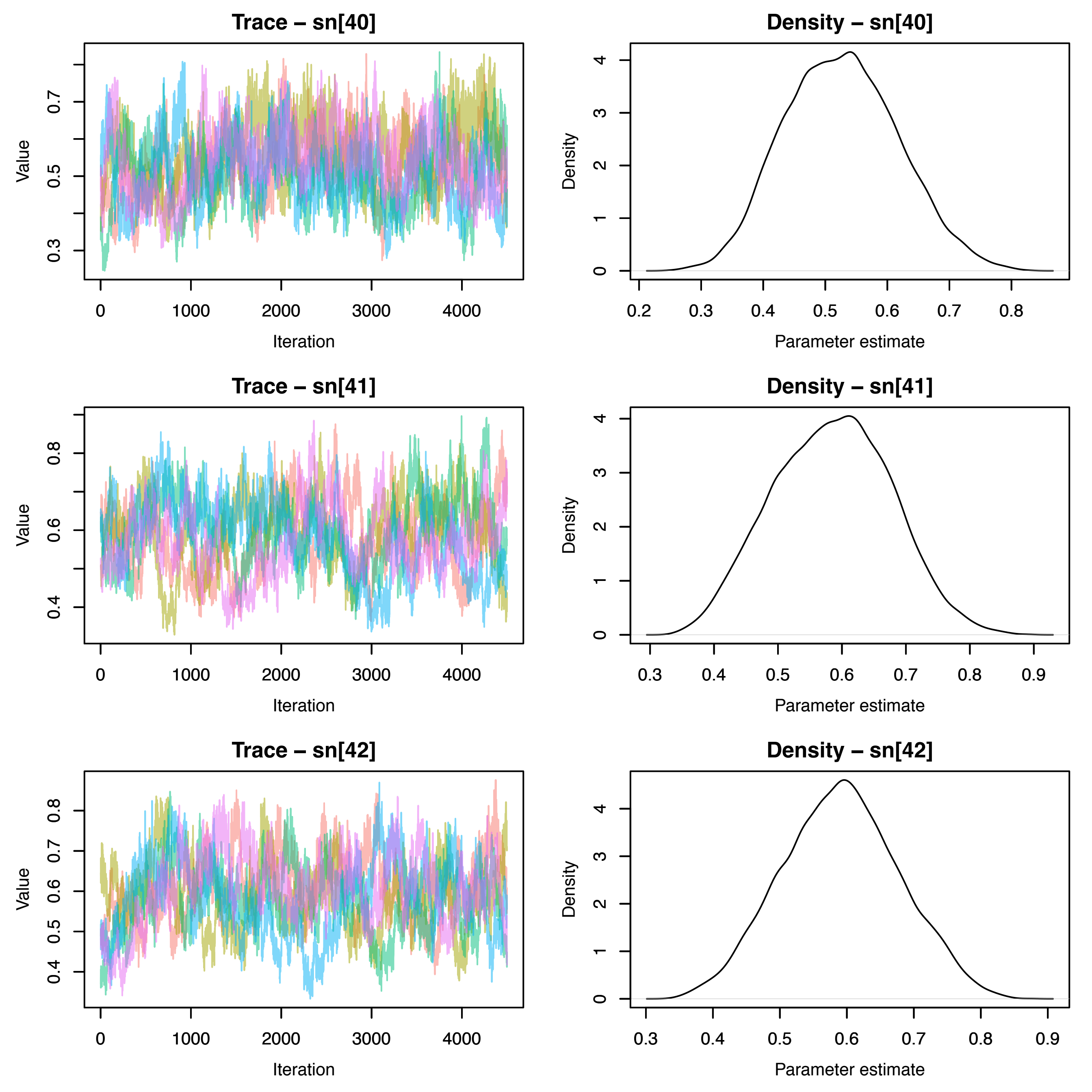
Trace – sn[25]
Density – sn[25]
Trace – sn[26]
Density – sn[26]
Trace – sn[27]
Density – sn[27]
Value
Iteration
Density
Parameter estimate


Trace – sn[28]
Value
Iteration
Density – sn[28]
Density
Parameter estimate
Trace – sn[29]
Value
Iteration
Density – sn[29]
Density
Parameter estimate
Trace – sn[30]
Value
Iteration
Density – sn[30]
Density
Parameter estimate

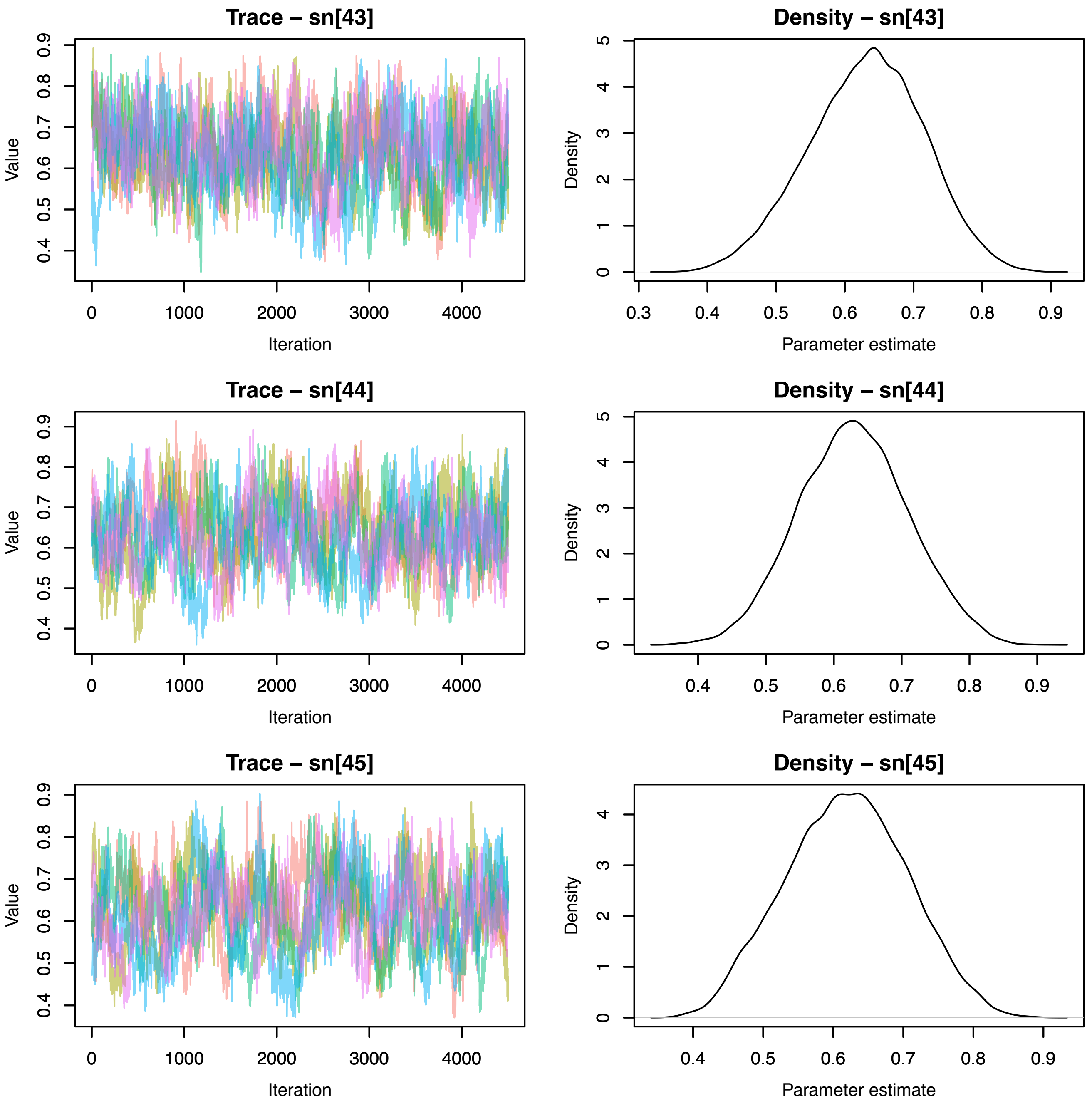

Trace – sn[31]
Value
Iteration
Density – sn[31]
Density
Parameter estimate
Trace – sn[32]
Value
Iteration
Density – sn[32]
Density
Parameter estimate
Trace – sn[33]
Value
Iteration
Density – sn[33]
Density
Parameter estimate

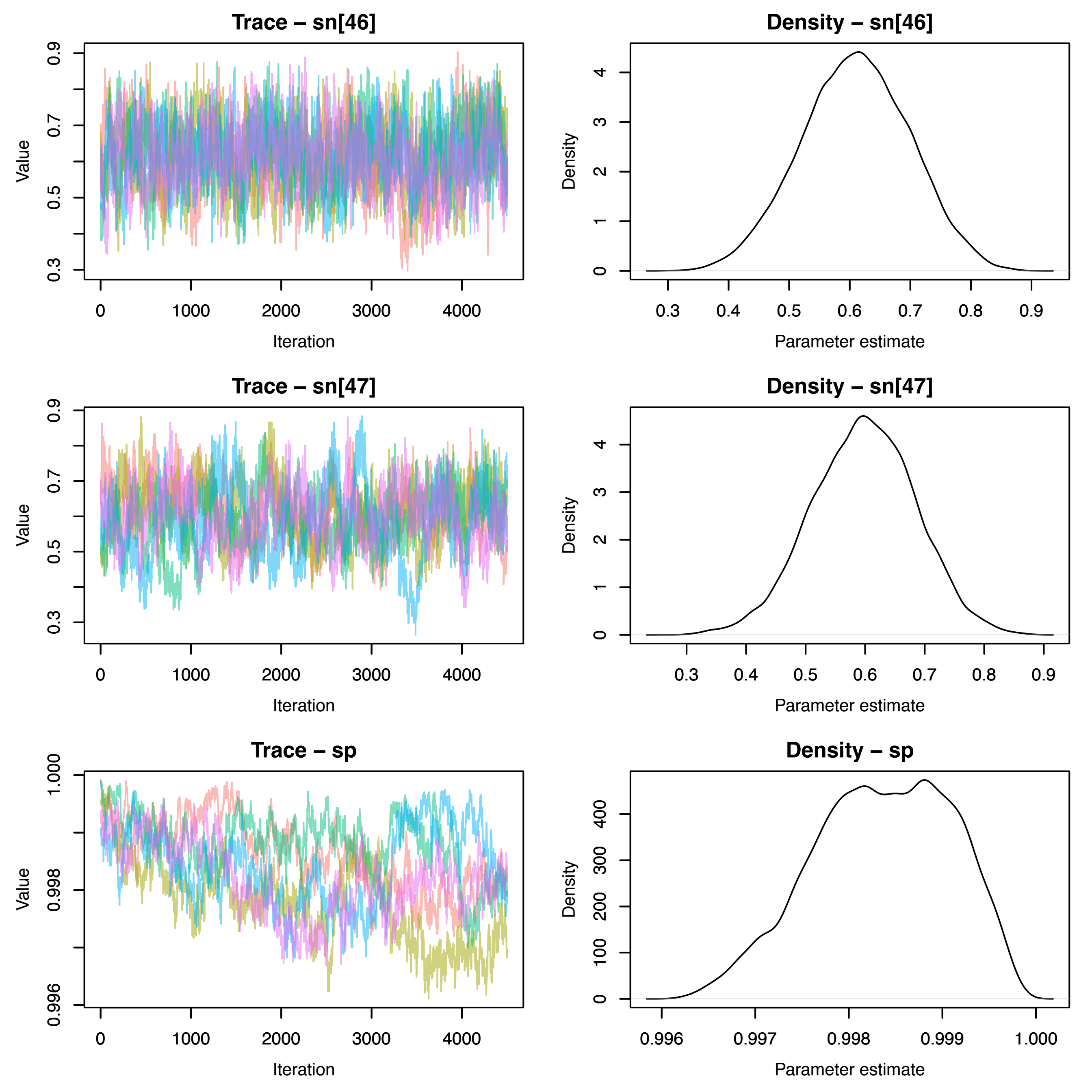

Trace – sn[34]
Density – sn[34]
Trace – sn[35]
Density – sn[35]
Trace – sn[36]
Density – sn[36]
Value
Iteration
Density
Parameter estimate
0
1000
2000
3000
4000
0.3
0.4
0.5
0.6
0.7
0.8
0.9


Trace – sn[37]
Value
Iteration
Density – sn[37]
Density
Parameter estimate
Trace – sn[38]
Value
Iteration
Density – sn[38]
Density
Parameter estimate
Trace – sn[39]
Value
Iteration
Density – sn[39]
Density
Parameter estimate

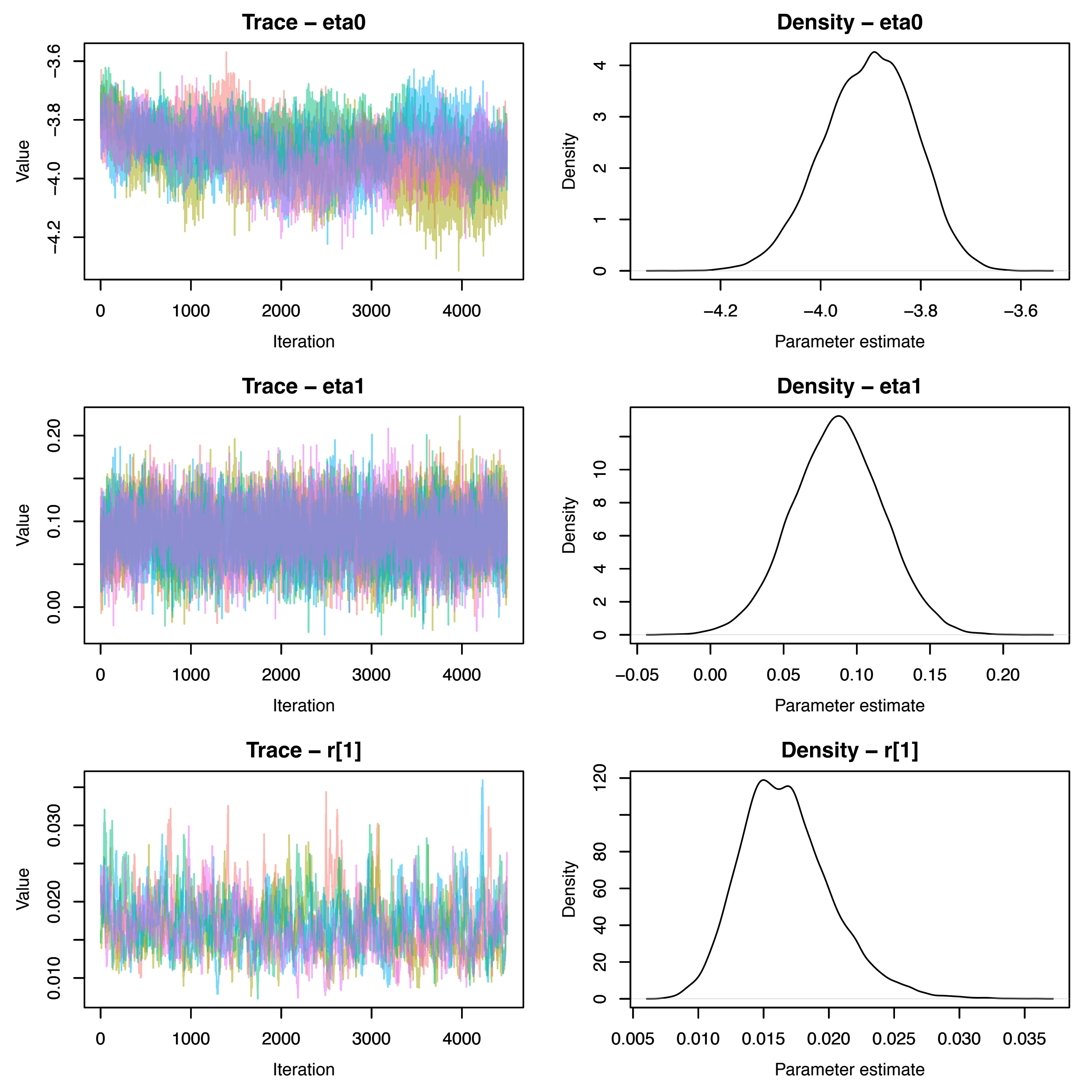


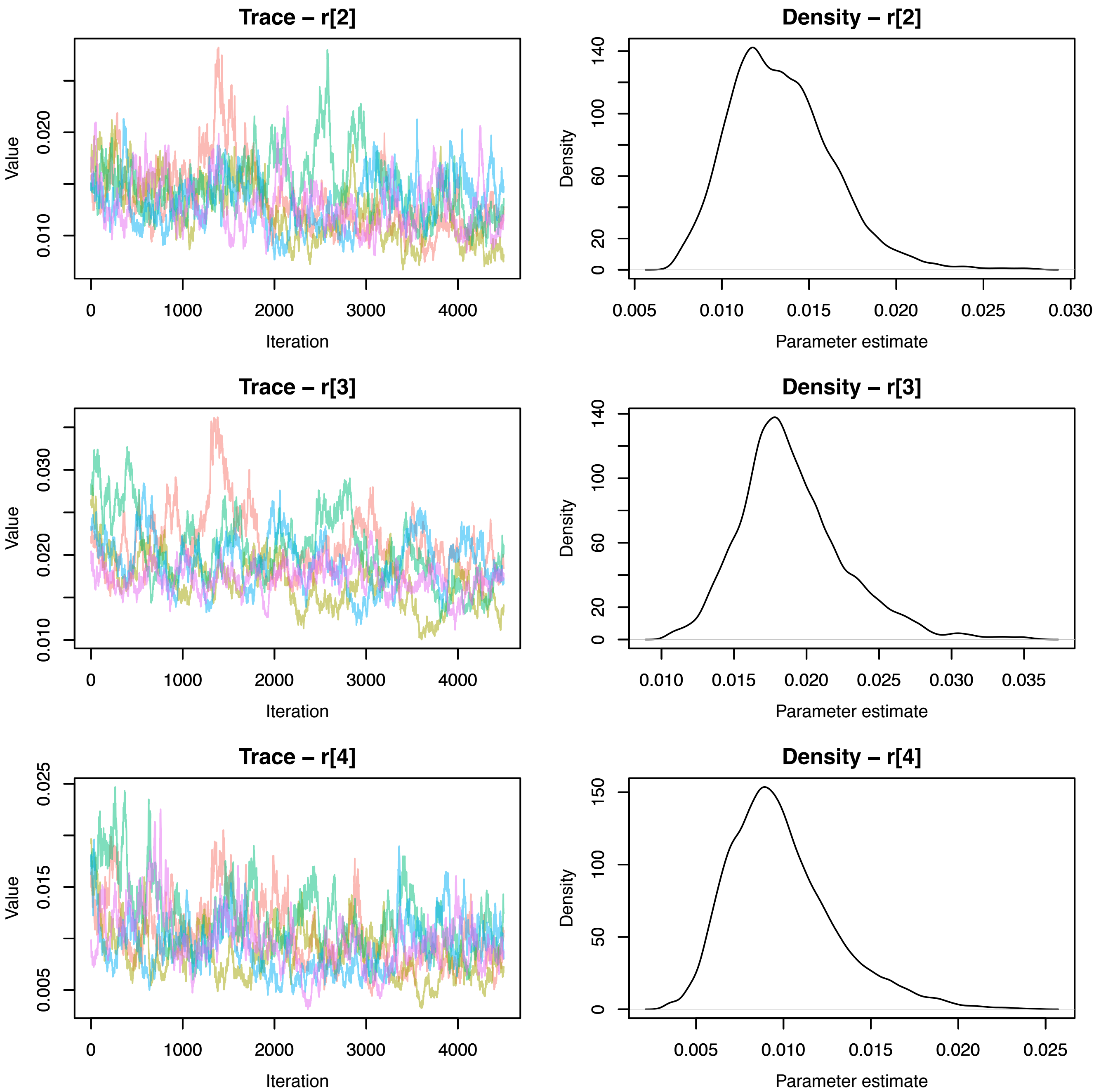
Trace – sn[40]
Density – sn[40]
Trace – sn[41]
Density – sn[41]
Trace – sn[42]
Density – sn[42]
Value
Iteration
Density
Parameter estimate

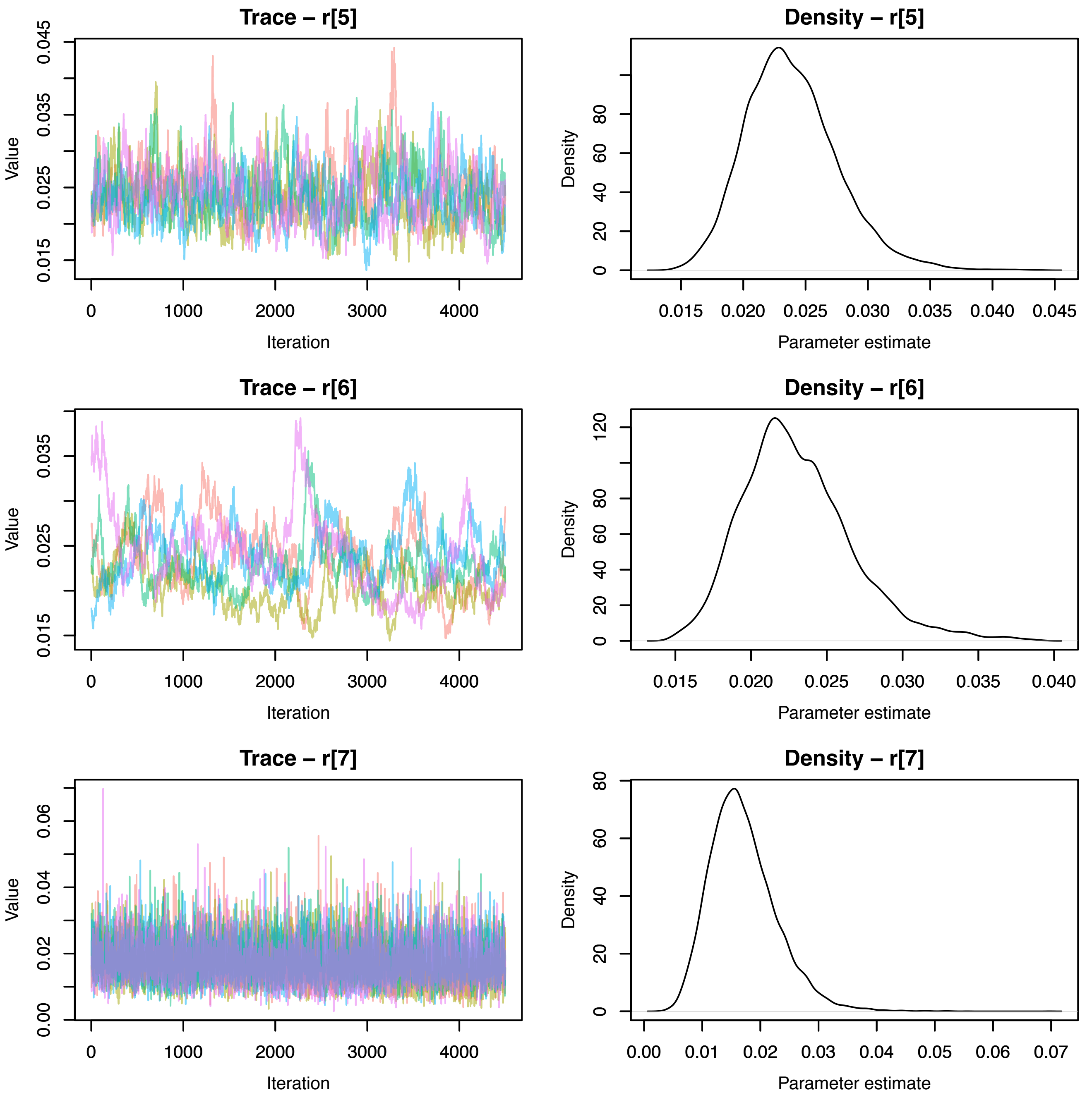

Trace – sn[43]
Value
Iteration
Density – sn[43]
Density
Parameter estimate
Trace – sn[44]
Value
Iteration
Density – sn[44]
Density
Parameter estimate
Trace – sn[45]
Value
Iteration
Density – sn[45]
Density
Parameter estimate



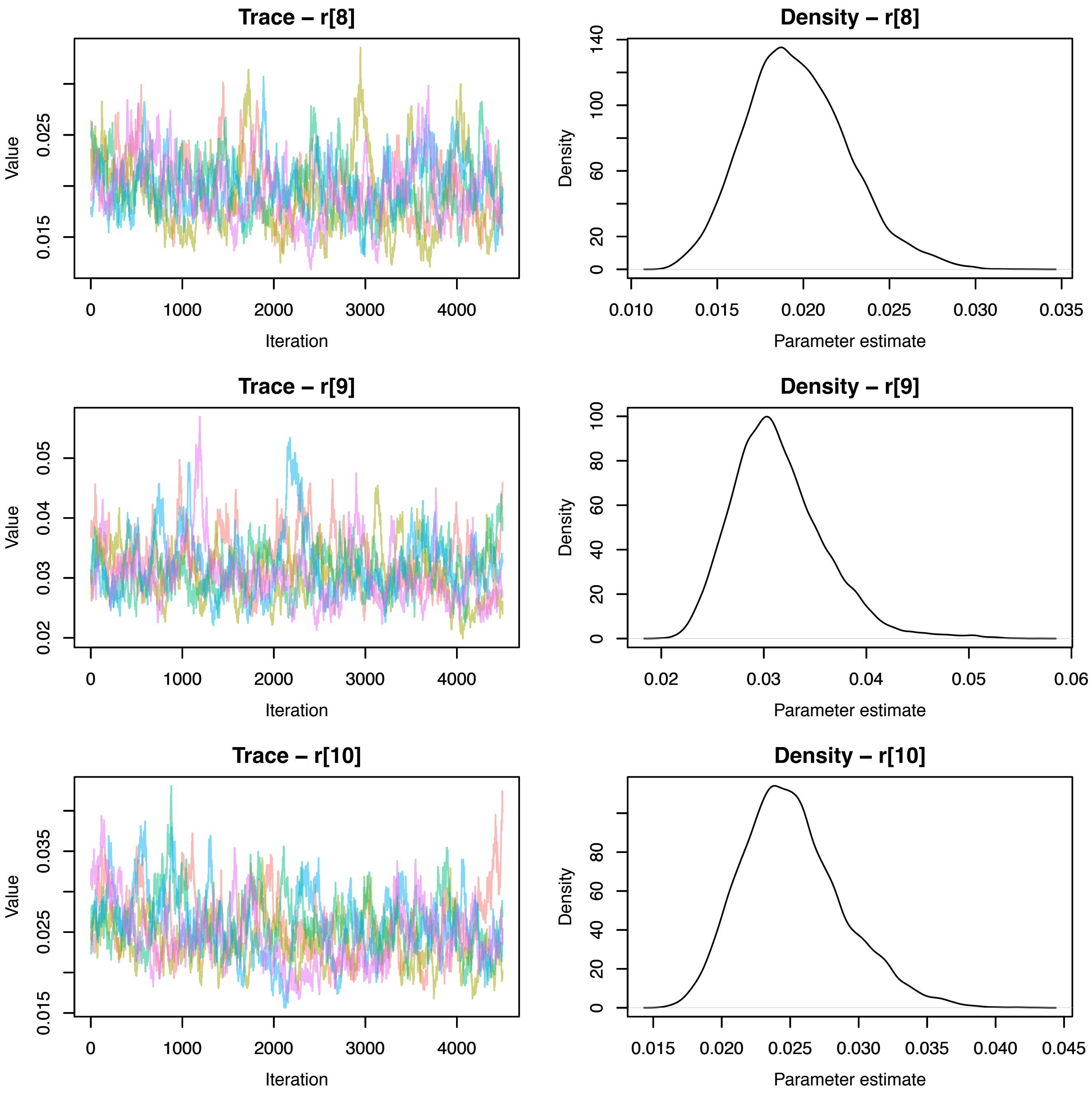
Trace – sn[46]
Value
0.3
0.5
0.7
0.9
0
1000
2000
3000
4000
Iteration
Density – sn[46]
Density
0
1
2
3
4
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Parameter estimate
Trace – sn[47]
Value
0.3
0.5
0.7
0.9
0
1000
2000
3000
4000
Iteration
Density – sn[47]
Density
0
1
2
3
4
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Parameter estimate
Trace – sp
Value
0.996
0.998
1.000
0
1000
2000
3000
4000
Iteration
Density – sp
Density
0
100
200
300
400
0.996
0.997
0.998
0.999
1.000
Parameter estimate


Trace – eta0
Value
Iteration
Density – eta0
Density
Parameter estimate
Trace – eta1
Value
Iteration
Density – eta1
Density
Parameter estimate
Trace – r[1]
Value
Iteration
Density – r[1]
Density
Parameter estimate

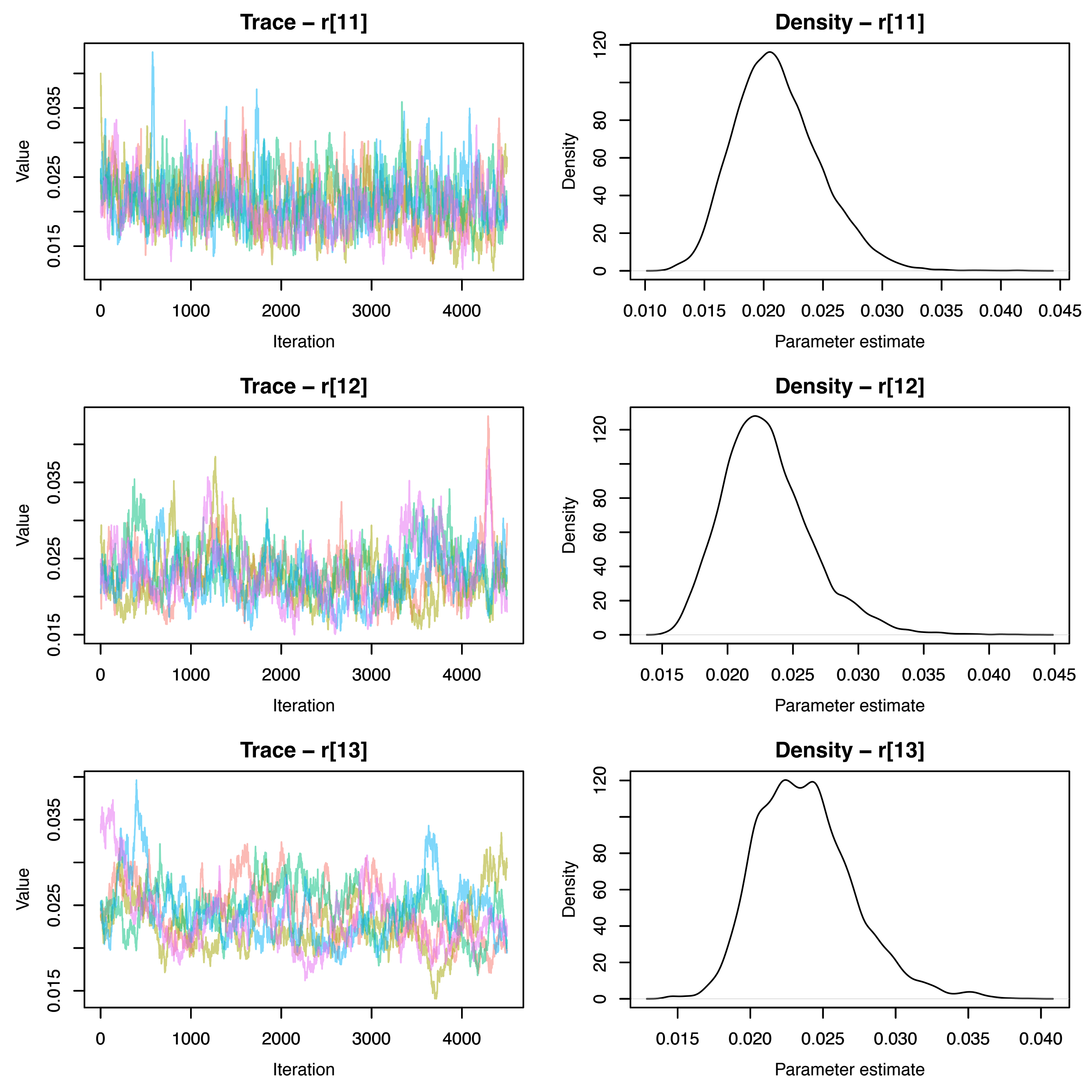

Trace – r[2]
Value
0.010
0.020
Iteration
0
1000
2000
3000
4000
Density – r[2]
Density
0
20
60
100
140
Parameter estimate
0.005
0.010
0.015
0.020
0.025
0.030
Trace – r[3]
Value
0.010
0.020
0.030
Iteration
0
1000
2000
3000
4000
Density – r[3]
Density
0
20
60
100
140
Parameter estimate
0.010
0.015
0.020
0.025
0.030
0.035
Trace – r[4]
Value
0.005
0.015
0.025
Iteration
0
1000
2000
3000
4000
Density – r[4]
Density
0
50
100
150
Parameter estimate
0.005
0.010
0.015
0.020
0.025



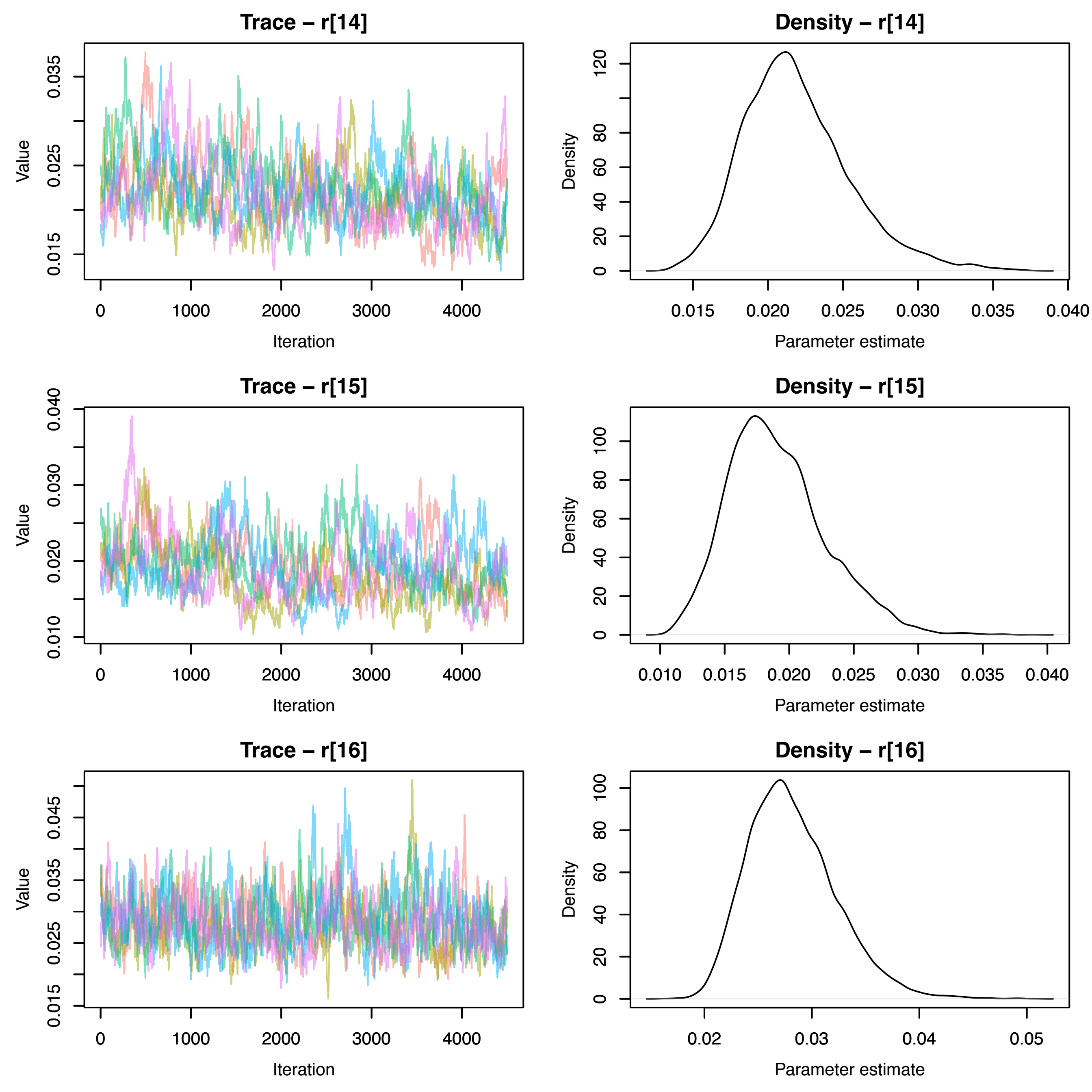
Trace – r[5]
Value
Iteration
Density – r[5]
Density
Parameter estimate
Trace – r[6]
Value
Iteration
Density – r[6]
Density
Parameter estimate
Trace – r[7]
Value
Iteration
Density – r[7]
Density
Parameter estimate

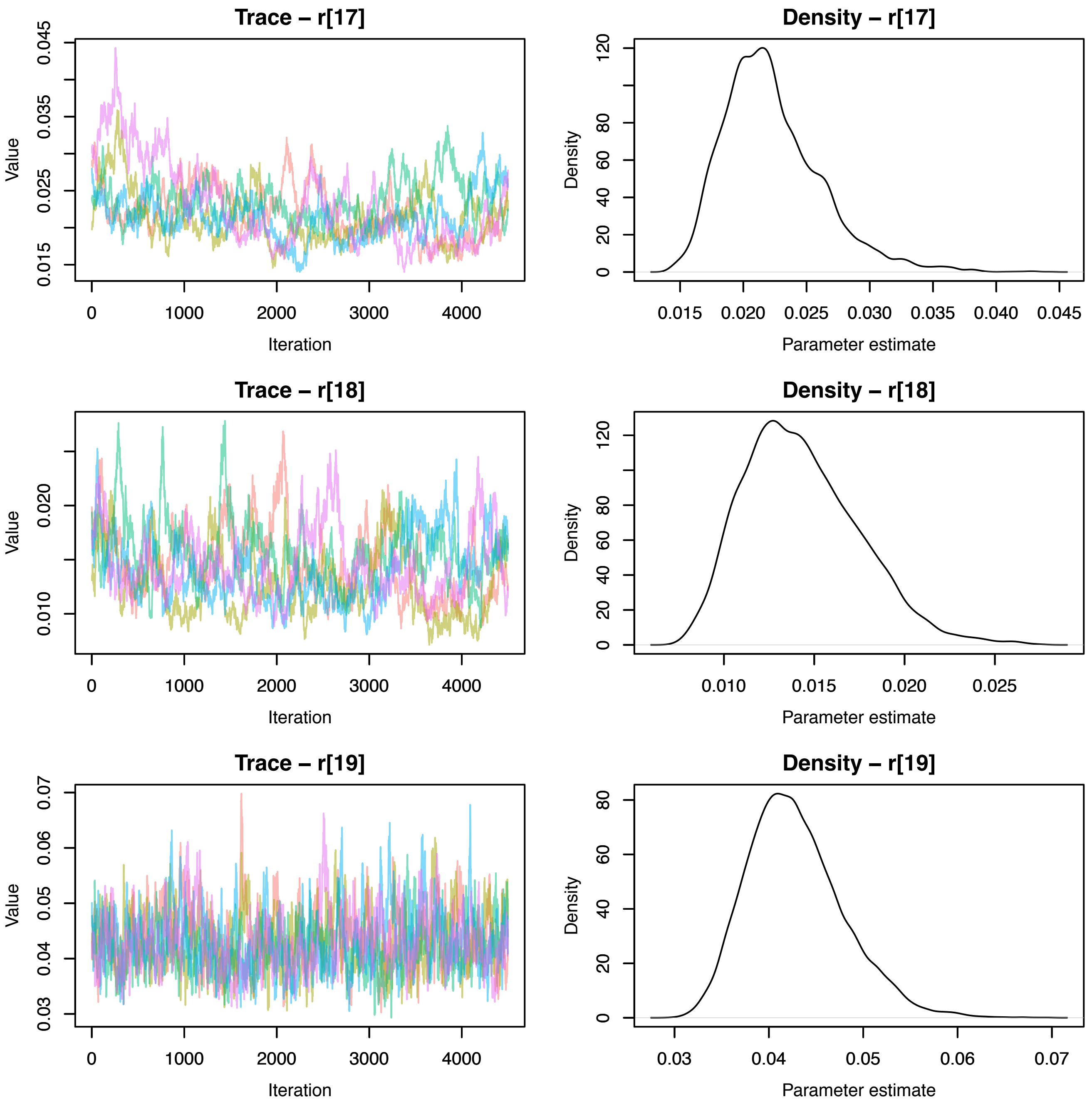


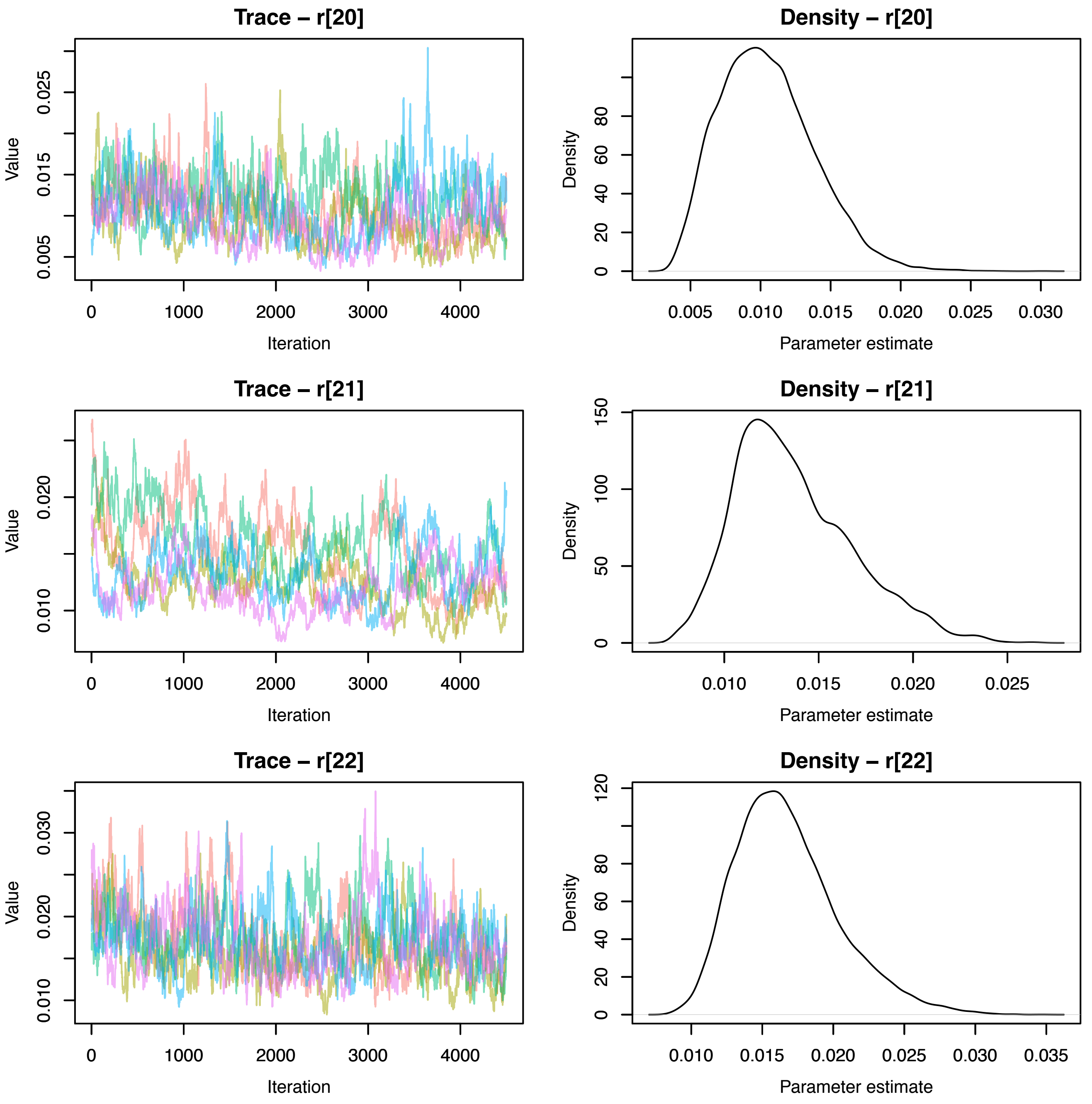
Trace – r[8]
Value
Iteration
Density – r[8]
Density
Parameter estimate
Trace – r[9]
Value
Iteration
Density – r[9]
Density
Parameter estimate
Trace – r[10]
Value
Iteration
Density – r[10]
Density
Parameter estimate

