Canonical Correlation Analysis for Elliptical Copulas Supplementary Materials

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1. ADDITIONAL SIMULATION RESULTS

1.1 Empirical bias and standard deviation of CCA with robust covariance estimation

Tables S1 through S5 report bias and standard deviation for estimates for transelliptical CCA estimated using the transformed Kendall's estimator, standard CCA using the sample covariance estimate, and robust CCA based on the re-weighted MCD estimator using a maximum proportion of 0.75 of the observations and S estimator with a breakdown point of 0.75. In addition two different rank based correlation estimators are used. One based on Spearman's correlation and one based on Blomqvist's beta. Further details on these can be found in the main text. The simulation set-ups can be found in section 3.1 of the main text, along with details on how the bias and standard deviation are calculated. The results for p=q=8 and n=200 can be found in Table 1 of the main text. In all cases only the bias and standard deviation for the four non-zero canonical correlations and the associated directions are reported. As in Table 1 of the main text only the bias and standard deviation for the X direction are reported with the results for the Y direction being nearly identical. When data are multivariate normal CCA based using the standard covariance matrix estimator performs the best. When data are simulated from a multivariate Cauchy distribution the S estimator performs the best, while the transelliptical CCA using the transformed Kendall's estimator and the re-weighted MCD estimator perform similarly. Standard CCA estimates have high bias and variance due to the lack of moments. When data have a multivariate lognormal distribution only the rank based methods consistent for canonical correlations and directions. All other estimators underestimate the strength of the canonical correlations due to the non-linear transformation of the marginal distributions. However the estimator using Blomqvist's Beta performs much worse than the other two rank based methods. When the data have t distributions with 5 degrees of freedom transelliptical CCA using the S estimator performs the best, followed by the transformed Kendall's estimator. When data have a t distribution with 10 degrees of freedom the S estimator again performs best, followed by the standard estimator and the transformed Kendall's estimator. The transformed Kendall's tau estimator is the only estimator considered that performs well across all transelliptical distributions. The standard, re-weighted MCD, and S estimators all perform poorly when data are transelliptical and not elliptical, while the method based on transformed Spearman's correlation only works for Gaussian copulas, and the estimator based on Blomqvist's Beta has poor finite sample performance relative to the other robust estimators considered. For a fixed sample size as dimension increases the bias and standard deviation of both the correlation and direction estimates increases for all estimators. In particular for the first canonical correlation the positive finite sample bias increases as dimension increases.

Table S1: Average empirical bias (standard deviation) from 1000 simulations of the estimates of the first four canonical correlations and directions. Six different estimation techniques used with data simulated from five different elliptical and transelliptical distributions with p=q=4 and n=200.

		Canonical Correlations				
		Normal	Cauchy	Lognormal	t5	t10
Cor 1	Standard	0.02 (0.07)	1.42 (1.06)	-0.14 (0.20)	0.05 (0.10)	0.02 (0.08)
	Kendall	0.02 (0.08)	0.08 (0.13)	0.02 (0.08)	0.03 (0.09)	0.03 (0.08)
	S	0.02 (0.08)	0.04 (0.11)	-0.20 (0.11)	0.03 (0.08)	0.03 (0.08)
	MCD	0.03 (0.08)	0.06 (0.12)	-0.20 (0.13)	0.03 (0.09)	0.04 (0.09)
	Spearman	0.01 (0.08)	-	0.01 (0.08)	-	-
	Blomqvist	0.17 (0.24)	0.17 (0.26)	0.17 (0.25)	0.18 (0.27)	0.17 (0.25)
Cor 2	Standard	0.05 (0.06)	1.01 (0.58)	-0.04 (0.13)	0.09 (0.09)	0.06 (0.07)
	Kendall	0.05 (0.06)	0.09 (0.09)	0.05 (0.07)	0.06 (0.07)	0.06 (0.07)
	S	0.05 (0.06)	0.09 (0.09)	-0.07 (0.08)	0.06 (0.07)	0.06 (0.07)
	MCD	0.06 (0.07)	0.12 (0.10)	-0.06 (0.09)	0.08 (0.08)	0.07 (0.07)
	Spearman	0.05 (0.06)	-	0.05 (0.07)	-	-
	Blomqvist	0.13 (0.11)	0.12 (0.11)	0.12 (0.11)	0.13 (0.11)	0.13 (0.12)
Cor 3	Standard	0.02 (0.06)	0.40 (0.35)	-0.10 (0.08)	0.03 (0.08)	0.02 (0.07)
	Kendall	0.02 (0.06)	0.03 (0.08)	0.02 (0.06)	0.02 (0.06)	0.02 (0.06)
	S	0.02 (0.06)	0.03 (0.08)	-0.09 (0.07)	0.02 (0.06)	0.02 (0.06)
	MCD	0.02 (0.06)	0.04 (0.09)	-0.09 (0.08)	0.03 (0.07)	0.02 (0.07)
	Spearman	0.02 (0.06)	-	0.02 (0.06)	-	-
	Blomqvist	0.04 (0.09)	0.03 (0.09)	0.03 (0.09)	0.03 (0.09)	0.04 (0.09)
Cor 4	Standard	-0.04 (0.06)	-0.08 (0.21)	-0.16 (0.07)	-0.07 (0.08)	-0.05 (0.07)
	Kendall	-0.04 (0.06)	-0.07 (0.08)	-0.04 (0.06)	-0.05 (0.07)	-0.04 (0.07)
	S	-0.04 (0.06)	-0.07 (0.08)	-0.15 (0.07)	-0.05 (0.07)	-0.05 (0.07)
	MCD	-0.05 (0.07)	-0.09 (0.10)	-0.17 (0.08)	-0.06 (0.08)	-0.06 (0.07)
	Spearman	-0.04 (0.06)	-	-0.04 (0.06)	-	-
	Blomqvist	-0.08 (0.09)	-0.08 (0.09)	-0.09 (0.09)	-0.09 (0.09)	-0.08 (0.09)
		Canonical Directions				
Dir 1	Standard	0.07 (0.03)	0.53 (0.43)	0.08 (0.06)	0.09 (0.05)	0.08 (0.03)
	Kendall	0.08 (0.03)	0.13 (0.06)	0.08 (0.03)	0.09 (0.04)	0.08 (0.03)
	S	0.07 (0.03)	0.10 (0.05)	0.09 (0.04)	0.08 (0.03)	0.08 (0.03)
	MCD	0.08 (0.03)	0.11 (0.05)	0.10 (0.05)	0.09 (0.04)	0.08 (0.04)
	Spearman	0.08 (0.03)	-	0.07 (0.03)	-	-
	Blomqvist	0.20 (0.09)	0.19 (0.09)	0.20 (0.09)	0.20 (0.09)	0.19 (0.09)
Dir 2	Standard	0.46 (0.30)	1.07 (0.34)	0.69 (0.48)	0.62 (0.38)	0.52 (0.34)
	Kendall	0.49 (0.31)	0.64 (0.36)	0.50 (0.31)	0.53 (0.33)	0.52 (0.34)
	S	0.50 (0.32)	0.63 (0.36)	0.61 (0.39)	0.52 (0.32)	0.51 (0.33)
	MCD	0.52 (0.33)	0.67 (0.37)	0.66 (0.39)	0.57 (0.35)	0.55 (0.34)
	Spearman	0.49 (0.31)	-	0.50 (0.32)	-	-
	Blomqvist	0.69 (0.38)	0.71 (0.37)	0.69 (0.36)	0.69 (0.36)	0.68 (0.37)
Dir 3	Standard	0.71 (0.39)	1.07 (0.31)	0.91 (0.44)	0.84 (0.39)	0.77 (0.39)
	Kendall	0.74 (0.39)	0.85 (0.39)	0.75 (0.39)	0.76 (0.37)	0.76 (0.40)
	S	0.75 (0.39)	0.85 (0.39)	0.83 (0.41)	0.75 (0.38)	0.76 (0.39)
	MCD	0.76 (0.39)	0.89 (0.38)	0.86 (0.40)	0.80 (0.38)	0.80 (0.39)
	Spearman	0.74 (0.39)	-	0.75 (0.39)	-	-
	Blomqvist	0.90 (0.39)	0.90 (0.39)	0.89 (0.38)	0.89 (0.39)	0.90 (0.39)
Dir 4	Standard	0.60 (0.37)	1.04 (0.30)	0.74 (0.46)	0.68 (0.37)	0.64 (0.37)
	Kendall	0.62 (0.37)	0.70 (0.37)	0.63 (0.36)	0.63 (0.37)	0.63 (0.37)
	S	0.62 (0.37)	0.72 (0.38)	0.69 (0.41)	0.63 (0.36)	0.63 (0.36)
	MCD	0.63 (0.37)	0.76 (0.38)	0.74 (0.40)	0.68 (0.36)	0.68 (0.38)
	Spearman	0.62 (0.37)	-	0.63 (0.36)	-	-
	Blomqvist	0.74 (0.39)	0.76 (0.38)	0.75 (0.38)	0.74 (0.38)	0.78 (0.39)

