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Supplementary Information for

Rational policymaking during a pandemic

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Methods supplement to Figure 2: deciding about school closures and their length during COVID-19 pandemic

Overview

This Appendix supports the case study presented in Figure 2 of the manuscript. Its purpose is to show how decision theory could be used in a context where distinct model projections exist. Using a simple example of a decision problem of school closures during the COVID-19 pandemic, we highlight what are the resulting recommendations from different formal decision rules in terms of policy responses.

It is important to note that the decision problem presented in this case study is necessarily simplistic and should be used for demonstrative purposes only.

1. Background

Context By end of April 2020, 191 countries had implemented national school closures in response to the COVID-19 pandemic (1). Yet the effectiveness of such a measure is highly uncertain, due to the lack of data on the relative contribution of school closures to transmission control, and conflicting modelling results (2).

The policymaker's problem Decisions about closures and their length involve a series of trade-offs. The policy assessment thus involves weighting benefits and costs of alternative courses of action. On the one hand, school closures can slow the pandemic and its impact by reducing child-child transmission, thus delaying the pandemic peak that overwhelms health care services, and therefore ultimately reducing morbidity and associated mortality. If this is the case, such interventions bring clear health benefits for the society and avoid unsustainable demands on the health system. On the other hand, school closures can have high direct and indirect health and socio-economic costs. For example, they may increase child-adult transmission, reduce the ability of healthcare and key workers to work and thus reduce the capacity of healthcare (3, 4). Economic costs of lengthy school closures are also high (5–7), generated for example through absenteeism by working parents, loss of education, etc.

Uncertainty The evidence supporting national closure of schools in the COVID-19 pandemic context was very weak. In particular, evidence of COVID-19 transmission through child-child contact or through schools was not available at the time of decision (2). As a consequence, it was unclear whether school closures would be effective in the COVID-19 pandemic (4).

Framework We use a framework that decomposes uncertainty into distinct layers of analysis: (i) uncertainty *within* models (also called *risk*, *aleatory* uncertainty, or *physical* uncertainty), (ii) uncertainty *across* models (also called *model ambiguity*, or *model uncertainty*), and (iii) uncertainty *about* models (also called *model misspecification*).[*]

We consider a general decision problem in which consequences depend on the states of the environment that are viewed as realizations of an underlying economic or physical generative mechanism (10). A *model* is a probability distribution induced by such a mechanism. It describes states' variability by combining a structural component based on theoretical knowledge (e.g. economic or physical) and a random component coming from, for example, measurement errors or minor omitted explanatory variables (13; 14). We assume that decision makers (DMs) posit a collection of such models. Uncertainty *across* model therefore results from the uncertainty about the true underlying mechanism: within the posited collection, there is uncertainty about which model actually governs states' realizations. However, even after a model is specified, there is still uncertainty *within* model, i.e. about which specific state will actually obtain; this is the notion of *risk* typically considered in economics. Finally, the third layer of uncertainty (*about* models), arises as the true model might not belong to the posited collection of models, reflecting the idea that all posited models have an inherent approximate nature.

2. Decision making under uncertainty

A. The structure of a decision problem. The general problem that a DM, in particular a policymaker, faces is to choose an *action* a within a set A of possible alternative actions, whose *consequences* $c \in C$ depend on the realization of a *state of the environment* $s \in S$ which is outside the DM's control. The relationship among consequences, actions and states is described by a consequence function $\rho : A \times S \to C$, where $c = \rho(a, s)$ is the consequence of action a when state s obtains. DMs have a (complete and transitive) *preference relation* $\succsim$ over actions that describes how they rank the different alternative actions.[†] The quintet $(A, S, C, \rho, \succsim)$ characterizes the decision problem under uncertainty. The aim of the DM is to select the action $\hat{a}$ that is *optimal* according to her preference, that is, such that $\hat{a} \succsim a$ for all actions $a \in A$. The preference $\succsim$ is assumed to admit a numerical representation via a *decision criterion* $V : A \to \mathbb{R}$, with

$$a \succsim b \Longleftrightarrow V(a) \geq V(b)$$

for all actions $a, b \in A$. This numerical representation permits to formulate the decision problem as an optimization problem

$$\max_a V(a) \quad \text{sub } a \in A. \qquad [1]$$

Optimal actions $\hat{a}$ are the solutions of this problem. To find an optimal action thus amounts to solve this optimization problem.