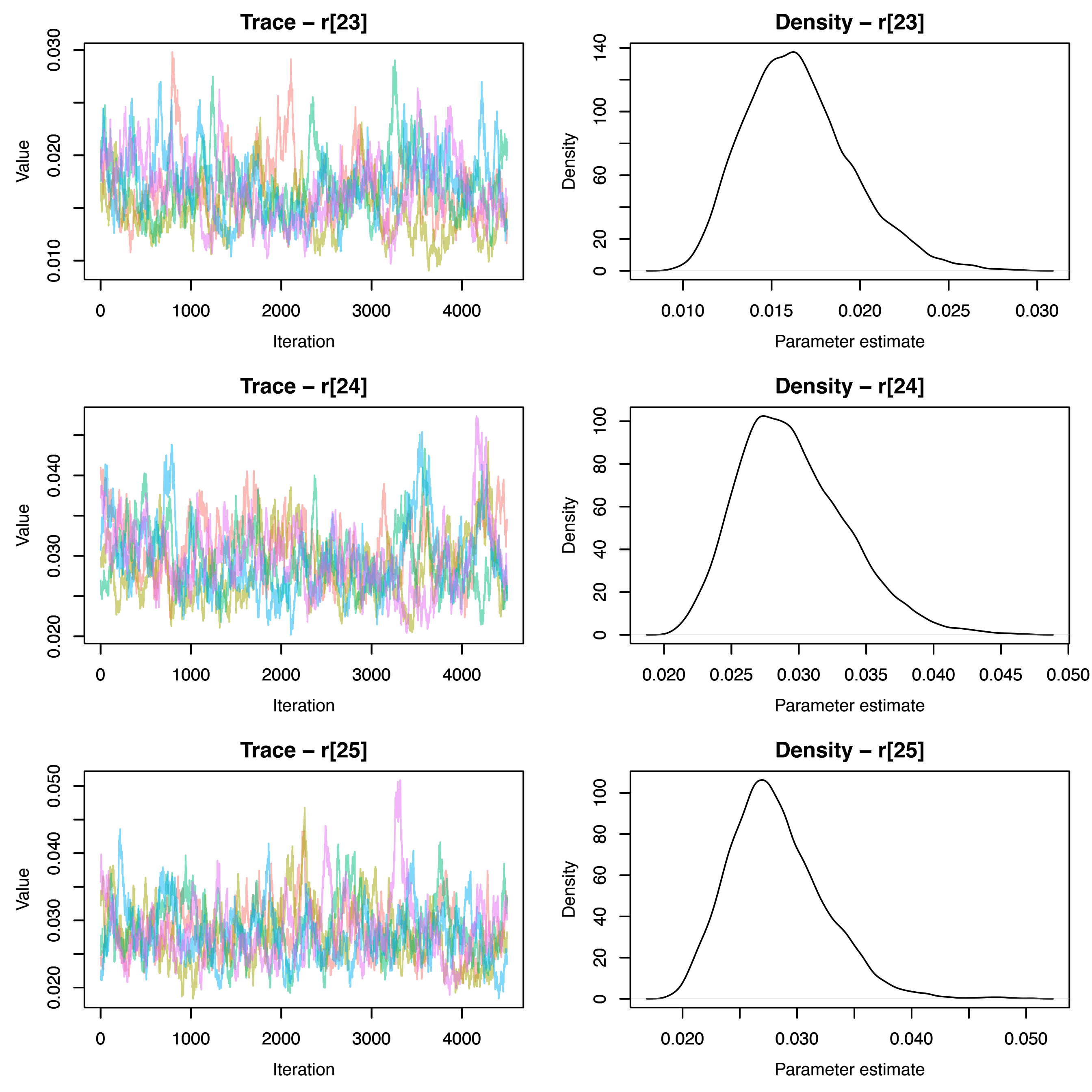


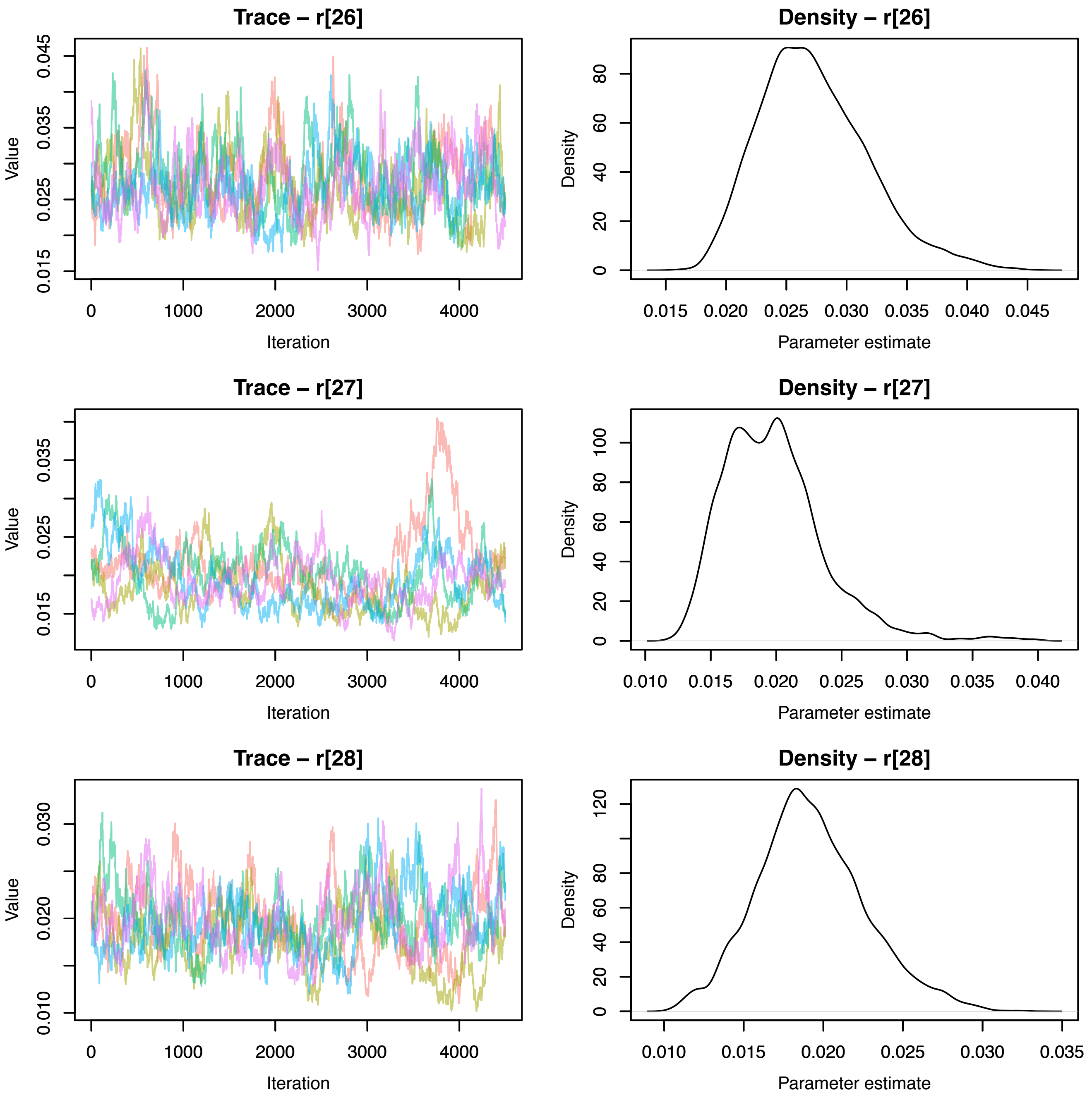
Trace – r[11]
Density – r[11]
Trace – r[12]
Density – r[12]
Trace – r[13]
Density – r[13]
Value
Iteration
Density
Parameter estimate

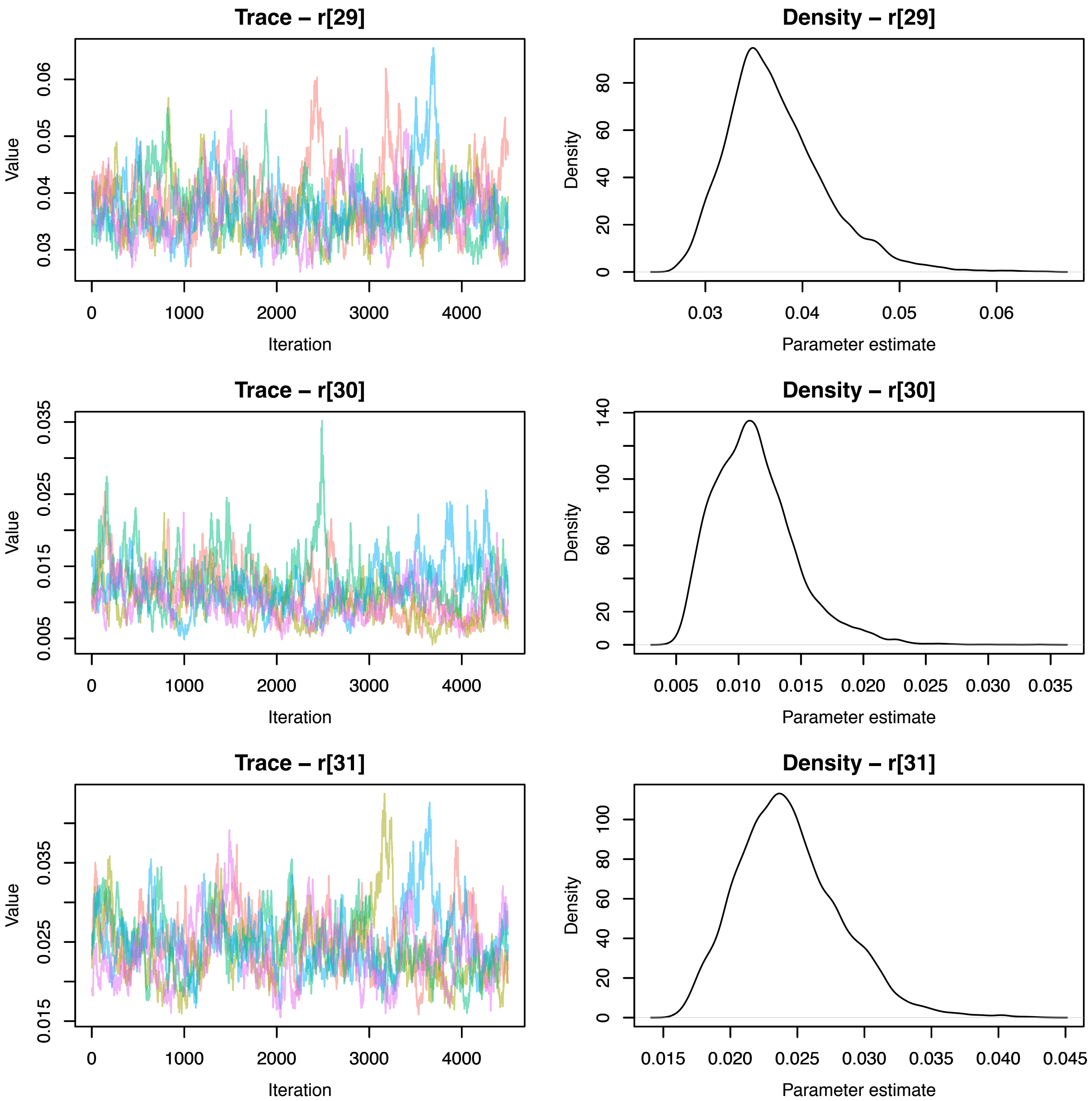

Trace – r[14]
Value
Iteration
0
1000
2000
3000
4000
0.015
0.025
0.035
Density – r[14]
Density
Parameter estimate
0.015
0.020
0.025
0.030
0.035
0.040
0
20
40
60
80
120
Trace – r[15]
Value
Iteration
0.010
0.020
0.030
0.040
Density – r[15]
Density
Parameter estimate
0.010
0.015
0.020
0.025
0.030
0.035
0.040
0
20
40
60
80
100
Trace – r[16]
Value
Iteration
0.015
0.025
0.035
0.045
Density – r[16]
Density
Parameter estimate
0.02
0.03
0.04
0.05
0
20
40
60
80
100


Trace – r[17]
Value
Iteration
Density – r[17]
Density
Parameter estimate
Trace – r[18]
Value
Iteration
Density – r[18]
Density
Parameter estimate
Trace – r[19]
Value
Iteration
Density – r[19]
Density
Parameter estimate

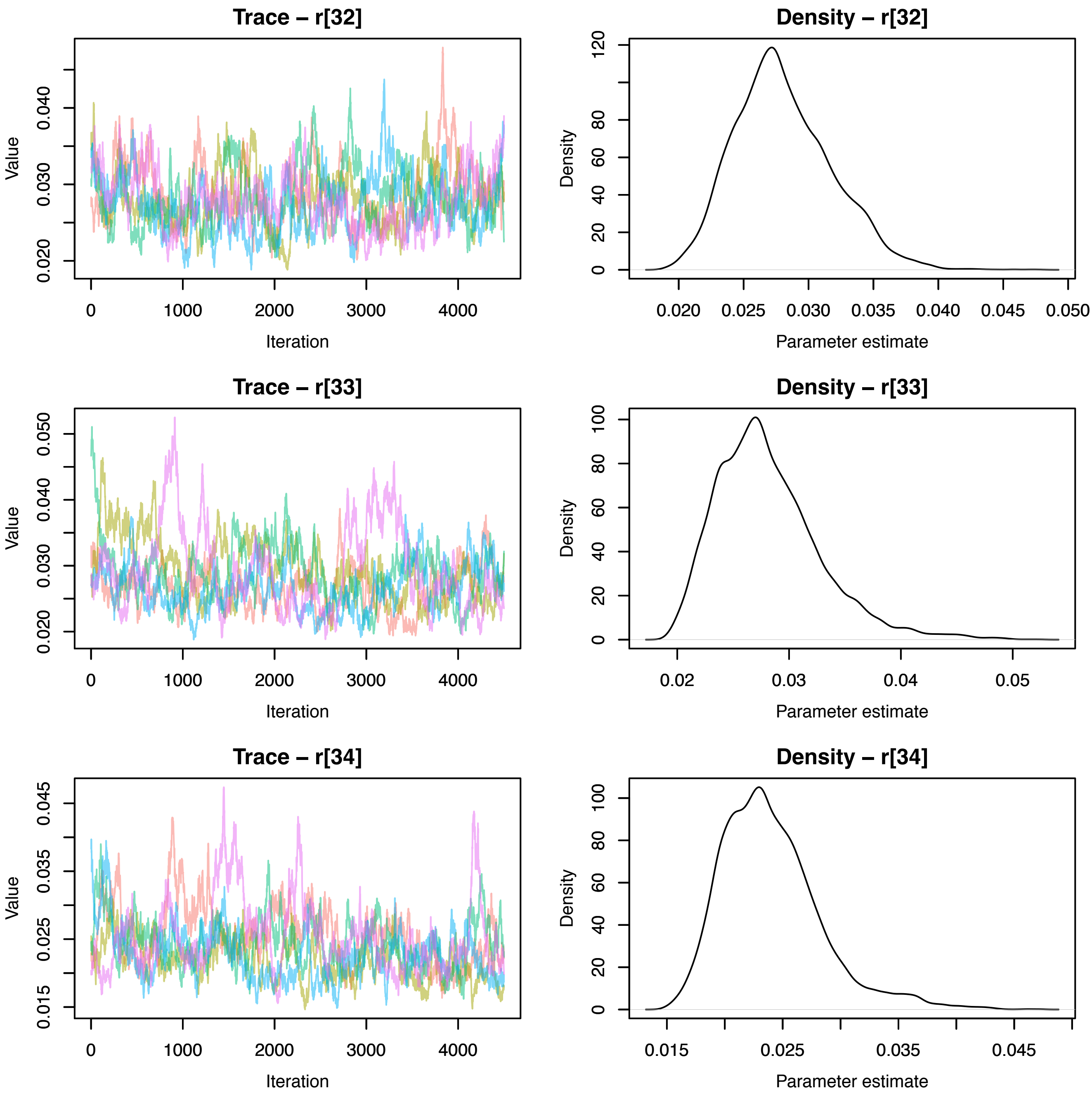

Trace – r[20]
Value
Iteration
Density – r[20]
Density
Parameter estimate
Trace – r[21]
Value
Iteration
Density – r[21]
Density
Parameter estimate
Trace – r[22]
Value
Iteration
Density – r[22]
Density
Parameter estimate

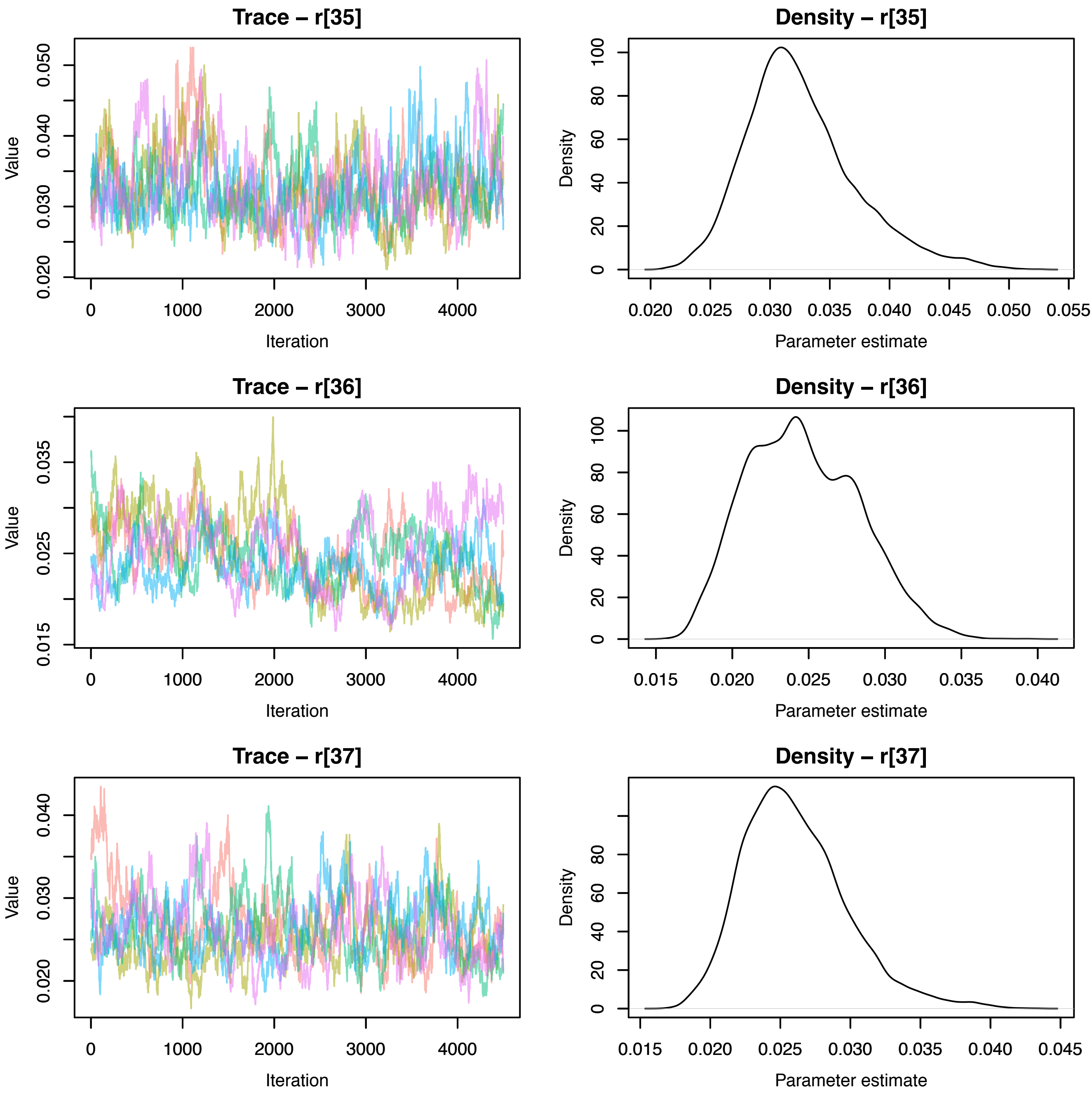

Trace – r[23]
Value
0.010
0.020
0.030
0
1000
2000
3000
4000
Iteration
Density – r[23]
Density
0
20
60
100
140
0.010
0.015
0.020
0.025
0.030
Parameter estimate
Trace – r[24]
Value
0.020
0.030
0.040
0
1000
2000
3000
4000
Iteration
Density – r[24]
Density
0
20
40
60
80
100
0.020
0.025
0.030
0.035
0.040
0.045
0.050
Parameter estimate
Trace – r[25]
Value
0.020
0.030
0.040
0.050
0
1000
2000
3000
4000
Iteration
Density – r[25]
Density
0
20
40
60
80
100
0.020
0.030
0.040
0.050
Parameter estimate


Trace – r[26]
Value
0.015
0.025
0.035
0.045
0
1000
2000
3000
4000
Iteration
Density – r[26]
Density
0
20
40
60
80
0.015
0.020
0.025
0.030
0.035
0.040
0.045
Parameter estimate
Trace – r[27]
Value
0.015
0.025
0.035
0
1000
2000
3000
4000
Iteration
Density – r[27]
Density
0
20
40
60
80
100
0.010
0.015
0.020
0.025
0.030
0.035
0.040
Parameter estimate
Trace – r[28]
Value
0.010
0.020
0.030
0
1000
2000
3000
4000
Iteration
Density – r[28]
Density
0
20
40
60
80
120
0.010
0.015
0.020
0.025
0.030
0.035
Parameter estimate

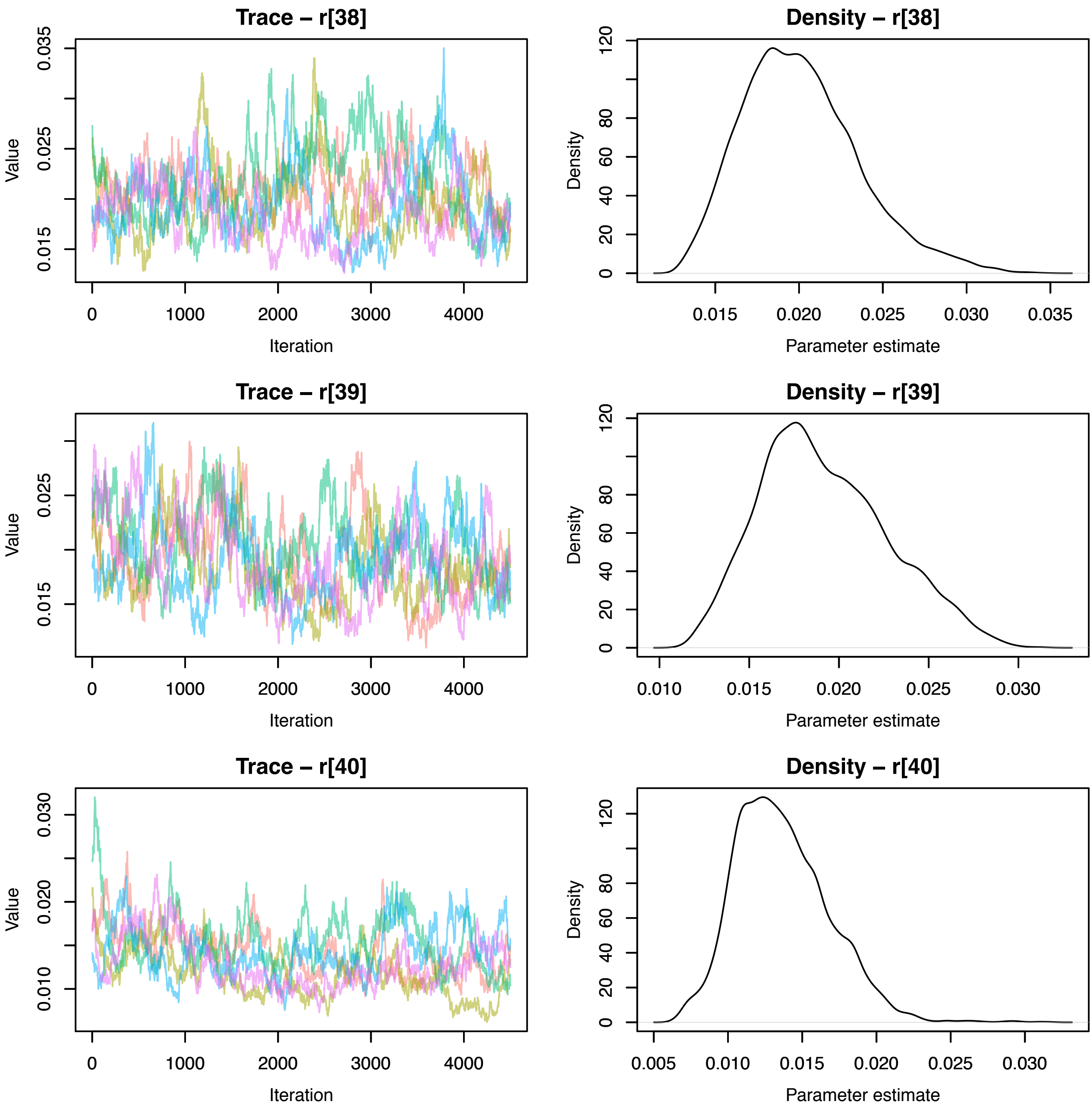


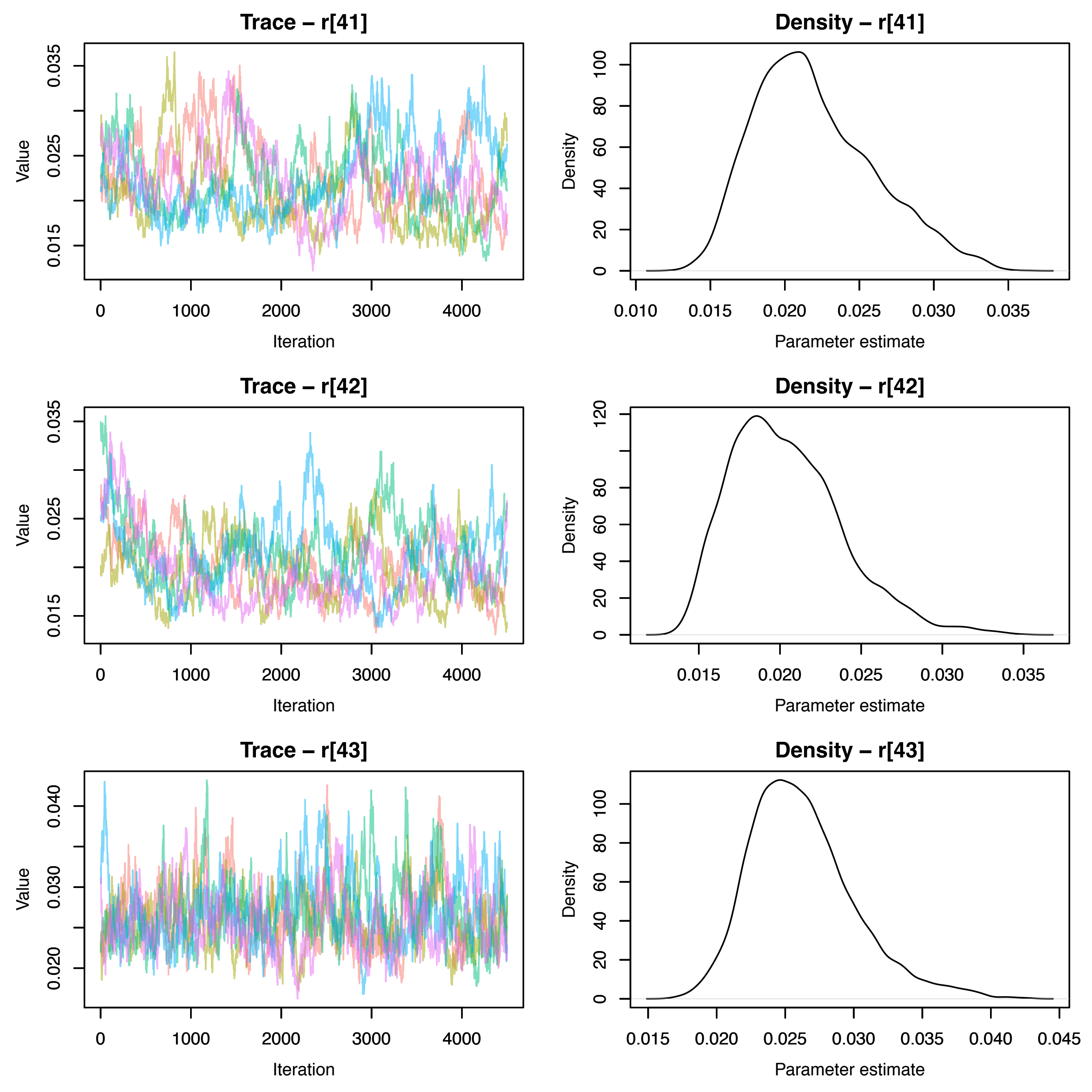
Trace – r[29]
Value
0.03
0.04
0.05
0.06
Iteration
0
1000
2000
3000
4000
Density – r[29]
Density
0
20
40
60
80
Parameter estimate
0.03
0.04
0.05
0.06
Trace – r[30]
Value
0.005
0.015
0.025
0.035
Iteration
0
1000
2000
3000
4000
Density – r[30]
Density
0
20
60
100
140
Parameter estimate
0.005
0.010
0.015
0.020
0.025
0.030
0.035
Trace – r[31]
Value
0.015
0.025
0.035
Iteration
0
1000
2000
3000
4000
Density – r[31]
Density
0
20
40
60
80
100
Parameter estimate
0.015
0.020
0.025
0.030
0.035
0.040
0.045


Trace – r[32]
Value
Iteration
Density – r[32]
Density
Parameter estimate
Trace – r[33]
Value
Iteration
Density – r[33]
Density
Parameter estimate
Trace – r[34]
Value
Iteration
Density – r[34]
Density
Parameter estimate

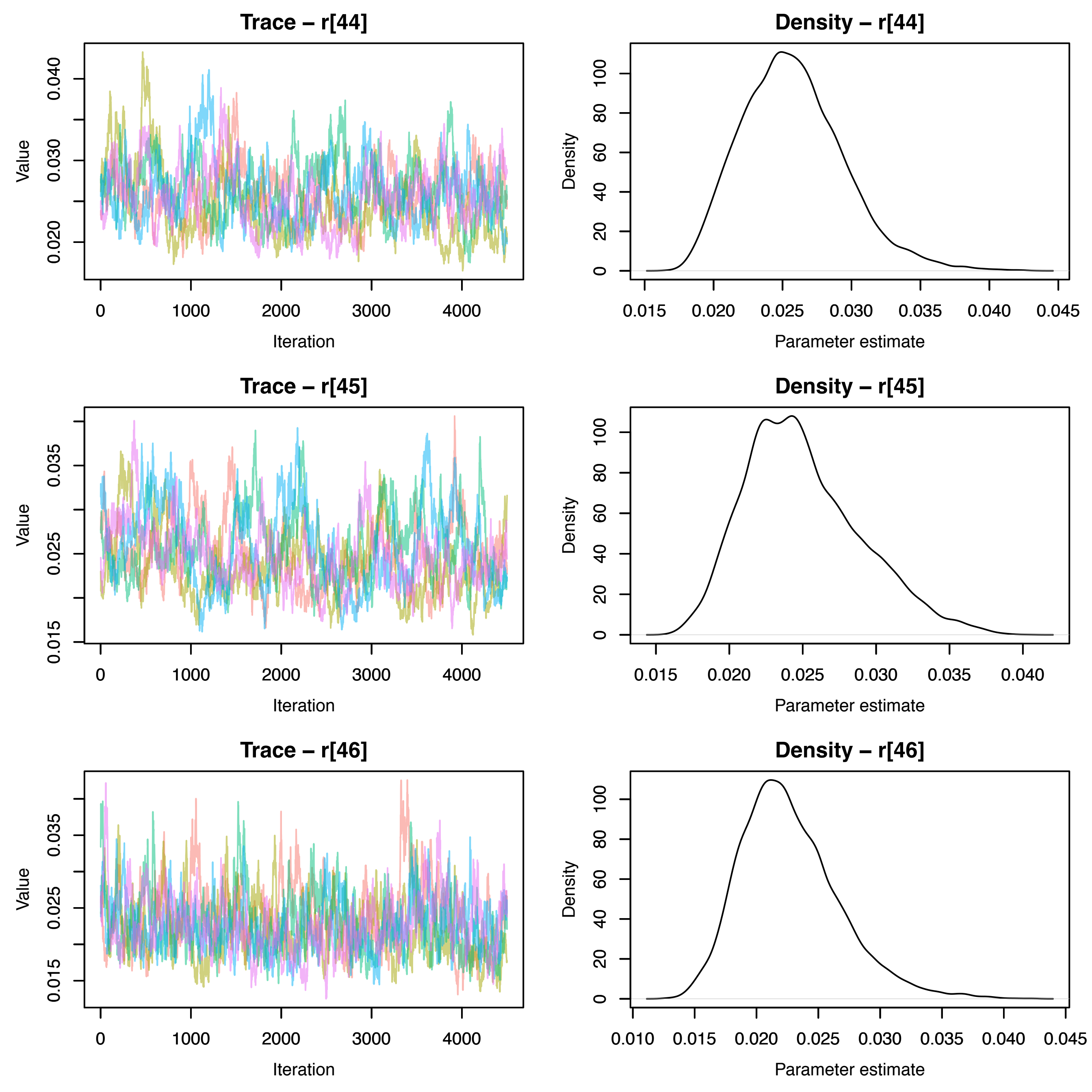


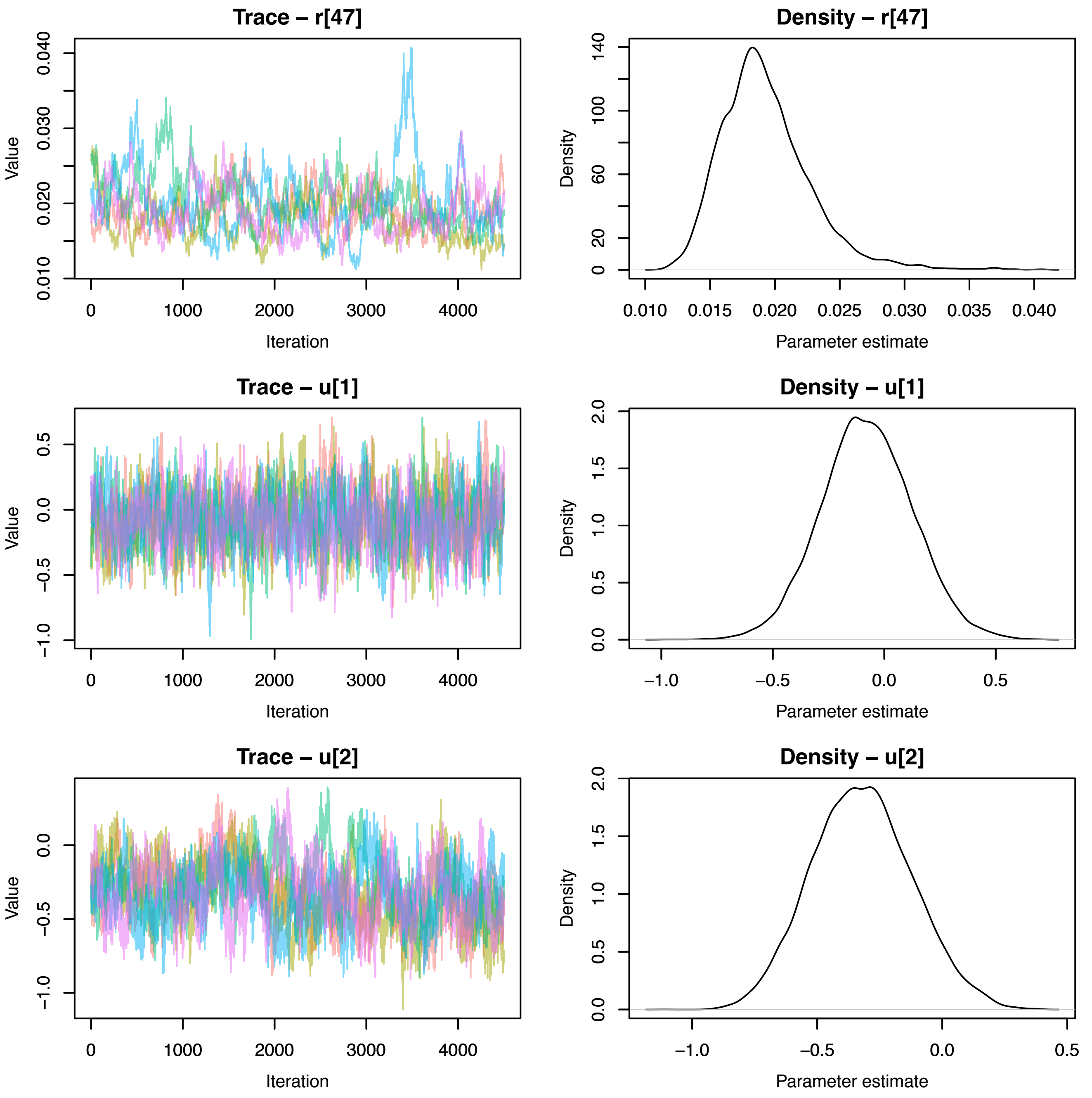
Trace – r[35]
Value
Iteration
0
1000
2000
3000
4000
0.020
0.030
0.040
0.050
Density – r[35]
Density
Parameter estimate
0
20
40
60
80
100
0.020
0.025
0.030
0.035
0.040
0.045
0.050
0.055
Trace – r[36]
Value
Iteration
0.015
0.025
0.035
Density – r[36]
Density
Parameter estimate
0.015
0.020
0.025
0.030
0.035
0.040
Trace – r[37]
Value
Iteration
0.020
0.030
0.040
Density – r[37]
Density
Parameter estimate
0.015
0.020
0.025
0.030
0.035
0.040
0.045

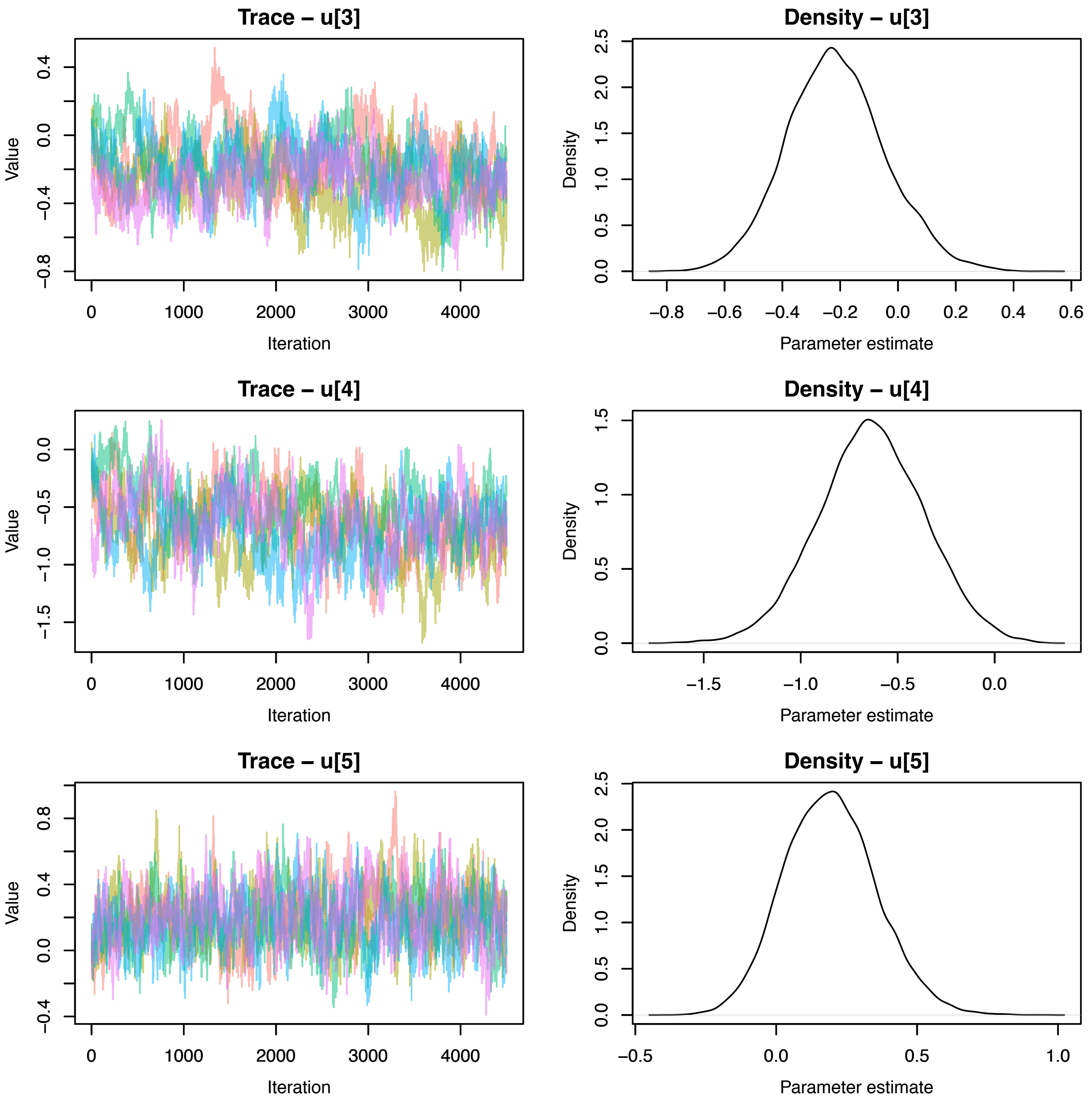

Trace – r[38]
Value
0.015
0.025
0.035
0
1000
2000
3000
4000
Iteration
Density – r[38]
Density
0
20
40
60
80
120
0.015
0.020
0.025
0.030
0.035
Parameter estimate
Trace – r[39]
Value
0.015
0.025
0
1000
2000
3000
4000
Iteration
Density – r[39]
Density
0
20
40
60
80
120
0.010
0.015
0.020
0.025
0.030
Parameter estimate
Trace – r[40]
Value
0.010
0.020
0.030
0
1000
2000
3000
4000
Iteration
Density – r[40]
Density
0
20
40
60
80
120
0.005
0.010
0.015
0.020
0.025
0.030
Parameter estimate