Table S2: Average empirical bias (standard deviation) from 1000 simulations of the estimates of the first four canonical correlations and directions. Six different estimation techniques used with data simulated from five different elliptical and transelliptical distributions with p=q=4 and n=1000.

		Canonical Correlations				
		Normal	Cauchy	Lognormal	t5	t10
Cor 1	Standard	0.00 (0.03)	1.40 (1.12)	-0.19 (0.11)	0.01 (0.05)	0.01 (0.04)
	Kendall	0.01 (0.03)	0.01 (0.06)	0.01 (0.03)	0.01 (0.04)	0.01 (0.04)
	S	0.00 (0.03)	0.01 (0.04)	-0.22 (0.05)	0.01 (0.04)	0.01 (0.04)
	MCD	0.01 (0.03)	0.01 (0.05)	-0.24 (0.06)	0.01 (0.04)	0.01 (0.04)
	Spearman	0.00 (0.04)	-	0.00 (0.04)	-	-
	Blomqvist	0.03 (0.08)	0.03 (0.09)	0.03 (0.08)	0.03 (0.08)	0.03 (0.08)
Cor 2	Standard	0.01 (0.03)	1.03 (0.58)	-0.13 (0.07)	0.02 (0.05)	0.01 (0.03)
	Kendall	0.01 (0.03)	0.02 (0.04)	0.01 (0.03)	0.01 (0.03)	0.01 (0.03)
	S	0.01 (0.03)	0.02 (0.04)	-0.13 (0.04)	0.01 (0.04)	0.01 (0.03)
	MCD	0.01 (0.03)	0.02 (0.05)	-0.13 (0.05)	0.01 (0.04)	0.01 (0.04)
	Spearman	0.01 (0.03)	-	0.01 (0.03)	-	-
	Blomqvist	0.03 (0.05)	0.03 (0.05)	0.03 (0.05)	0.02 (0.05)	0.02 (0.05)
Cor 3	Standard	0.00 (0.03)	0.39 (0.34)	-0.11 (0.05)	0.01 (0.04)	0.00 (0.03)
	Kendall	0.01 (0.03)	0.01 (0.04)	0.01 (0.03)	0.01 (0.03)	0.00 (0.03)
	S	0.01 (0.03)	0.01 (0.04)	-0.11 (0.03)	0.01 (0.03)	0.01 (0.03)
	MCD	0.01 (0.03)	0.01 (0.04)	-0.12 (0.04)	0.01 (0.04)	0.01 (0.04)
	Spearman	0.00 (0.03)	-	0.00 (0.03)	-	-
	Blomqvist	0.01 (0.04)	0.01 (0.04)	0.01 (0.05)	0.01 (0.04)	0.01 (0.05)
Cor 4	Standard	-0.01 (0.03)	-0.09 (0.20)	-0.12 (0.04)	-0.02 (0.04)	-0.01 (0.03)
	Kendall	-0.01 (0.03)	-0.01 (0.04)	-0.01 (0.03)	-0.01 (0.03)	-0.01 (0.03)
	S	-0.01 (0.03)	-0.02 (0.04)	-0.11 (0.03)	-0.01 (0.03)	-0.01 (0.03)
	MCD	-0.01 (0.03)	-0.02 (0.05)	-0.13 (0.04)	-0.01 (0.04)	-0.01 (0.03)
	Spearman	-0.01 (0.03)	-	-0.01 (0.03)	-	-
	Blomqvist	-0.02 (0.05)	-0.02 (0.05)	-0.02 (0.04)	-0.02 (0.05)	-0.02 (0.04)
		Canonical Directions				
Dir 1	Standard	0.03 (0.01)	0.54 (0.42)	0.03 (0.02)	0.04 (0.02)	0.03 (0.01)
	Kendall	0.03 (0.01)	0.06 (0.02)	0.03 (0.01)	0.04 (0.02)	0.03 (0.01)
	S	0.03 (0.01)	0.04 (0.02)	0.04 (0.02)	0.03 (0.01)	0.03 (0.01)
	MCD	0.03 (0.01)	0.05 (0.02)	0.04 (0.02)	0.04 (0.02)	0.04 (0.02)
	Spearman	0.03 (0.01)	-	0.03 (0.01)	-	-
	Blomqvist	0.08 (0.03)	0.08 (0.04)	0.08 (0.04)	0.08 (0.03)	0.08 (0.03)
Dir 2	Standard	0.20 (0.12)	1.08 (0.34)	0.39 (0.39)	0.33 (0.23)	0.23 (0.14)
	Kendall	0.21 (0.14)	0.30 (0.20)	0.21 (0.13)	0.25 (0.17)	0.22 (0.14)
	S	0.21 (0.14)	0.29 (0.20)	0.30 (0.24)	0.24 (0.17)	0.22 (0.14)
	MCD	0.21 (0.13)	0.35 (0.24)	0.40 (0.29)	0.28 (0.21)	0.25 (0.17)
	Spearman	0.21 (0.14)	-	0.21 (0.13)	-	-
	Blomqvist	0.37 (0.25)	0.37 (0.25)	0.37 (0.26)	0.37 (0.26)	0.36 (0.25)
Dir 3	Standard	0.35 (0.24)	1.08 (0.30)	0.67 (0.46)	0.54 (0.33)	0.39 (0.26)
	Kendall	0.37 (0.25)	0.50 (0.32)	0.37 (0.24)	0.42 (0.27)	0.38 (0.26)
	S	0.38 (0.27)	0.49 (0.31)	0.52 (0.35)	0.42 (0.27)	0.38 (0.26)
	MCD	0.38 (0.27)	0.57 (0.34)	0.63 (0.36)	0.49 (0.32)	0.41 (0.28)
	Spearman	0.37 (0.26)	-	0.37 (0.25)	-	-
	Blomqvist	0.58 (0.35)	0.58 (0.35)	0.60 (0.36)	0.59 (0.35)	0.57 (0.35)
Dir 4	Standard	0.32 (0.24)	1.02 (0.30)	0.53 (0.44)	0.46 (0.32)	0.34 (0.25)
	Kendall	0.33 (0.25)	0.44 (0.31)	0.33 (0.24)	0.36 (0.27)	0.33 (0.25)
	S	0.34 (0.26)	0.42 (0.29)	0.44 (0.34)	0.36 (0.27)	0.33 (0.25)
	MCD	0.34 (0.26)	0.48 (0.33)	0.52 (0.33)	0.42 (0.31)	0.36 (0.26)
	Spearman	0.33 (0.25)	-	0.33 (0.24)	-	-
	Blomqvist	0.49 (0.34)	0.49 (0.34)	0.51 (0.35)	0.49 (0.33)	0.47 (0.33)

Table S3: Average empirical bias (standard deviation) from 1000 simulations of the estimates of the first four canonical correlations and directions. Six different estimation techniques used with data simulated from five different elliptical and transelliptical distributions with p=q=8 and n=1000.