[*] See (8–11) for a discussion, and (12) for empirical evidence on the distinction between these layers.

[†] As is usual, we write $a \succsim b$ if the DM prefers action a to action b (i.e., either strictly prefers action a to action b, $a \succ b$, or is indifferent between the two, $a \sim b$).

The DM may address, especially in policy problems, state uncertainty through the guise of models. Based on ex ante scientific and socio-economic information, the DM might be able to posit a set of probability models $M \subseteq \Delta(S)$ describing the likelihoods of the different states. This set of models is taken as a datum of the decision problem, which is now characterized by a sextet $(A, S, C, \rho, \succsim, M)$. It is often assumed, following (15), that the correct model belongs to the set of models that the DM posits, thus abstracting from model misspecification issues.

B. Decision criteria. The form of the decision criterion V determines the nature of decision problem [1]. Different possible criteria have been proposed in the literature. The ones we consider are: the subjective expected utility (SEU) criterion, which dates back to the seminal works of (15–18), and has recently been revisited by (19) to accommodate explicitly the presence of uncertainty across models; the maxmin criterion of (15); the smooth ambiguity criterion, developed by (20); and the multiple priors criterion proposed by (21). For the sake of brevity, the more general α-versions of the maxmin criterion (22) and of the multiple priors criterion (23) are only discussed in the main text.

C. Making decisions in a pandemic.

C.1. States and consequences. With the letter R we denote a rate of contagion within a given population, i.e., the average number of individuals infected per single case. The baseline rate of contagion, denoted by R_0, is called *basic reproduction number*. It applies to a population never exposed to the virus, where everyone is susceptible,[‡] and depends on the biology of the virus as well as on the natural (pre-pandemic) socio-economic structure that characterizes the population (24). The biology of the virus determines its ability to infect (i.e., the probability of infection per interaction) and the duration of infectiousness.[§] The natural socio-economic structure determines the natural social distancing and, through it, the average number of interactions per individual (25, 26). For instance, natural social distancing might be higher in Northern than in Southern European countries.

These natural factors, biological and socio-economic, determine R_0. It is the natural, *ex ante*, rate at which the pandemic progresses, without private and public decisions that respond to it. Ex post, after these decisions are put in place and affect the biological and socio-economic factors that determine R_0, the relevant rate of contagion becomes the *effective reproduction* (or *reproductive*) *number* R_e (27). For example, school closure is a public decision that may increase social distancing (a socio-economic factor), while a diligent use of protective gear is a private decision that may decrease the virus ability to infect (a biological factor).

Here we focus on public decisions, policies, and assume that private ones are subsumed by them.[¶] A policy translates a basic reproduction number R_0 into an effective one R_e. Yet, how this translation occurs often remains uncertain. For instance, evidence on the effectiveness of school closure policies for COVID-19 comes from influenza outbreaks, but the ability of children to transmit the disease greatly varies across coronaviruses (28). For this reason, we represent how a policy a maps R_0 into R_e via the relation $R_e = f(a, R_0, \theta_r, \varepsilon_r)$, where θ_r is a structural parameter and ε_r is a shock.[‖] We assume that $\partial f/\partial a \leq 0$ and $\partial f/\partial R_0 > 0$. For example, if the relation is linear we have

$$R_e = \theta_{r,1} a + \theta_{r,2} R_0 + \varepsilon_r, \qquad [2]$$

with $\theta_{r,1} \leq 0$ and $\theta_{r,2} > 0$.[**] In the baseline scenario without policy intervention – i.e., when $a = 0$ – the effective reproduction number R_e is determined by: (i) the natural proportion $\theta_{r,2}$ of the population that is susceptible, (ii) the basic reproduction number R_0 that summarizes the biological and socio-economic factors previously discussed, (iii) a shock ε_r that accounts for minor omitted variables. The economic damage D, in monetary terms (e.g., loss of GDP)[††], associated with the pandemic is determined by the rate of contagion R_e via a function $D = g(R_e, \theta_d, \varepsilon_d)$, where θ_d is a structural parameter and ε_d is a shock. This function represents the ability of health and economic systems to cope with the pandemic. We assume that $\partial g/\partial R_e > 0$. For example, assuming a quadratic damage function we have:

$$D = \theta_{d,1} R_e^2 + \theta_{d,2} R_e + \varepsilon_d, \qquad [3]$$

with $2\theta_{d,1} R_e + \theta_{d,2} > 0$. The economic damage D associated with a policy a is then $D = g(R_e, \theta_d, \varepsilon_d) = g(f(a, R_0, \theta_r, \varepsilon_r), \theta_d, \varepsilon_d)$. A policy affects, according to relation f, the effective reproduction number and, through it, determines an economic damage according to relation g.