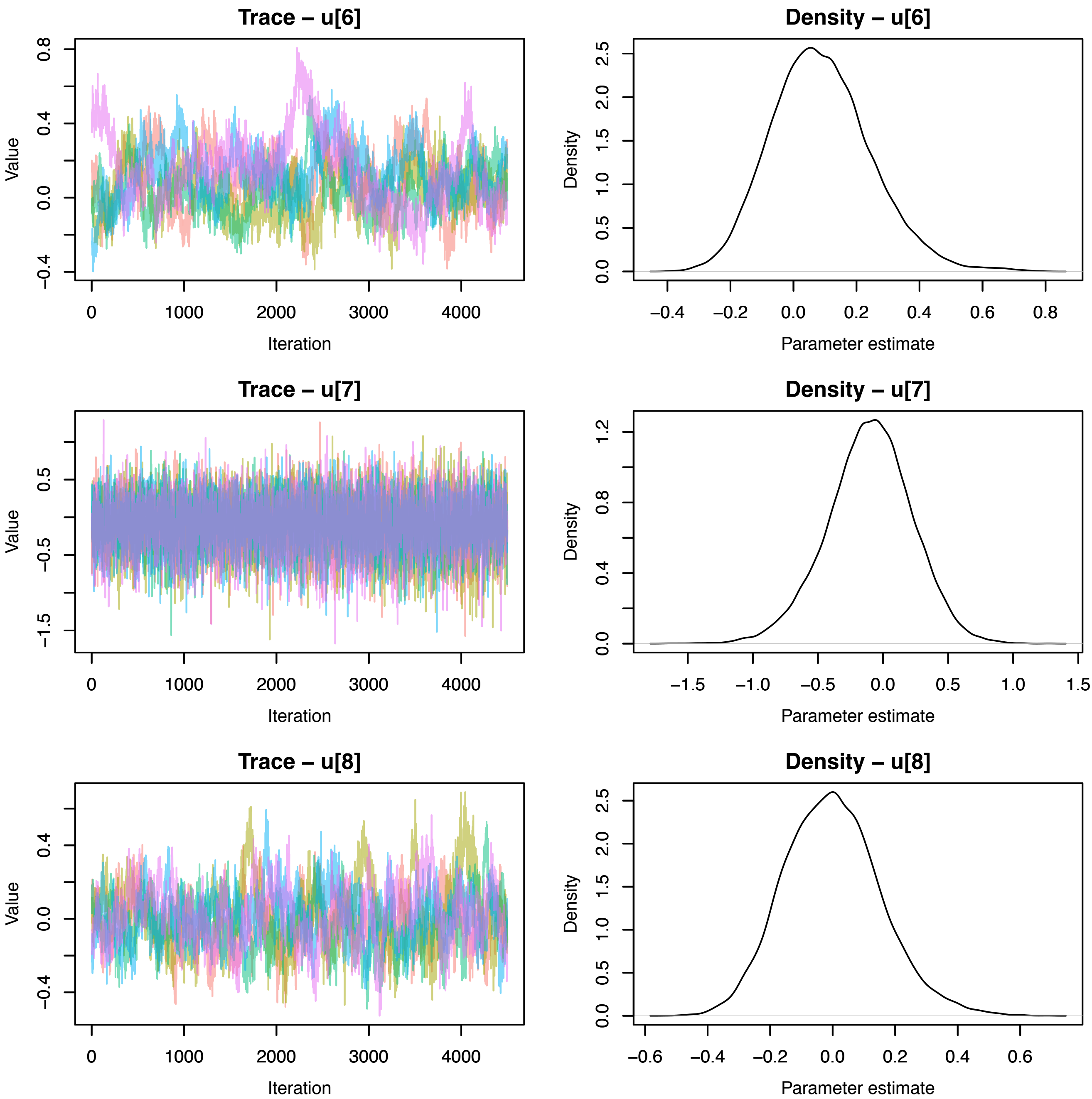


Trace – r[41]
Value
Iteration
Density – r[41]
Density
Parameter estimate
Trace – r[42]
Value
Iteration
Density – r[42]
Density
Parameter estimate
Trace – r[43]
Value
Iteration
Density – r[43]
Density
Parameter estimate

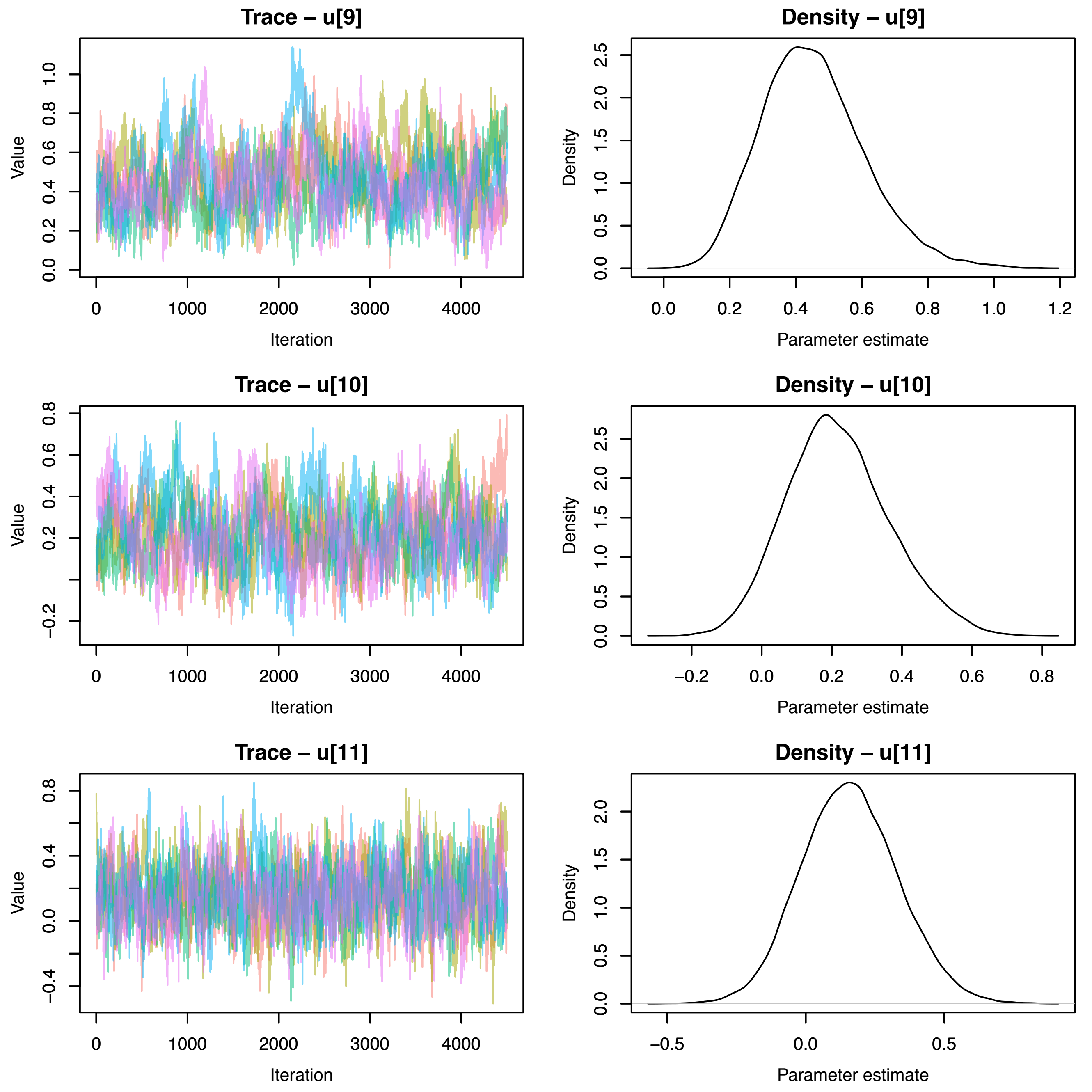

Trace – r[44]
Value
0.020
0.030
0.040
Iteration
0
1000
2000
3000
4000
Density – r[44]
Density
0
20
40
60
80
100
Parameter estimate
0.015
0.020
0.025
0.030
0.035
0.040
0.045
Trace – r[45]
Value
0.015
0.025
0.035
Iteration
0
1000
2000
3000
4000
Density – r[45]
Density
0
20
40
60
80
100
Parameter estimate
0.015
0.020
0.025
0.030
0.035
0.040
Trace – r[46]
Value
0.015
0.025
0.035
Iteration
0
1000
2000
3000
4000
Density – r[46]
Density
0
20
40
60
80
100
Parameter estimate
0.010
0.015
0.020
0.025
0.030
0.035
0.040
0.045

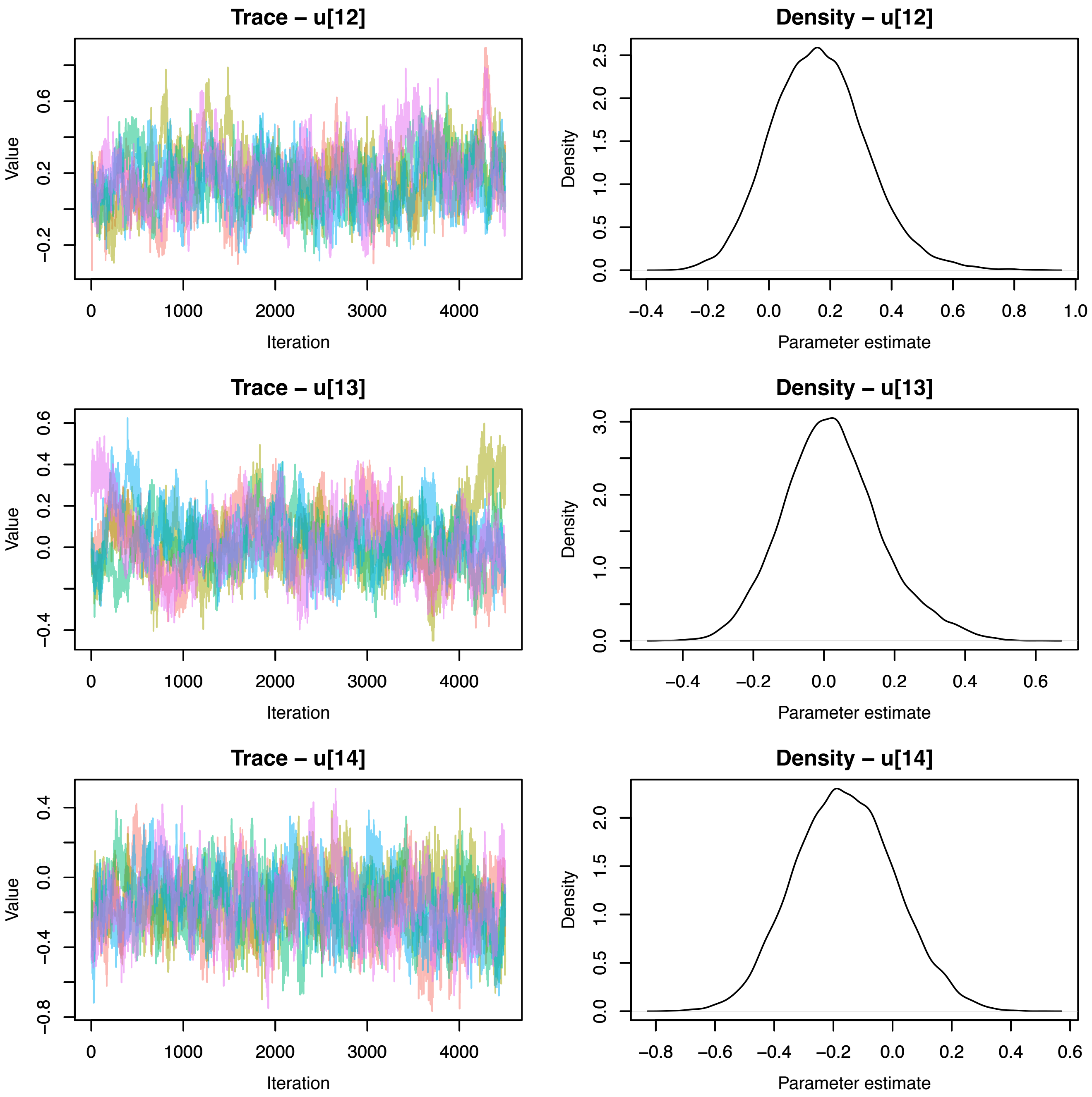

Trace – r[47]
Value
Iteration
Density – r[47]
Density
Parameter estimate
Trace – u[1]
Value
Iteration
Density – u[1]
Density
Parameter estimate
Trace – u[2]
Value
Iteration
Density – u[2]
Density
Parameter estimate

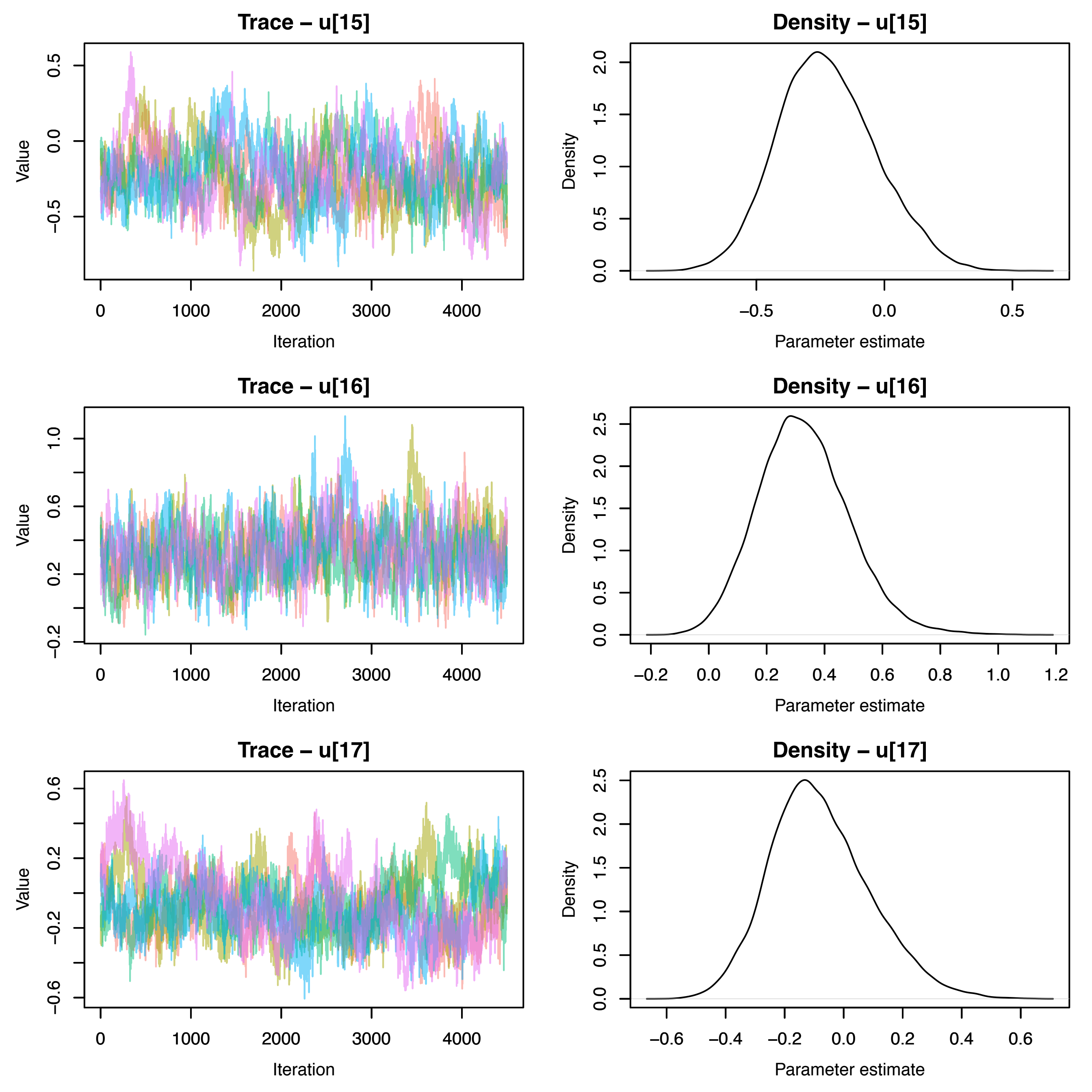

Trace – u[3]
Value
Iteration
0 1000 2000 3000 4000
0.4 0.0 −0.4 −0.8
Density – u[3]
Density
Parameter estimate
−0.8 −0.6 −0.4 −0.2 0.0 0.2 0.4 0.6
0.0 0.5 1.0 1.5 2.0 2.5
Trace – u[4]
Value
Iteration
0 1000 2000 3000 4000
0.0 −0.5 −1.0 −1.5
Density – u[4]
Density
Parameter estimate
−1.5 −1.0 −0.5 0.0
0.0 0.5 1.0 1.5
Trace – u[5]
Value
Iteration
0 1000 2000 3000 4000
0.8 0.4 0.0 −0.4
Density – u[5]
Density
Parameter estimate
−0.5 0.0 0.5 1.0
0.0 0.5 1.0 1.5 2.0 2.5

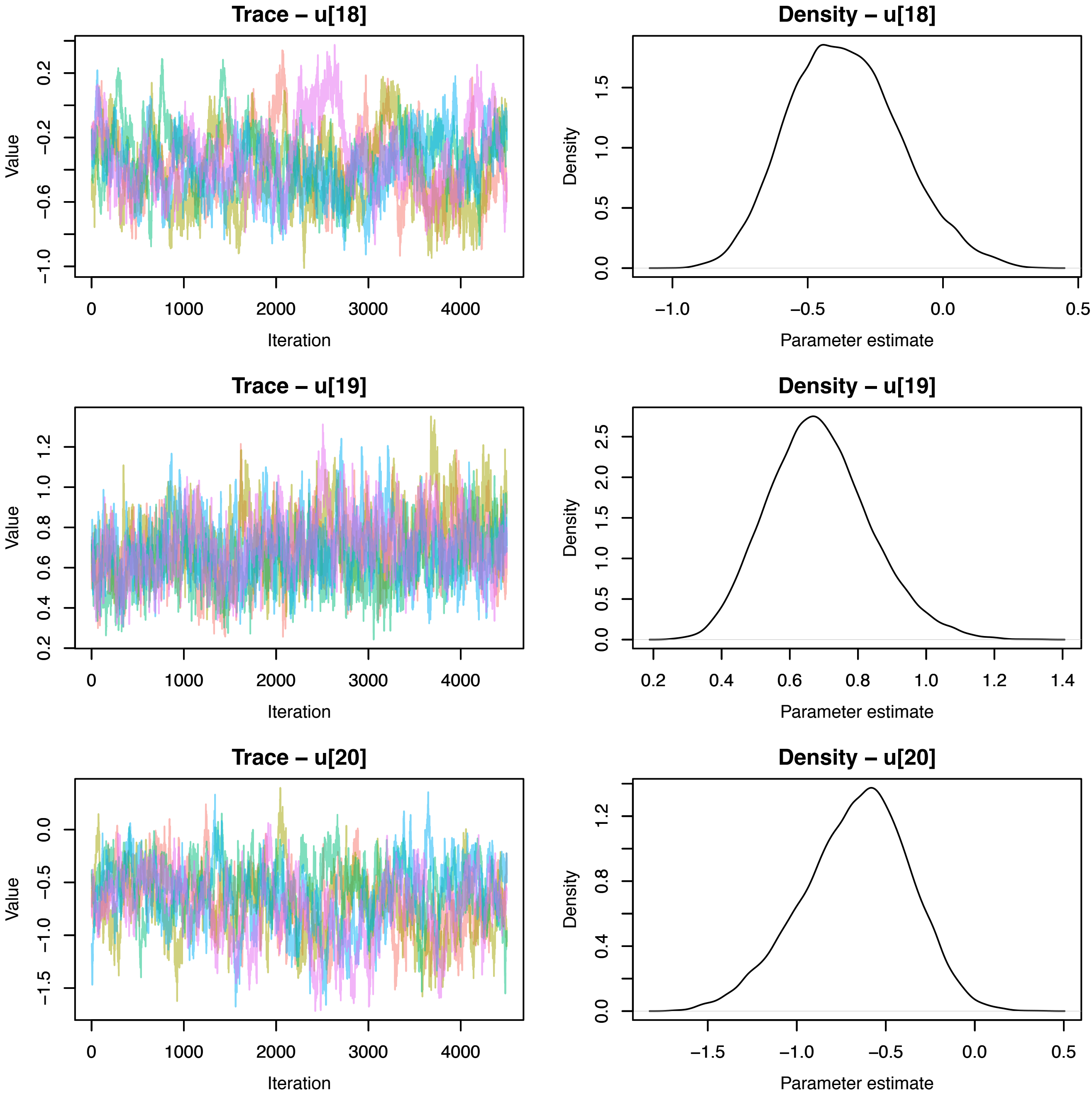


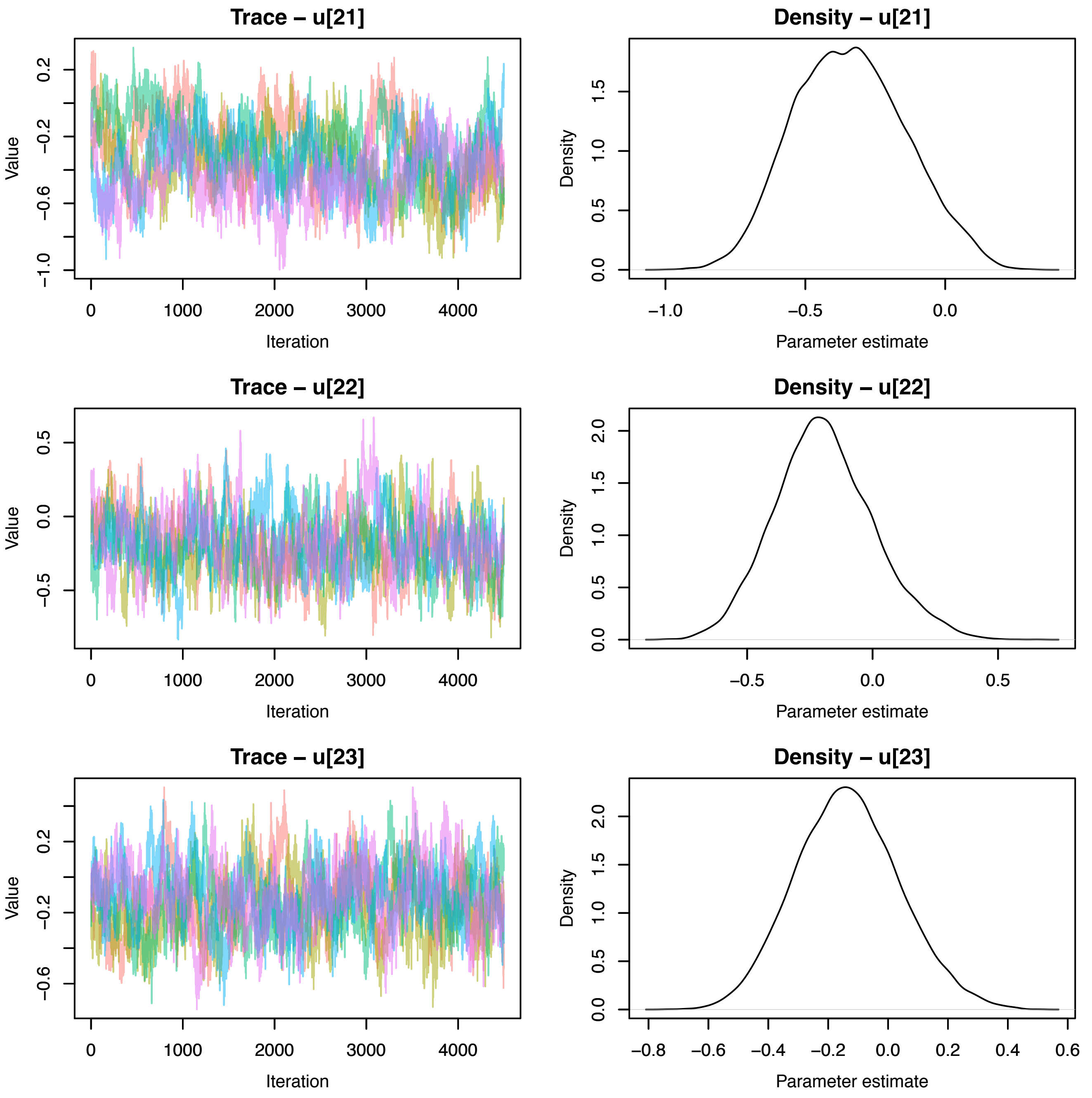
Trace – u[6]
Value
Iteration
0.8
0.4
0.0
−0.4
0
1000
2000
3000
4000
Density – u[6]
Density
Parameter estimate
2.5
2.0
1.5
1.0
0.5
0.0
−0.4
−0.2
0.0
0.2
0.4
0.6
0.8
Trace – u[7]
Value
Iteration
0.5
−0.5
−1.5
0
1000
2000
3000
4000
Density – u[7]
Density
Parameter estimate
1.2
0.8
0.4
0.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
Trace – u[8]
Value
Iteration
0.4
0.0
−0.4
0
1000
2000
3000
4000
Density – u[8]
Density
Parameter estimate
2.5
2.0
1.5
1.0
0.5
0.0
−0.6
−0.4
−0.2
0.0
0.2
0.4
0.6


Trace – u[9]
Value
0.0
0.2
0.4
0.6
0.8
1.0
0
1000
2000
3000
4000
Iteration
Density – u[9]
Density
0.0
0.5
1.0
1.5
2.0
2.5
0.0
0.2
0.4
0.6
0.8
1.0
1.2
Parameter estimate
Trace – u[10]
Value
−0.2
0.2
0.4
0.6
0.8
0
1000
2000
3000
4000
Iteration
Density – u[10]
Density
0.0
0.5
1.0
1.5
2.0
2.5
−0.2
0.0
0.2
0.4
0.6
0.8
Parameter estimate
Trace – u[11]
Value
−0.4
0.0
0.4
0.8
0
1000
2000
3000
4000
Iteration
Density – u[11]
Density
0.0
0.5
1.0
1.5
2.0
−0.5
0.0
0.5
Parameter estimate

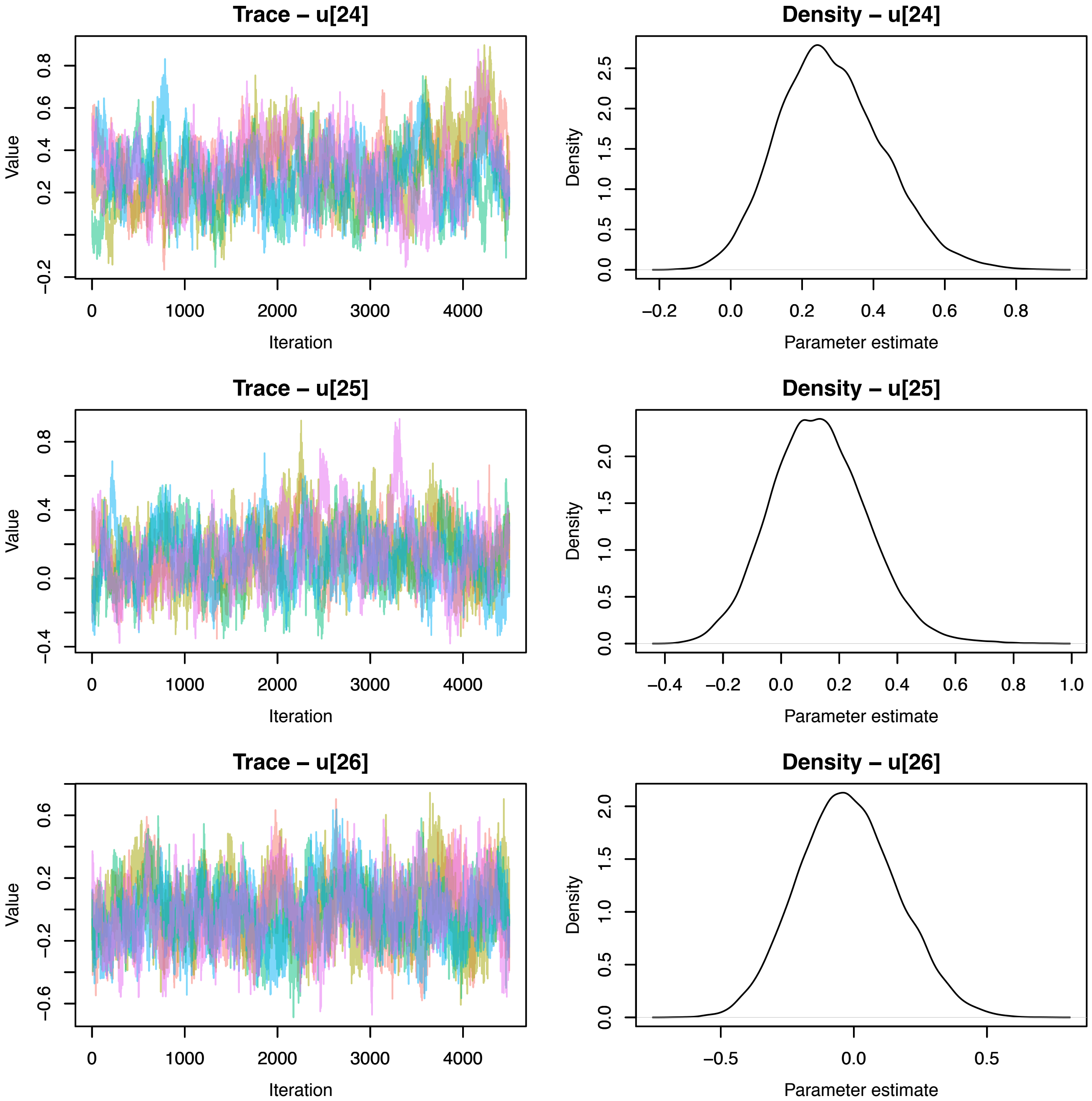

Trace – u[12]
Value
Iteration
Density – u[12]
Density
Parameter estimate
Trace – u[13]
Value
Iteration
Density – u[13]
Density
Parameter estimate
Trace – u[14]
Value
Iteration
Density – u[14]
Density
Parameter estimate

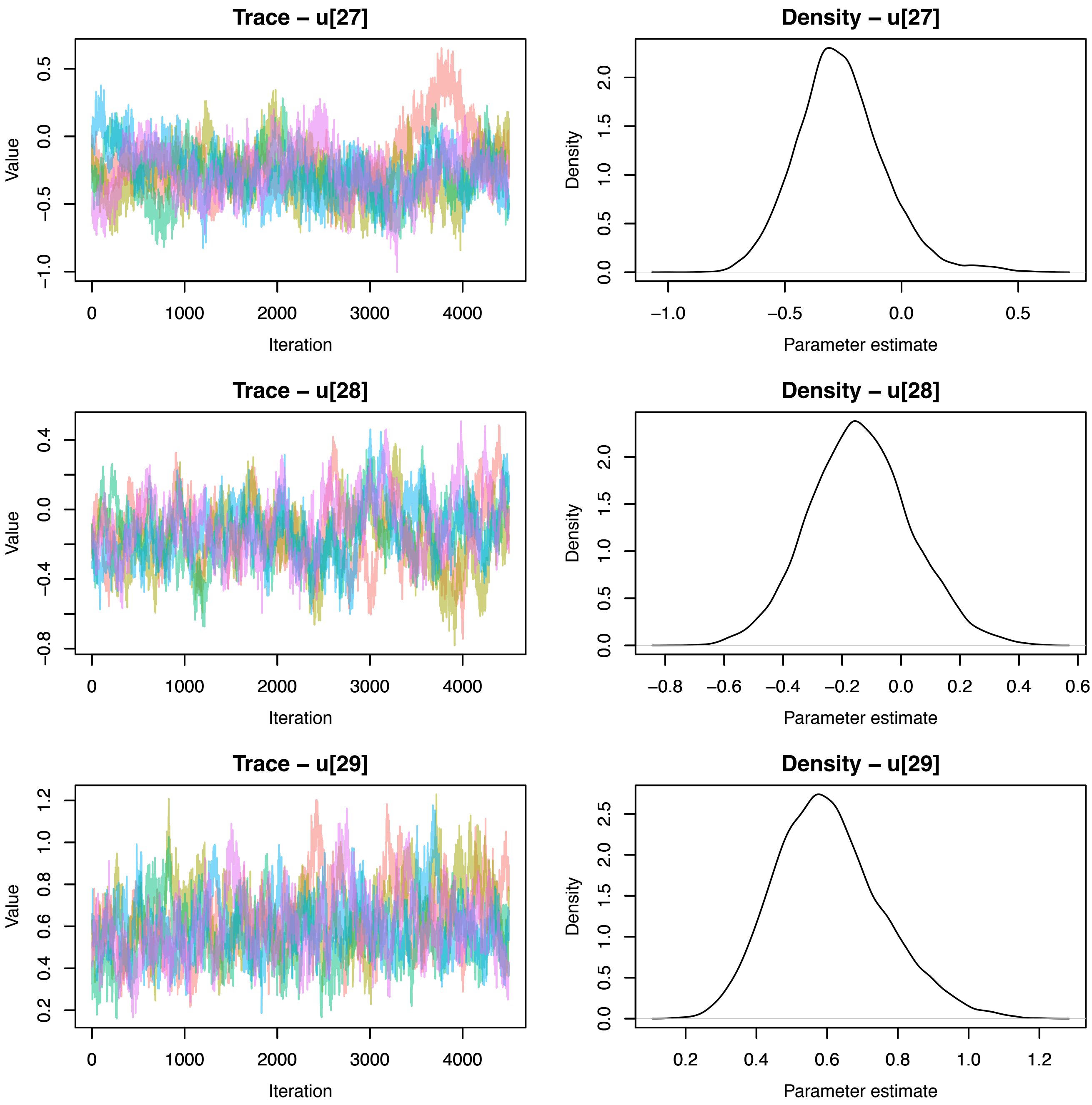

Trace – u[15]
Density – u[15]
Trace – u[16]
Density – u[16]
Trace – u[17]
Density – u[17]
Value
Iteration
Density
Parameter estimate
0
1000
2000
3000
4000

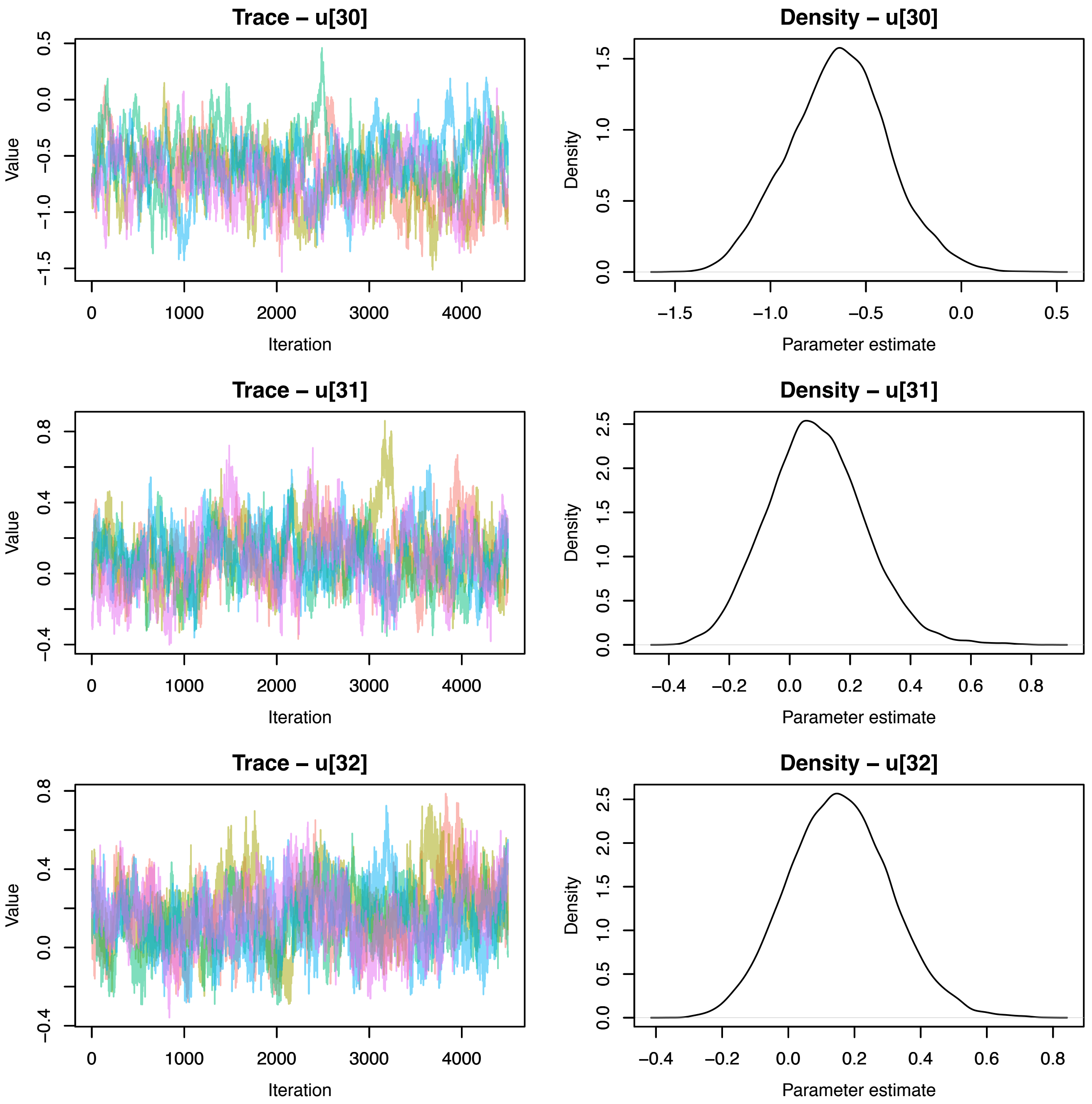

Trace – u[18]
Value
Iteration
Density – u[18]
Density
Parameter estimate
Trace – u[19]
Value
Iteration
Density – u[19]
Density
Parameter estimate
Trace – u[20]
Value
Iteration
Density – u[20]
Density
Parameter estimate



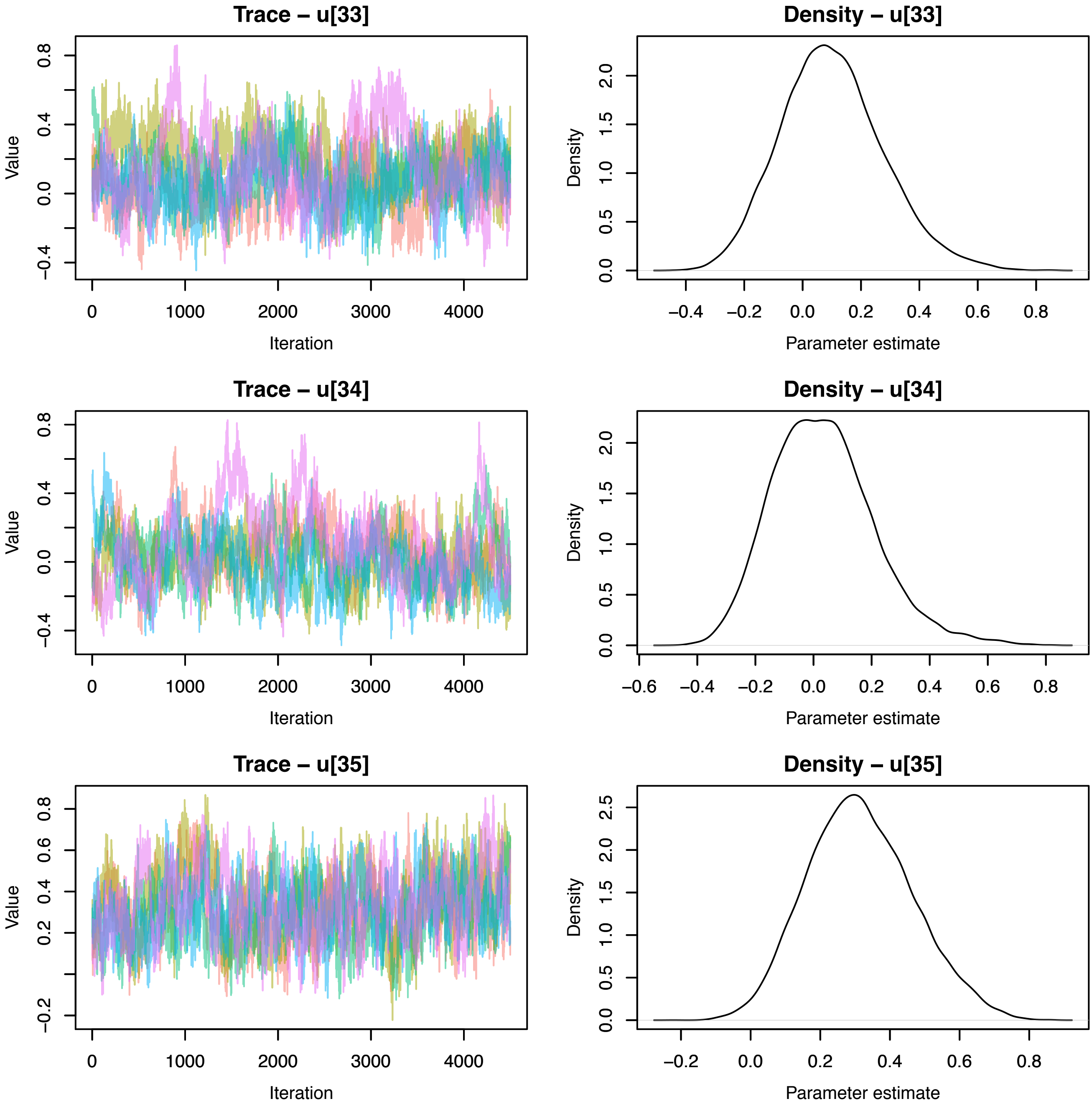
Trace – u[21]
Value
Iteration
0
1000
2000
3000
4000
0.2
−0.2
−0.6
−1.0
Density – u[21]
Density
Parameter estimate
−1.0
−0.5
0.0
0.0
0.5
1.0
1.5
Trace – u[22]
Value
Iteration
0
1000
2000
3000
4000
0.5
0.0
−0.5
Density – u[22]
Density
Parameter estimate
−0.5
0.0
0.5
0.0
0.5
1.0
1.5
2.0
Trace – u[23]
Value
Iteration
0
1000
2000
3000
4000
0.2
−0.2
−0.6
Density – u[23]
Density
Parameter estimate
−0.8
−0.6
−0.4
−0.2
0.0
0.2
0.4
0.6
0.0
0.5
1.0
1.5
2.0

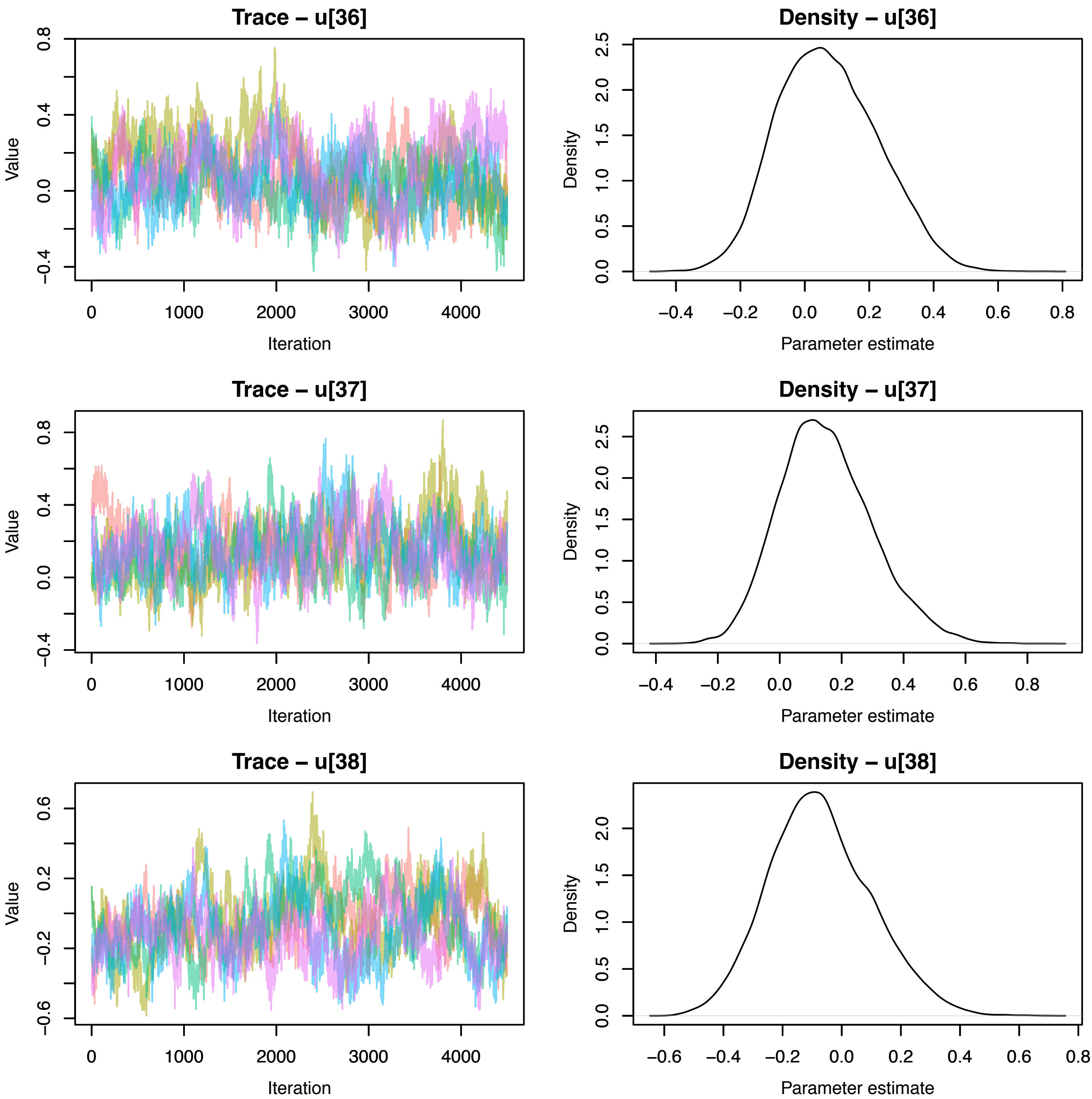

Trace – u[24]
Density – u[24]
Value
Iteration
Density
Parameter estimate
Trace – u[25]
Density – u[25]
Value
Iteration
Density
Parameter estimate
Trace – u[26]
Density – u[26]
Value
Iteration
Density
Parameter estimate