		Canonical Correlations				
		Normal	Cauchy	Lognormal	t5	t10
Cor 1	Standard	0.01 (0.03)	1.91 (1.18)	-0.19 (0.11)	0.02 (0.05)	0.01 (0.04)
	Kendall	0.01 (0.03)	0.03 (0.06)	0.01 (0.03)	0.01 (0.04)	0.01 (0.04)
	S	0.01 (0.03)	0.02 (0.04)	-0.21 (0.05)	0.01 (0.04)	0.01 (0.03)
	MCD	0.01 (0.03)	0.02 (0.05)	-0.21 (0.06)	0.01 (0.04)	0.01 (0.04)
	Spearman	0.01 (0.04)	-	0.01 (0.04)	-	-
	Blomqvist	0.06 (0.09)	0.06 (0.09)	0.06 (0.08)	0.06 (0.09)	0.06 (0.08)
Cor 2	Standard	0.02 (0.03)	1.66 (0.66)	-0.11 (0.06)	0.04 (0.05)	0.02 (0.03)
	Kendall	0.02 (0.03)	0.03 (0.04)	0.02 (0.03)	0.02 (0.04)	0.02 (0.03)
	S	0.02 (0.03)	0.03 (0.04)	-0.11 (0.04)	0.02 (0.03)	0.02 (0.03)
	MCD	0.02 (0.03)	0.04 (0.05)	-0.11 (0.04)	0.03 (0.04)	0.02 (0.04)
	Spearman	0.02 (0.03)	-	0.02 (0.03)	-	-
	Blomqvist	0.05 (0.05)	0.05 (0.05)	0.05 (0.05)	0.05 (0.05)	0.05 (0.05)
Cor 3	Standard	0.02 (0.03)	1.17 (0.46)	-0.10 (0.05)	0.03 (0.04)	0.02 (0.03)
	Kendall	0.02 (0.03)	0.03 (0.04)	0.01 (0.03)	0.02 (0.03)	0.02 (0.03)
	S	0.02 (0.03)	0.03 (0.04)	-0.10 (0.03)	0.02 (0.03)	0.02 (0.03)
	MCD	0.02 (0.03)	0.04 (0.04)	-0.09 (0.04)	0.03 (0.04)	0.02 (0.03)
	Spearman	0.02 (0.03)	-	0.01 (0.03)	-	-
	Blomqvist	0.04 (0.04)	0.04 (0.05)	0.04 (0.04)	0.04 (0.04)	0.04 (0.04)
Cor 4	Standard	0.00 (0.03)	0.82 (0.35)	-0.11 (0.04)	0.01 (0.04)	0.00 (0.03)
	Kendall	0.00 (0.03)	0.01 (0.04)	0.00 (0.03)	0.00 (0.03)	0.00 (0.03)
	S	0.00 (0.03)	0.01 (0.04)	-0.09 (0.03)	0.00 (0.03)	0.00 (0.03)
	MCD	0.00 (0.03)	0.01 (0.04)	-0.10 (0.04)	0.01 (0.04)	0.01 (0.03)
	Spearman	0.00 (0.03)	-	0.00 (0.03)	-	-
	Blomqvist	0.01 (0.04)	0.01 (0.04)	0.01 (0.04)	0.01 (0.04)	0.01 (0.04)
		Canonical Directions				
Dir 1	Standard	0.04 (0.01)	0.64 (0.43)	0.05 (0.02)	0.06 (0.02)	0.05 (0.01)
	Kendall	0.05 (0.01)	0.08 (0.02)	0.05 (0.01)	0.05 (0.01)	0.05 (0.01)
	S	0.04 (0.01)	0.06 (0.02)	0.05 (0.01)	0.05 (0.01)	0.05 (0.01)
	MCD	0.05 (0.01)	0.07 (0.02)	0.06 (0.02)	0.06 (0.02)	0.05 (0.01)
	Spearman	0.05 (0.01)	-	0.05 (0.01)	-	-
	Blomqvist	0.12 (0.03)	0.12 (0.03)	0.12 (0.03)	0.11 (0.03)	0.12 (0.03)
Dir 2	Standard	0.24 (0.11)	1.25 (0.27)	0.41 (0.35)	0.38 (0.24)	0.27 (0.13)
	Kendall	0.25 (0.12)	0.35 (0.18)	0.25 (0.14)	0.27 (0.14)	0.26 (0.13)
	S	0.24 (0.13)	0.34 (0.19)	0.31 (0.19)	0.26 (0.13)	0.25 (0.13)
	MCD	0.25 (0.13)	0.40 (0.23)	0.40 (0.24)	0.32 (0.18)	0.30 (0.17)
	Spearman	0.26 (0.13)	-	0.25 (0.14)	-	-
	Blomqvist	0.46 (0.26)	0.43 (0.24)	0.43 (0.24)	0.43 (0.23)	0.45 (0.26)
Dir 3	Standard	0.40 (0.23)	1.25 (0.25)	0.67 (0.42)	0.59 (0.32)	0.45 (0.26)
	Kendall	0.42 (0.24)	0.56 (0.30)	0.41 (0.24)	0.45 (0.26)	0.44 (0.25)
	S	0.41 (0.23)	0.55 (0.30)	0.54 (0.33)	0.43 (0.24)	0.41 (0.23)
	MCD	0.43 (0.25)	0.63 (0.32)	0.66 (0.33)	0.54 (0.29)	0.50 (0.28)
	Spearman	0.42 (0.24)	-	0.41 (0.24)	-	-
	Blomqvist	0.67 (0.33)	0.65 (0.32)	0.64 (0.32)	0.65 (0.32)	0.67 (0.33)
Dir 4	Standard	0.38 (0.22)	1.26 (0.24)	0.60 (0.37)	0.56 (0.29)	0.43 (0.25)
	Kendall	0.39 (0.23)	0.53 (0.28)	0.39 (0.22)	0.43 (0.24)	0.42 (0.24)
	S	0.39 (0.22)	0.52 (0.28)	0.51 (0.30)	0.41 (0.23)	0.39 (0.22)
	MCD	0.41 (0.24)	0.59 (0.29)	0.63 (0.30)	0.51 (0.27)	0.46 (0.26)
	Spearman	0.40 (0.23)	-	0.39 (0.23)	-	-
	Blomqvist	0.60 (0.30)	0.61 (0.30)	0.59 (0.28)	0.61 (0.29)	0.62 (0.30)

Table S4: Average empirical bias (standard deviation) from 1000 simulations of the estimates of the first four canonical correlations and directions. Six different estimation techniques used with data simulated from five different elliptical and transelliptical distributions with p=q=16 and n=200.