In the linear-quadratic example, we have

$$D = \kappa_1 a^2 + \kappa_2 a + \kappa_3, \qquad [4]$$

where

$$\kappa_1 = \theta_{d,1} \theta_{r,1}^2 \qquad [5]$$

$$\kappa_2 = \theta_{r,1} \left(2\theta_{d,1} \left(\theta_{r,2} R_0 + \varepsilon_r\right) + \theta_{d,2}\right) \qquad [6]$$

$$\kappa_3 = \theta_{d,1} \left(\theta_{r,2} R_0 + \varepsilon_r\right)^2 + \theta_{d,2} \left(\theta_{r,2} R_0 + \varepsilon_r\right) + \varepsilon_d. \qquad [7]$$

[‡] An individual is *susceptible* if has no immune protection against the virus.

[§] By *interaction* we mean a contact amenable to virus transmission (in terms of closeness and duration).

[¶] A highly non-trivial assumption that, for instance, requires people to use diligently protective gear if asked by local or national authorities.

[‖] Throughout, shocks have zero mean and unit variance.

[**] For simplicity, we allow R_e to be negative (otherwise, we should add constraints that preserve the positivity of R_e, something that we prefer to abstract from).

[††] The scalar D lumps together economic consequences and health effects (through a monetary measure, e.g. GDP loss). Yet, in principle D may be a multidimensional vector with different components, say health and economic ones, that the decision criterion then trades off.

Policy a has an uncertain implementation cost C that, for instance, for school closures includes, as previously mentioned, absenteeism by working parents and loss of education. This cost is represented by a function $\mathsf{C} = h(a, \theta_{\mathsf{c}}, \varepsilon_{\mathsf{c}})$. We assume that costs grow more than proportionally, so that $\partial h/\partial a > 0$ and $\partial^2 h/\partial a^2 > 0$ (e.g., the cost of school closures grows more than proportionally with its duration). For example, a quadratic cost function is

$$\mathsf{C} = \theta_{\mathsf{c},1}a^2 + \theta_{\mathsf{c},2}a + \varepsilon_{\mathsf{c}}, \tag{8}$$

with $2\theta_{\mathsf{c},1}a + \theta_{\mathsf{c},2} > 0$ and $\theta_{\mathsf{c},1} > 0$. We also assume that the policymaker knows the functional forms of the relations f, g and h (e.g., whether they are linear or quadratic) but not their structural parameters. This lack of knowledge, along with that of the basic reproduction number R_0 and of the shocks' value ε, prevents the DM to know the actions' consequences. States thus have the form $s = (R_0, \varepsilon, \theta) \in S$ with both random and structural components. In particular, the vector $\varepsilon = (\varepsilon_r, \varepsilon_d, \varepsilon_{\mathsf{c}}) \in E$ represents the shocks affecting the health and economic systems, while the vector $\theta = (\theta_r, \theta_d, \theta_{\mathsf{c}}) \in \Theta$ specifies the structural coefficients parametrizing a model population. If we denote by $B = -D$ the benefit of policy a as its ability to reduce the economic damages due to the pandemic, its consequence is the difference $B - \mathsf{C}$ between its benefits and costs. The consequence function is then $\rho(a, \varepsilon, \theta) = -g(f(a, R_0, \theta_r, \varepsilon_r), \theta_d, \varepsilon_d) - h(a, \theta_{\mathsf{c}}, \varepsilon_{\mathsf{c}})$. In the linear-quadratic example, it becomes

$$\rho(a, \varepsilon, \theta) = -(\kappa_1 + \theta_{\mathsf{c},1})a^2 - (\kappa_2 + \theta_{\mathsf{c},2})a - \kappa_3 + \varepsilon_{\mathsf{c}}$$