Trace – u[27]
Value
Iteration
0 1000 2000 3000 4000
−1.0 −0.5 0.0 0.5
Density – u[27]
Density
Parameter estimate
−1.0 −0.5 0.0 0.5
0.0 0.5 1.0 1.5 2.0
Trace – u[28]
Value
Iteration
0 1000 2000 3000 4000
−0.8 −0.4 0.0 0.4
Density – u[28]
Density
Parameter estimate
−0.8 −0.6 −0.4 −0.2 0.0 0.2 0.4 0.6
0.0 0.5 1.0 1.5 2.0
Trace – u[29]
Value
Iteration
0 1000 2000 3000 4000
0.2 0.4 0.6 0.8 1.0 1.2
Density – u[29]
Density
Parameter estimate
0.2 0.4 0.6 0.8 1.0 1.2
0.0 0.5 1.0 1.5 2.0 2.5

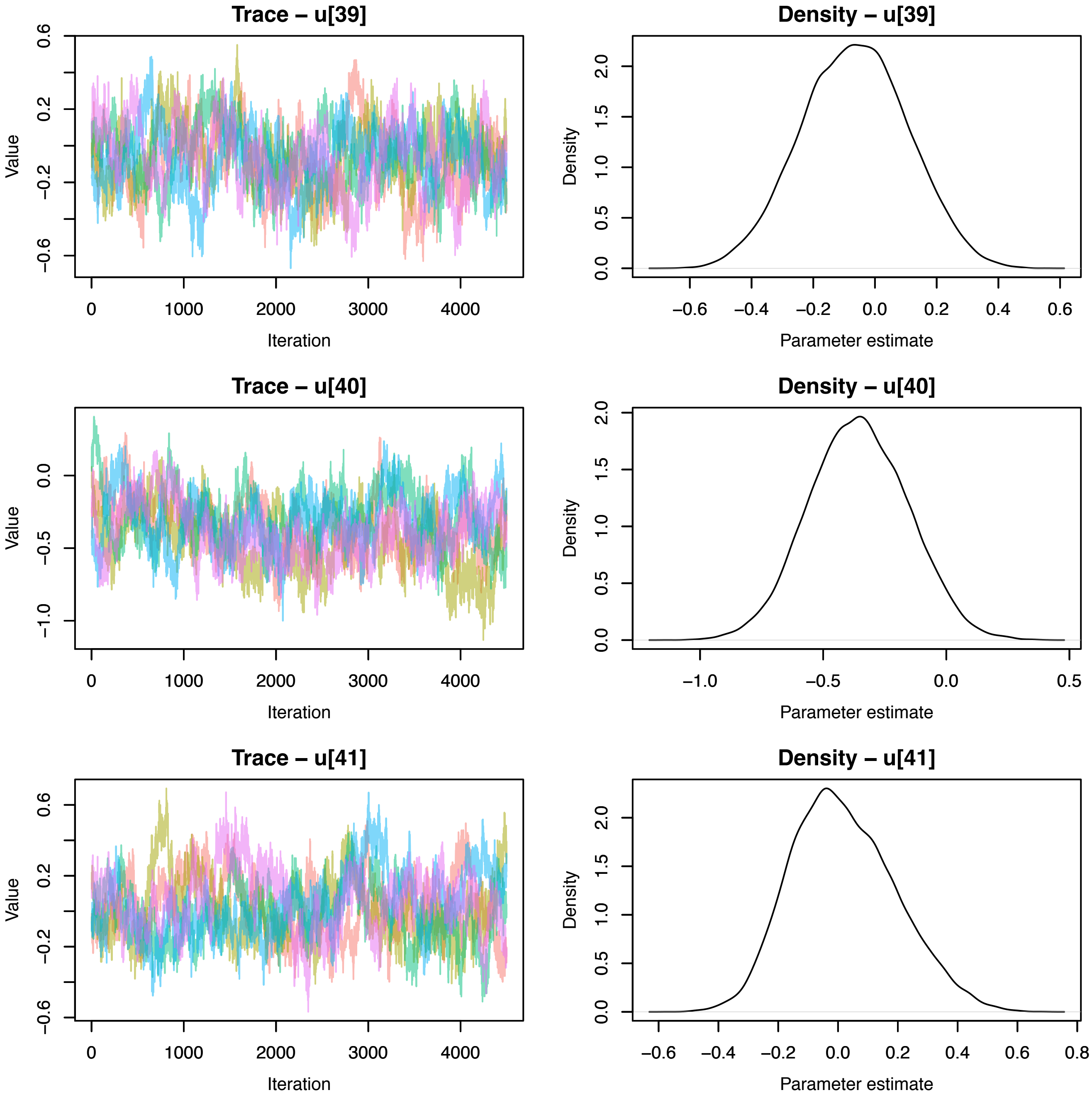


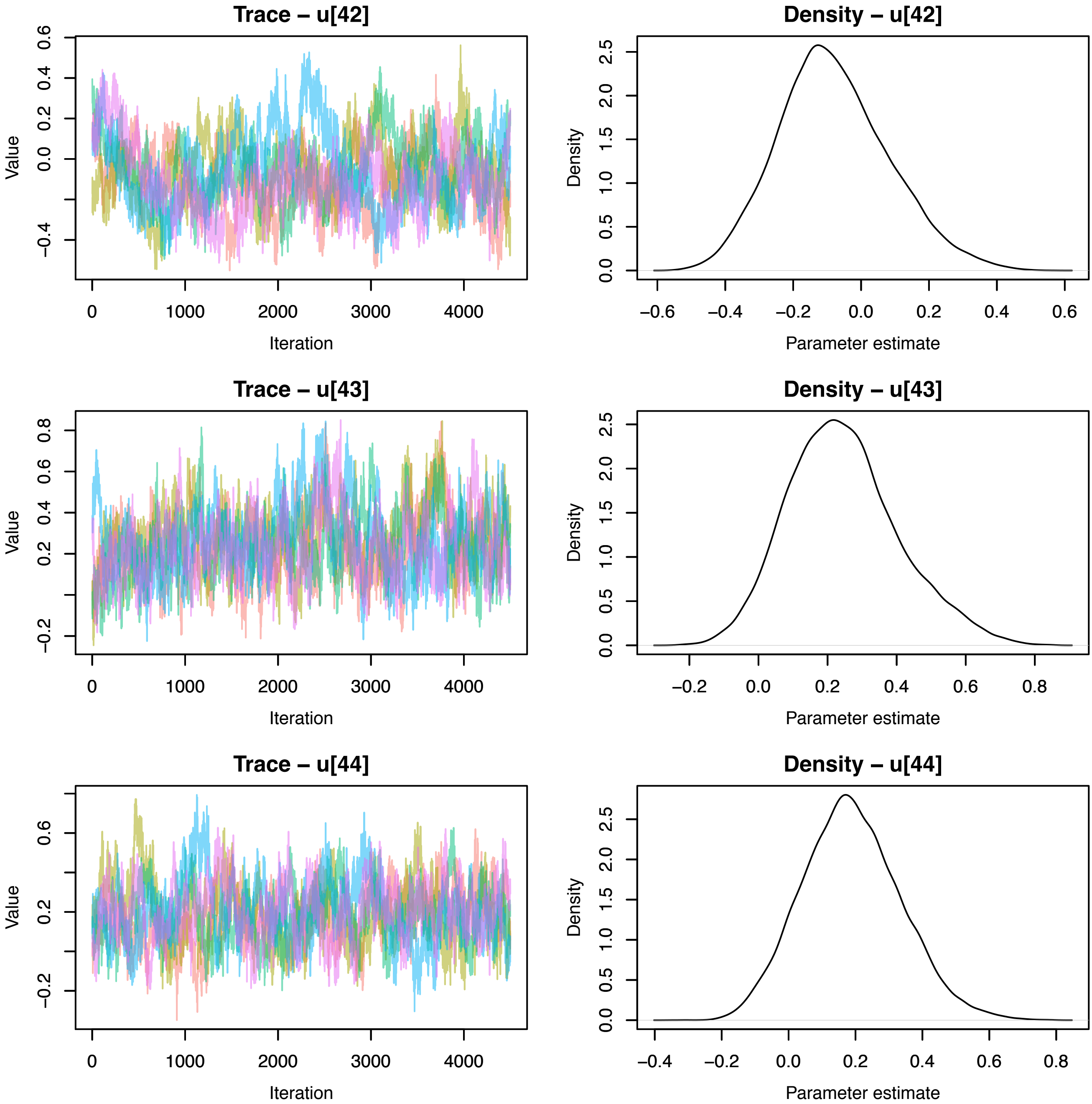
Trace – u[30]
Value
Iteration
0
1000
2000
3000
4000
0.5
0.0
−0.5
−1.0
−1.5
Density – u[30]
Density
Parameter estimate
−1.5
−1.0
−0.5
0.0
0.5
1.5
1.0
0.5
0.0
Trace – u[31]
Value
Iteration
0.8
0.4
0.0
−0.4
Density – u[31]
Density
Parameter estimate
−0.4
−0.2
0.0
0.2
0.4
0.6
0.8
2.5
2.0
1.5
1.0
0.5
0.0
Trace – u[32]
Value
Iteration
0.8
0.4
0.0
−0.4
Density – u[32]
Density
Parameter estimate
−0.4
−0.2
0.0
0.2
0.4
0.6
0.8
2.5
2.0
1.5
1.0
0.5
0.0

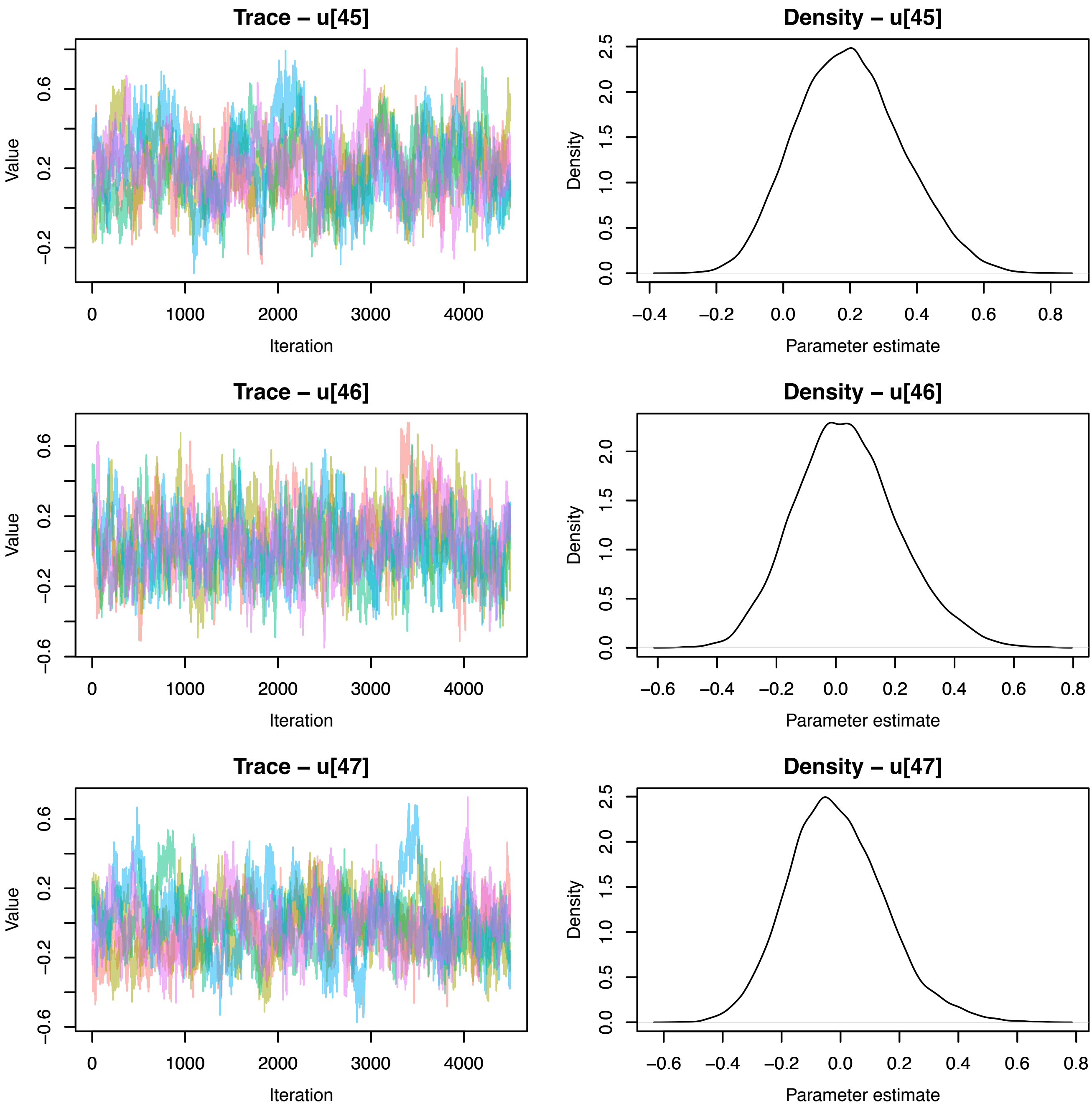

Trace – u[33]
Value
Iteration
0
1000
2000
3000
4000
−0.4
0.0
0.4
0.8
Density – u[33]
Density
Parameter estimate
−0.4
−0.2
0.0
0.2
0.4
0.6
0.8
0.0
0.5
1.0
1.5
2.0
Trace – u[34]
Value
Iteration
Density – u[34]
Density
Parameter estimate
−0.6
Trace – u[35]
Value
Iteration
−0.2
0.2
0.4
0.6
0.8
Density – u[35]
Density
Parameter estimate
2.5

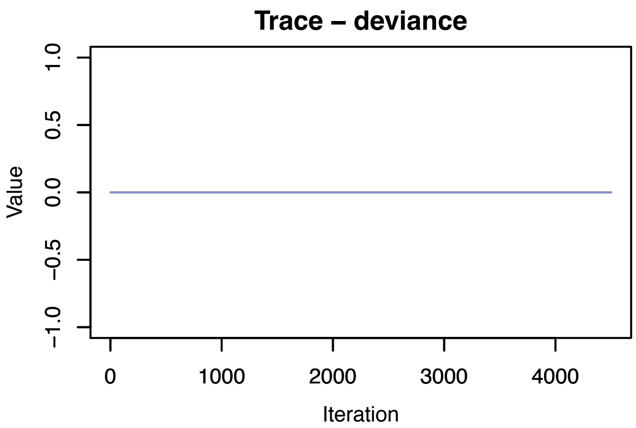


Trace – u[36]
Value
Iteration
0
1000
2000
3000
4000
−0.4
0.0
0.4
0.8
Density – u[36]
Density
Parameter estimate
−0.4
−0.2
0.0
0.2
0.4
0.6
0.8
0.0
0.5
1.0
1.5
2.0
2.5
Trace – u[37]
Value
Iteration
0
1000
2000
3000
4000
−0.4
0.0
0.4
0.8
Density – u[37]
Density
Parameter estimate
−0.4
−0.2
0.0
0.2
0.4
0.6
0.8
0.0
0.5
1.0
1.5
2.0
2.5
Trace – u[38]
Value
Iteration
0
1000
2000
3000
4000
−0.6
−0.2
0.2
0.6
Density – u[38]
Density
Parameter estimate
−0.6
−0.4
−0.2
0.0
0.2
0.4
0.6
0.8
0.0
0.5
1.0
1.5
2.0

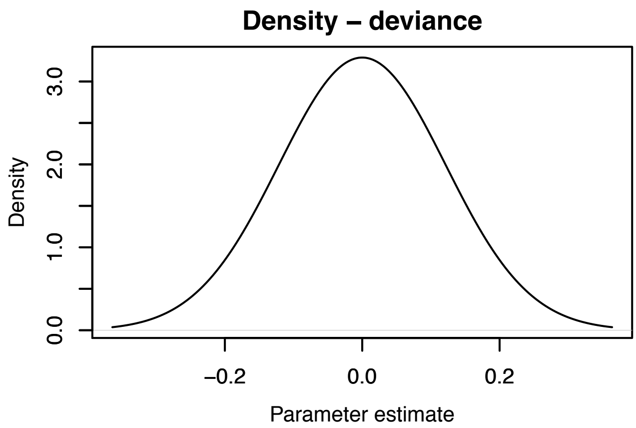

Trace – u[39]
Value
Iteration
Density – u[39]
Density
Parameter estimate
Trace – u[40]
Value
Iteration
Density – u[40]
Density
Parameter estimate
Trace – u[41]
Value
Iteration
Density – u[41]
Density
Parameter estimate


Trace – u[42]
Value
Iteration
0
1000
2000
3000
4000
0.6
0.4
0.2
0.0
−0.4
Density – u[42]
Density
Parameter estimate
−0.6
−0.4
−0.2
0.0
0.2
0.4
0.6
2.5
2.0
1.5
1.0
0.5
0.0
Trace – u[43]
Value
Iteration
0.8
0.6
0.4
0.2
−0.2
Density – u[43]
Density
Parameter estimate
−0.2
0.0
0.2
0.4
0.6
0.8
Trace – u[44]
Value
Iteration
0.6
0.2
−0.2
Density – u[44]
Density
Parameter estimate
−0.4
−0.2
0.0
0.2
0.4
0.6
0.8


Trace – u[45]
Value
Iteration
Density – u[45]
Density
Parameter estimate
Trace – u[46]
Value
Iteration
Density – u[46]
Density
Parameter estimate
Trace – u[47]
Value
Iteration
Density – u[47]
Density
Parameter estimate

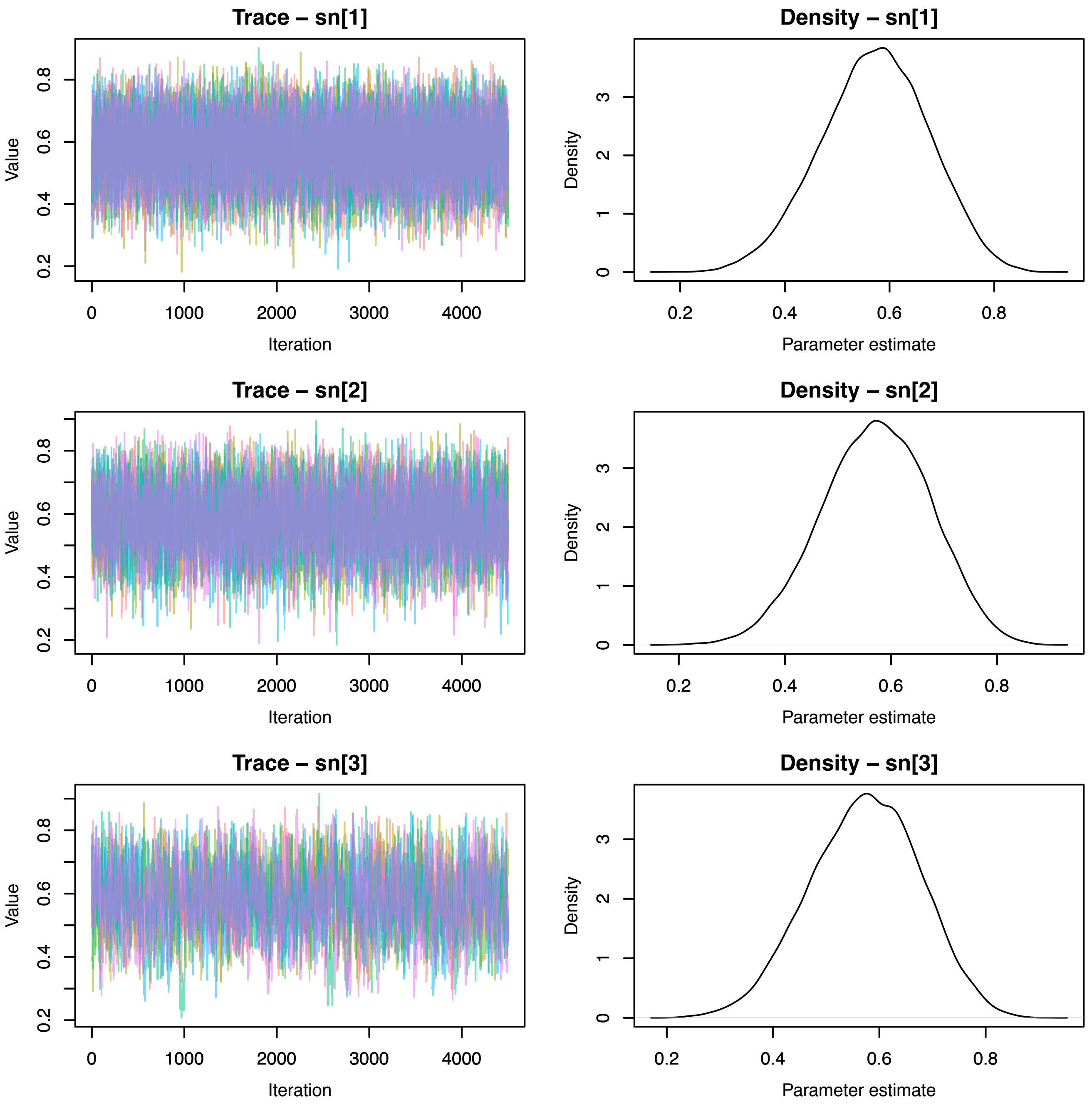


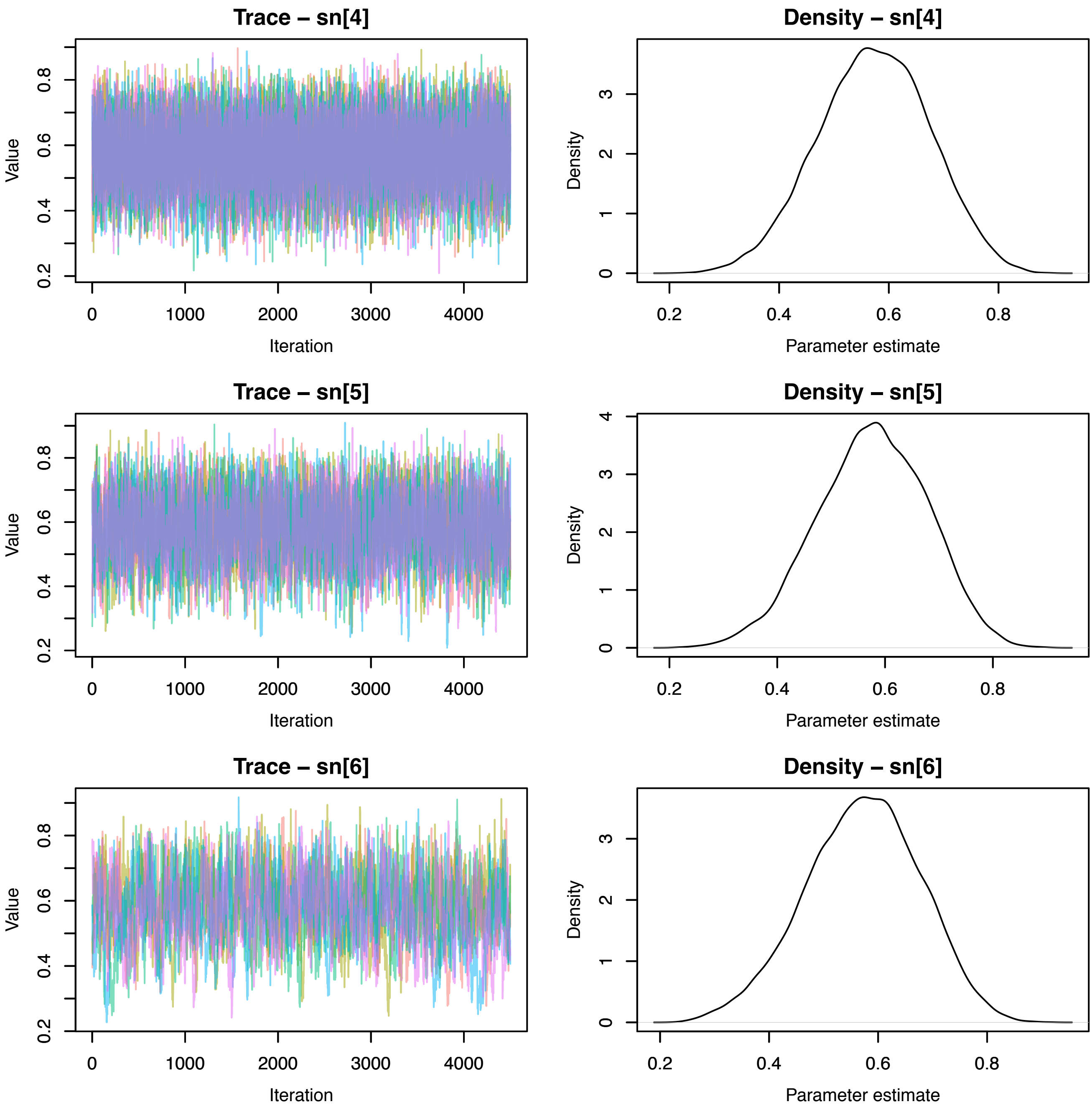
Trace – deviance
Value
1.0
0.5
0.0
−0.5
−1.0
0
1000
2000
3000
4000
Iteration

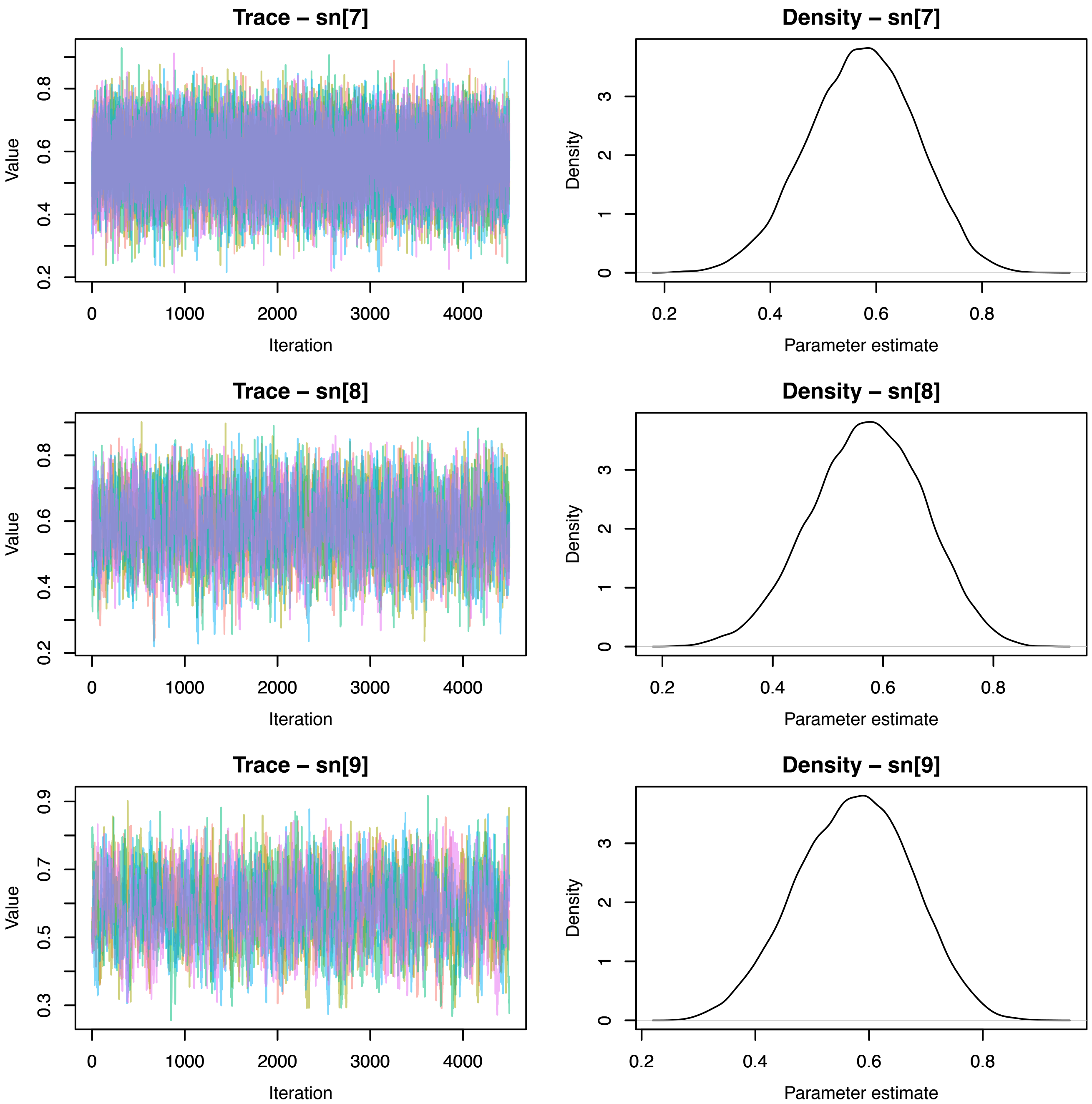



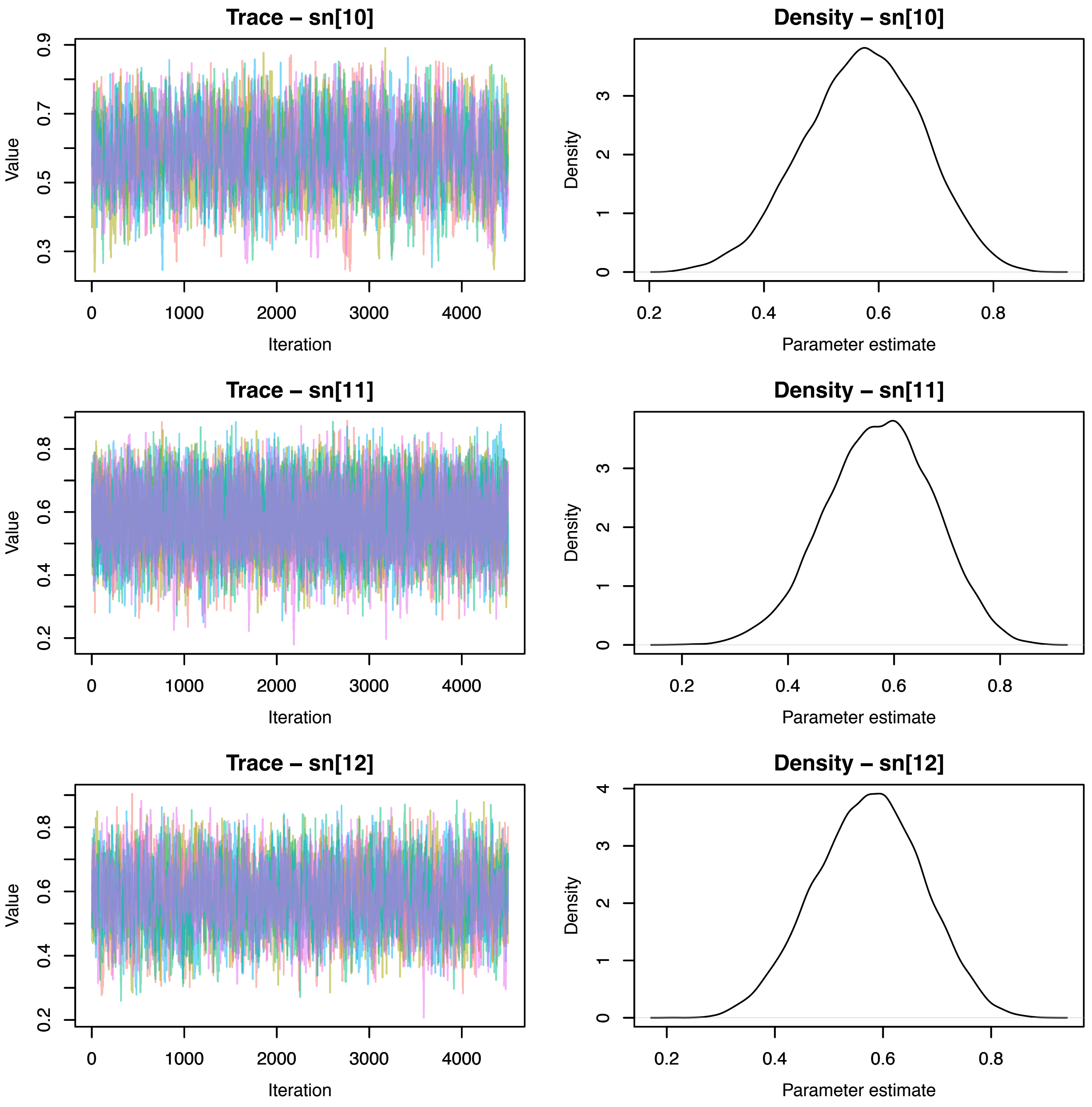
Density – deviance
Density
3.0
2.0
1.0
0.0
−0.2
0.0
0.2
Parameter estimate

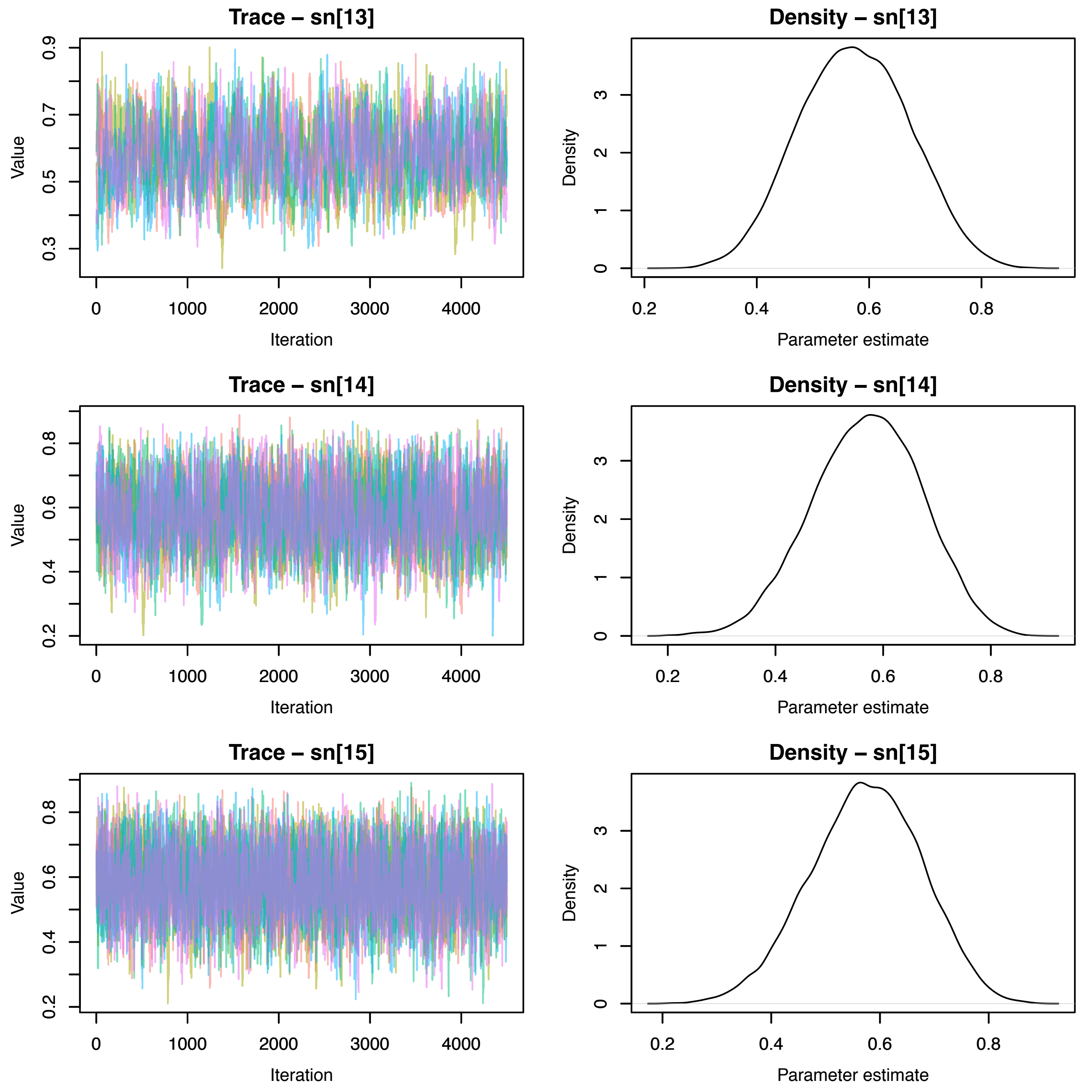
*Appendix 4: Diagnostic plots of select parameters from Model #2 (Eq. 6)*


Trace – sn[1]
Density – sn[1]
Trace – sn[2]
Density – sn[2]
Trace – sn[3]
Density – sn[3]
Value
Iteration
Density
Parameter estimate
0
1000
2000
3000
4000
0.2
0.4
0.6
0.8
0
1
2
3

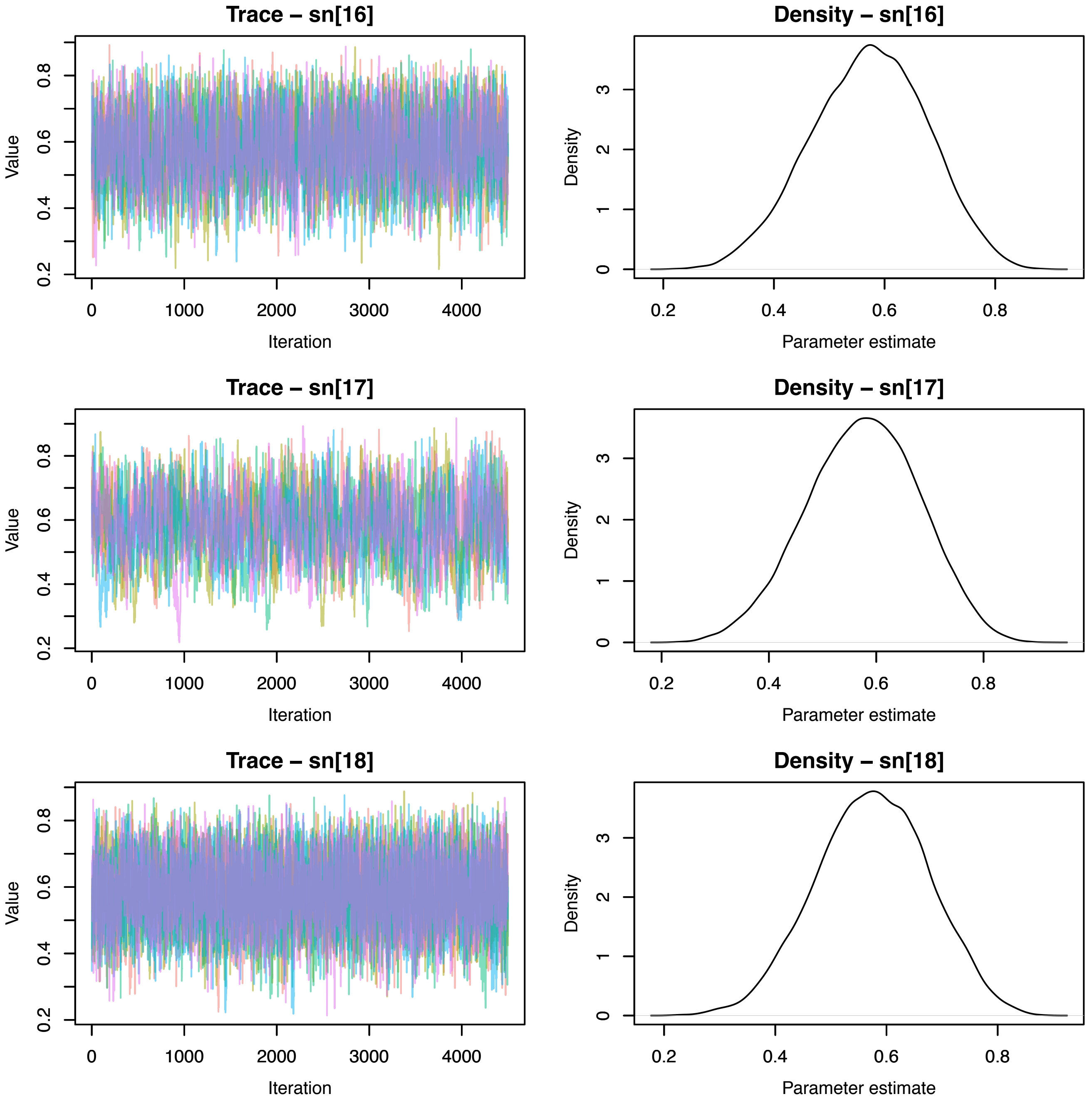

Trace – sn[4]
Density – sn[4]
Trace – sn[5]
Density – sn[5]
Trace – sn[6]
Density – sn[6]
Value
Iteration
Density
Parameter estimate
0
1000
2000
3000
4000
0.2
0.4
0.6
0.8
0
1
2
3
4