		Canonical Correlations				
		Normal	Cauchy	Lognormal	t5	t10
Cor 1	Standard	0.10 (0.07)	2.43 (1.13)	-0.08 (0.18)	0.17 (0.10)	0.12 (0.09)
	Kendall	0.13 (0.08)	0.41 (0.20)	0.12 (0.08)	0.16 (0.09)	0.14 (0.09)
	S	0.10 (0.07)	0.19 (0.11)	-0.10 (0.12)	0.12 (0.08)	0.10 (0.08)
	MCD	0.14 (0.09)	0.25 (0.12)	-0.07 (0.14)	0.16 (0.09)	0.15 (0.10)
	Spearman	0.11 (0.09)	-	0.11 (0.08)	-	-
	Blomqvist	2.11 (0.39)	2.11 (0.40)	2.09 (0.43)	2.13 (0.39)	2.11 (0.41)
Cor 2	Standard	0.18 (0.06)	2.33 (0.69)	0.13 (0.10)	0.33 (0.10)	0.22 (0.06)
	Kendall	0.21 (0.06)	0.42 (0.09)	0.21 (0.06)	0.26 (0.07)	0.23 (0.06)
	S	0.19 (0.06)	0.36 (0.08)	0.10 (0.07)	0.22 (0.06)	0.20 (0.06)
	MCD	0.25 (0.07)	0.45 (0.08)	0.18 (0.07)	0.29 (0.07)	0.28 (0.07)
	Spearman	0.21 (0.06)	-	0.20 (0.06)	-	-
	Blomqvist	1.13 (0.58)	1.14 (0.58)	1.13 (0.58)	1.14 (0.59)	1.12 (0.57)
Cor 3	Standard	0.18 (0.05)	1.94 (0.52)	0.13 (0.06)	0.31 (0.06)	0.22 (0.05)
	Kendall	0.21 (0.05)	0.39 (0.07)	0.21 (0.05)	0.25 (0.05)	0.23 (0.05)
	S	0.19 (0.05)	0.36 (0.06)	0.12 (0.05)	0.22 (0.05)	0.20 (0.05)
	MCD	0.25 (0.05)	0.44 (0.07)	0.19 (0.05)	0.29 (0.05)	0.28 (0.05)
	Spearman	0.21 (0.05)	-	0.20 (0.05)	-	-
	Blomqvist	0.72 (0.16)	0.70 (0.15)	0.71 (0.17)	0.71 (0.16)	0.70 (0.14)
Cor 4	Standard	0.17 (0.04)	1.64 (0.40)	0.13 (0.05)	0.28 (0.05)	0.21 (0.04)
	Kendall	0.19 (0.04)	0.36 (0.05)	0.19 (0.04)	0.23 (0.04)	0.21 (0.04)
	S	0.17 (0.04)	0.34 (0.05)	0.12 (0.04)	0.20 (0.04)	0.18 (0.04)
	MCD	0.23 (0.04)	0.41 (0.06)	0.19 (0.04)	0.28 (0.05)	0.26 (0.04)
	Spearman	0.19 (0.04)	-	0.19 (0.04)	-	-
	Blomqvist	0.58 (0.09)	0.57 (0.09)	0.57 (0.09)	0.57 (0.10)	0.57 (0.09)
		Canonical Directions				
Dir 1	Standard	0.15 (0.03)	0.77 (0.42)	0.19 (0.08)	0.19 (0.05)	0.16 (0.04)
	Kendall	0.17 (0.04)	0.29 (0.07)	0.17 (0.03)	0.19 (0.04)	0.18 (0.04)
	S	0.15 (0.03)	0.22 (0.05)	0.18 (0.04)	0.16 (0.03)	0.16 (0.03)
	MCD	0.18 (0.04)	0.25 (0.06)	0.22 (0.06)	0.20 (0.04)	0.19 (0.04)
	Spearman	0.17 (0.03)	-	0.17 (0.03)	-	-
	Blomqvist	0.60 (0.19)	0.60 (0.19)	0.59 (0.19)	0.60 (0.19)	0.59 (0.18)
Dir 2	Standard	0.73 (0.28)	1.34 (0.20)	0.99 (0.37)	0.93 (0.30)	0.80 (0.28)
	Kendall	0.77 (0.28)	0.98 (0.27)	0.77 (0.28)	0.84 (0.29)	0.81 (0.28)
	S	0.73 (0.28)	0.97 (0.29)	0.92 (0.32)	0.80 (0.29)	0.76 (0.28)
	MCD	0.86 (0.29)	1.02 (0.28)	1.02 (0.31)	0.91 (0.29)	0.89 (0.29)
	Spearman	0.77 (0.29)	-	0.77 (0.28)	-	-
	Blomqvist	1.18 (0.23)	1.20 (0.22)	1.19 (0.24)	1.20 (0.24)	1.20 (0.23)
Dir 3	Standard	1.00 (0.29)	1.36 (0.18)	1.22 (0.25)	1.15 (0.26)	1.07 (0.28)
	Kendall	1.04 (0.28)	1.20 (0.24)	1.05 (0.29)	1.11 (0.26)	1.08 (0.27)
	S	1.01 (0.28)	1.20 (0.24)	1.18 (0.26)	1.08 (0.27)	1.04 (0.28)
	MCD	1.11 (0.26)	1.23 (0.23)	1.23 (0.23)	1.17 (0.26)	1.15 (0.26)
	Spearman	1.04 (0.28)	-	1.05 (0.29)	-	-
	Blomqvist	1.27 (0.20)	1.29 (0.20)	1.27 (0.20)	1.28 (0.20)	1.27 (0.20)
Dir 4	Standard	1.11 (0.26)	1.37 (0.18)	1.29 (0.21)	1.24 (0.22)	1.18 (0.25)
	Kendall	1.14 (0.25)	1.27 (0.21)	1.15 (0.25)	1.20 (0.24)	1.18 (0.24)
	S	1.11 (0.26)	1.26 (0.21)	1.27 (0.21)	1.19 (0.25)	1.15 (0.26)
	MCD	1.21 (0.23)	1.29 (0.20)	1.30 (0.20)	1.25 (0.22)	1.24 (0.22)
	Spearman	1.14 (0.25)	-	1.15 (0.25)	-	-
	Blomqvist	1.32 (0.18)	1.33 (0.17)	1.30 (0.19)	1.30 (0.19)	1.31 (0.19)

Table S5: Average empirical bias (standard deviation) from 1000 simulations of the estimates of the first four canonical correlations and directions. Six different estimation techniques used with data simulated from five different elliptical and transelliptical distributions with p=q=16 and n=1000.