C.2. Models and beliefs. Shocks have the form $\varepsilon_r = \sigma_r w_r$; $\varepsilon_d = \sigma_d w_d$; $\varepsilon_{\mathsf{c}} = \sigma_{\mathsf{c}} w_{\mathsf{c}}$, where w_r, w_d and w_{c} are uncorrelated "white noises" with zero mean and unit variance. The vector parameter $\sigma = (\sigma_r, \sigma_d, \sigma_{\mathsf{c}}) \in \Sigma$ then specifies the standard deviations of shocks. To ease the analysis, we assume that their distribution q_σ is known, up to their standard deviations σ. We also assume that the distribution p_ξ of the rate R_0 is indexed by a parameter $\xi \in \Xi$ that accounts for different epidemiological views on the quantification of the basic reproduction number. With this, the positive scalar $m(\varepsilon, \theta, R_0)$ gives the joint probability of shock ε, parameter θ and rate R_0 under a posited model $m \in M$. We adopt the model factorization $m = q_\sigma \times \delta_\theta \times p_\xi$, that is,[‡‡]

$$m\left(\varepsilon, \theta', R_0\right) = \begin{cases} q_\sigma(\varepsilon)\, p_\xi(R_0) & \text{if } \theta' = \theta \\ 0 & \text{else} \end{cases} \tag{9}$$

where $q_\sigma(\varepsilon)$ is the probability of shock ε under the standard deviation specification σ, while $p_\xi(R_0)$ is the probability that R_0 is the basic reproduction number according to epidemiological view ξ. We can thus index models as $m_{\theta,\sigma,\xi} = q_\sigma \times \delta_\theta \times p_\xi$ and denote by $M = \{m_{\theta,\sigma,\xi}\}$ the set of models that the policymaker posits. Because of the factorization, the policymaker's subjective belief $\mu(m)$ that m is the correct model is actually over the values of θ, σ and ξ and so has the form $\mu(\theta, \sigma, \xi)$. A convenient separable form is, with an abuse of notation, $\mu(\theta, \xi) = \mu(\theta, \sigma)\,\mu(\xi)$.

For example, consider the pandemic decision problem $(A, S, C, \rho, \succsim, M)$ and the set of models $M = \{m_{\theta,\sigma,\xi}\}$ that the policymaker posits. Assume that a von Neumann-Morgernstern utility function $u : C \to \mathbb{R}$ translates economic consequences, measured in monetary terms, into utility levels. This function captures attitudes toward risk (i.e. uncertainty within models). The *expected reward* of action a under model $m \in M$ is

$$\begin{aligned} \mathrm{R}(a, \theta, \sigma, \xi) &= \sum_{\varepsilon, R_0} u(\rho(a, \theta, \varepsilon, R_0))\, m_{\theta,\sigma,\xi}(\varepsilon, R_0) \\ &= \sum_{\varepsilon, R_0} u(\rho(a, \theta, \varepsilon, R_0))\, q_\sigma(\varepsilon)\, p_\xi(R_0). \end{aligned}$$

D. Numerical example. In the case study presented in Figure 2 of the manuscript, the policymaker must decide whether to and how long to close school for. Closing schools is costly (e.g. it increases child-adult transmission, reduces the ability of healthcare and key workers to work and the capacity of healthcare, generates economic costs through absenteeism by working parents, loss of education, etc.), but it helps slow the pandemic and its impact by reducing child-child transmission, thus delaying the pandemic peak that overwhelms health care services, and therefore ultimately reducing morbidity and associated mortality.

Here, we illustrate how different decision rules may be used in this specific example, in which there is only structural uncertainty about the benefits of school closures. The cost function is assumed to be known, so that there are only three different models in the set M.

- **Model 1** is based on the evidence coming from influenza outbreaks, for which the majority of transmission is between children (29). According to this model, closing schools would be the biggest contributor to reducing R_e to below 1 and it may be the only intervention that could do so. In this case the benefit would, for example, be proportional to the duration of school closure. In the linear-quadratic example, this would imply that $\kappa_1 = 0$ and $\kappa_2 < 0$, so that benefits positively depend on the action a: stronger measures reduce the effective reproductive number R_e, and thus the economic damage D of the pandemic.

- **Model 2**, instead, relies on some previous coronavirus outbreaks, for which evidence suggests minimal transmission between children (28). In this case, R_e cannot be reduced below 1, school closures do not affect the size of the epidemic, and therefore do not bring any benefits. This is for example the case if $\kappa_1 = \kappa_2 = 0$ in the linear-quadratic setup, so that the economic damages, and thus the benefits, are unaffected by the policy action a.