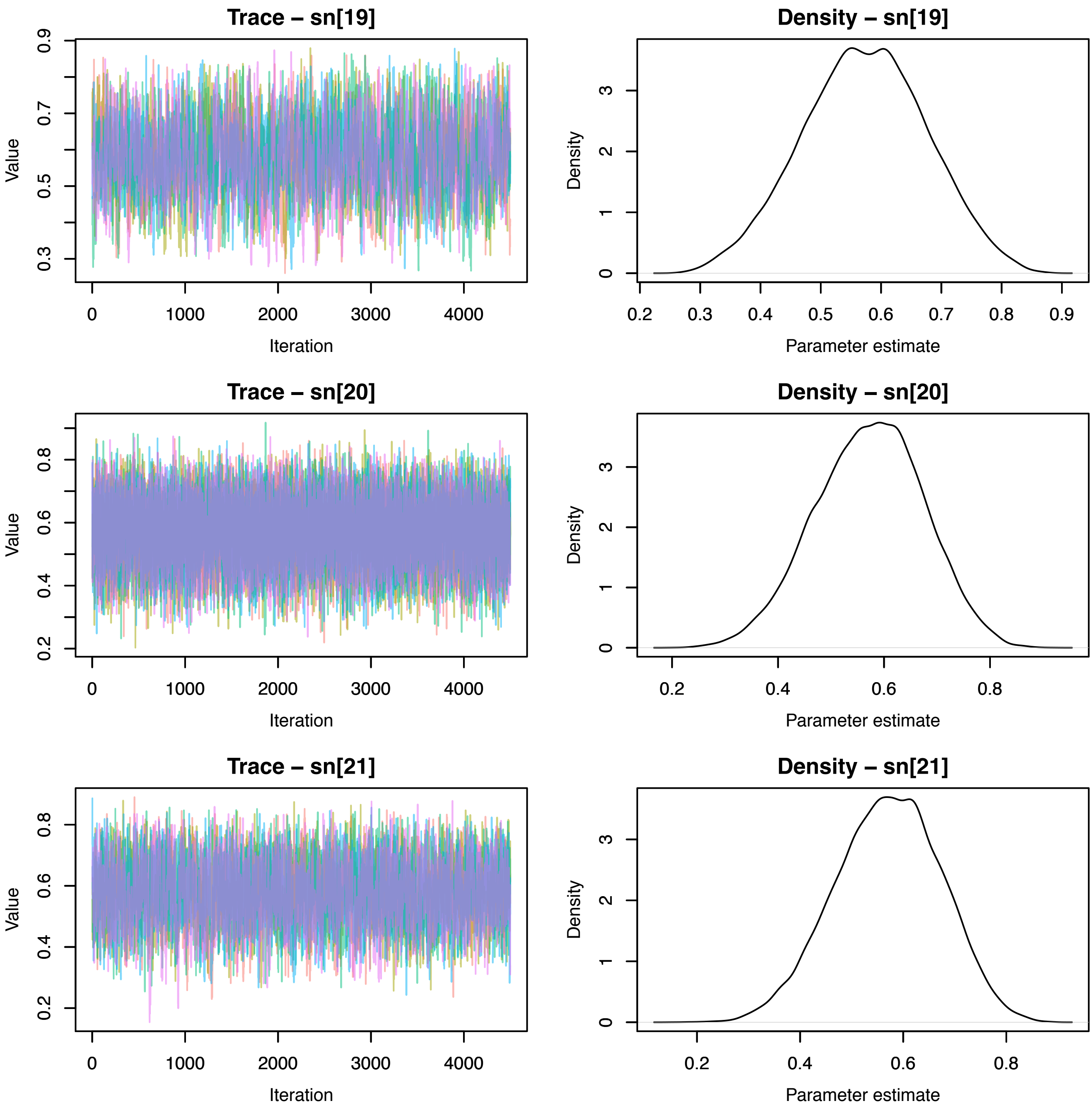

Trace – sn[7]
Value
Iteration
0
1000
2000
3000
4000
0.2
0.4
0.6
0.8
Density – sn[7]
Density
Parameter estimate
0
1
2
3
0.2
0.4
0.6
0.8
Trace – sn[8]
Value
Iteration
0
1000
2000
3000
4000
0.2
0.4
0.6
0.8
Density – sn[8]
Density
Parameter estimate
0
1
2
3
0.2
0.4
0.6
0.8
Trace – sn[9]
Value
Iteration
0
1000
2000
3000
4000
0.3
0.5
0.7
0.9
Density – sn[9]
Density
Parameter estimate
0
1
2
3
0.2
0.4
0.6
0.8

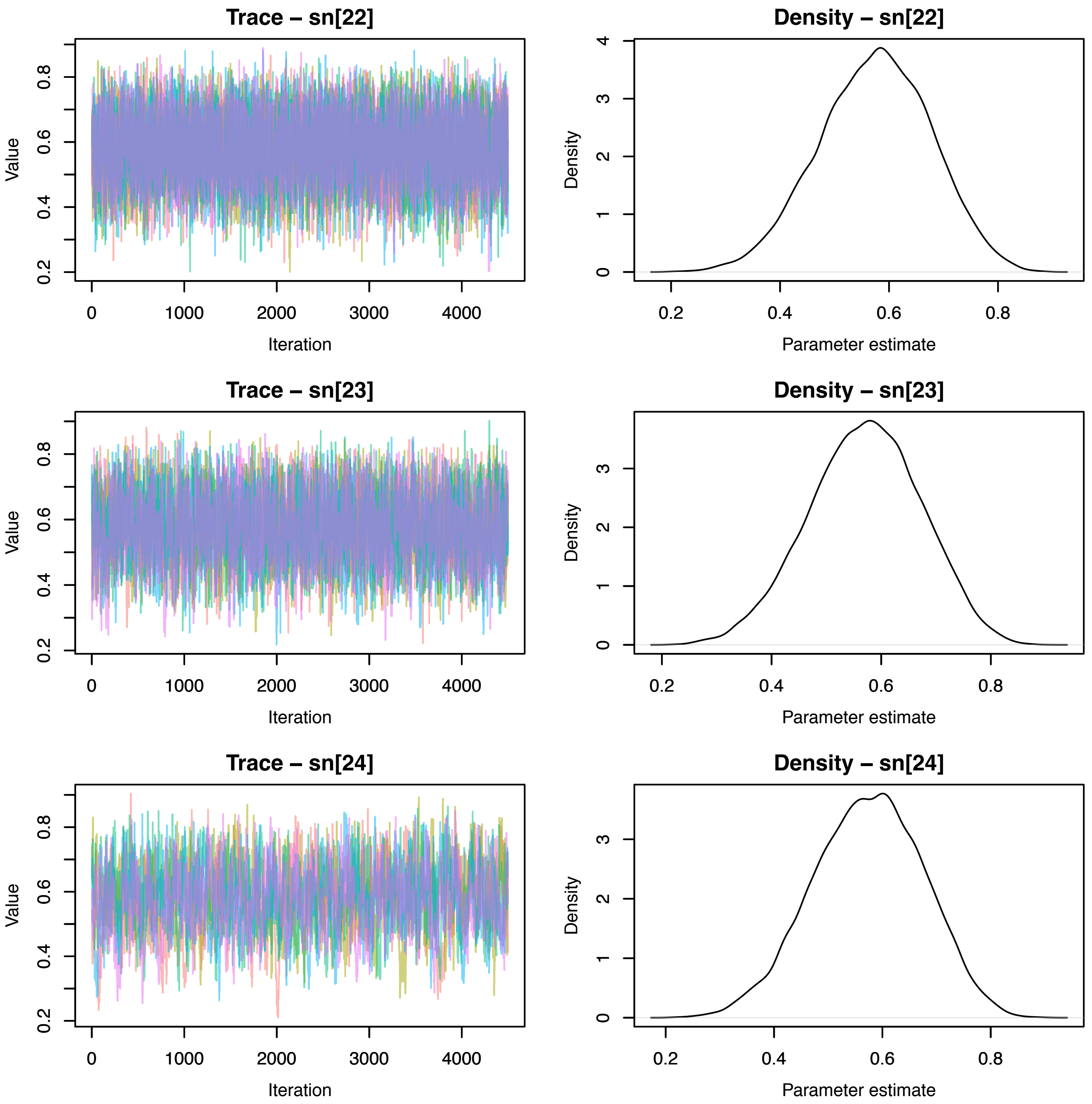

Trace – sn[10]
Value
Iteration
Density – sn[10]
Density
Parameter estimate
Trace – sn[11]
Value
Iteration
Density – sn[11]
Density
Parameter estimate
Trace – sn[12]
Value
Iteration
Density – sn[12]
Density
Parameter estimate

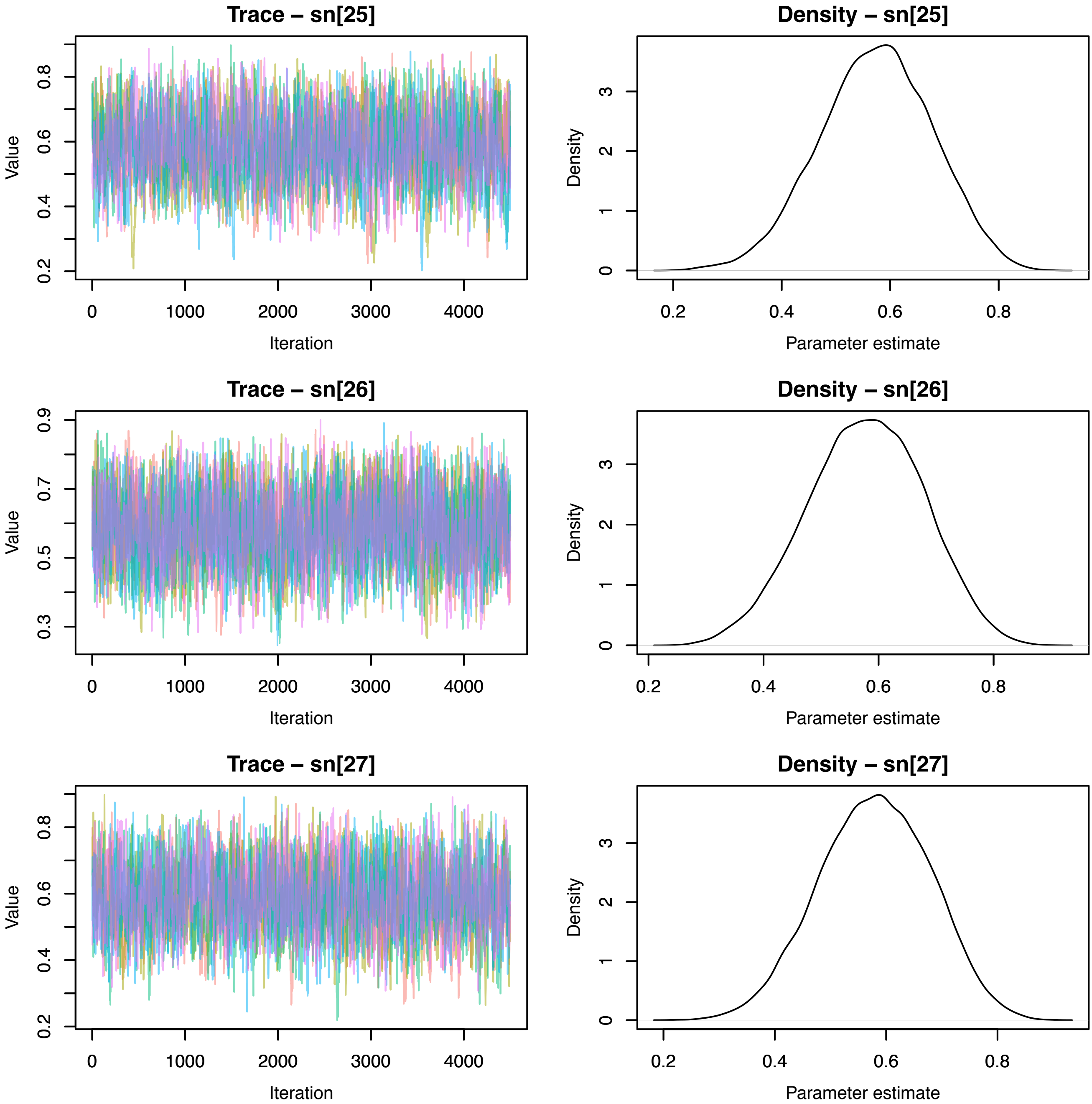


Trace – sn[13]
Value
Iteration
0.3
0.5
0.7
0.9
0
1000
2000
3000
4000
Density – sn[13]
Density
Parameter estimate
0
1
2
3
0.2
0.4
0.6
0.8
Trace – sn[14]
Value
Iteration
0.2
0.4
0.6
0.8
0
1000
2000
3000
4000
Density – sn[14]
Density
Parameter estimate
0
1
2
3
0.2
0.4
0.6
0.8
Trace – sn[15]
Value
Iteration
0.2
0.4
0.6
0.8
0
1000
2000
3000
4000
Density – sn[15]
Density
Parameter estimate
0
1
2
3
0.2
0.4
0.6
0.8

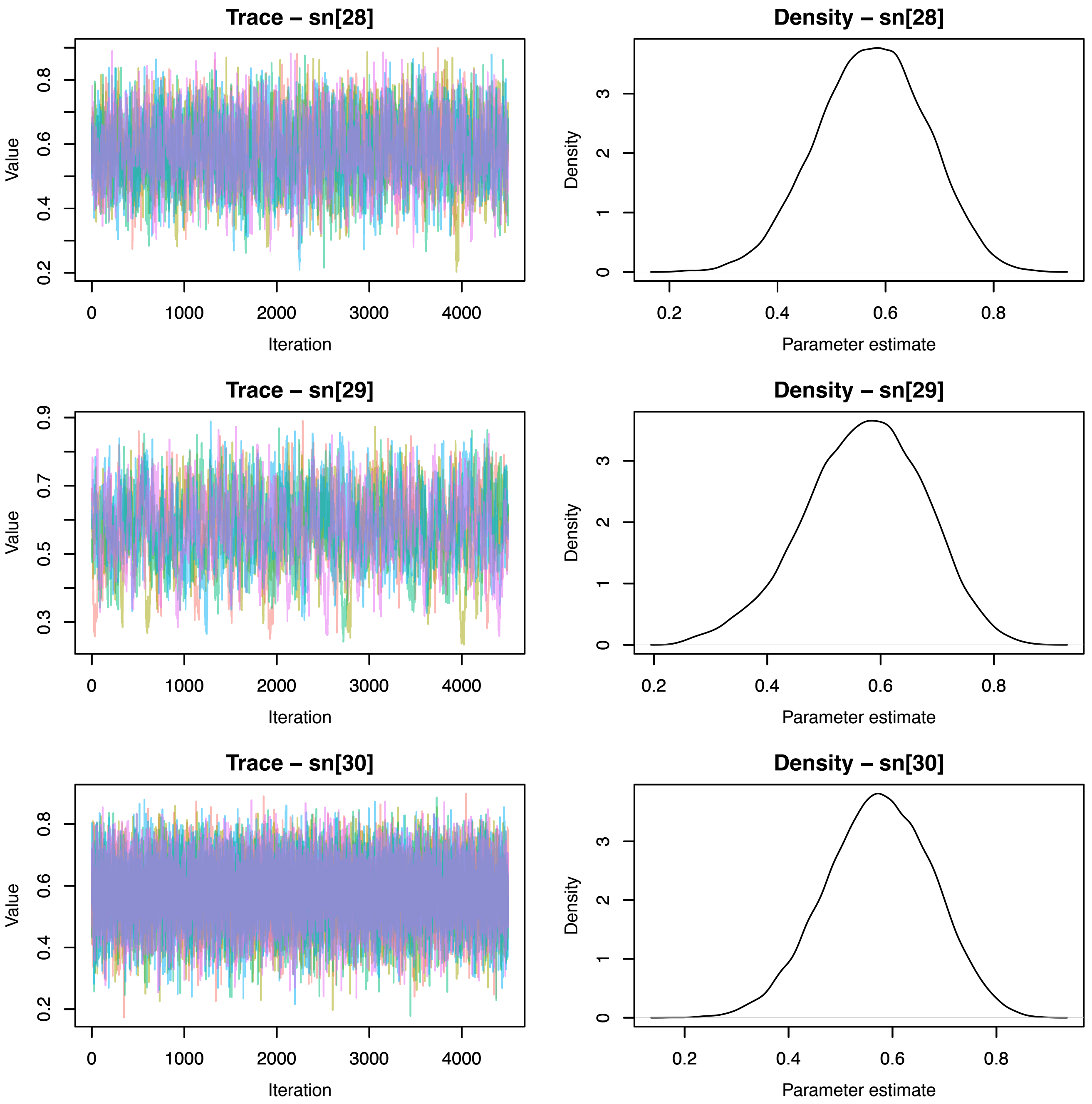

Trace – sn[16]
Value
Iteration
0
1000
2000
3000
4000
0.2
0.4
0.6
0.8
Density – sn[16]
Density
Parameter estimate
0
1
2
3
Trace – sn[17]
Value
Iteration
Density – sn[17]
Density
Parameter estimate
Trace – sn[18]
Value
Iteration
Density – sn[18]
Density
Parameter estimate

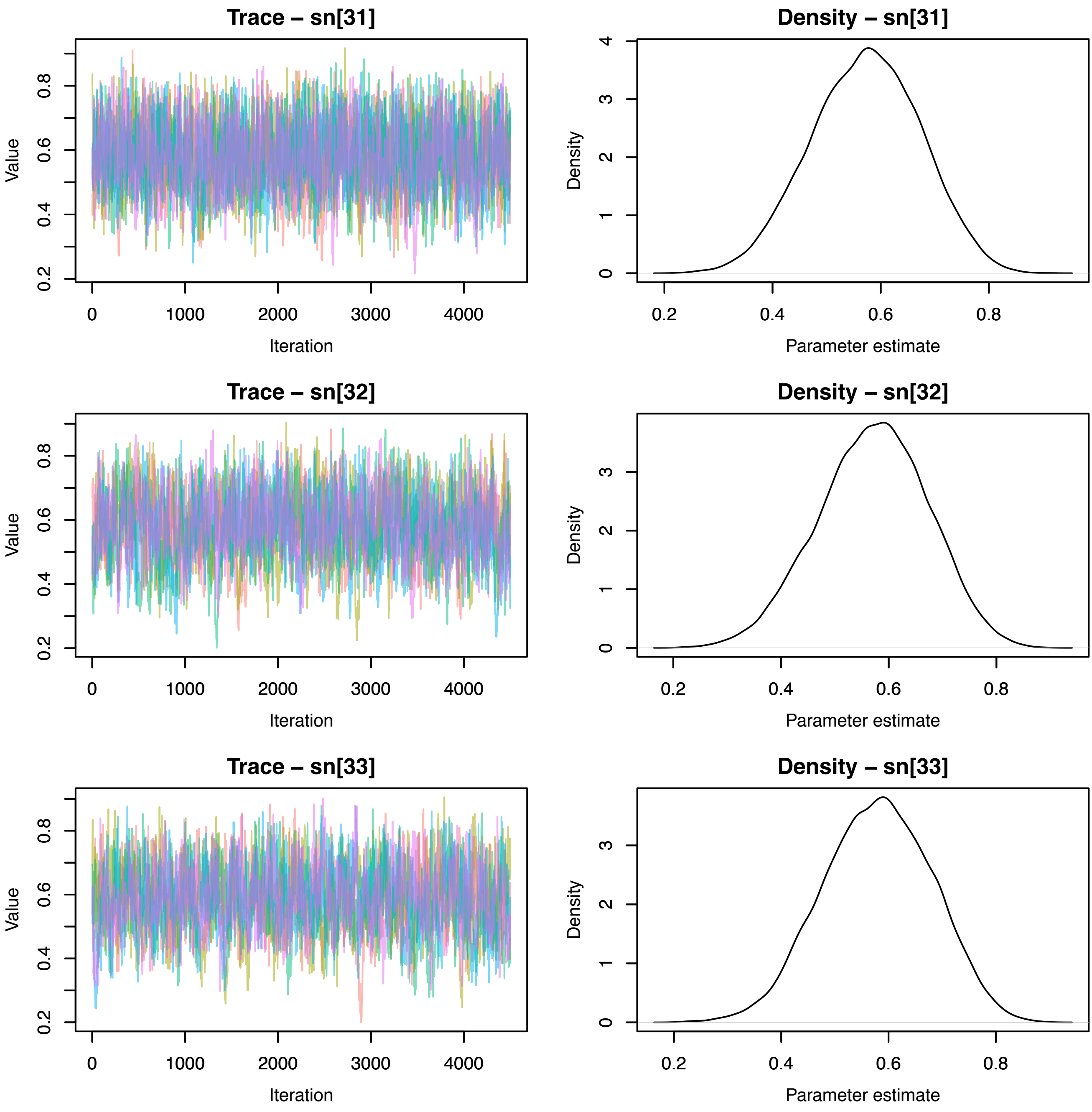

Trace – sn[19]
Value
Iteration
Density – sn[19]
Density
Parameter estimate
Trace – sn[20]
Value
Iteration
Density – sn[20]
Density
Parameter estimate
Trace – sn[21]
Value
Iteration
Density – sn[21]
Density
Parameter estimate

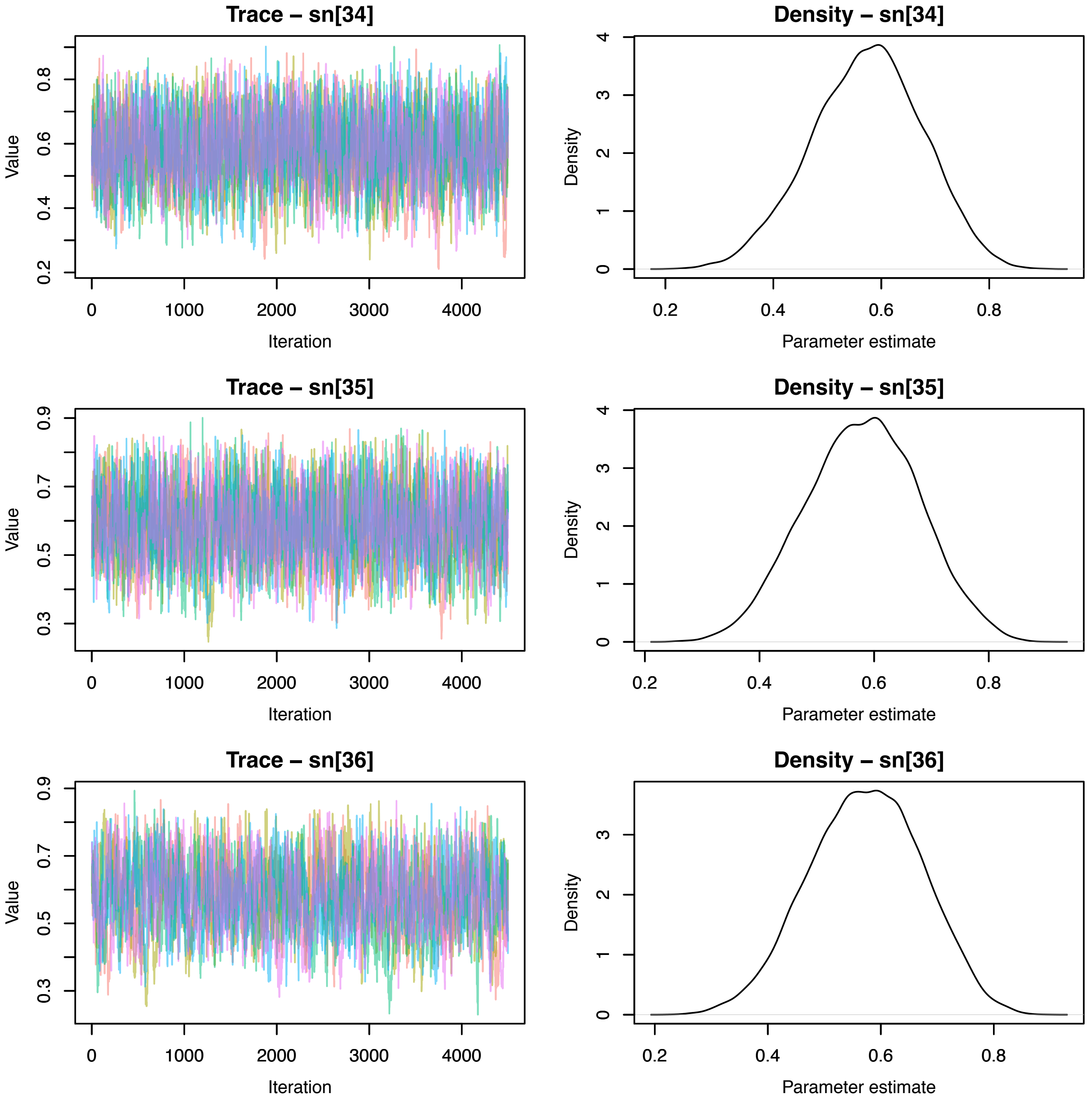

Trace – sn[22]
Value
Iteration
Density – sn[22]
Density
Parameter estimate
Trace – sn[23]
Value
Iteration
Density – sn[23]
Density
Parameter estimate
Trace – sn[24]
Value
Iteration
Density – sn[24]
Density
Parameter estimate

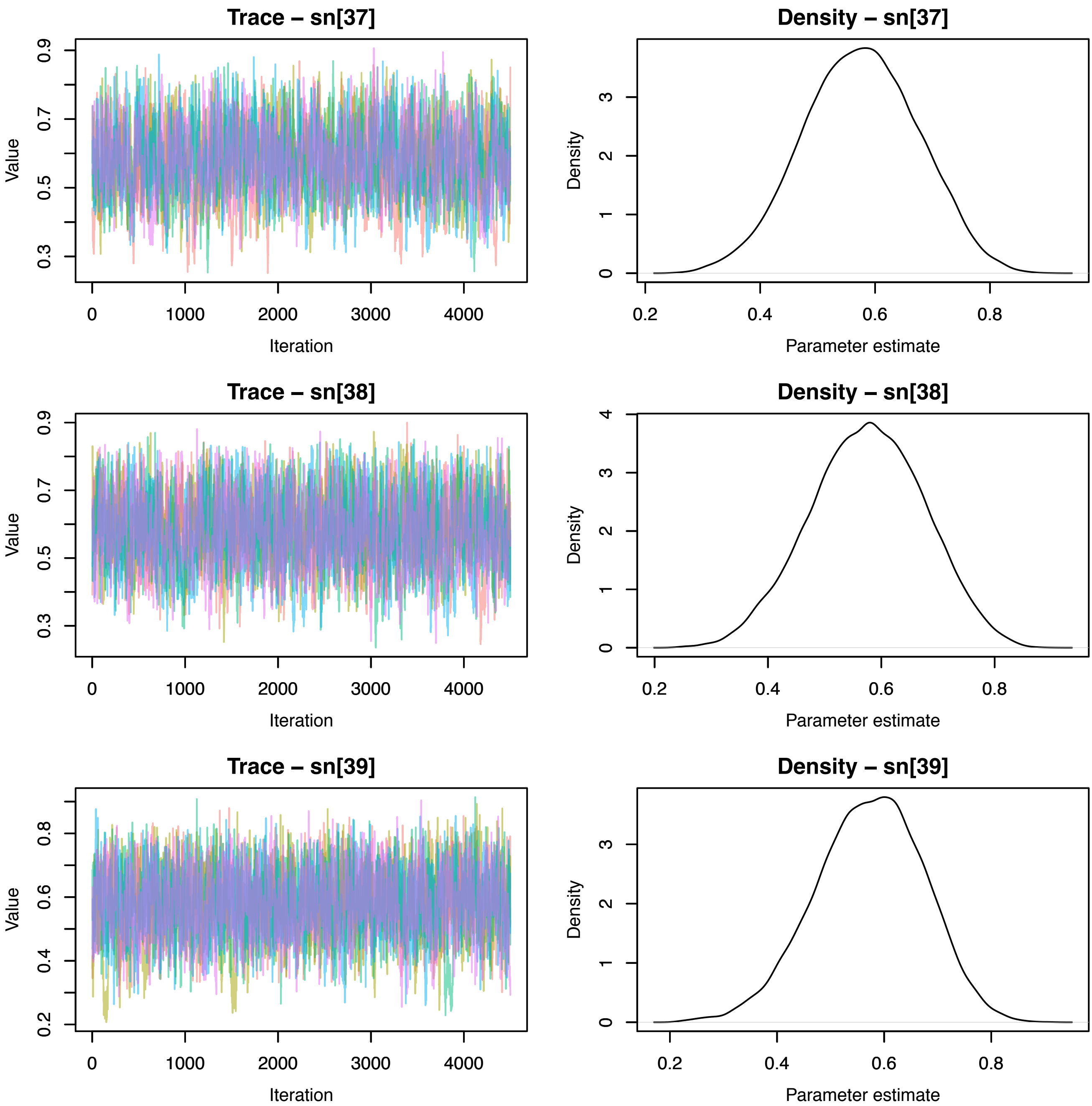


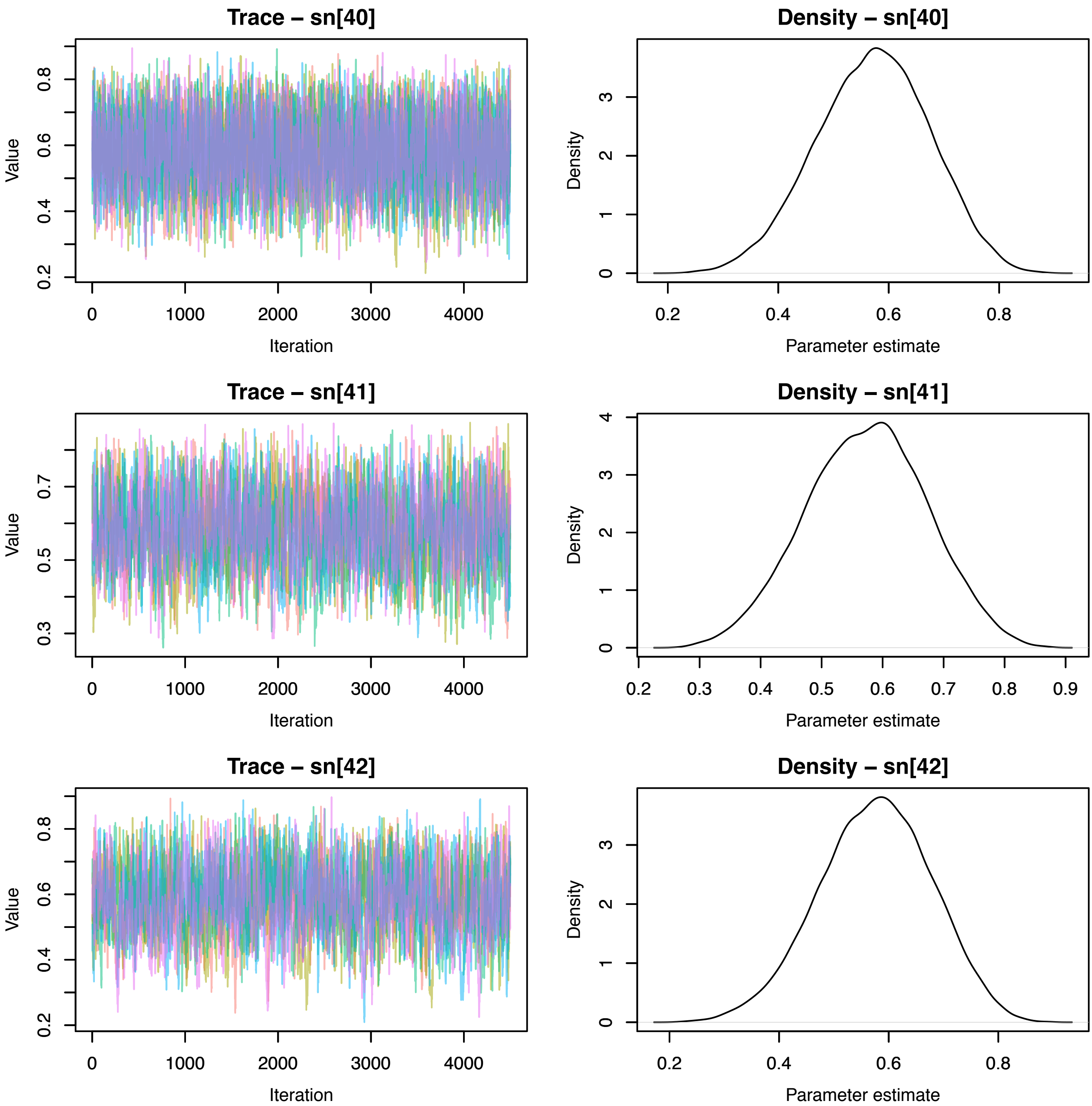
Trace – sn[25]
Density – sn[25]
Trace – sn[26]
Density – sn[26]
Trace – sn[27]
Density – sn[27]
Value
Iteration
Density
Parameter estimate
0
1000
2000
3000
4000
0.2
0.4
0.6
0.8
0.3
0.5
0.7
0.9
1
2
3

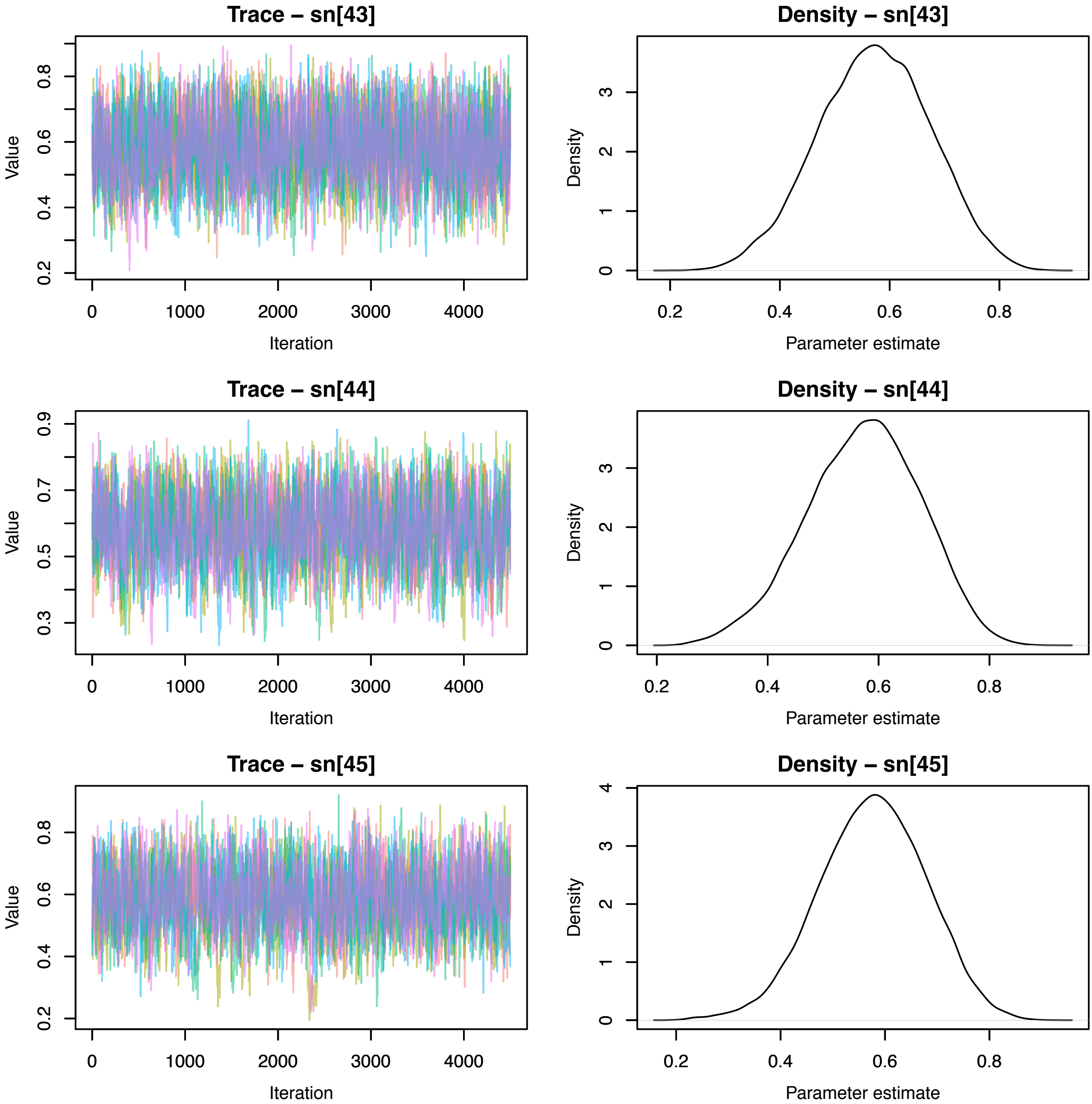

Trace – sn[28]
Density – sn[28]
Trace – sn[29]
Density – sn[29]
Trace – sn[30]
Density – sn[30]
Value
Iteration
Density
Parameter estimate
0
1000
2000
3000
4000
0.2
0.4
0.6
0.8
0.3
0.5
0.7
0.9
0
1
2
3

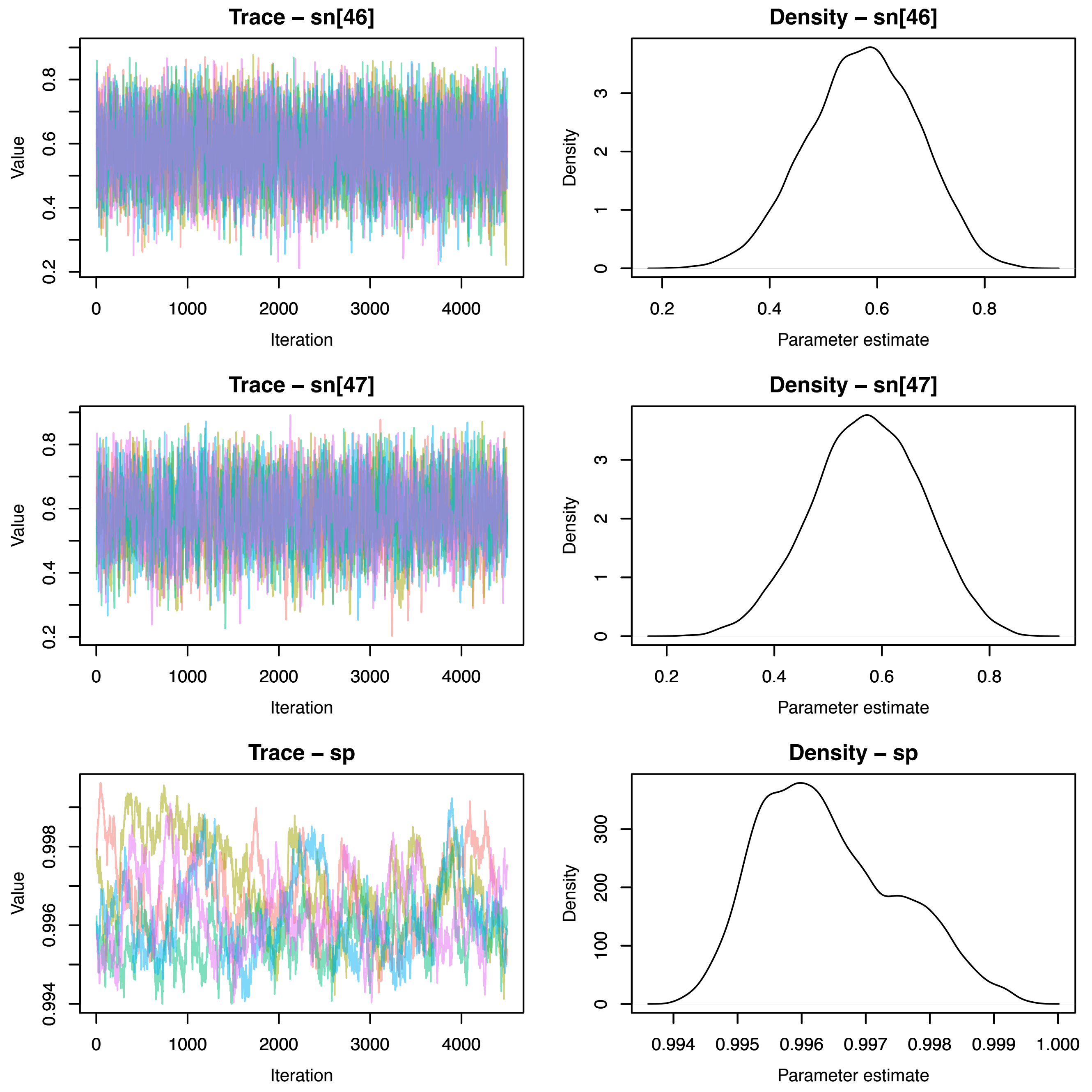

Trace – sn[31]
Density – sn[31]
Trace – sn[32]
Density – sn[32]
Trace – sn[33]
Density – sn[33]
Value
Iteration
Density
Parameter estimate
0
1000
2000
3000
4000
0.2
0.4
0.6
0.8
0
1
2
3
4

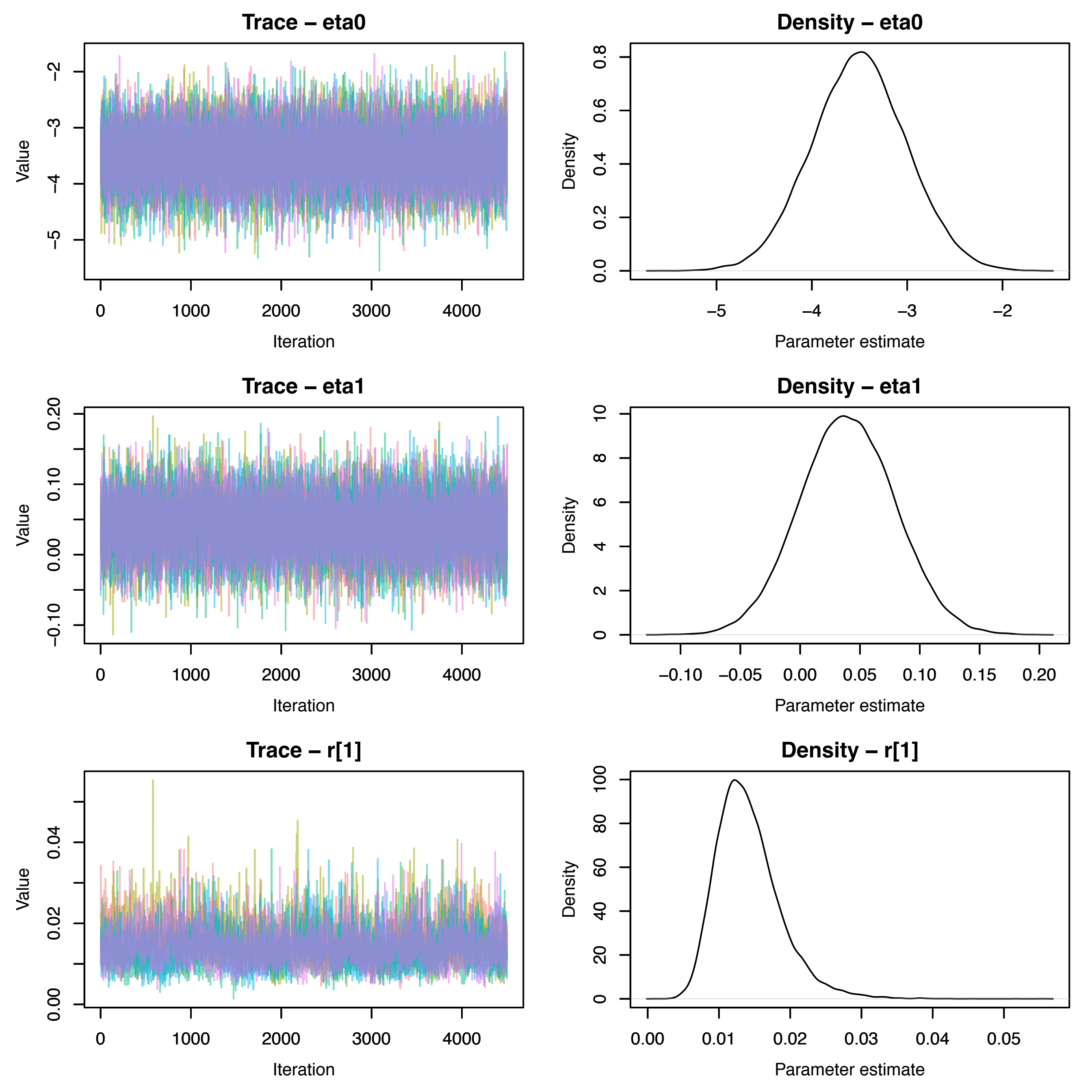

Trace – sn[34]
Value
Iteration
Density – sn[34]
Density
Parameter estimate
Trace – sn[35]
Value
Iteration
Density – sn[35]
Density
Parameter estimate
Trace – sn[36]
Value
Iteration
Density – sn[36]
Density
Parameter estimate


Trace – sn[37]
Value
0.3
0.5
0.7
0.9
0
1000
2000
3000
4000
Iteration
Density – sn[37]
Density
0
1
2
3
0.2
0.4
0.6
0.8
Parameter estimate
Trace – sn[38]
Value
0.3
0.5
0.7
0.9
0
1000
2000
3000
4000
Iteration
Density – sn[38]
Density
0
1
2
3
4
0.2
0.4
0.6
0.8
Parameter estimate
Trace – sn[39]
Value
0.2
0.4
0.6
0.8
0
1000
2000
3000
4000
Iteration
Density – sn[39]
Density
0
1
2
3
0.2
0.4
0.6
0.8
Parameter estimate