		Canonical Correlations				
		Normal	Cauchy	Lognormal	t5	t10
Cor 1	Standard	0.02 (0.03)	2.46 (1.23)	-0.18 (0.10)	0.04 (0.05)	0.02 (0.04)
	Kendall	0.02 (0.04)	0.06 (0.06)	0.02 (0.04)	0.03 (0.04)	0.02 (0.04)
	S	0.02 (0.03)	0.03 (0.05)	-0.19 (0.05)	0.02 (0.04)	0.02 (0.03)
	MCD	0.02 (0.03)	0.05 (0.05)	-0.19 (0.06)	0.03 (0.04)	0.03 (0.04)
	Spearman	0.02 (0.04)	-	0.02 (0.04)	-	-
	Blomqvist	0.14 (0.10)	0.14 (0.10)	0.14 (0.10)	0.14 (0.10)	0.14 (0.10)
Cor 2	Standard	0.03 (0.03)	2.31 (0.76)	-0.09 (0.06)	0.08 (0.05)	0.04 (0.03)
	Kendall	0.04 (0.03)	0.07 (0.04)	0.04 (0.03)	0.04 (0.04)	0.04 (0.03)
	S	0.03 (0.03)	0.06 (0.04)	-0.09 (0.04)	0.04 (0.03)	0.03 (0.03)
	MCD	0.04 (0.03)	0.09 (0.05)	-0.07 (0.04)	0.06 (0.04)	0.05 (0.04)
	Spearman	0.04 (0.03)	-	0.04 (0.03)	-	-
	Blomqvist	0.11 (0.05)	0.11 (0.05)	0.11 (0.05)	0.11 (0.05)	0.11 (0.05)
Cor 3	Standard	0.04 (0.03)	1.87 (0.54)	-0.07 (0.04)	0.08 (0.04)	0.05 (0.03)
	Kendall	0.04 (0.03)	0.07 (0.04)	0.04 (0.03)	0.05 (0.03)	0.04 (0.03)
	S	0.04 (0.03)	0.06 (0.04)	-0.07 (0.03)	0.04 (0.03)	0.04 (0.03)
	MCD	0.04 (0.03)	0.09 (0.04)	-0.05 (0.03)	0.06 (0.03)	0.06 (0.03)
	Spearman	0.04 (0.03)	-	0.04 (0.03)	-	-
	Blomqvist	0.10 (0.04)	0.10 (0.04)	0.10 (0.04)	0.10 (0.04)	0.10 (0.04)
Cor 4	Standard	0.03 (0.03)	1.56 (0.43)	-0.07 (0.03)	0.06 (0.04)	0.04 (0.03)
	Kendall	0.03 (0.03)	0.06 (0.04)	0.03 (0.03)	0.04 (0.03)	0.04 (0.03)
	S	0.03 (0.03)	0.05 (0.03)	-0.06 (0.03)	0.03 (0.03)	0.03 (0.03)
	MCD	0.03 (0.03)	0.08 (0.04)	-0.04 (0.03)	0.05 (0.03)	0.05 (0.03)
	Spearman	0.03 (0.03)	-	0.03 (0.03)	-	-
	Blomqvist	0.08 (0.04)	0.08 (0.04)	0.08 (0.04)	0.08 (0.04)	0.08 (0.04)
		Canonical Directions				
Dir 1	Standard	0.06 (0.01)	0.80 (0.44)	0.08 (0.02)	0.09 (0.02)	0.07 (0.01)
	Kendall	0.07 (0.01)	0.11 (0.02)	0.07 (0.01)	0.08 (0.01)	0.07 (0.01)
	S	0.06 (0.01)	0.08 (0.02)	0.07 (0.01)	0.07 (0.01)	0.06 (0.01)
	MCD	0.07 (0.01)	0.10 (0.02)	0.09 (0.02)	0.08 (0.02)	0.08 (0.02)
	Spearman	0.07 (0.01)	-	0.07 (0.01)	-	-
	Blomqvist	0.17 (0.04)	0.17 (0.03)	0.17 (0.03)	0.17 (0.03)	0.17 (0.03)
Dir 2	Standard	0.29 (0.10)	1.35 (0.20)	0.51 (0.35)	0.47 (0.24)	0.34 (0.16)
	Kendall	0.31 (0.12)	0.44 (0.17)	0.31 (0.11)	0.34 (0.14)	0.33 (0.14)
	S	0.29 (0.11)	0.41 (0.18)	0.40 (0.22)	0.31 (0.13)	0.30 (0.13)
	MCD	0.31 (0.11)	0.50 (0.22)	0.51 (0.26)	0.40 (0.18)	0.37 (0.16)
	Spearman	0.31 (0.12)	-	0.31 (0.12)	-	-
	Blomqvist	0.54 (0.22)	0.55 (0.23)	0.54 (0.23)	0.54 (0.22)	0.54 (0.22)
Dir 3	Standard	0.46 (0.21)	1.37 (0.18)	0.80 (0.38)	0.73 (0.31)	0.53 (0.24)
	Kendall	0.50 (0.23)	0.67 (0.28)	0.49 (0.22)	0.55 (0.26)	0.51 (0.23)
	S	0.46 (0.21)	0.65 (0.29)	0.64 (0.32)	0.51 (0.24)	0.48 (0.22)
	MCD	0.49 (0.22)	0.76 (0.29)	0.79 (0.32)	0.64 (0.28)	0.59 (0.27)
	Spearman	0.50 (0.23)	-	0.49 (0.23)	-	-
	Blomqvist	0.79 (0.30)	0.80 (0.30)	0.79 (0.30)	0.79 (0.30)	0.78 (0.29)
Dir 4	Standard	0.48 (0.19)	1.37 (0.18)	0.82 (0.33)	0.75 (0.29)	0.54 (0.21)
	Kendall	0.52 (0.21)	0.69 (0.25)	0.51 (0.21)	0.57 (0.24)	0.52 (0.21)
	S	0.48 (0.19)	0.68 (0.25)	0.67 (0.27)	0.52 (0.22)	0.49 (0.20)
	MCD	0.51 (0.20)	0.80 (0.26)	0.86 (0.28)	0.65 (0.25)	0.61 (0.24)
	Spearman	0.51 (0.21)	-	0.51 (0.21)	-	-
	Blomqvist	0.83 (0.26)	0.81 (0.26)	0.82 (0.27)	0.81 (0.27)	0.80 (0.26)

1.2 Confidence intervals for canonical correlations and canonical direction loadings

This section reports confidence interval coverages for transelliptical CCA correlations and direction loadings. The simulation set-ups are the same as section 3.1 of the main text. Tables S6, and S7 report the coverages for bootstrap and asymptotic confidence intervals for the canonical correlations. For Table S6 p=q=4, and for Table S7 p=q=16, while the results for p=q=8 can be found in Table 2 of the main text. When p=q=4 the asymptotic confidence intervals perform comparably the bootstrap confidence intervals, while for p=q=16 the asymptotic confidence intervals perform poorly, particularly when n=200. This poor performance in the higher dimension setting is due to the finite sample bias increasing as dimension increases. Normal approximation bootstrap confidence intervals are able to help control for this bias. Tables S8 through S12 report the bootstrap and asymptotic confidence interval coverages for canonical direction loadings. Only the coverages for the X direction are reported with the coverages for the Y direction being nearly identical. In addition results for t distribution with 5 and 10 degrees of freedom are not reported, and are similar to results for the multivariate Cauchy and multivariate normal distribution. Table S8 reports coverages when p=q=4 and data are simulated from a multivariate normal distribution, and Table S9 reports coverages when p=q=4 and data are simulated from a multivariate Cauchy distribution. In both cases there is undercoverage for loadings higher order directions, which improves as sample size increases. Table S10 reports coverages when p=q=8 and data are simulated from a multivariate Cauchy distribution, while Table 3 in the main text reports coverages when p=q=8 and data are simulated from a multivariate normal distribution. Table S11 reports coverages when p=q=16 and data are simulated from a multivariate normal distribution, and Table S12 reports coverages when p=q=16 and data are simulated from a multivariate Cauchy distribution. An increase in dimension does not have a large effect on the confidence interval coverages for the direction loadings, unlike the confidence interval coverages for the canonical correlations which get much worse when p=q=16.

Table S6: Proportion of 1000 simulations in which the estimated confidence interval for the transelliptical canonical correlation contains the true transelliptical canonical correlation. Calculated for bootstrap and asymptotic confidence intervals using the transformed Kendall's estimator for first four canonical correlations with data simulated from five different elliptical and transelliptical distributions with p=q=4 and n=200 and 1000.