[‡‡] Here δ_θ is the probability distribution concentrated on θ, i.e., $\delta_\theta(\theta) = 1$ and $\delta_\theta\left(\theta'\right) = 0$ if $\theta' \neq \theta$.

- **Model 3** projects that some child to child transmission happens so that closing schools contributes to reducing R_e to below 1 and reduces the size of the epidemic. However, this only works in combination with other measures (30). Without it, R_e would be above 1 but as an isolated measure school closures do not have such a big effect (31). Under this scenario, the effectiveness of school closures is important at the beginning, but declines as time goes on. In the linear-quadratic example, this would imply that $\kappa_1 > 0$ and $\kappa_2 < 0$.

For simplicity, we assume that there is no uncertainty within models ($\varepsilon_r = \varepsilon_d = \varepsilon_{\mathsf{C}} = 0$).

The illustrative benefit and cost functions we used are the following:

- $B(a) = 4a + 20$ in the case of model 1,
- $B(a) = 100$ in the case of model 2,
- $B(a) = -0.1a^2 + 4a + 70$ in the case of model 3,
- $\mathsf{C}(a) = 0.1a^2 + 10$.

Consider the decision problem $(A, S, C, \rho, \succsim, M)$. In our case, we restrict the action space so that $A = [0, 20]$. For each of these 3 models $m_{\theta,\sigma,\xi}$, it is possible to compute the expected reward $\mathrm{R}(a, \theta, \sigma, \xi)$ associated with a school closure policy. The policymaker, however, does not know which is the correct one. The expected reward is, in that sense, itself uncertain because it depends on the values of the different structural parameters used. For each particular model representing the net overall monetary benefits of the school closure policy, it is possible to determine the optimal action to put in place. Table S1 presents the expected rewards, together with their associated optimal actions $\hat{a}$ in the case of linear utility u.

Table S1. Example of expected rewards and their associated optimal actions with linear utility u

	$\mathrm{R}(a, \theta, \sigma, \xi)$	$\hat{a}$
Model 1	$-0.1a^2 + 4a + 10$	20
Model 2	$-0.1a^2 + 90$	0
Model 3	$-0.2a^2 + 4a + 60$	10

If the policymaker considers uncertainty within and across models in the same way, she aggregates the expected rewards by taking a weighted average over them, where the weights represent the degrees of belief in each specific model. The decision criterion in this case is the classical SEU criterion of (17). For example, under a uniform prior over the possible models, i.e. if $\mu(m) = 1/3$ for all m, the optimal decision is a school closure policy $\hat{a}_{seu} = 10$. It therefore means that, putting the same weight on the three different models given by three different sources, or a single model (such as model 3) on which all experts would agree, would lead exactly to the same optimal school closure policy. Instead, if the policymaker decides to behave extremely precautionary by taking into account only the model providing the lowest expected reward, she only considers Model 1 and decides to close schools for the maximum length $\hat{a}_{mxm} = 20$. This policymaker is extremely averse to uncertainty across models, and in consequence, uses the maxmin decision rule of (15). Alternatively, if the policymaker is averse to uncertainty in the sense of disliking more uncertainty across than within models but is not as precautionary as a maxmin policymaker, she may follow the smooth ambiguity criterion of (20). In such a case, the optimal length of school closures is longer than under expected utility. It approximately corresponds to 12, when the ambiguity function ϕ is logarithmic. Finally, if the policymaker has multiple prior probability measures over the models, she can computes the expected utility for each of them, and considers only the one providing the lowest level of subjective expected utility. For example, imagine two distinct priors: the uniform prior, in which the 3 models are weighted equally, and the prior that considers model 2 as implausible, but models 1 and 3 as equally likely (i.e., this prior puts a weight 0 on model 2 and a weight 0.5 over the two other models). The optimal length of school closures under the multiple priors model of (21) in this situation is higher than under subjective expected utility. It corresponds to closing schools for approximately 13 weeks. Table S2 summarizes the optimal decisions for each of these decision rules.

Table S2. Example of optimal policies depending on the decision rules followed

Decision rules (criterion)	Optimal policy
V_{seu}	$\hat{a}_{seu} = 10$
V_{mxm}	$\hat{a}_{mxm} = 20$
V_{smt}	$\hat{a}_{smt} = 11.65$
V_{mp}	$\hat{a}_{mp} = 13.33$

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