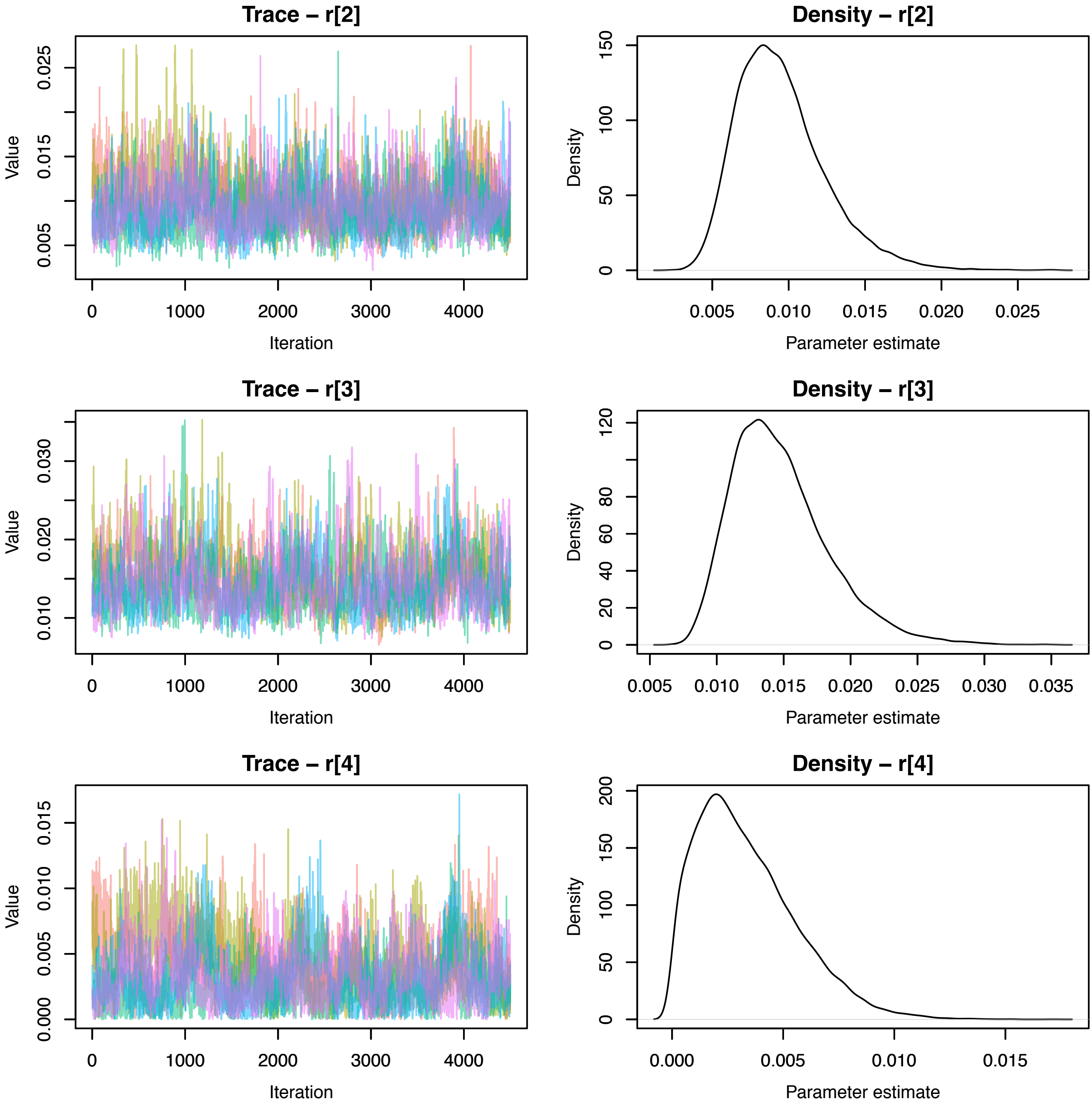

Trace – sn[40]
Value
Iteration
Density – sn[40]
Density
Parameter estimate
Trace – sn[41]
Value
Iteration
Density – sn[41]
Density
Parameter estimate
Trace – sn[42]
Value
Iteration
Density – sn[42]
Density
Parameter estimate

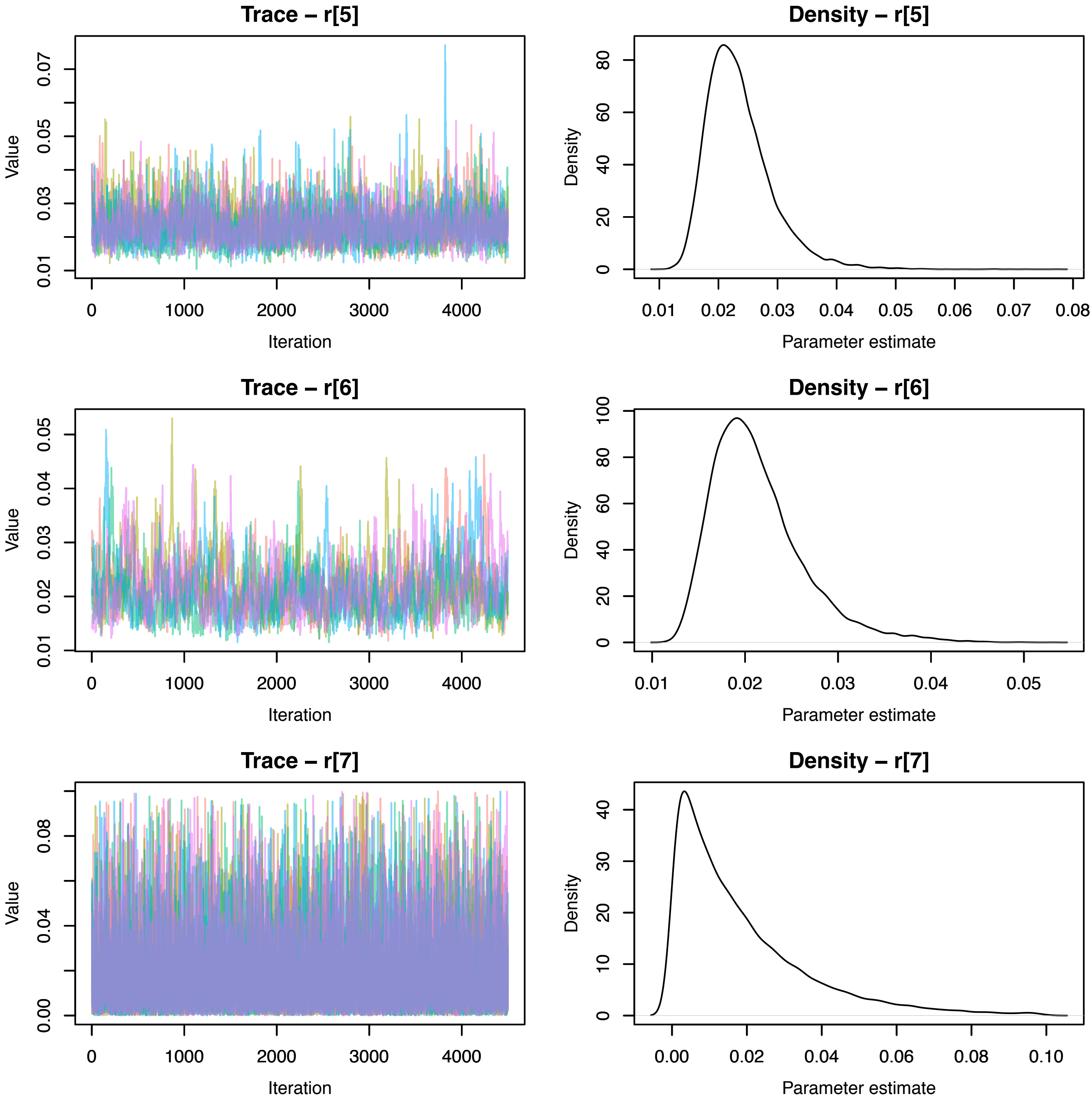

Trace – sn[43]
Value
Iteration
Density – sn[43]
Density
Parameter estimate
Trace – sn[44]
Value
Iteration
Density – sn[44]
Density
Parameter estimate
Trace – sn[45]
Value
Iteration
Density – sn[45]
Density
Parameter estimate

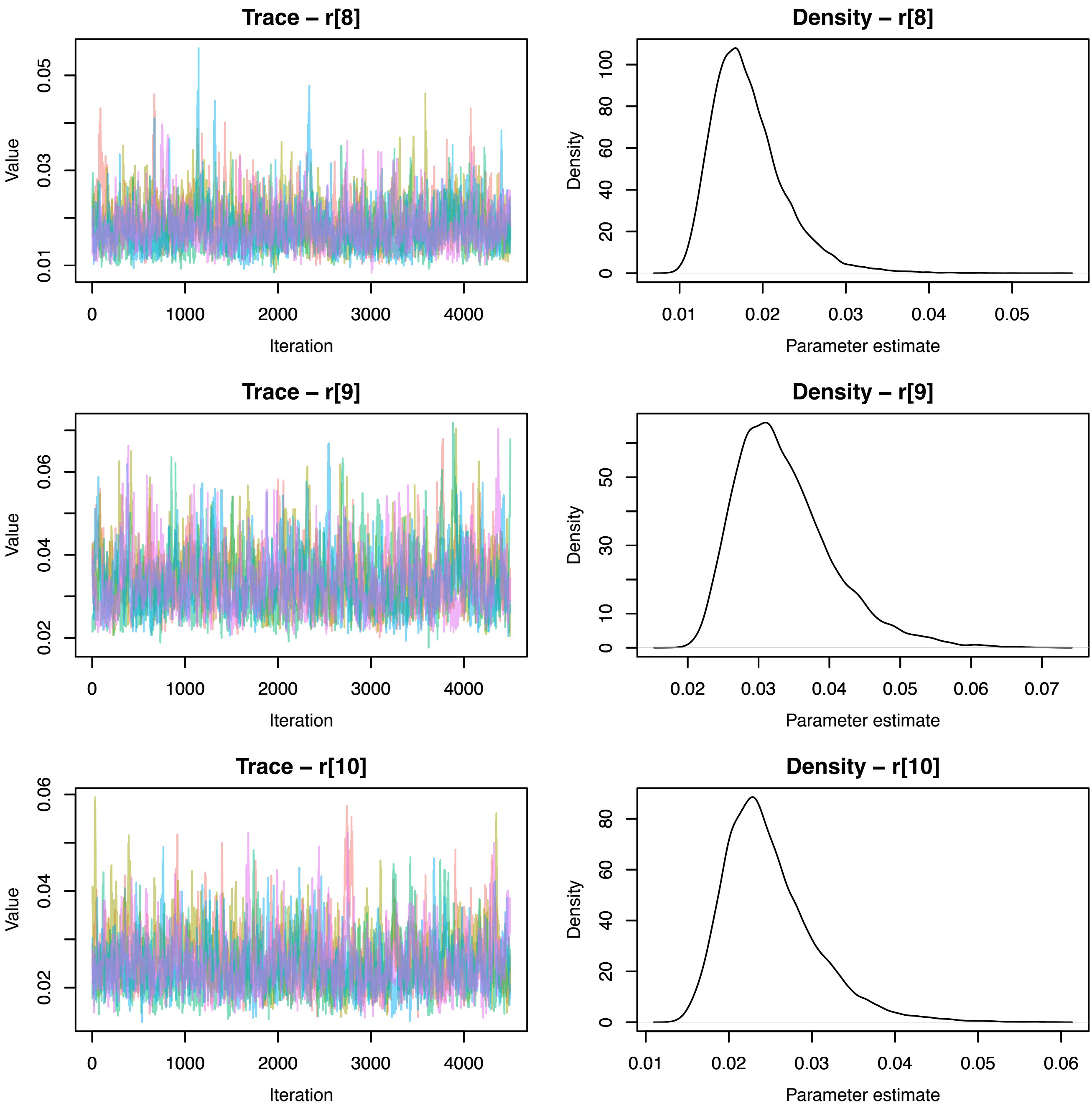

Trace – sn[46]
Value
Iteration
Density – sn[46]
Density
Parameter estimate
Trace – sn[47]
Value
Iteration
Density – sn[47]
Density
Parameter estimate
Trace – sp
Value
Iteration
Density – sp
Density
Parameter estimate

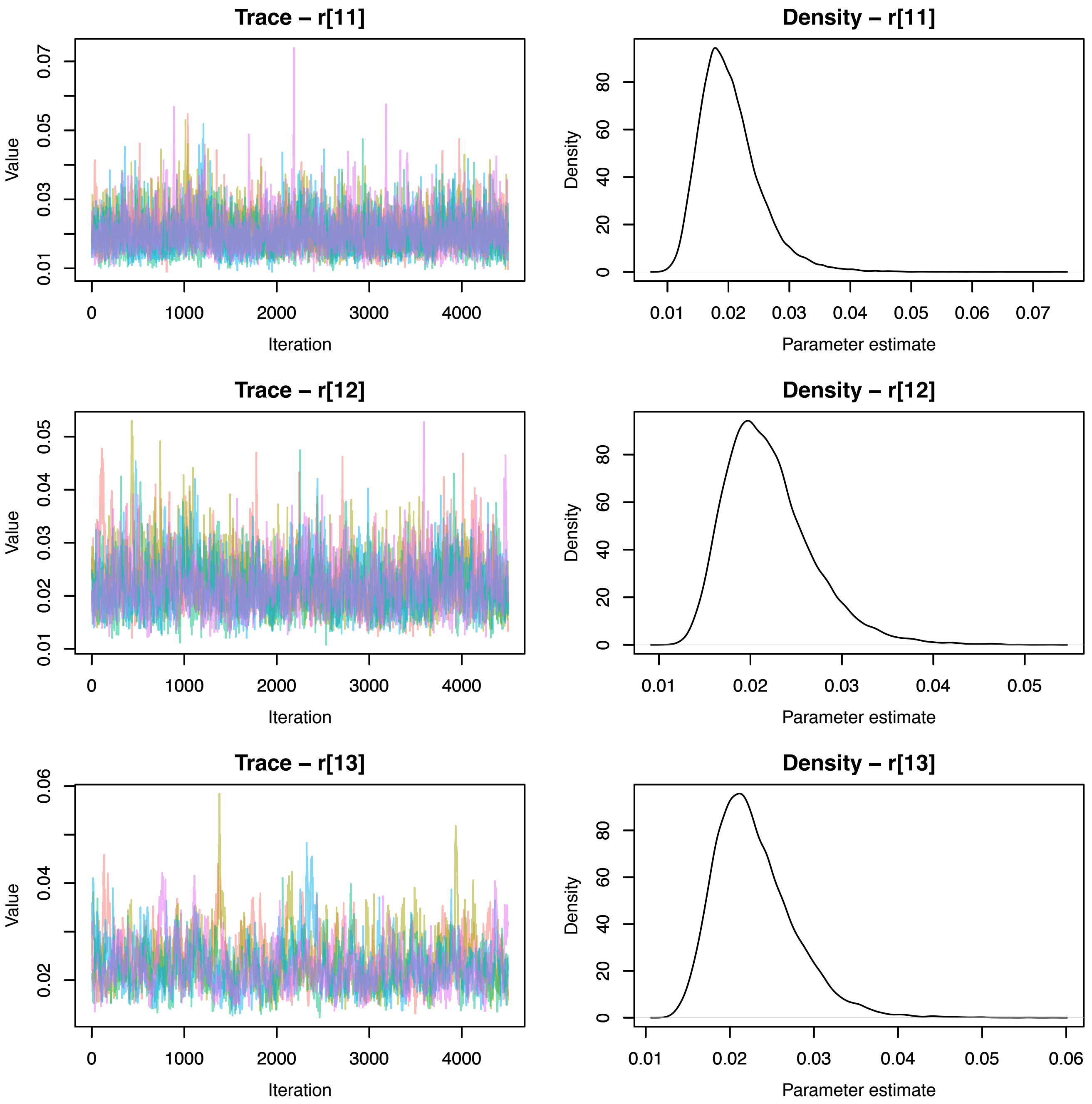


Trace – eta0
Value
Iteration
Density – eta0
Density
Parameter estimate
Trace – eta1
Value
Iteration
Density – eta1
Density
Parameter estimate
Trace – r[1]
Value
Iteration
Density – r[1]
Density
Parameter estimate

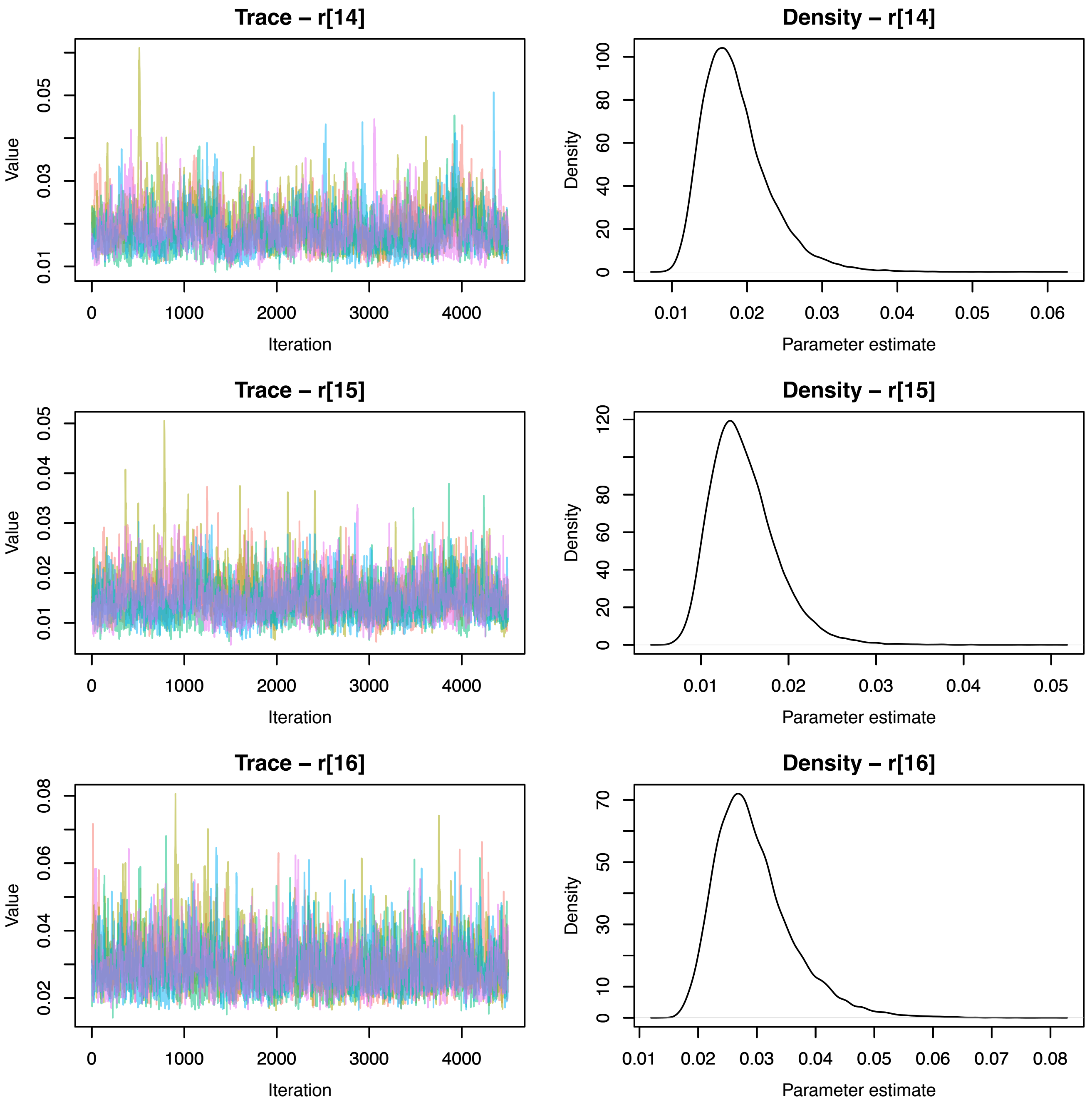

Trace – r[2]
Value
0.005
0.015
0.025
0
1000
2000
3000
4000
Iteration
Density – r[2]
Density
0
50
100
150
0.005
0.010
0.015
0.020
0.025
Parameter estimate
Trace – r[3]
Value
0.010
0.020
0.030
0
1000
2000
3000
4000
Iteration
Density – r[3]
Density
0
20
40
60
80
120
0.005
0.010
0.015
0.020
0.025
0.030
0.035
Parameter estimate
Trace – r[4]
Value
0.000
0.005
0.010
0.015
0
1000
2000
3000
4000
Iteration
Density – r[4]
Density
0
50
100
150
200
0.000
0.005
0.010
0.015
Parameter estimate

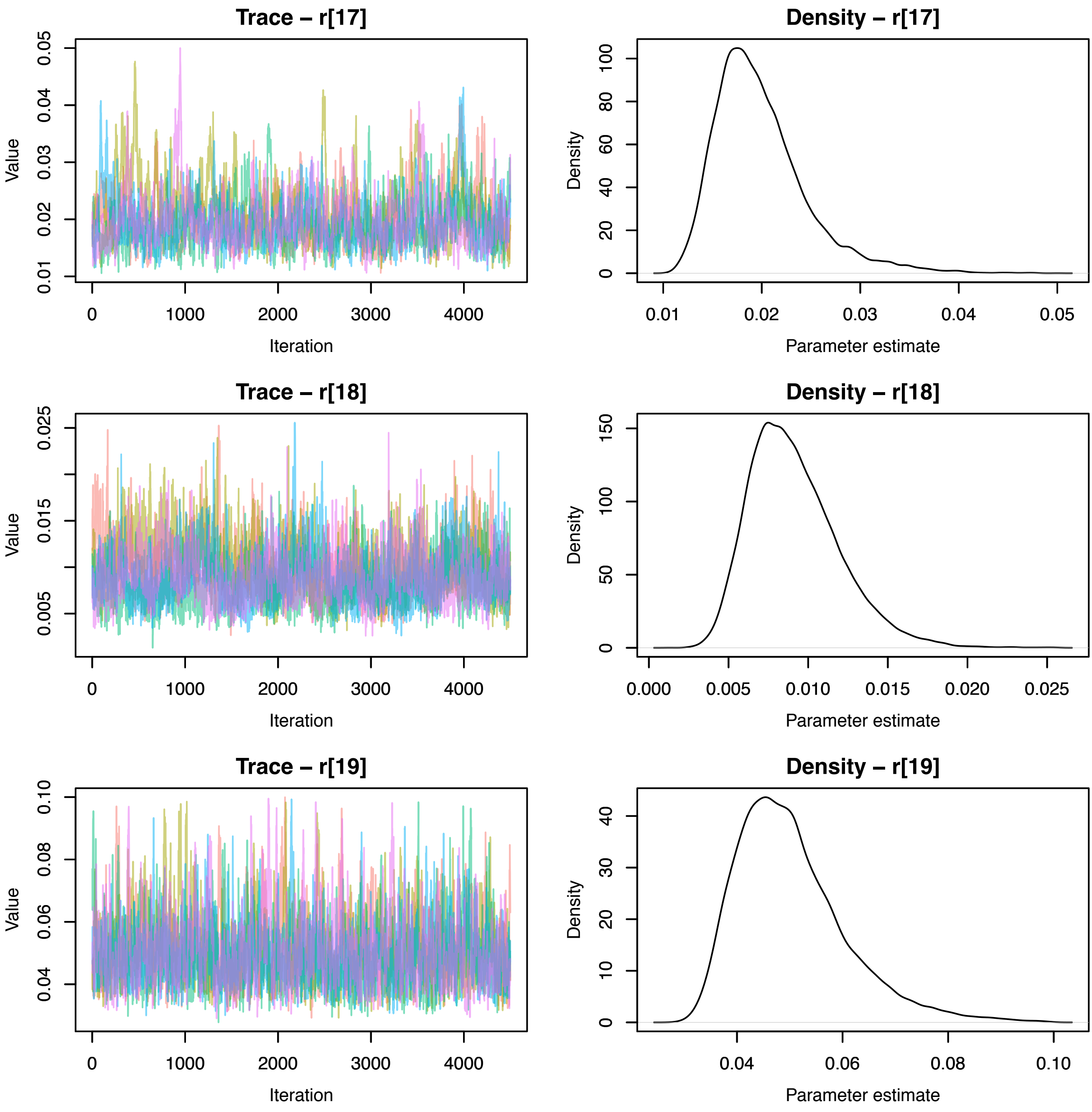

Trace – r[5]
Value
Iteration
Density – r[5]
Density
Parameter estimate
Trace – r[6]
Value
Iteration
Density – r[6]
Density
Parameter estimate
Trace – r[7]
Value
Iteration
Density – r[7]
Density
Parameter estimate


Trace – r[8]
Density – r[8]
Trace – r[9]
Density – r[9]
Trace – r[10]
Density – r[10]
Value
Iteration
Density
Parameter estimate

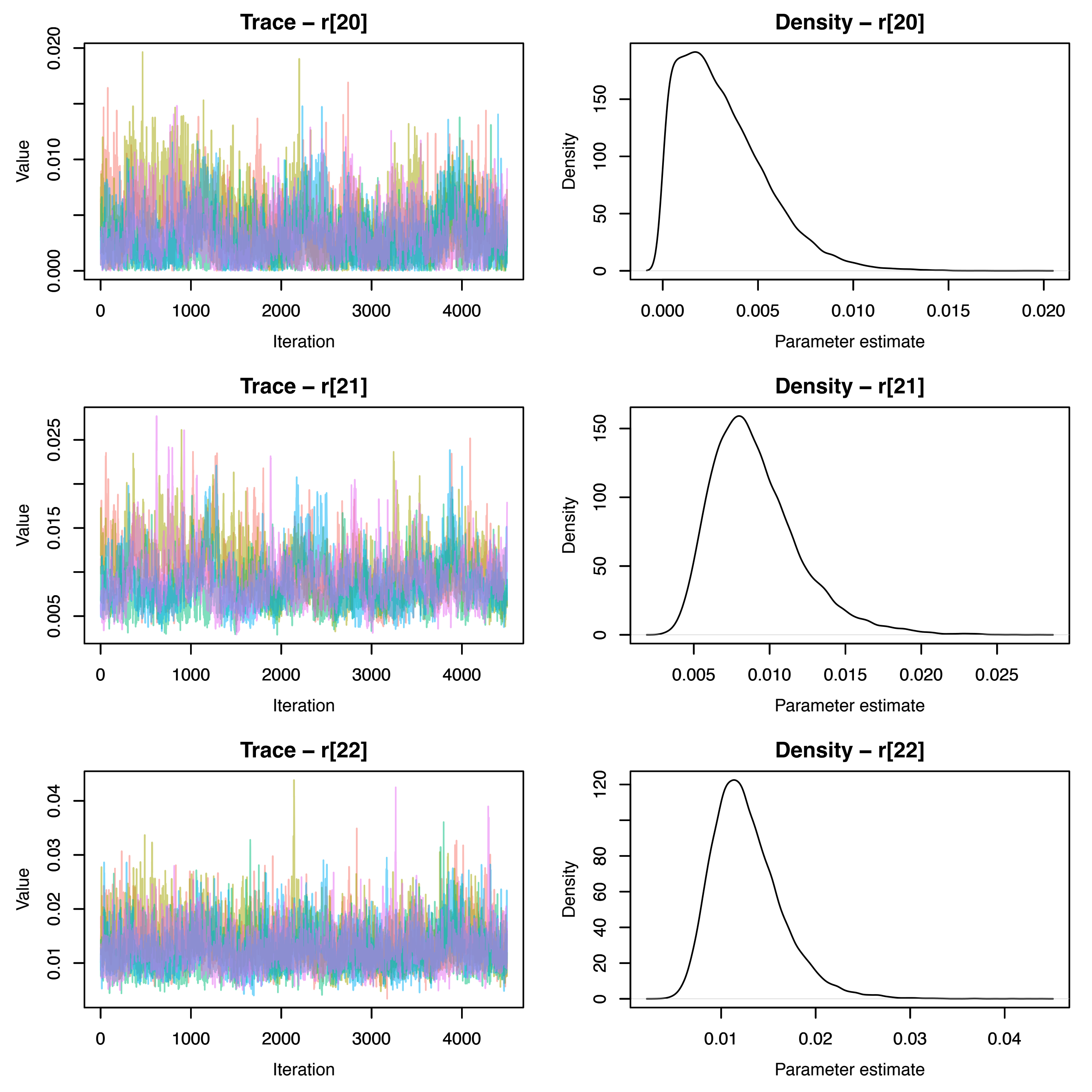


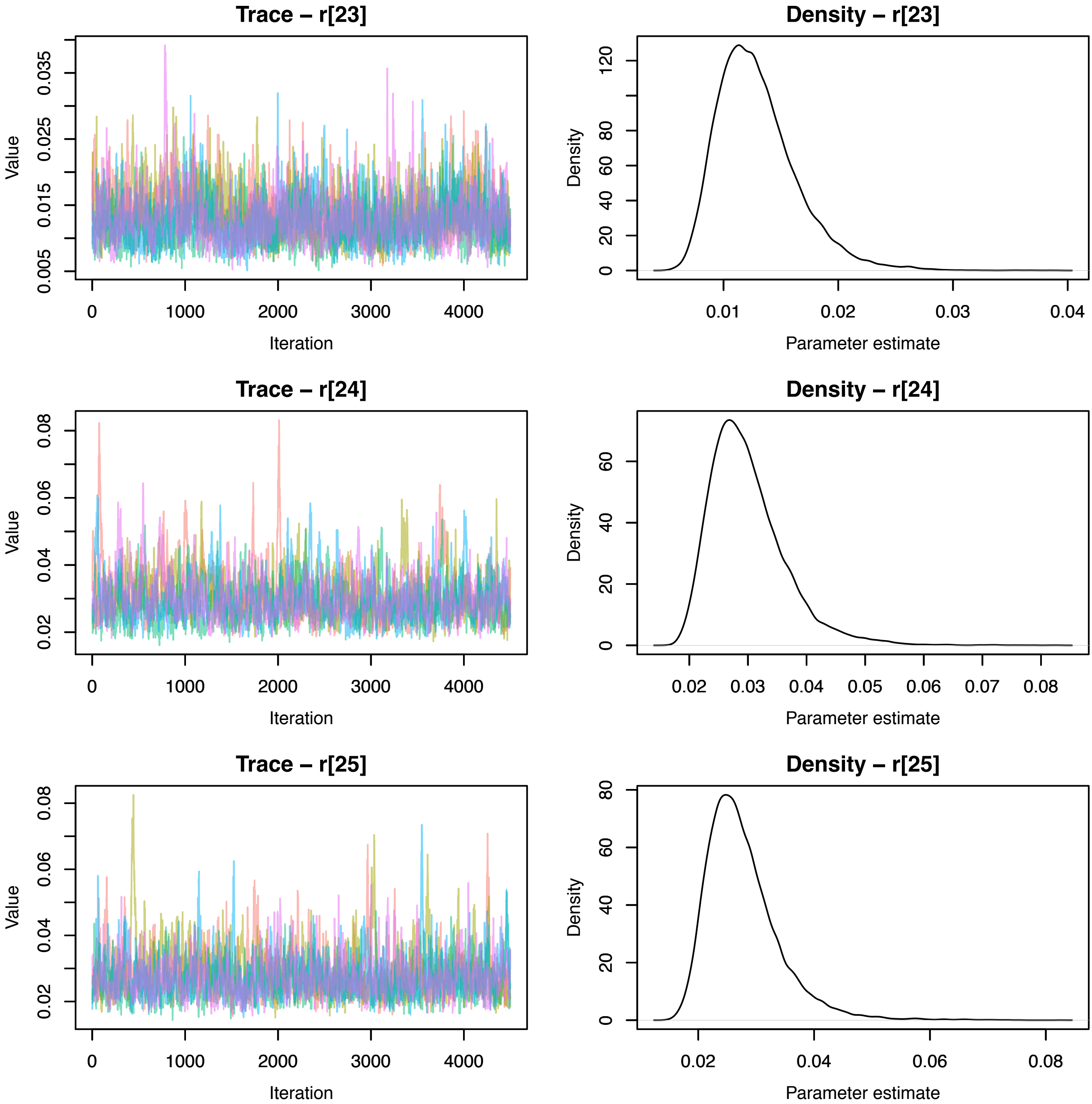
Trace – r[11]
Value
Iteration
Density – r[11]
Density
Parameter estimate
Trace – r[12]
Value
Iteration
Density – r[12]
Density
Parameter estimate
Trace – r[13]
Value
Iteration
Density – r[13]
Density
Parameter estimate


Trace – r[14]
Value
Iteration
0.01
0.03
0.05
0
1000
2000
3000
4000
Density – r[14]
Density
Parameter estimate
0
20
40
60
80
100
0.01
0.02
0.03
0.04
0.05
0.06
Trace – r[15]
Value
Iteration
0.01
0.02
0.03
0.04
0.05
0
1000
2000
3000
4000
Density – r[15]
Density
Parameter estimate
0
20
40
60
80
120
0.01
0.02
0.03
0.04
0.05
Trace – r[16]
Value
Iteration
0.02
0.04
0.06
0.08
0
1000
2000
3000
4000
Density – r[16]
Density
Parameter estimate
0
10
30
50
70
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08

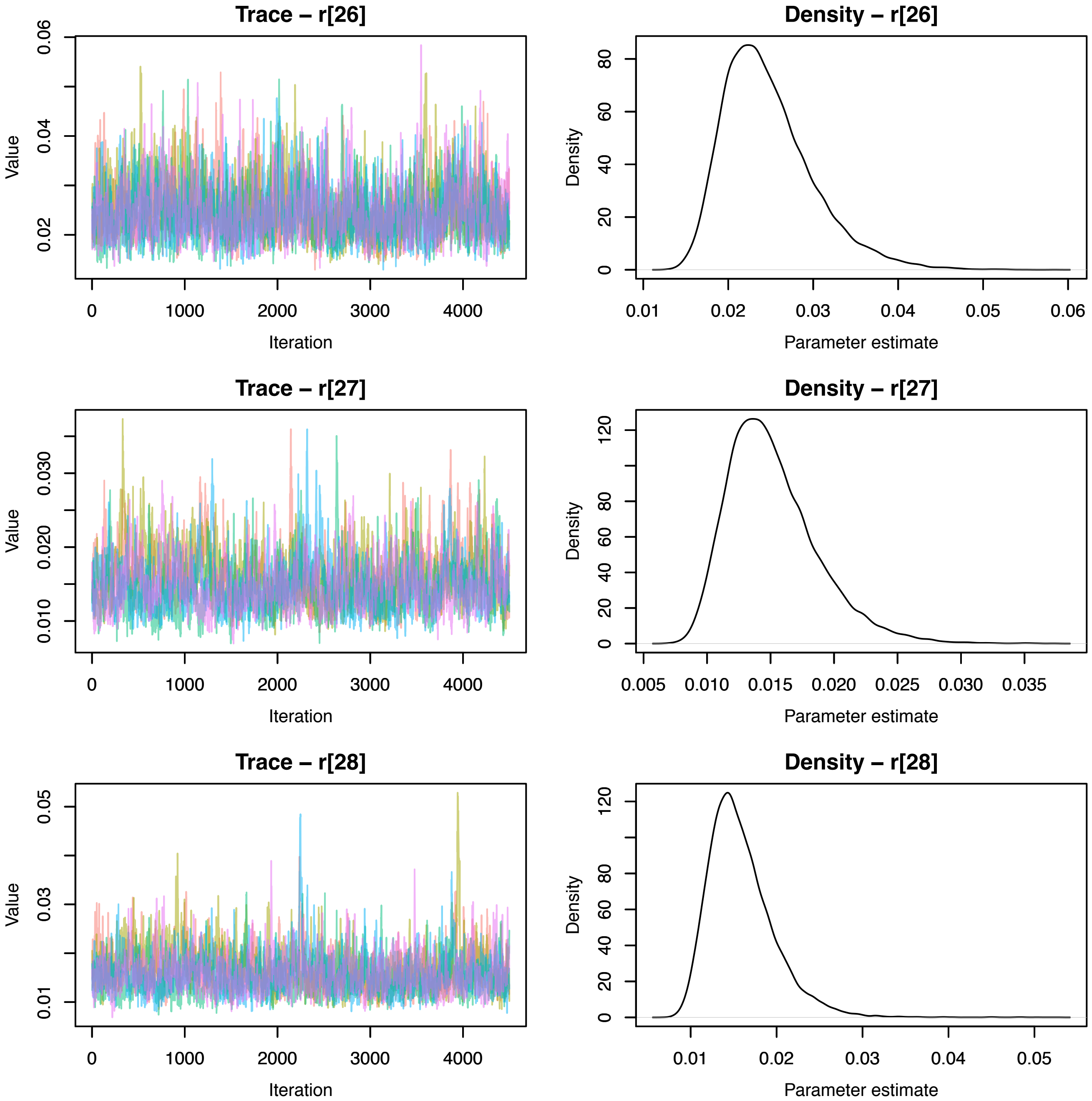


Trace – r[17]
Value
0.01
0.02
0.03
0.04
0.05
0
1000
2000
3000
4000
Iteration
Density – r[17]
Density
0
20
40
60
80
100
0.01
0.02
0.03
0.04
0.05
Parameter estimate
Trace – r[18]
Value
0.005
0.015
0.025
0
1000
2000
3000
4000
Iteration
Density – r[18]
Density
0
50
100
150
0.000
0.005
0.010
0.015
0.020
0.025
Parameter estimate
Trace – r[19]
Value
0.04
0.06
0.08
0.10
0
1000
2000
3000
4000
Iteration
Density – r[19]
Density
0
10
20
30
40
0.04
0.06
0.08
0.10
Parameter estimate

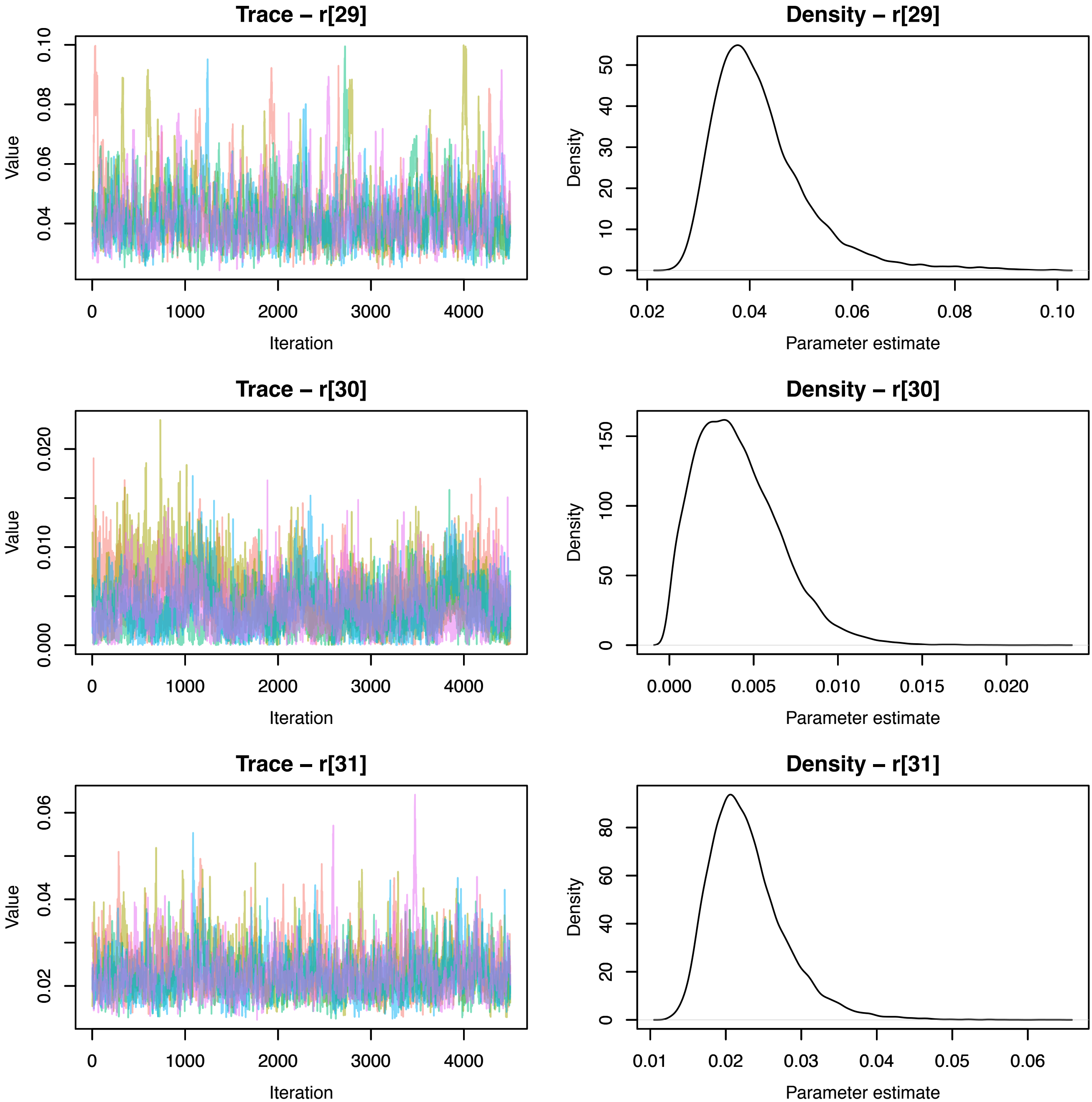

Trace – r[20]
Value
Iteration
Density – r[20]
Density
Parameter estimate
Trace – r[21]
Value
Iteration
Density – r[21]
Density
Parameter estimate
Trace – r[22]
Value
Iteration
Density – r[22]
Density
Parameter estimate

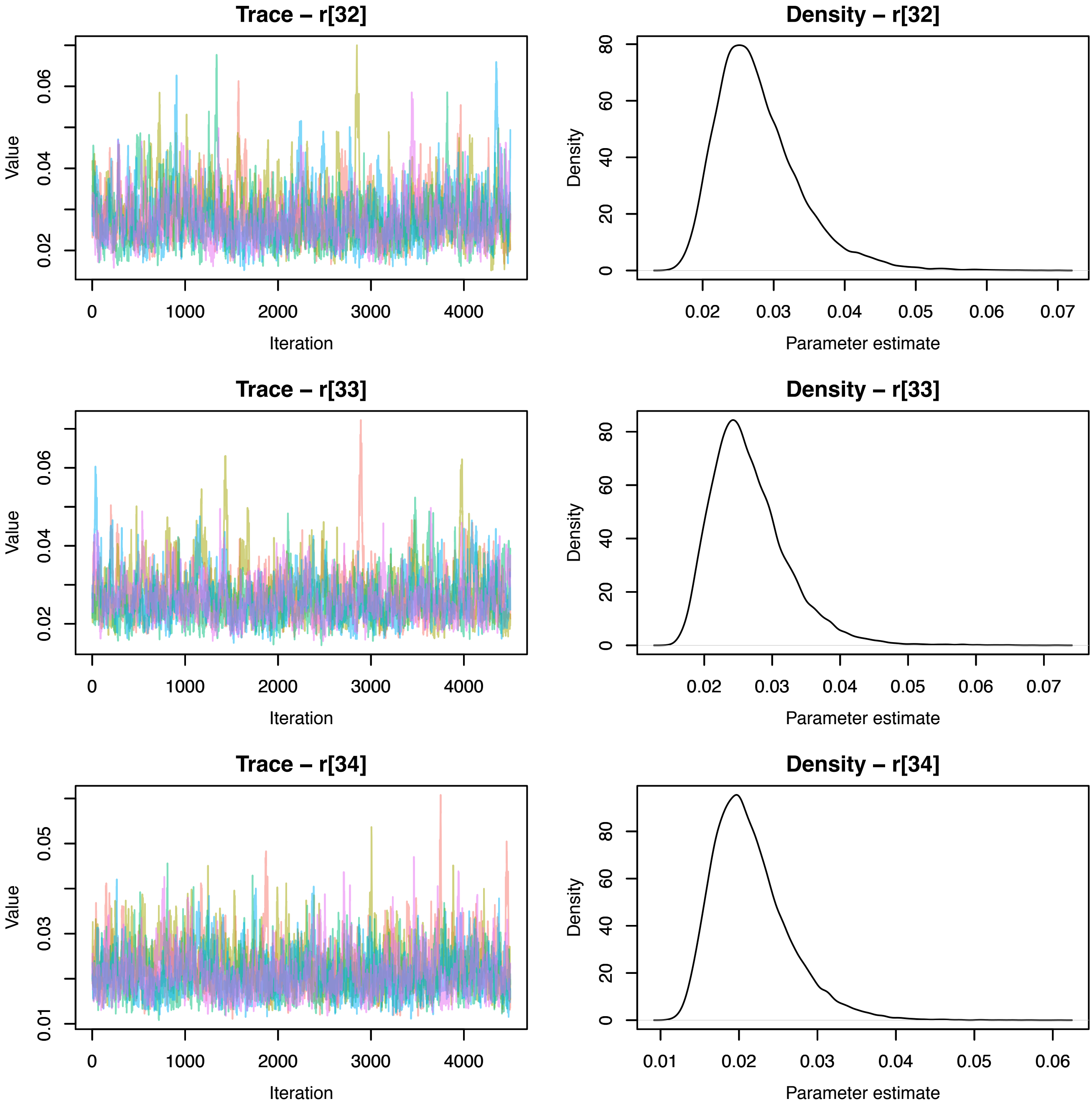


Trace – r[23]
Value
Iteration
Density – r[23]
Density
Parameter estimate
Trace – r[24]
Value
Iteration
Density – r[24]
Density
Parameter estimate
Trace – r[25]
Value
Iteration
Density – r[25]
Density
Parameter estimate