	Bootstrap Coverages					Asymptotic Coverages				
	Normal	Cauchy	Lognormal	t5	t10	Normal	Cauchy	Lognormal	t5	t10
Canonical Correlation	n=200									
1	0.94	0.88	0.94	0.91	0.93	0.94	0.83	0.92	0.89	0.90
2	0.92	0.92	0.92	0.91	0.91	0.89	0.86	0.91	0.88	0.90
3	0.90	0.92	0.93	0.91	0.91	0.97	0.98	0.97	0.97	0.97
4	0.84	0.76	0.84	0.80	0.81	0.94	0.95	0.96	0.96	0.95
	n=1000									
1	0.94	0.94	0.95	0.95	0.95	0.95	0.92	0.94	0.92	0.95
2	0.93	0.91	0.93	0.93	0.92	0.95	0.94	0.96	0.94	0.92
3	0.90	0.90	0.91	0.92	0.90	0.96	0.97	0.96	0.97	0.96
4	0.90	0.90	0.89	0.90	0.90	0.96	0.96	0.96	0.96	0.96

Table S7: Proportion of 1000 simulations in which the estimated confidence interval for the transelliptical canonical correlation contains the true transelliptical canonical correlation. Calculated for bootstrap and asymptotic confidence intervals using the transformed Kendall's estimator for first four canonical correlations with data simulated from five different elliptical and transelliptical distributions with p=q=16 and n=200 and 1000.

	Bootstrap Coverages					Asymptotic Coverages				
	Normal	Cauchy	Lognormal	t5	t10	Normal	Cauchy	Lognormal	t5	t10
Canonical Correlation	n=200									
1	0.86	0.53	0.86	0.80	0.83	0.56	0.18	0.56	0.47	0.53
2	0.84	0.81	0.83	0.83	0.82	0.14	0.00	0.14	0.05	0.08
3	0.82	0.87	0.87	0.88	0.84	0.07	0.00	0.07	0.03	0.06
4	0.86	0.91	0.86	0.90	0.87	0.11	0.00	0.11	0.04	0.06
	n=1000									
1	0.92	0.89	0.93	0.92	0.93	0.87	0.73	0.87	0.86	0.86
2	0.90	0.90	0.92	0.91	0.91	0.79	0.62	0.79	0.77	0.77
3	0.89	0.86	0.89	0.89	0.89	0.77	0.64	0.77	0.76	0.78
4	0.87	0.82	0.89	0.88	0.88	0.88	0.79	0.88	0.84	0.86

Table S8: Proportion of 1000 simulations in which the estimated confidence interval for the transelliptical canonical direction loading contains the true canonical direction loading. Calculated for bootstrap and asymptotic confidence intervals using the transformed Kendall's estimator for the first four canonical directions with data simulated from the multivariate normal distribution with p=q=4 and n=200 and 1000.

	Bootstrap Coverages				Asymptotic Coverages			
	Dir 1	Dir 2	Dir 3	Dir 4	Dir 1	Dir 2	Dir 3	Dir 4
Variable	n=200							
1	1.00	0.95	0.93	0.95	1.00	0.94	0.95	0.95
2	0.96	0.98	0.80	0.89	0.95	0.96	0.90	0.79
3	0.96	0.78	0.97	0.69	0.97	0.89	0.94	0.68
4	0.96	0.89	0.72	0.97	0.97	0.84	0.75	0.94
	n=1000							
1	1.00	0.95	0.95	0.95	1.00	0.95	0.94	0.95
2	0.94	1.00	0.92	0.95	0.95	1.00	0.94	0.92
3	0.96	0.92	0.99	0.85	0.97	0.93	0.99	0.86
4	0.96	0.94	0.86	0.99	0.97	0.93	0.87	0.99

Table S9: Proportion of 1000 simulations in which the estimated confidence interval for the transelliptical canonical direction loading contains the true canonical direction loading. Calculated for bootstrap and asymptotic confidence intervals using the transformed Kendall's estimator for the first four canonical directions with data simulated from the multivariate Cauchy distribution with p=q=4 and n=200 and 1000.

	Bootstrap Coverages				Asymptotic Coverages			
	Dir 1	Dir 2	Dir 3	Dir 4	Dir 1	Dir 2	Dir 3	Dir 4
Variable	n=200							
1	1.00	0.96	0.96	0.96	1.00	0.94	0.94	0.94
2	0.96	0.96	0.73	0.85	0.97	0.94	0.83	0.77
3	0.95	0.71	0.95	0.62	0.95	0.88	0.92	0.66
4	0.95	0.81	0.65	0.95	0.96	0.81	0.76	0.94
	n=1000							
1	1.00	0.96	0.95	0.94	1.00	0.94	0.94	0.94
2	0.95	1.00	0.88	0.93	0.96	0.99	0.94	0.89
3	0.95	0.88	0.99	0.77	0.95	0.94	0.98	0.79
4	0.95	0.92	0.80	0.98	0.96	0.90	0.83	0.98

Table S10: Proportion of 1000 simulations in which the estimated confidence interval for the transelliptical canonical direction loading contains the true canonical direction loading. Calculated for bootstrap and asymptotic confidence intervals using the transformed Kendall's estimator for the first four canonical directions with data simulated from the multivariate Cauchy distribution with p=q=8 and n=200 and 1000.

	Bootstrap Coverages				Asymptotic Coverages			
	Dir 1	Dir 2	Dir 3	Dir 4	Dir 1	Dir 2	Dir 3	Dir 4
Variable	n=200							
1	1.00	0.95	0.94	0.91	1.00	0.93	0.93	0.95
2	0.97	0.93	0.70	0.82	0.97	0.89	0.84	0.74
3	0.97	0.66	0.89	0.60	0.99	0.93	0.92	0.85
4	0.97	0.80	0.61	0.90	0.98	0.84	0.81	0.89
5	0.96	0.92	0.87	0.81	0.99	0.98	0.96	0.97
6	0.96	0.92	0.86	0.84	0.99	0.99	0.98	0.99
7	0.96	0.91	0.88	0.85	1.00	0.99	0.98	0.98
8	0.96	0.89	0.88	0.82	0.99	0.99	0.97	0.97
	n=1000							
1	1.00	0.94	0.95	0.95	1.00	0.95	0.95	0.94
2	0.95	1.00	0.89	0.89	0.96	0.99	0.93	0.89
3	0.94	0.88	0.99	0.77	0.97	0.93	0.98	0.80
4	0.95	0.90	0.78	0.99	0.98	0.92	0.82	0.98
5	0.95	0.95	0.93	0.96	0.99	0.99	0.99	0.99
6	0.96	0.95	0.94	0.96	0.99	0.99	0.99	1.00
7	0.95	0.95	0.95	0.95	0.99	1.00	0.99	1.00
8	0.94	0.94	0.95	0.96	0.99	0.99	0.99	0.99

Table S11: Proportion of 1000 simulations in which the estimated confidence interval for the transelliptical canonical direction loading contains the true canonical direction loading. Calculated for bootstrap and asymptotic confidence intervals using the transformed Kendall's estimator for the first four canonical directions with data simulated from the multivariate normal distribution with p=q=16 and n=200 and 1000.

	Bootstrap Coverages				Asymptotic Coverages			
	Dir 1	Dir 2	Dir 3	Dir 4	Dir 1	Dir 2	Dir 3	Dir 4
Variable	n=200							
1	1.00	0.96	0.94	0.91	0.98	0.94	0.93	0.94
2	0.98	0.93	0.69	0.83	0.98	0.83	0.88	0.76
3	0.97	0.67	0.80	0.65	0.99	0.98	0.85	0.90
4	0.98	0.79	0.59	0.71	0.99	0.87	0.83	0.83
5	0.98	0.91	0.86	0.80	1.00	0.99	0.97	0.99
6	0.98	0.92	0.87	0.79	1.00	1.00	0.99	1.00
7	0.98	0.92	0.84	0.79	1.00	1.00	1.00	1.00
8	0.97	0.92	0.84	0.80	1.00	1.00	1.00	1.00
9	0.98	0.94	0.89	0.77	1.00	1.00	1.00	1.00
10	0.97	0.92	0.86	0.80	1.00	1.00	1.00	1.00
11	0.98	0.92	0.88	0.78	1.00	1.00	1.00	1.00
12	0.98	0.93	0.88	0.77	1.00	1.00	1.00	1.00
13	0.98	0.91	0.87	0.75	1.00	1.00	1.00	1.00
14	0.97	0.93	0.85	0.80	1.00	1.00	1.00	1.00
15	0.98	0.92	0.88	0.80	1.00	1.00	1.00	1.00
16	0.98	0.93	0.87	0.79	1.00	1.00	1.00	1.00
	n=1000							
1	1.00	0.96	0.95	0.95	0.99	0.95	0.95	0.94
2	0.96	1.00	0.92	0.93	0.96	0.96	0.96	0.92
3	0.96	0.92	0.99	0.86	0.96	0.95	0.87	0.97
4	0.95	0.95	0.85	0.99	0.97	0.96	0.94	0.86
5	0.95	0.94	0.94	0.95	0.99	0.99	1.00	1.00
6	0.96	0.95	0.95	0.96	1.00	1.00	1.00	1.00
7	0.95	0.94	0.95	0.96	1.00	1.00	1.00	1.00
8	0.95	0.94	0.95	0.94	1.00	1.00	1.00	1.00
9	0.95	0.95	0.95	0.95	1.00	1.00	1.00	1.00
10	0.96	0.95	0.95	0.95	1.00	1.00	1.00	1.00
11	0.95	0.96	0.95	0.95	1.00	1.00	1.00	1.00
12	0.96	0.95	0.94	0.96	1.00	1.00	1.00	1.00
13	0.96	0.96	0.94	0.94	1.00	1.00	1.00	1.00
14	0.95	0.95	0.94	0.95	1.00	1.00	1.00	1.00
15	0.95	0.95	0.95	0.95	1.00	1.00	1.00	1.00
16	0.96	0.94	0.96	0.95	1.00	1.00	1.00	1.00

Table S12: Proportion of 1000 simulations in which the estimated confidence interval for the transelliptical canonical direction loading contains the true canonical direction loading. Calculated for bootstrap and asymptotic confidence intervals using the transformed Kendall's estimator for the first four canonical directions with data simulated from the multivariate Cauchy distribution with p=q=16 and n=200 and 1000.

	Bootstrap Coverages				Asymptotic Coverages			
	Dir 1	Dir 2	Dir 3	Dir 4	Dir 1	Dir 2	Dir 3	Dir 4
Variable	n=200							
1	1.00	0.99	0.96	0.93	0.98	0.91	0.91	0.91
2	0.99	0.86	0.66	0.78	0.97	0.68	0.81	0.74
3	1.00	0.64	0.73	0.66	1.00	0.97	0.77	0.94
4	1.00	0.72	0.65	0.58	0.99	0.89	0.88	0.77
5	1.00	0.88	0.82	0.76	1.00	0.99	0.98	0.99
6	1.00	0.86	0.82	0.77	1.00	1.00	0.98	0.99
7	1.00	0.89	0.81	0.77	1.00	1.00	0.99	1.00
8	1.00	0.87	0.84	0.77	1.00	1.00	1.00	1.00
9	0.99	0.88	0.82	0.78	1.00	1.00	1.00	1.00
10	1.00	0.90	0.84	0.76	1.00	1.00	1.00	1.00
11	1.00	0.89	0.82	0.76	1.00	1.00	1.00	1.00
12	0.99	0.89	0.83	0.77	1.00	1.00	1.00	1.00
13	1.00	0.88	0.83	0.76	1.00	1.00	1.00	1.00
14	0.99	0.89	0.83	0.80	1.00	1.00	1.00	1.00
15	1.00	0.91	0.82	0.77	1.00	1.00	1.00	1.00
16	1.00	0.88	0.82	0.76	1.00	1.00	1.00	1.00
	n=1000							
1	1.00	0.95	0.93	0.94	1.00	0.94	0.94	0.93
2	0.97	0.99	0.87	0.90	0.96	0.96	0.93	0.89
3	0.95	0.86	0.98	0.71	0.98	0.95	0.92	0.84
4	0.95	0.92	0.74	0.98	0.98	0.92	0.84	0.94
5	0.95	0.95	0.92	0.94	0.99	1.00	0.99	1.00
6	0.94	0.95	0.94	0.93	1.00	1.00	1.00	1.00
7	0.96	0.94	0.92	0.94	1.00	1.00	1.00	1.00
8	0.95	0.94	0.93	0.94	1.00	1.00	1.00	1.00
9	0.95	0.94	0.93	0.94	1.00	1.00	1.00	1.00
10	0.94	0.94	0.92	0.94	1.00	1.00	1.00	1.00
11	0.94	0.94	0.94	0.93	1.00	1.00	1.00	1.00
12	0.93	0.94	0.93	0.92	1.00	1.00	1.00	1.00
13	0.94	0.94	0.93	0.93	1.00	1.00	1.00	1.00
14	0.96	0.93	0.95	0.93	1.00	1.00	1.00	1.00
15	0.95	0.94	0.93	0.93	1.00	1.00	1.00	1.00
16	0.96	0.93	0.91	0.95	1.00	1.00	1.00	1.00

2. ADDITIONAL RESULTS FOR WHITE MATTER TRACTOGRAPHY DATA AND EXECUTIVE FUNCTION IN SIX YEAR OLD CHILDREN

This section reports additional results using transelliptical CCA and standard CCA to investigate the relationship between white matter structure and executive function in six year old children. A description of the data set and the white matter tracts and executive function (EF) tests that are used can be found in Section 5 of the main text. Table S13 reports the first canonical direction and correlation using average fractal anisotropy (FA) for each of the white matter tracts. Analysis methods are the same to the results reported in the main text using average radial diffusivity (RD) instead of average FA. The results for transelliptical CCA using average RD values can be found in Table 6 of the main text. The sign of the direction loadings for all of the EF tests except CANTAB Stockings of Cambridge (SOC) are the same between transelliptical CCA using average FA values for the white matter tracts and average RD values. In both cases the Stanford Binet Verbal Fluid Reasoning Score (SB V) has the loading with the highest absolute value. The transelliptical CCA direction loadings for average FA values have the opposite sign of the first direction loadings for average RD values in 11 of the 16 tracts. This is to be expected because RD tends to decrease as myelination increases, while FA tends to increase as myelination increases, so we would expect the effects of FA and RD to go in opposite directions. The loadings for the EF variables and average FA values are similar between transelliptical CCA and standard CCA, and the jackknife corrected correlation for both is close to 0.35. Table S14 reports the results for the first canonical correlation and direction using average AD for each of the white matter tracts. The same methods are used as the analysis using average RD and average FA. Transelliptical CCA did not find a significant direction, and the first direction using standard CCA is only marginally significant (p=0.06).