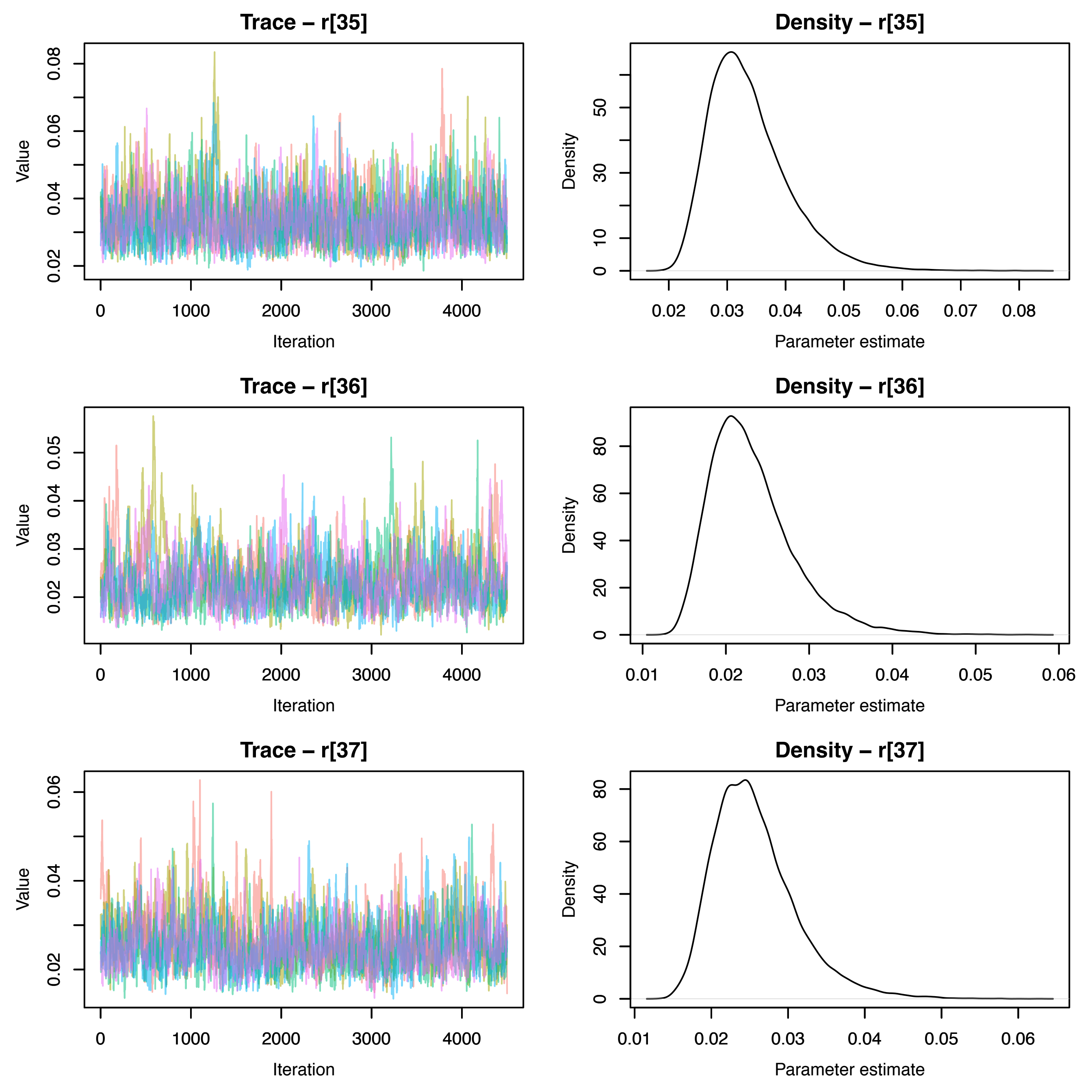


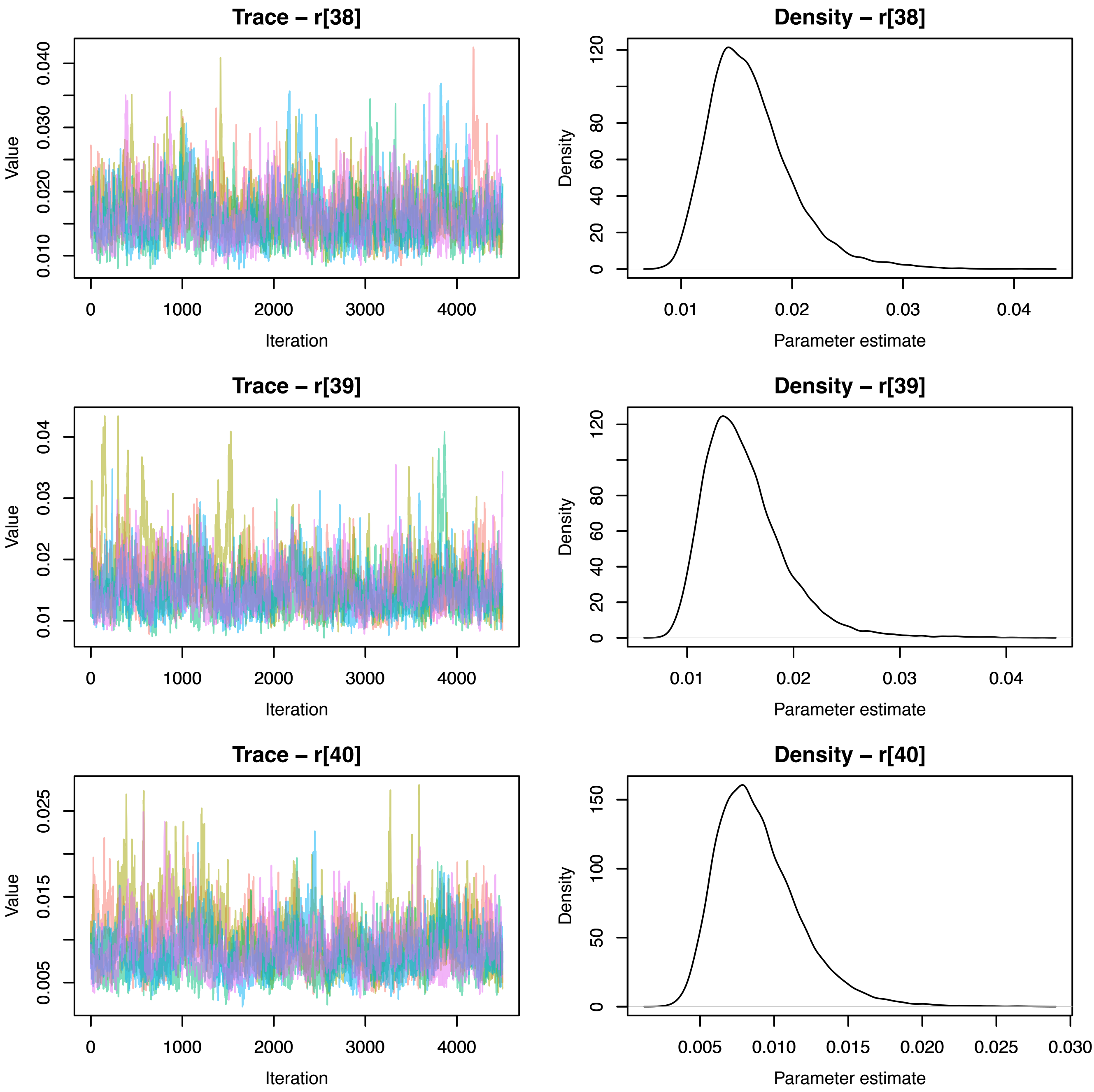
Trace – r[26]
Value
0.02
0.04
0.06
0
1000
2000
3000
4000
Iteration
Density – r[26]
Density
0
20
40
60
80
0.01
0.02
0.03
0.04
0.05
0.06
Parameter estimate
Trace – r[27]
Value
0.010
0.020
0.030
0
1000
2000
3000
4000
Iteration
Density – r[27]
Density
0
20
40
60
80
120
0.005
0.010
0.015
0.020
0.025
0.030
0.035
Parameter estimate
Trace – r[28]
Value
0.01
0.03
0.05
0
1000
2000
3000
4000
Iteration
Density – r[28]
Density
0
20
40
60
80
120
0.01
0.02
0.03
0.04
0.05
Parameter estimate


Trace – r[29]
Value
0.04
0.06
0.08
0.10
0
1000
2000
3000
4000
Iteration
Density – r[29]
Density
0
10
20
30
40
50
0.02
0.04
0.06
0.08
0.10
Parameter estimate
Trace – r[30]
Value
0.000
0.010
0.020
0
1000
2000
3000
4000
Iteration
Density – r[30]
Density
0
50
100
150
0.000
0.005
0.010
0.015
0.020
Parameter estimate
Trace – r[31]
Value
0.02
0.04
0.06
0
1000
2000
3000
4000
Iteration
Density – r[31]
Density
0
20
40
60
80
0.01
0.02
0.03
0.04
0.05
0.06
Parameter estimate

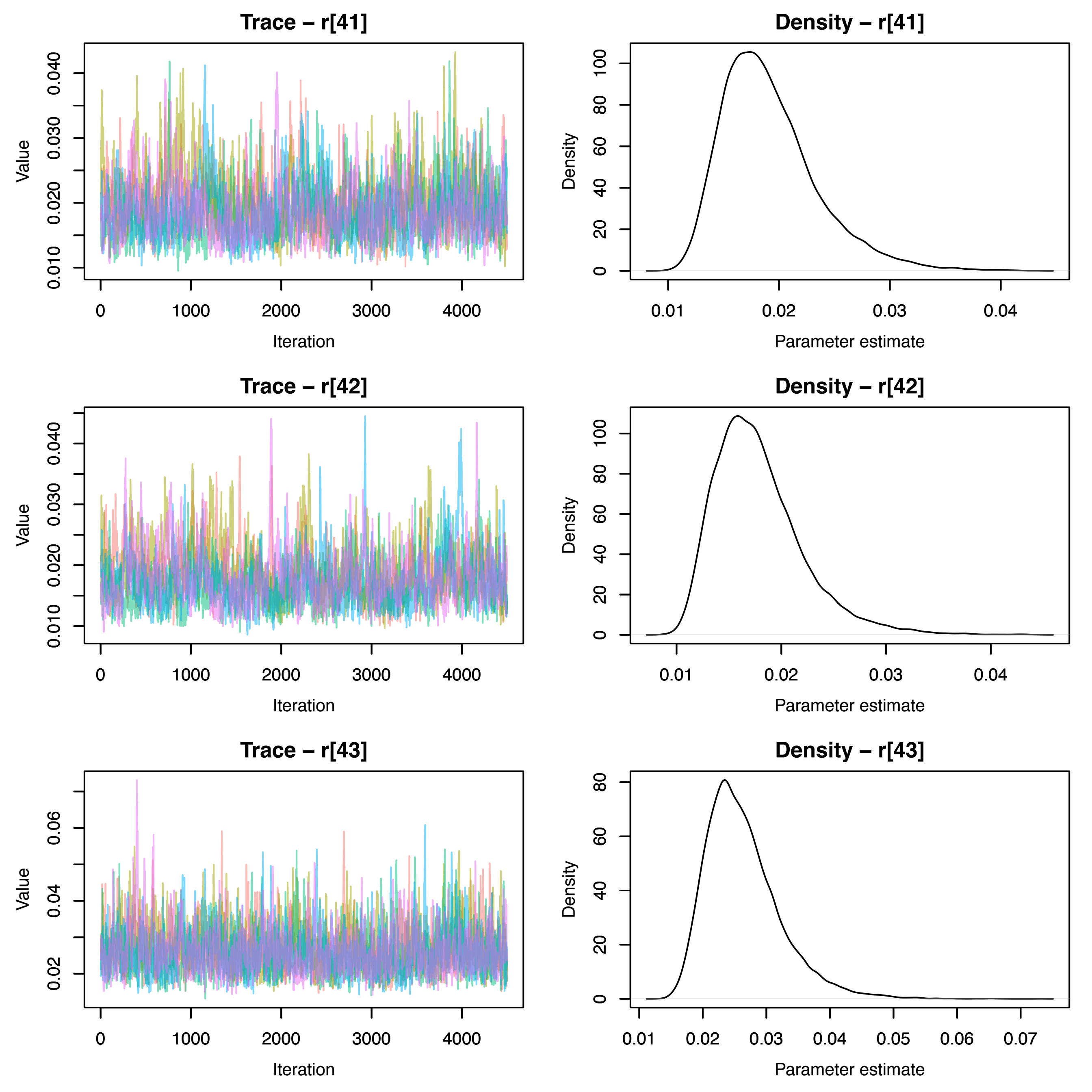


Trace – r[32]
Value
0.02
0.04
0.06
Iteration
0
1000
2000
3000
4000
Density – r[32]
Density
0
20
40
60
80
Parameter estimate
0.02
0.03
0.04
0.05
0.06
0.07
Trace – r[33]
Value
0.02
0.04
0.06
Iteration
0
1000
2000
3000
4000
Density – r[33]
Density
0
20
40
60
80
Parameter estimate
0.02
0.03
0.04
0.05
0.06
0.07
Trace – r[34]
Value
0.01
0.03
0.05
Iteration
0
1000
2000
3000
4000
Density – r[34]
Density
0
20
40
60
80
Parameter estimate
0.01
0.02
0.03
0.04
0.05
0.06

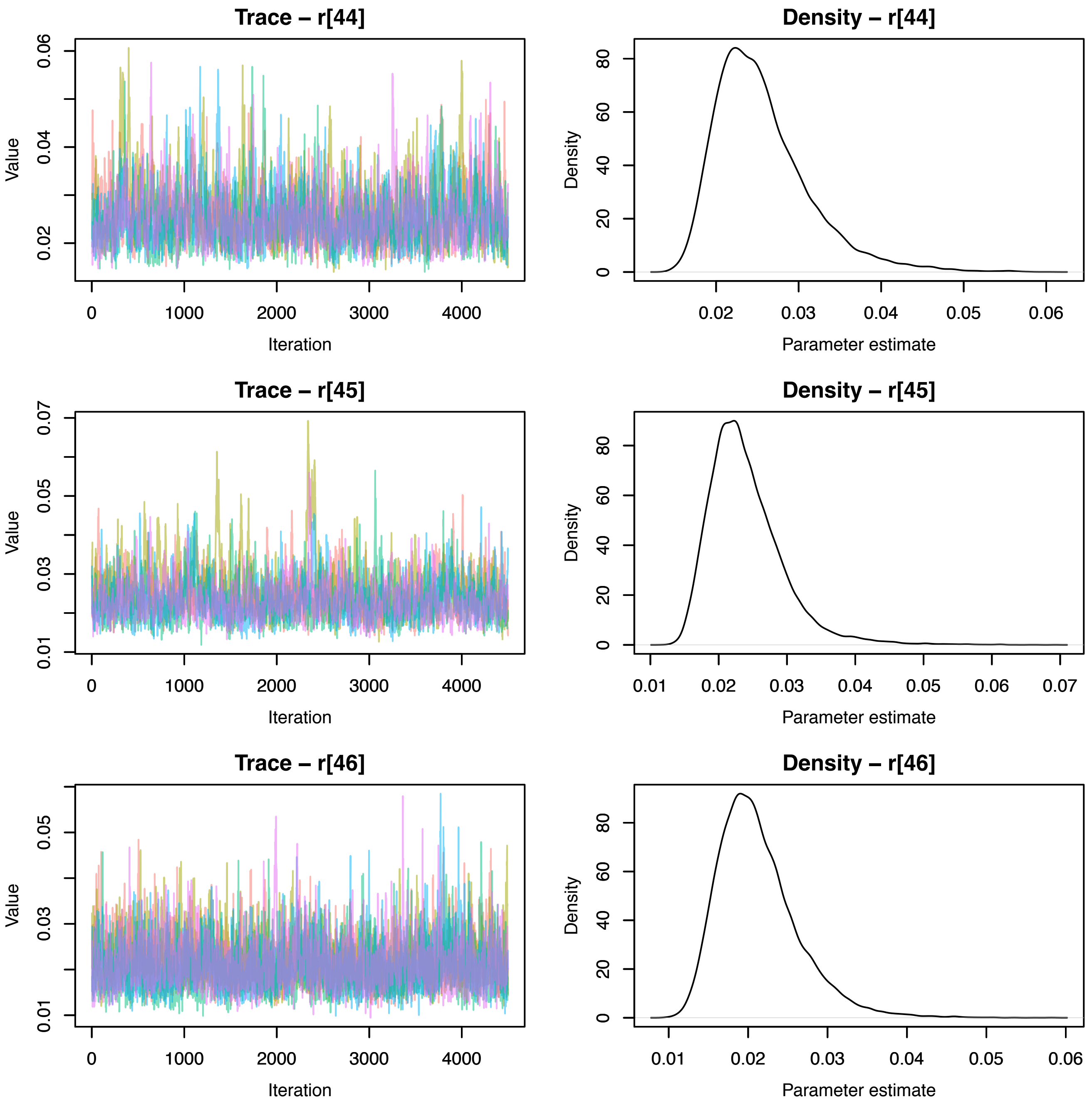

Trace – r[35]
Density – r[35]
Trace – r[36]
Density – r[36]
Trace – r[37]
Density – r[37]
Value
Iteration
Density
Parameter estimate
0
1000
2000
3000
4000
0.02
0.04
0.06
0.08
0.03
0.05
0.07
0.01
0
10
20
30
40
50
60
80

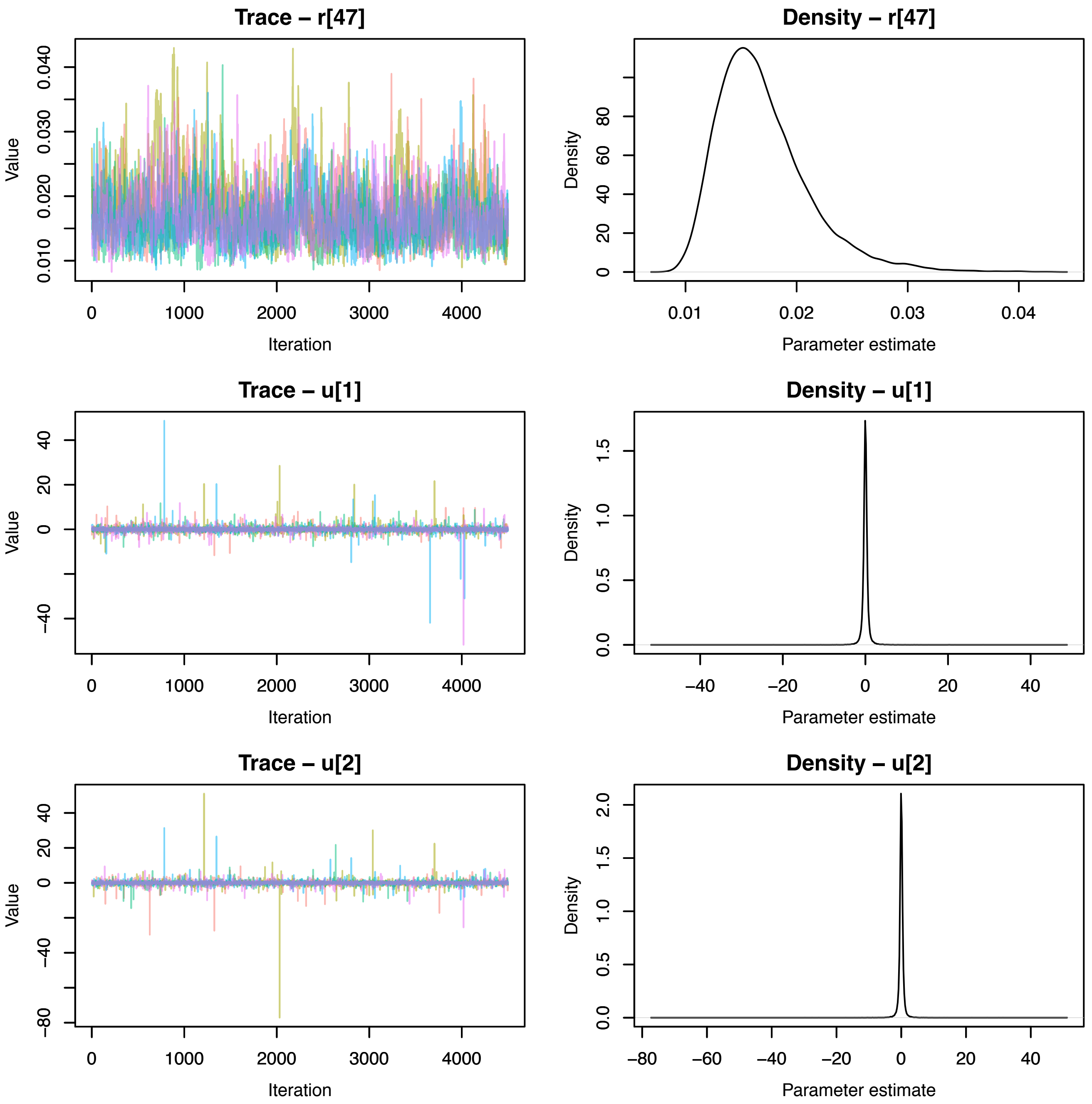


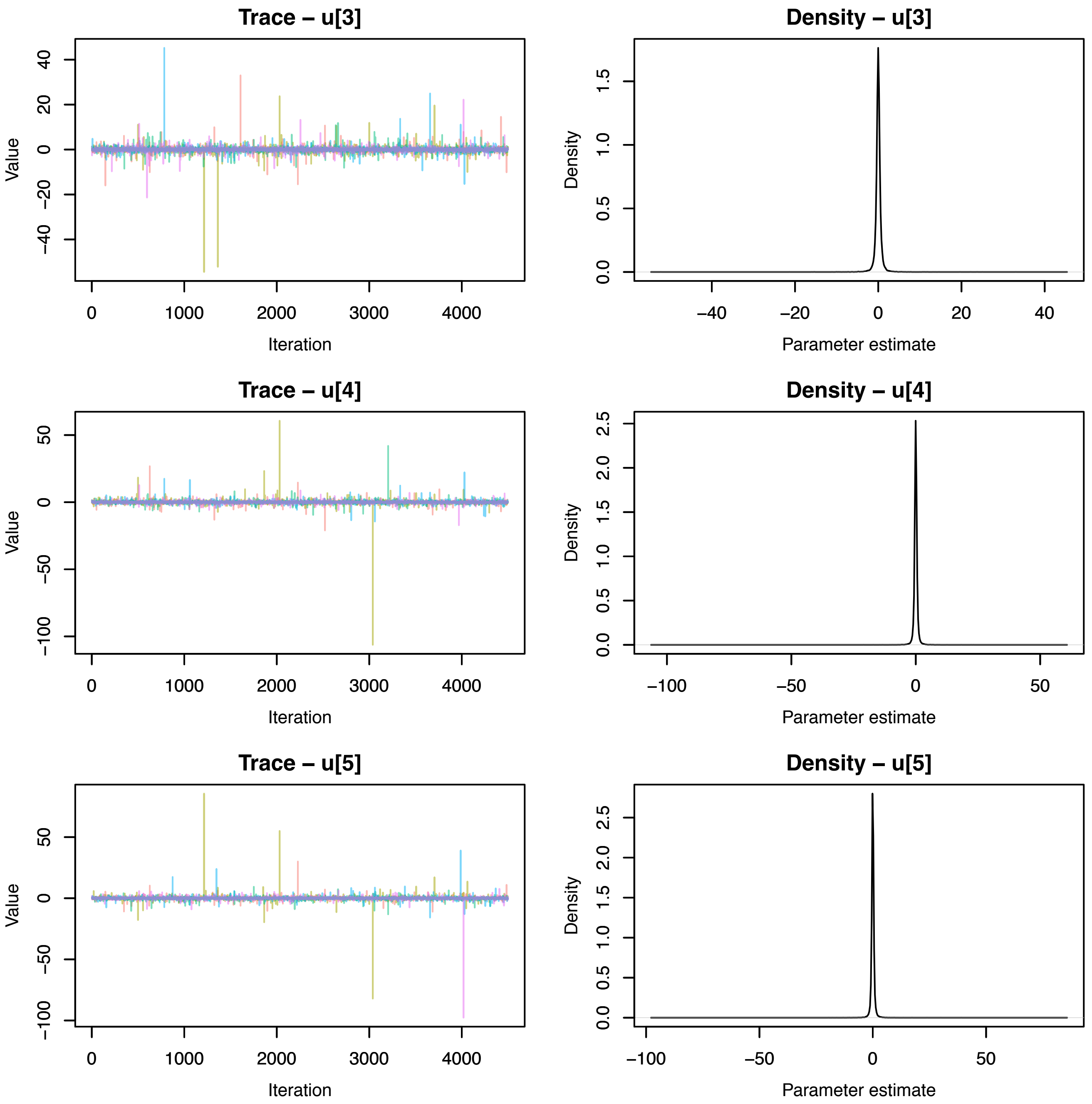
Trace – r[38]
Value
0.010
0.020
0.030
0.040
Iteration
0
1000
2000
3000
4000
Density – r[38]
Density
0
20
40
60
80
120
0.01
0.02
0.03
0.04
Parameter estimate
Trace – r[39]
Value
0.01
0.02
0.03
0.04
Iteration
0
1000
2000
3000
4000
Density – r[39]
Density
0
20
40
60
80
120
0.01
0.02
0.03
0.04
Parameter estimate
Trace – r[40]
Value
0.005
0.015
0.025
Iteration
0
1000
2000
3000
4000
Density – r[40]
Density
0
50
100
150
0.005
0.010
0.015
0.020
0.025
0.030
Parameter estimate



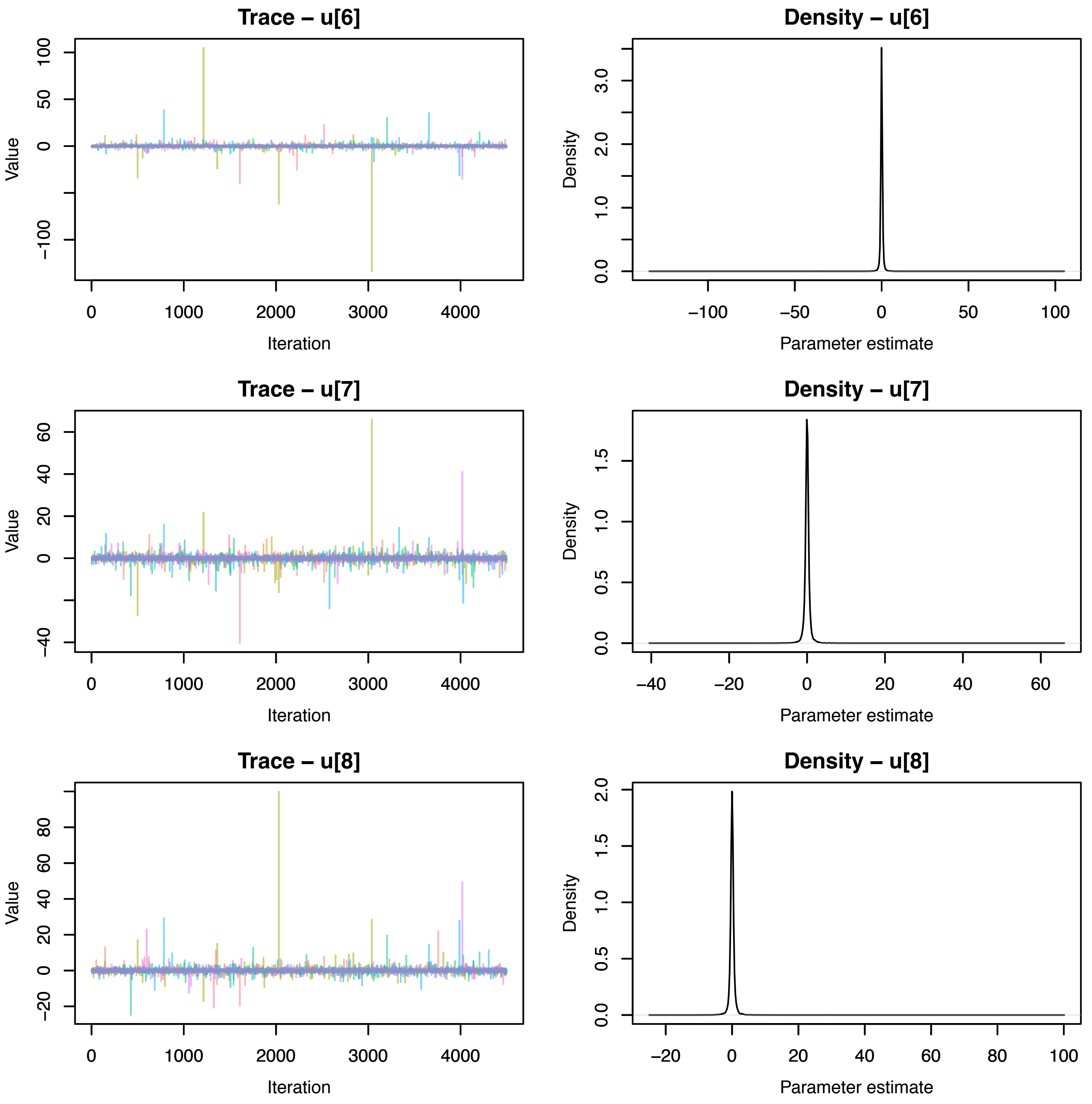
Trace – r[41]
Value
Iteration
Density – r[41]
Density
Parameter estimate
Trace – r[42]
Value
Iteration
Density – r[42]
Density
Parameter estimate
Trace – r[43]
Value
Iteration
Density – r[43]
Density
Parameter estimate

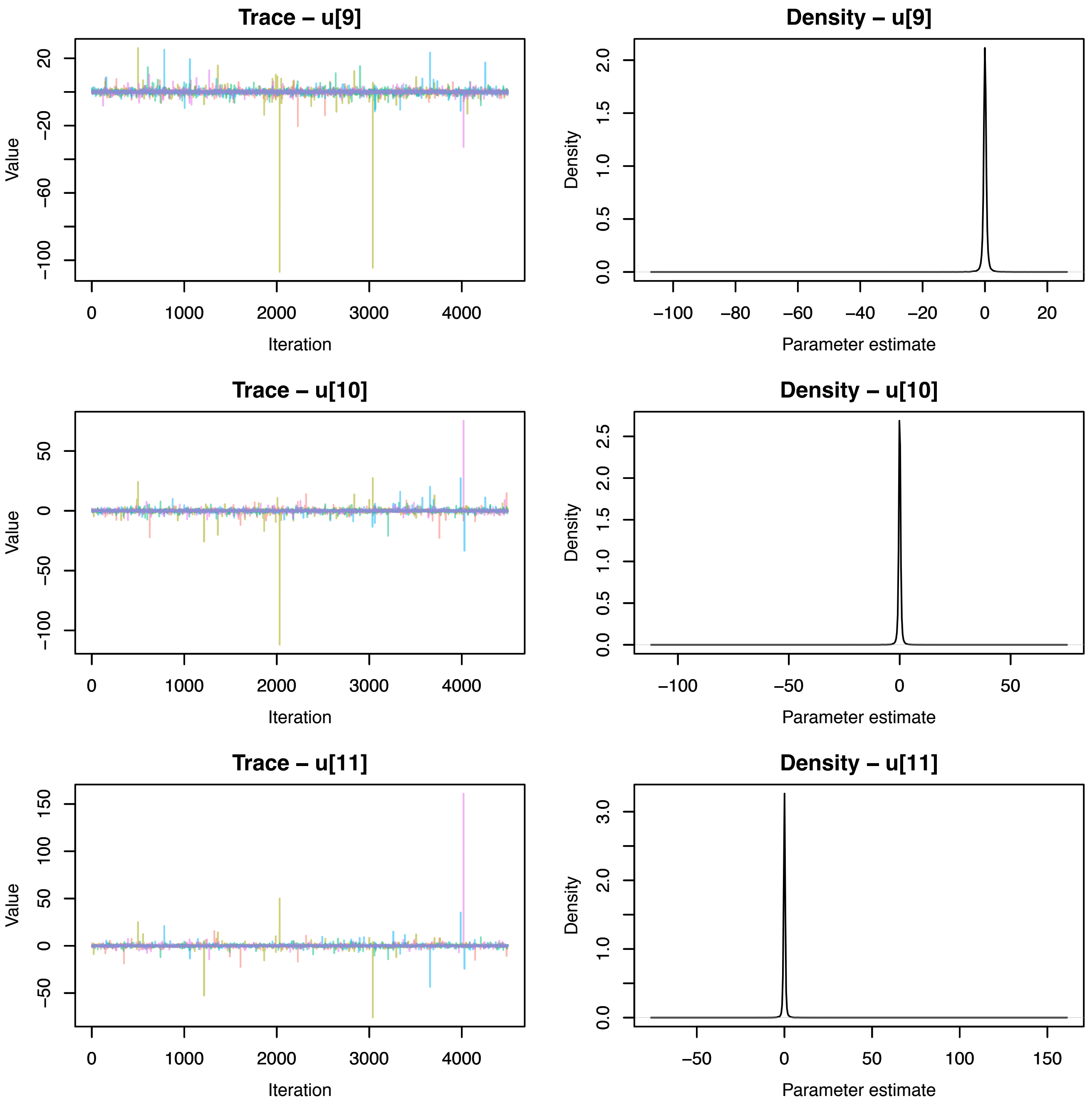

Trace – r[44]
Value
Iteration
Density – r[44]
Density
Parameter estimate
Trace – r[45]
Value
Iteration
Density – r[45]
Density
Parameter estimate
Trace – r[46]
Value
Iteration
Density – r[46]
Density
Parameter estimate


Trace – r[47]
Value
0.010
0.020
0.030
0.040
0
1000
2000
3000
4000
Iteration
Density – r[47]
Density
0
20
40
60
80
0.01
0.02
0.03
0.04
Parameter estimate
Trace – u[1]
Value
−40
0
20
40
0
1000
2000
3000
4000
Iteration
Density – u[1]
Density
0.0
0.5
1.0
1.5
−40
−20
0
20
40
Parameter estimate
Trace – u[2]
Value
−80
−40
0
20
40
0
1000
2000
3000
4000
Iteration
Density – u[2]
Density
0.0
0.5
1.0
1.5
2.0
−80
−60
−40
−20
0
20
40
Parameter estimate

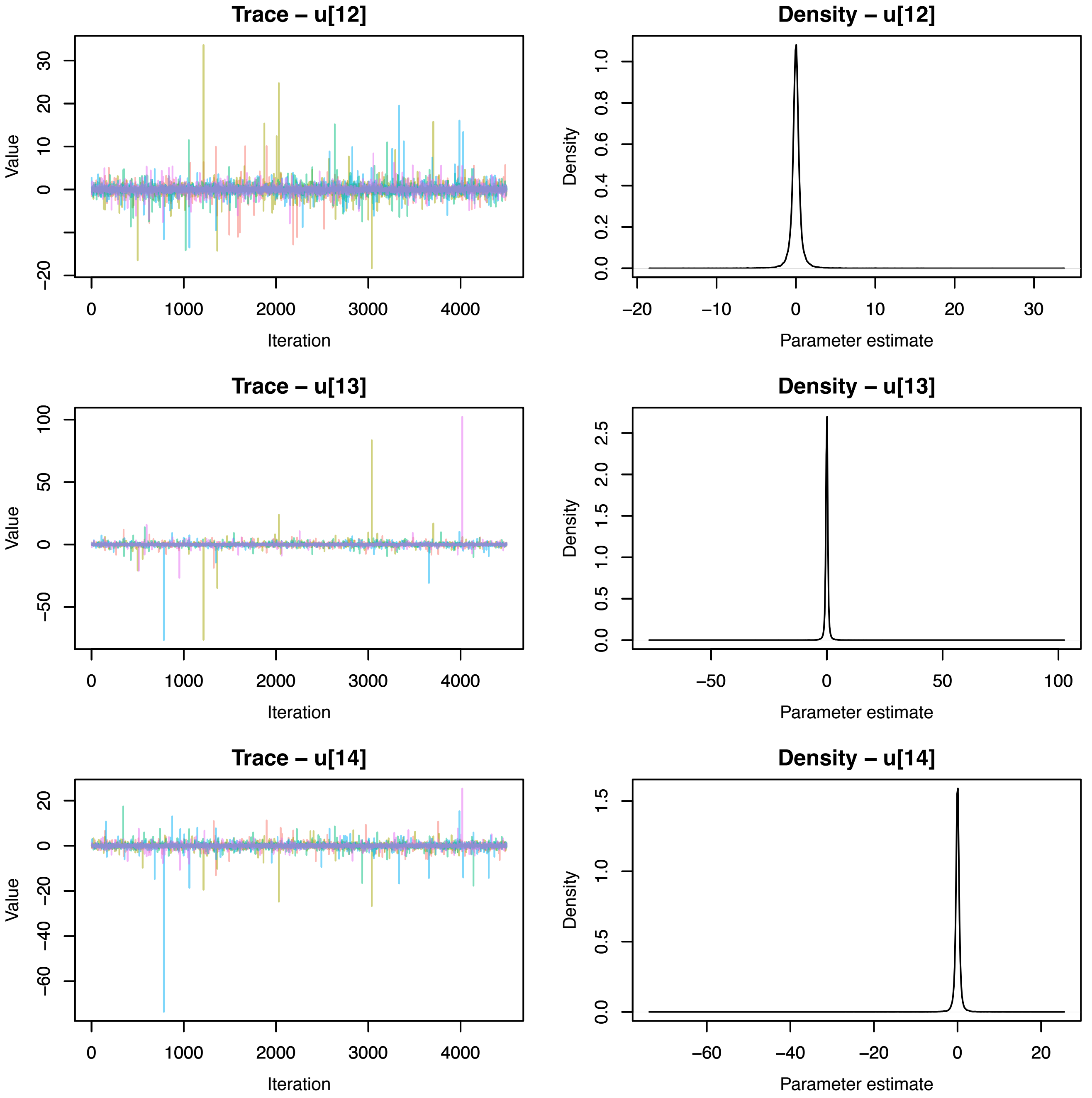


Trace – u[3]
Value
Iteration
Density – u[3]
Density
Parameter estimate
Trace – u[4]
Value
Iteration
Density – u[4]
Density
Parameter estimate
Trace – u[5]
Value
Iteration
Density – u[5]
Density
Parameter estimate

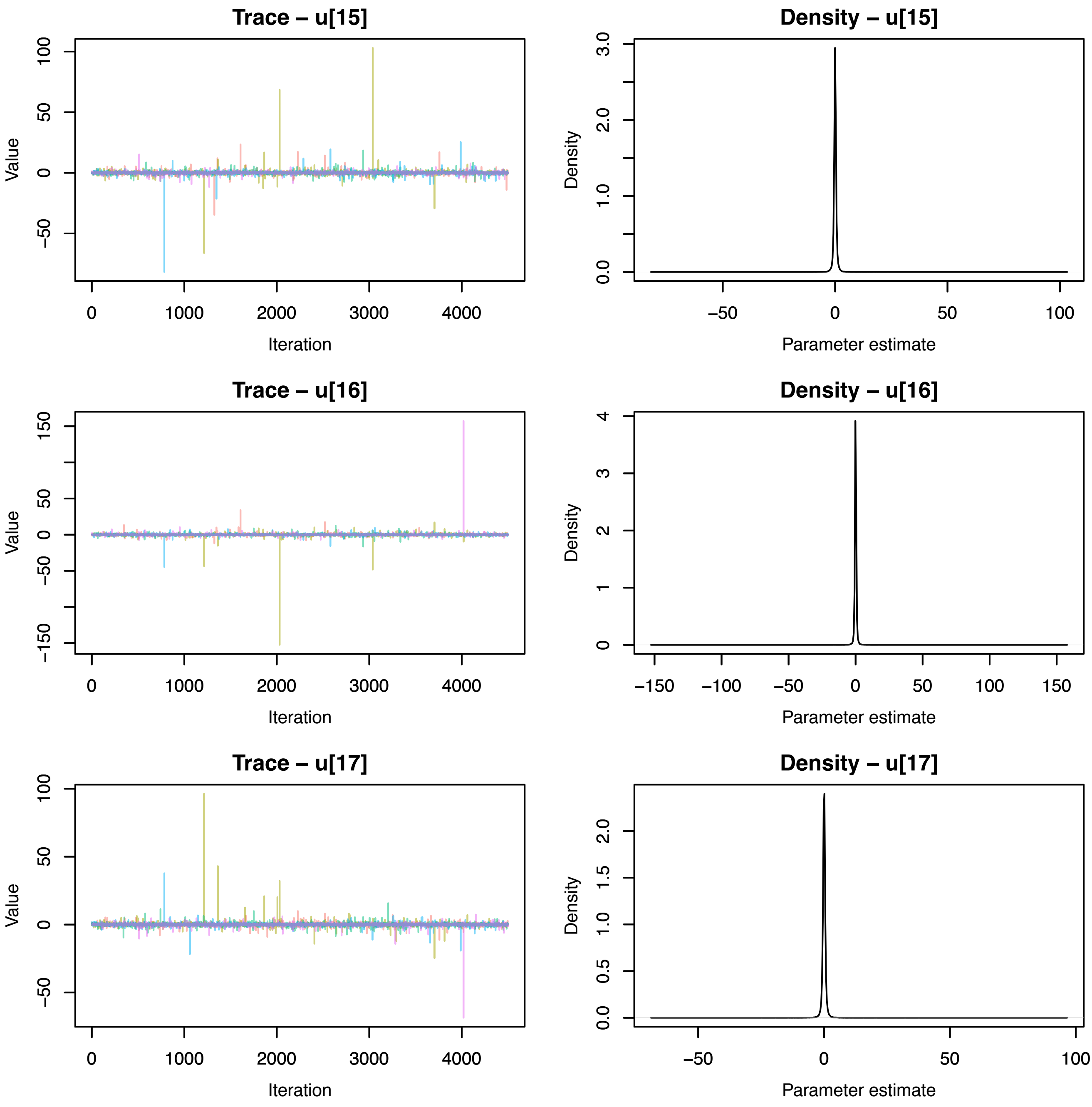

Trace – u[6]
Density – u[6]
Trace – u[7]
Density – u[7]
Trace – u[8]
Density – u[8]
Value
Iteration
Density
Parameter estimate