Table S13: Estimates for first canonical correlation direction loadings and correlations along with jackknife corrected correlation and bootstrap p-values for canonical correlation analysis between fractional anisotropy measures for 20 white matter tracts and five executive function tests in six-year-old children. Estimated for transelliptical canonical correlation analysis using the transformed Kendall's estimator as well as standard canonical correlation analysis.

DTI Vars	Transelliptical CCA Loadings	Boot CI	Asymp CI	Standard CCA Loadings	Boot CI
ARC FT Left	-0.57	(-1.50, 0.03)	(-1.15, 0.00)	-0.11	(-0.74, 0.47)
ARC FT Right	-0.05	(-1.04, 0.95)	(-0.99, 0.89)	-0.08	(-0.80, 0.59)
ARC FP Left	0.13	(-0.42, 0.72)	(-0.36, 0.63)	-0.08	(-0.64, 0.42)
ARC FP Right	-0.27	(-1.07, 0.27)	(-0.89, 0.35)	-0.38	(-0.94, -0.08)
ARC TP Left	0.15	(-0.28, 0.64)	(-0.25, 0.55)	-0.10	(-0.57, 0.26)
ARC TP Right	0.10	(-0.41, 0.71)	(-0.39, 0.58)	0.14	(-0.23, 0.61)
CGC Left	-0.09	(-0.84, 0.51)	(-0.71, 0.53)	-0.44	(-1.30, 0.11)
CGC Right	0.05	(-0.50, 0.68)	(-0.44, 0.55)	0.36	(-0.10, 1.07)
CTPF Left	0.01	(-0.62, 0.57)	(-0.50, 0.52)	-0.23	(-0.79, 0.17)
CTPF Right	-0.29	(-1.02, 0.29)	(-0.81, 0.23)	-0.25	(-0.82, 0.16)
Genu	0.16	(-0.55, 1.00)	(-0.50, 0.81)	0.42	(-0.07, 1.26)
ILF Left	-0.06	(-1.01, 0.79)	(-0.85, 0.74)	0.18	(-0.40, 0.86)
ILF Right	1.02	(0.41, 2.26)	(0.28, 1.76)	0.51	(-0.03, 1.36)
IFOF Left	-0.09	(-1.02, 0.76)	(-0.90, 0.71)	-0.52	(-1.38, -0.01)
IFOF Right	-0.55	(-1.79, 0.53)	(-1.66, 0.57)	0.10	(-0.70, 0.91)
SLF Left	0.10	(-0.42, 0.70)	(-0.38, 0.57)	0.06	(-0.39, 0.55)
SLF Right	0.09	(-0.52, 0.85)	(-0.59, 0.77)	0.24	(-0.29, 0.99)
Splenium	0.68	(0.38, 1.51)	(0.29, 1.08)	0.72	(0.47, 1.43)
UNC Left	-0.22	(-0.99, 0.39)	(-0.76, 0.32)	0.07	(-0.48, 0.65)
UNC Right	0.04	(-0.83, 0.78)	(-0.74, 0.81)	-0.25	(-0.89, 0.28)
EF Vars					
SB V	0.98	(0.64, 1.87)	(0.74, 1.22)	0.85	(0.48, 1.55)
SB NV	-0.70	(-1.43, -0.28)	(-1.08, -0.31)	-0.38	(-1.05, 0.15)
Brief	-0.38	(-0.96, -0.05)	(-0.72, -0.04)	-0.52	(-1.19, -0.14)
SOC	0.09	(-0.51, 0.73)	(-0.39, 0.57)	0.27	(-0.22, 0.92)
SSPSpan	-0.17	(-1.05, 0.74)	(-0.95, 0.61)	-0.21	(-0.90, 0.40)
Cor	0.55			0.47	
Jackknife Cor	0.34			0.31	
Pval	0.04			0.01	
N	214			214	

Table S14: Estimates for first canonical correlation direction loadings and correlations along with jackknife corrected correlation and bootstrap p-values for canonical correlation analysis between axial diffusivity measures for 20 white matter tracts and five executive function tests in six-year-old children. Estimated for transelliptical canonical correlation analysis using the transformed Kendall's estimator as well as standard canonical correlation analysis.

DTI Vars	Transelliptical CCA Loadings	Boot CI	Asymp CI	Standard CCA Loadings	Boot CI
ARC FT Left	0.33	(-0.45, 1.34)	(-0.42, 1.08)	0.14	(-0.40, 0.77)
ARC FT Right	0.22	(-0.79, 1.56)	(-0.89, 1.33)	0.00	(-0.67, 0.68)
ARC FP Left	-0.42	(-1.76, 0.35)	(-1.50, 0.66)	-0.11	(-0.74, 0.42)
ARC FP Right	-0.29	(-1.29, 0.25)	(-0.88, 0.29)	-0.40	(-1.12, 0.02)
ARC TP Left	0.19	(-0.44, 1.09)	(-0.38, 0.75)	0.11	(-0.27, 0.59)
ARC TP Right	-0.16	(-1.04, 0.46)	(-0.60, 0.28)	-0.24	(-0.73, 0.02)
CGC Left	-0.50	(-1.54, 0.28)	(-1.26, 0.27)	-0.62	(-1.45, -0.33)
CGC Right	0.21	(-0.49, 1.06)	(-0.62, 1.03)	0.42	(0.02, 1.20)
CTPF Left	-0.01	(-0.72, 0.91)	(-0.65, 0.63)	0.22	(-0.13, 0.75)
CTPF Right	0.32	(-0.93, 1.57)	(-0.71, 1.35)	-0.15	(-0.71, 0.33)
Genu	0.45	(-0.43, 1.95)	(-0.77, 1.68)	0.72	(0.43, 1.59)
ILF Left	-0.47	(-2.13, 1.76)	(-1.26, 0.33)	0.05	(-0.51, 0.63)
ILF Right	0.75	(-1.19, 2.54)	(-0.46, 1.97)	0.33	(-0.07, 0.99)
IFOF Left	0.55	(-1.70, 2.29)	(-0.25, 1.36)	-0.08	(-0.69, 0.49)
IFOF Right	-1.23	(-3.44, 0.68)	(-2.42, -0.05)	-0.66	(-1.49, -0.39)
SLF Left	-0.18	(-1.12, 0.59)	(-0.95, 0.59)	-0.30	(-0.85, 0.00)
SLF Right	0.49	(0.12, 1.44)	(-0.06, 1.05)	0.59	(0.39, 1.27)
Splenium	0.03	(-0.69, 0.83)	(-0.74, 0.80)	0.00	(-0.42, 0.42)
UNC Left	-0.42	(-1.36, 0.67)	(-1.05, 0.22)	0.26	(-0.21, 0.94)
UNC Right	0.26	(-1.08, 1.39)	(-0.46, 0.98)	0.07	(-0.40, 0.58)
EF Vars					
SB V	0.42	(-0.30, 1.35)	(-0.45, 1.29)	0.40	(-0.13, 1.13)
SB NV	-0.34	(-1.14, 0.37)	(-1.00, 0.33)	-0.13	(-0.87, 0.54)
Brief	-0.38	(-1.27, 0.20)	(-1.49, 0.73)	-0.78	(-1.70, -0.39)
SOC	0.22	(-0.32, 1.05)	(-0.43, 0.86)	0.14	(-0.38, 0.76)
SSPSpan	0.73	(0.08, 2.01)	(-0.08, 1.54)	0.30	(-0.41, 1.21)
Cor	0.53			0.44	
Jackknife Cor	0.00			0.22	
Pval	0.60			0.06	
N	214			214	