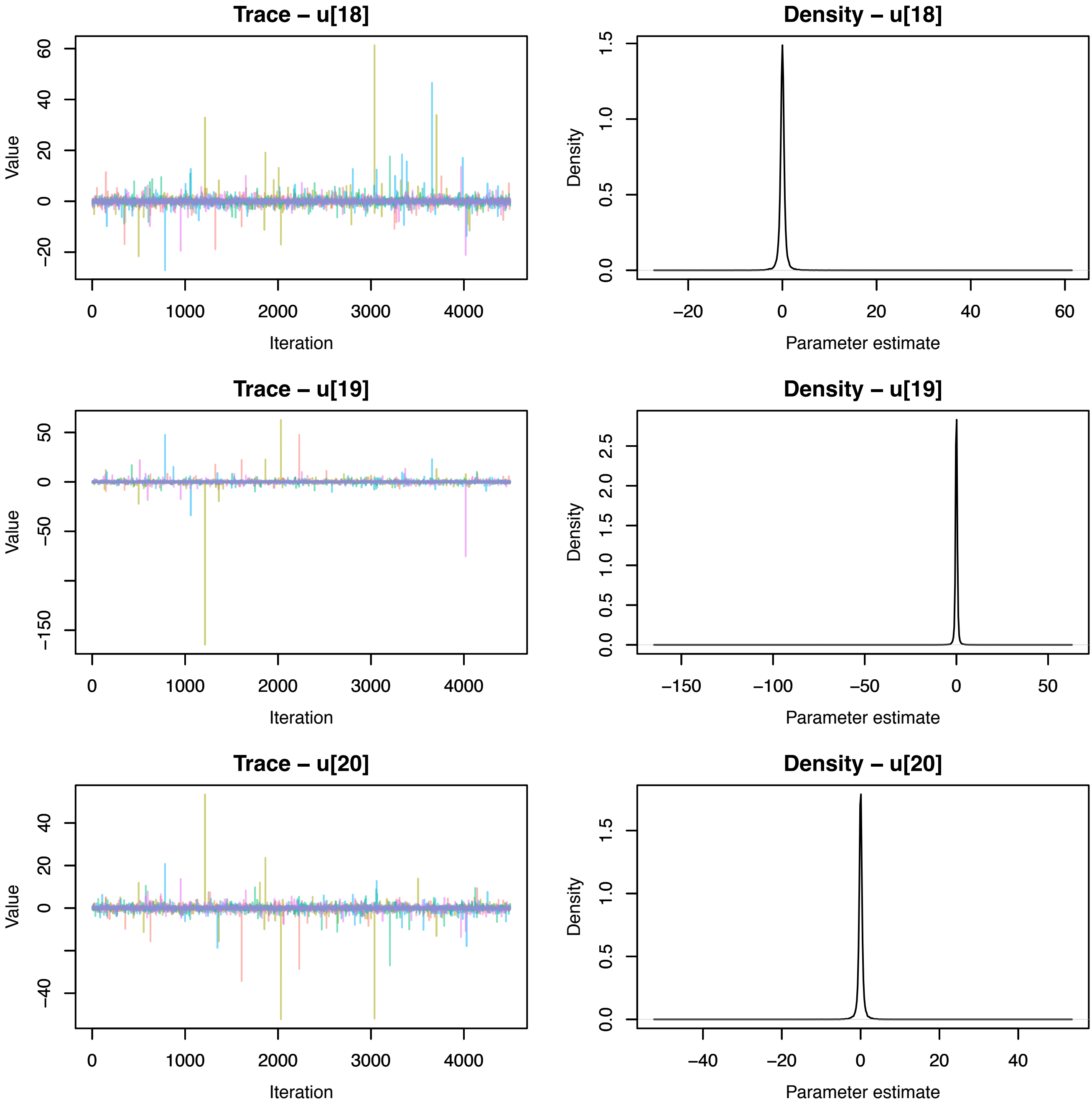
Trace – u[9]
Value
Iteration
Density – u[9]
Density
Parameter estimate
Trace – u[10]
Value
Iteration
Density – u[10]
Density
Parameter estimate
Trace – u[11]
Value
Iteration
Density – u[11]
Density
Parameter estimate



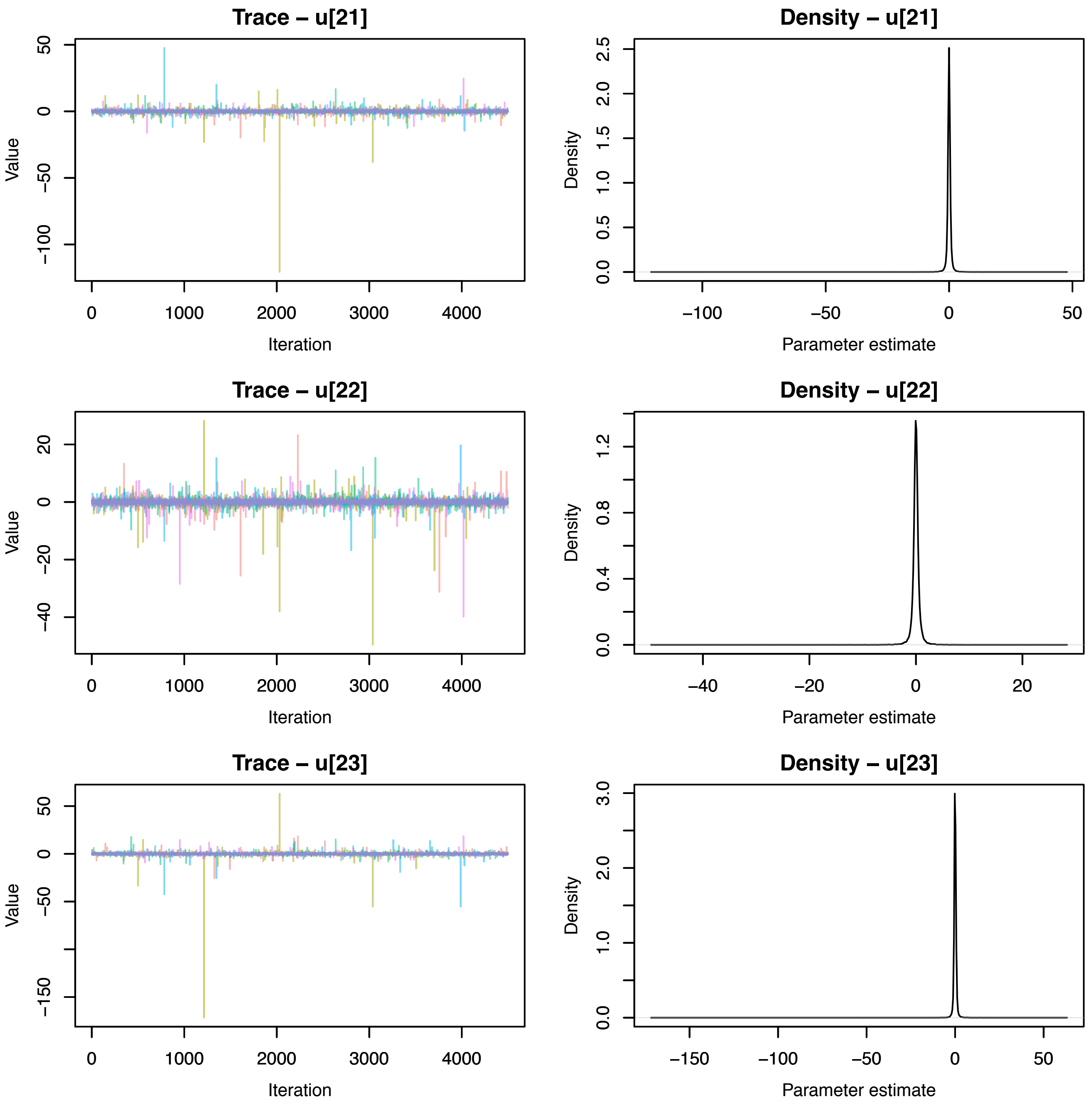
Trace – u[12]
Value
30
20
10
0
−20
0
1000
2000
3000
4000
Iteration
Density – u[12]
Density
1.0
0.8
0.6
0.4
0.2
0.0
−20
−10
0
10
20
30
Parameter estimate
Trace – u[13]
Value
100
50
0
−50
0
1000
2000
3000
4000
Iteration
Density – u[13]
Density
2.5
2.0
1.5
1.0
0.5
0.0
−50
0
50
100
Parameter estimate
Trace – u[14]
Value
20
0
−20
−40
−60
0
1000
2000
3000
4000
Iteration
Density – u[14]
Density
1.5
1.0
0.5
0.0
−60
−40
−20
0
20
Parameter estimate



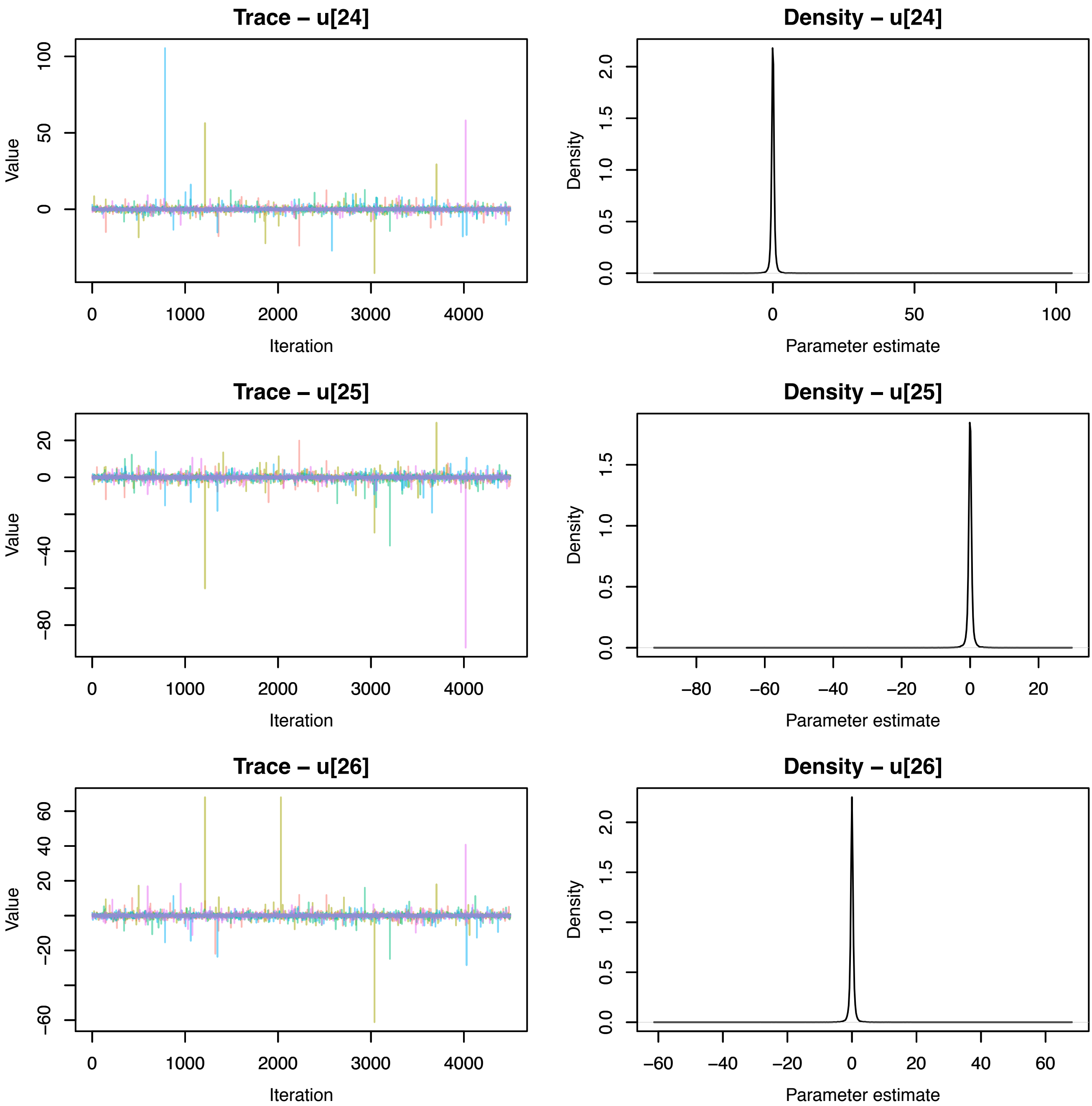
Trace – u[15]
Value
Iteration
0
1000
2000
3000
4000
100
50
0
−50
Density – u[15]
Density
Parameter estimate
3.0
2.0
1.0
0.0
−50
0
50
100
Trace – u[16]
Value
Iteration
150
50
0
−50
−150
Density – u[16]
Density
Parameter estimate
4
3
2
1
0
−150
−100
−50
0
50
100
150
Trace – u[17]
Value
Iteration
100
50
0
−50
Density – u[17]
Density
Parameter estimate
2.0
1.5
1.0
0.5
0.0
−50
0
50
100

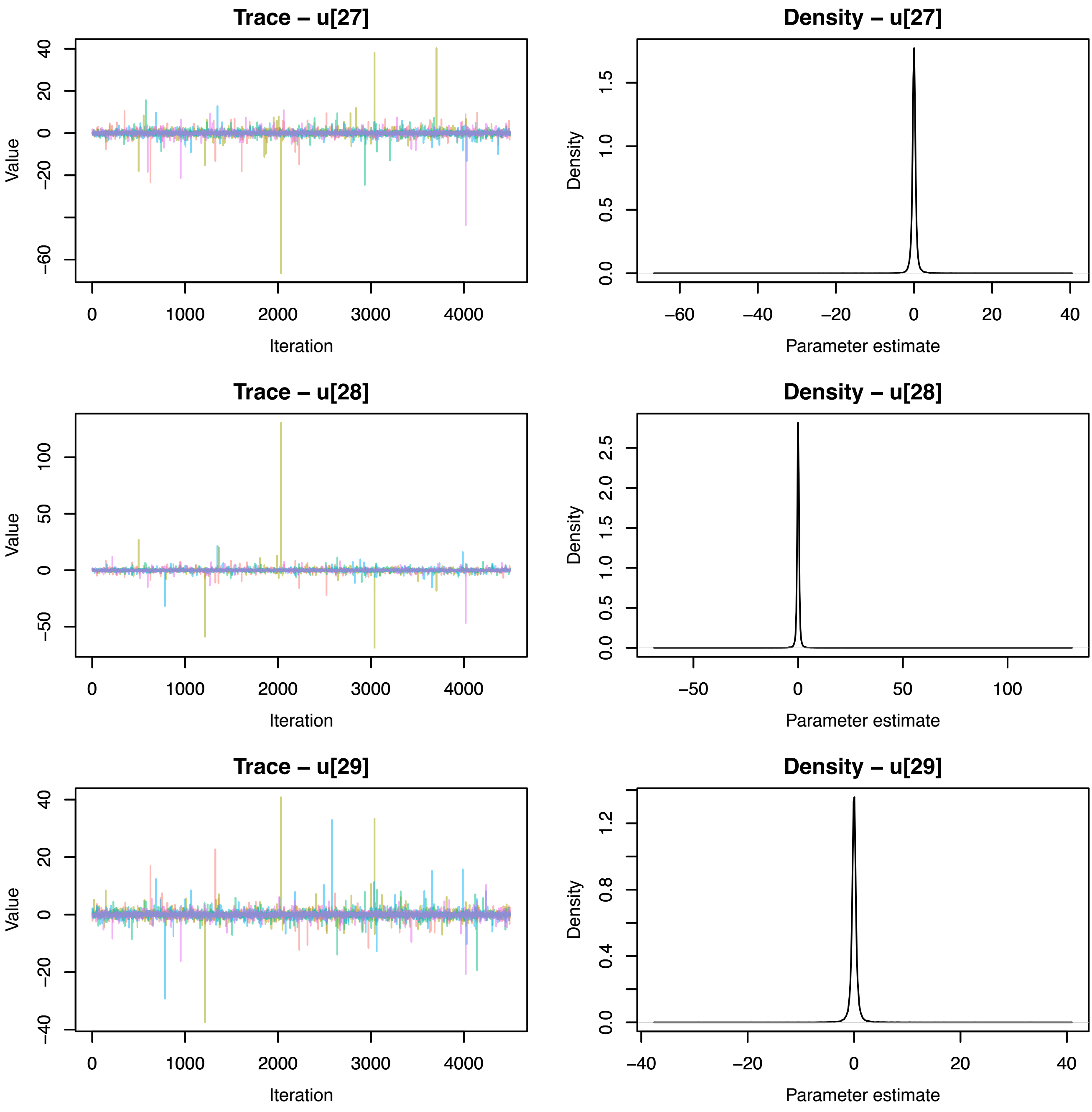


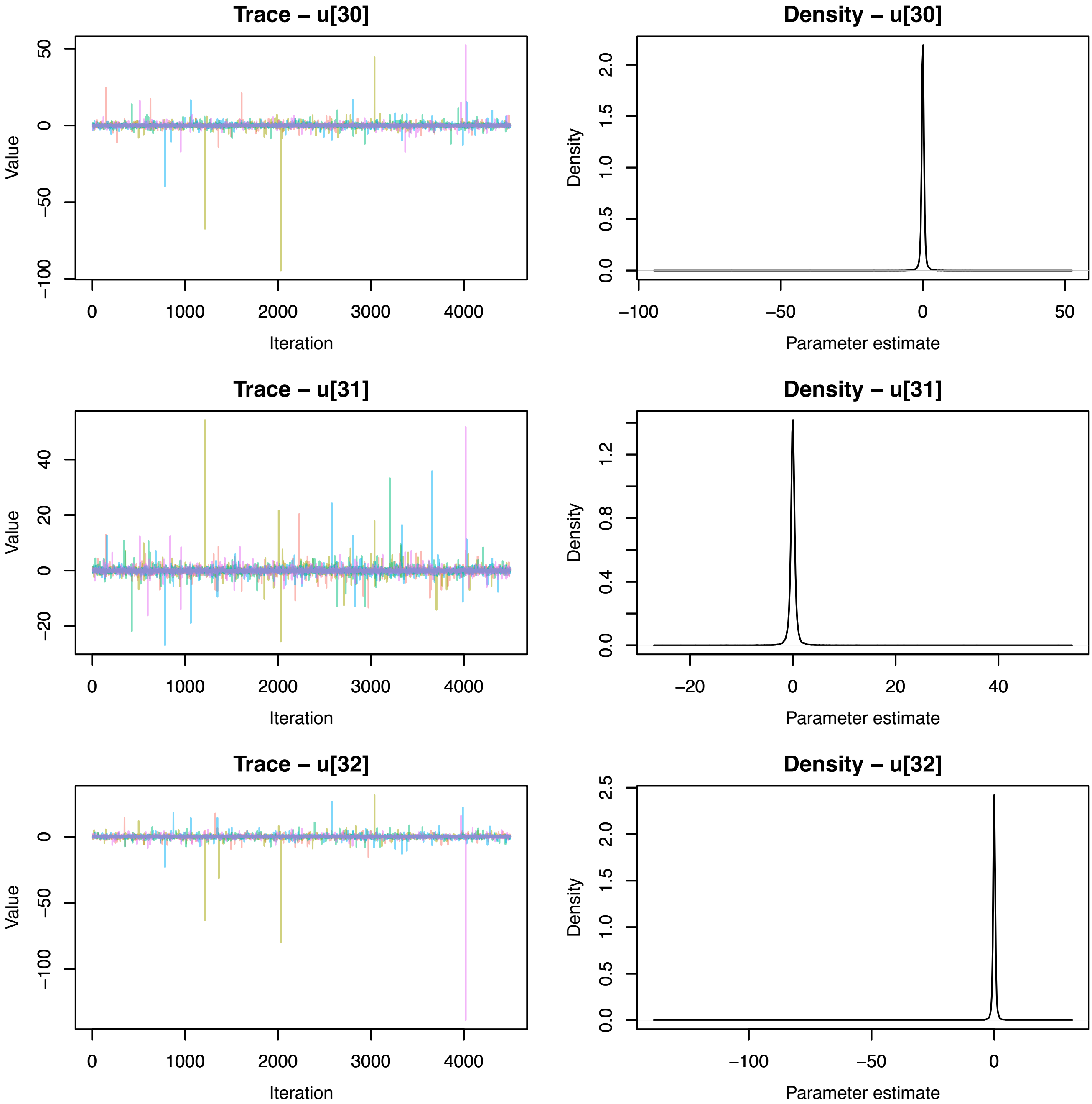
Trace – u[18]
Value
Iteration
Density – u[18]
Density
Parameter estimate
Trace – u[19]
Value
Iteration
Density – u[19]
Density
Parameter estimate
Trace – u[20]
Value
Iteration
Density – u[20]
Density
Parameter estimate



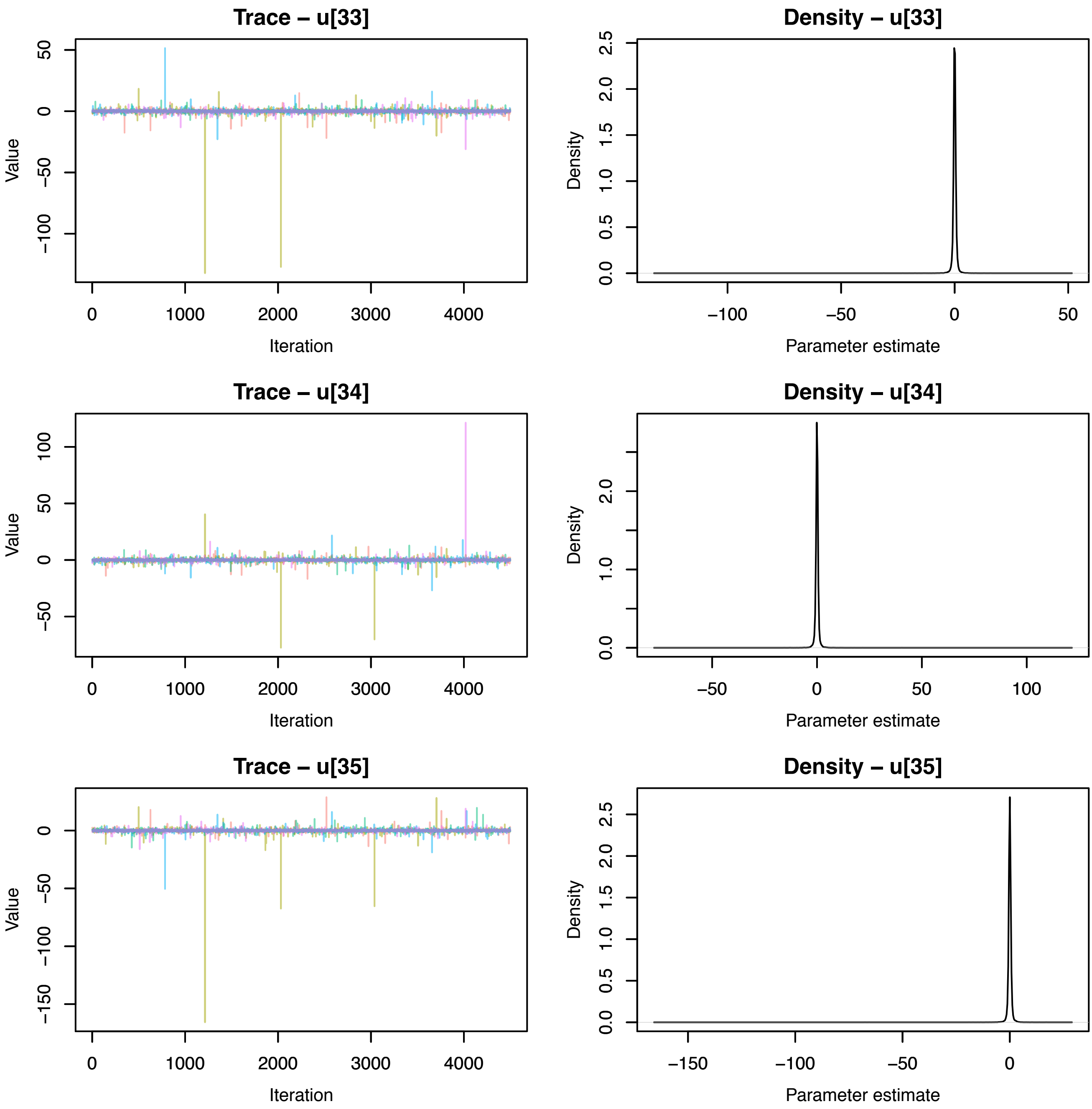
Trace – u[21]
Value
Iteration
Density – u[21]
Density
Parameter estimate
Trace – u[22]
Value
Iteration
Density – u[22]
Density
Parameter estimate
Trace – u[23]
Value
Iteration
Density – u[23]
Density
Parameter estimate

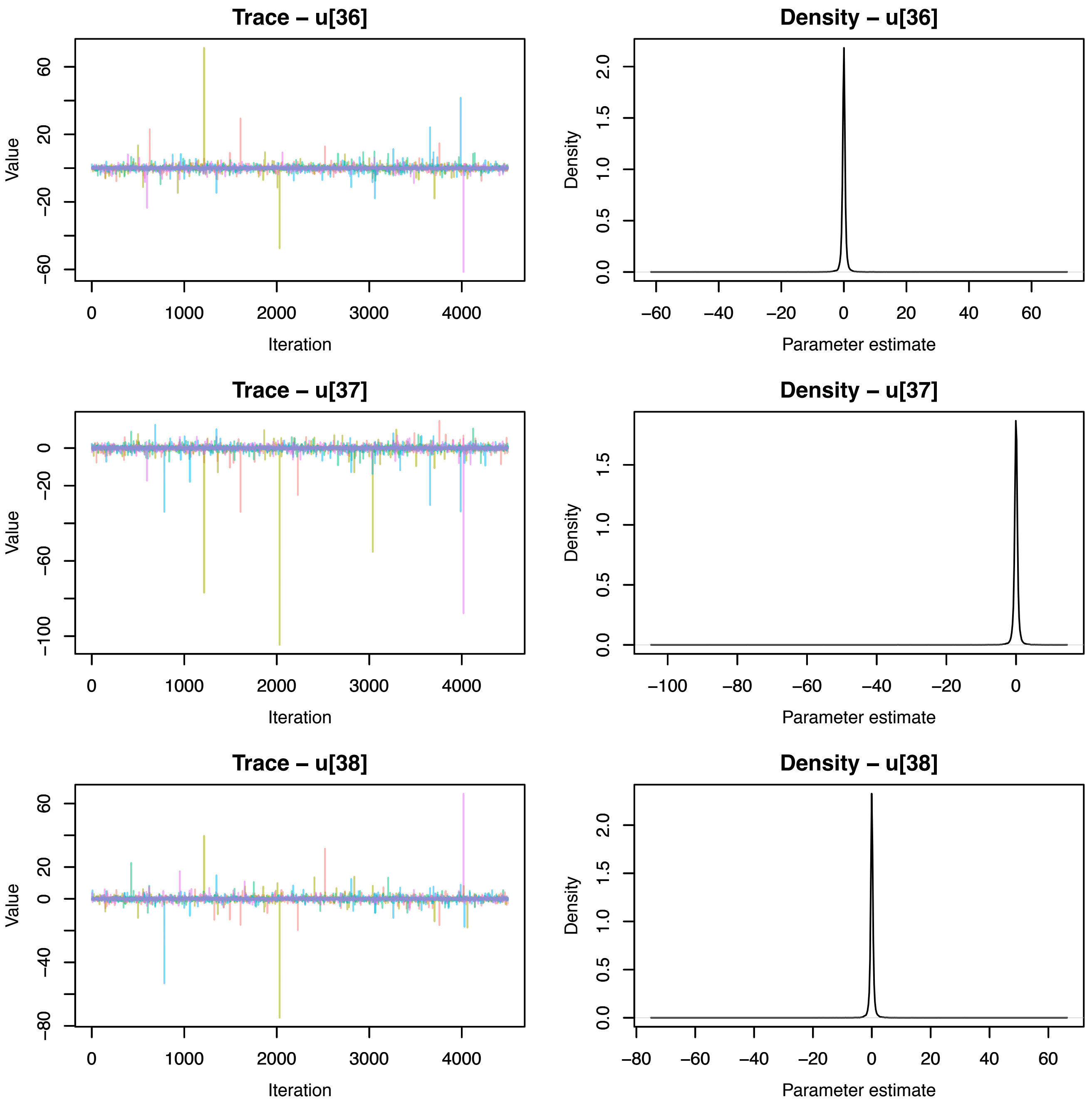

Trace – u[24]
Value
100
50
0
0
1000
2000
3000
4000
Iteration
Density – u[24]
Density
2.0
1.5
1.0
0.5
0.0
0
50
100
Parameter estimate
Trace – u[25]
Value
20
0
−40
−80
0
1000
2000
3000
4000
Iteration
Density – u[25]
Density
1.5
1.0
0.5
0.0
−80
−60
−40
−20
0
20
Parameter estimate
Trace – u[26]
Value
60
40
20
−20
−60
0
1000
2000
3000
4000
Iteration
Density – u[26]
Density
2.0
1.5
1.0
0.5
0.0
−60
−40
−20
0
20
40
60
Parameter estimate

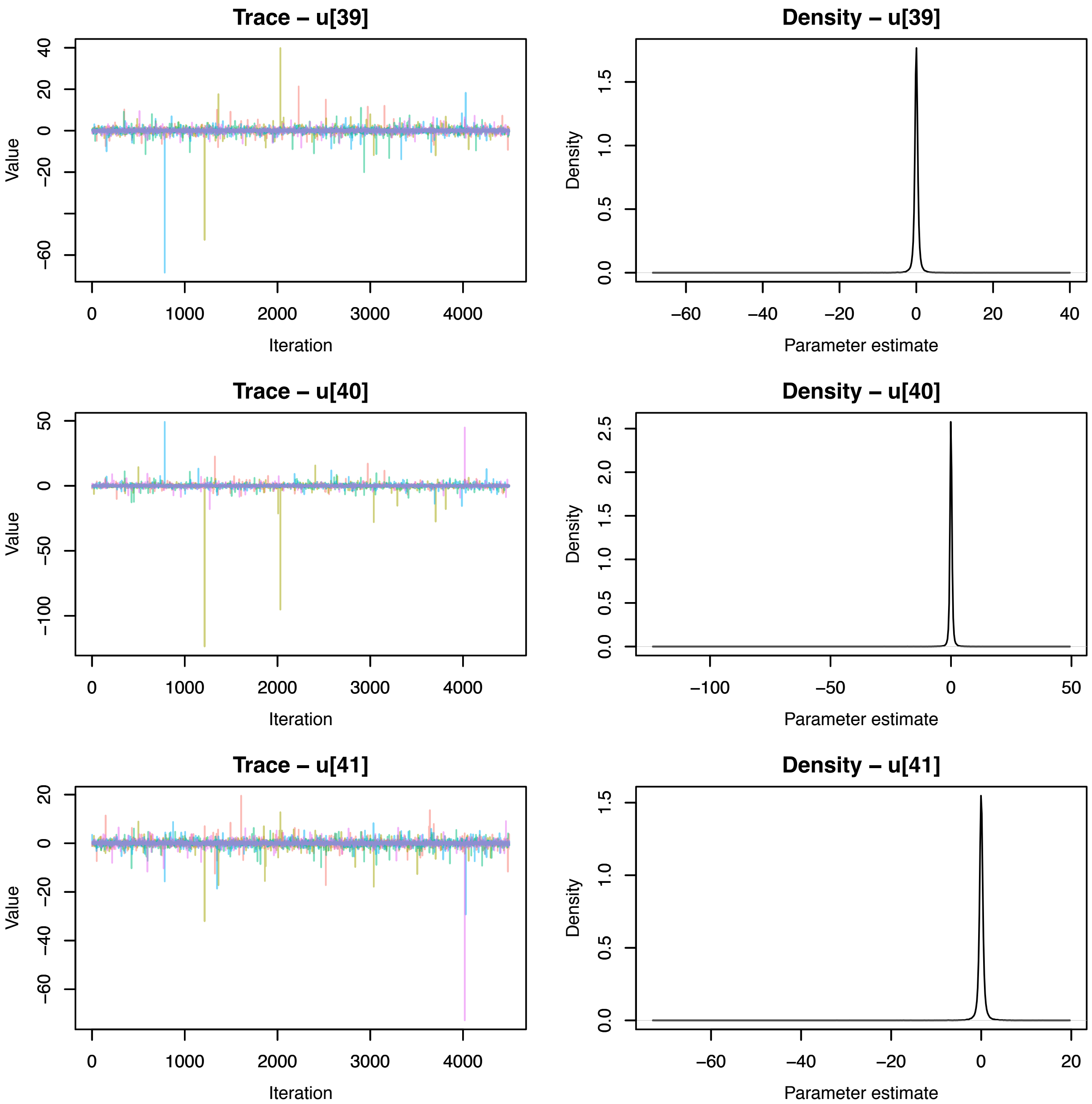

Trace – u[27]
Value
Iteration
Density – u[27]
Density
Parameter estimate
Trace – u[28]
Value
Iteration
Density – u[28]
Density
Parameter estimate
Trace – u[29]
Value
Iteration
Density – u[29]
Density
Parameter estimate

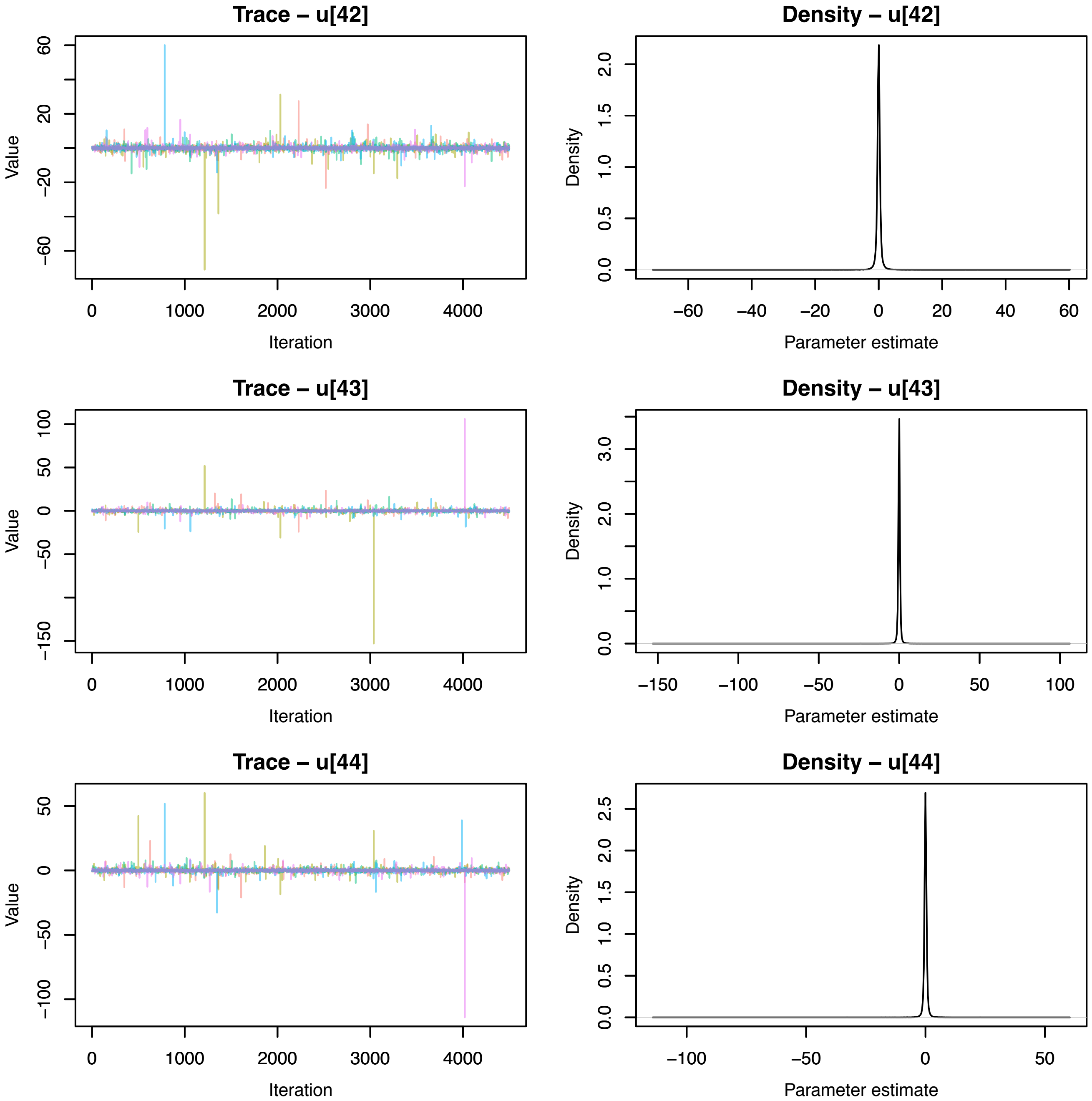


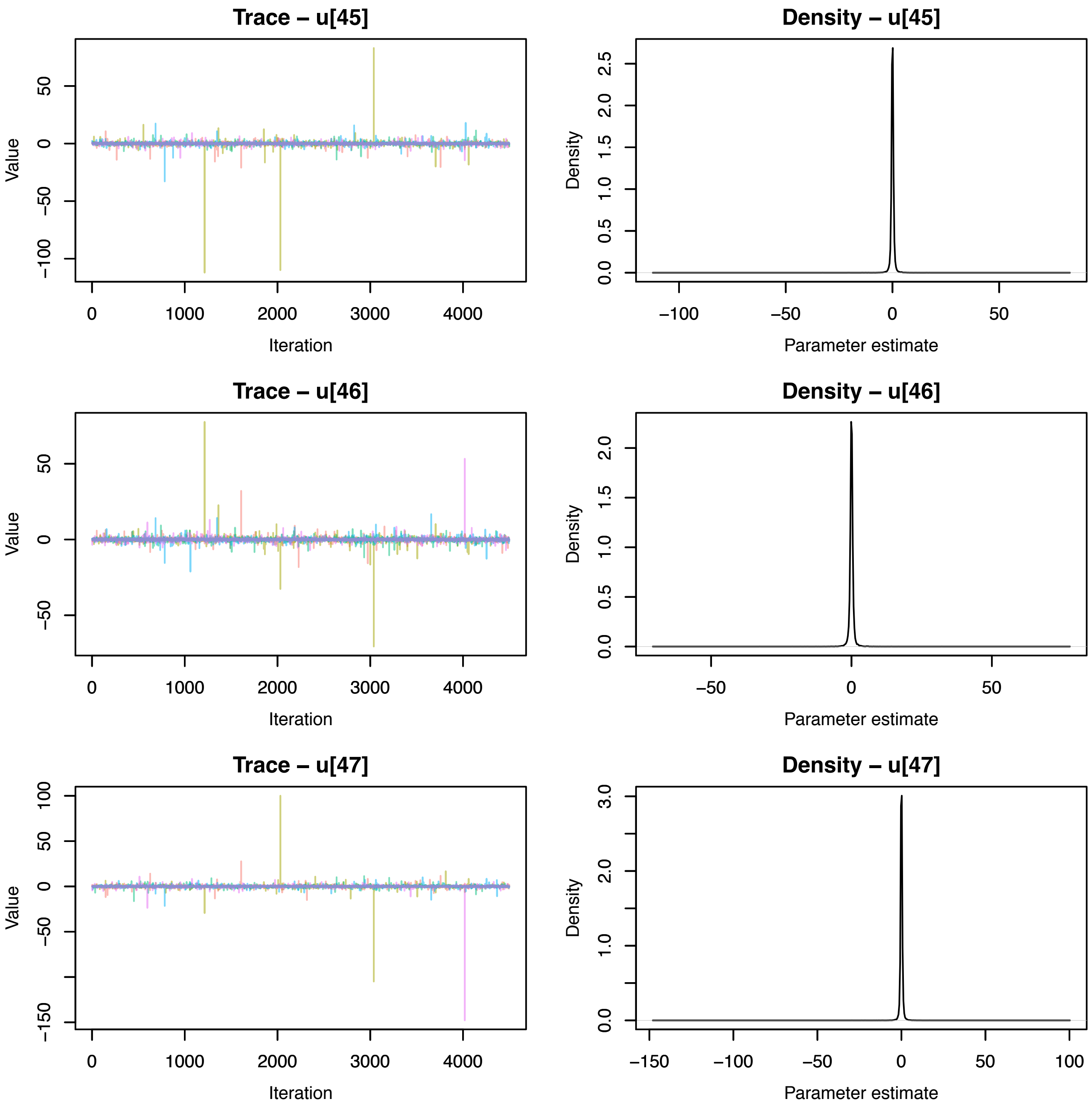
Trace – u[30]
Value
Iteration
Density – u[30]
Density
Parameter estimate
Trace – u[31]
Value
Iteration
Density – u[31]
Density
Parameter estimate
Trace – u[32]
Value
Iteration
Density – u[32]
Density
Parameter estimate


Trace – u[33]
Value
Iteration
0
1000
2000
3000
4000
50
0
−50
−100
Density – u[33]
Density
Parameter estimate
0.0
0.5
1.0
1.5
2.0
2.5
−100
−50
0
50
Trace – u[34]
Value
Iteration
100
50
0
−50
0
1000
2000
3000
4000
Density – u[34]
Density
Parameter estimate
0.0
1.0
2.0
−50
0
50
100
Trace – u[35]
Value
Iteration
0
−50
−100
−150
0
1000
2000
3000
4000
Density – u[35]
Density
Parameter estimate
0.0
0.5
1.0
1.5
2.0
2.5
−150
−100
−50
0

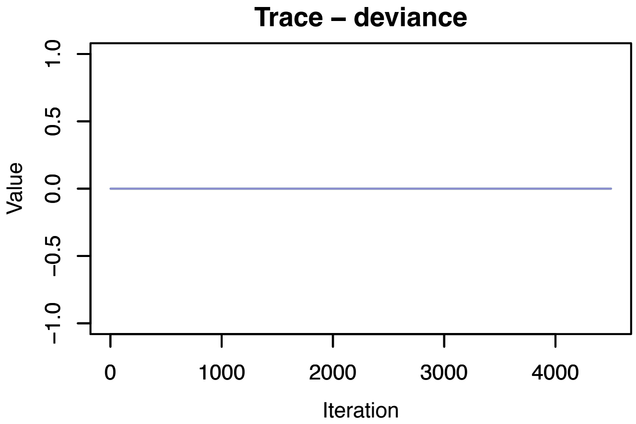

Trace – u[36]
Density – u[36]
Trace – u[37]
Density – u[37]
Trace – u[38]
Density – u[38]
Value
Iteration
Density
Parameter estimate
0
1000
2000
3000
4000
60
20
−20
−60
0
−20
−60
−100
60
20
0
−40
−80
0.0
0.5
1.0
1.5
2.0
−60
−40
−20
0
20
40
60
−100
−80
−60
−40
−20
0
−80

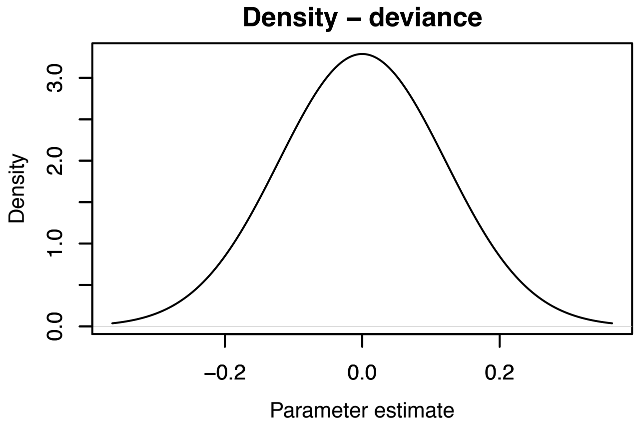

Trace – u[39]
Density – u[39]
Value
Iteration
Density
Parameter estimate
Trace – u[40]
Density – u[40]
Value
Iteration
Density
Parameter estimate
Trace – u[41]
Density – u[41]
Value
Iteration
Density
Parameter estimate


Trace – u[42]
Value
60
20
−20
−60
0
1000
2000
3000
4000
Iteration
Density – u[42]
Density
2.0
1.5
1.0
0.5
0.0
−60
−40
−20
0
20
40
60
Parameter estimate
Trace – u[43]
Value
100
50
0
−50
−150
0
1000
2000
3000
4000
Iteration
Density – u[43]
Density
3.0
2.0
1.0
0.0
−150
−100
−50
0
50
100
Parameter estimate
Trace – u[44]
Value
50
0
−50
−100
0
1000
2000
3000
4000
Iteration
Density – u[44]
Density
2.5
2.0
1.5
1.0
0.5
0.0
−100
−50
0
50
Parameter estimate



Trace – u[45]
Value
Iteration
Density – u[45]
Density
Parameter estimate
Trace – u[46]
Value
Iteration
Density – u[46]
Density
Parameter estimate
Trace – u[47]
Value
Iteration
Density – u[47]
Density
Parameter estimate


Trace – deviance
Value
1.0
0.5
0.0
−0.5
−1.0
0
1000
2000
3000
4000
Iteration


Density – deviance
Density
3.0
2.0
1.0
0.0
−0.2
0.0
0.2
Parameter estimate