Supporting Information

Junho Lee, Donggu Lee, Sean Lawler, Yangjin Kim

S2: Nondimensionalization of the mathematical model

The governing equations for the variables in the main text in the presence of TGF-β antibody (A) are as follows:

$$\begin{aligned}
\frac{\partial n}{\partial t} &= \nabla \cdot \left(D_n \nabla n - \chi_E n \frac{\nabla E}{\delta_E + \sigma_E |\nabla E|} - \chi_\rho I_S n \frac{\nabla \rho}{\delta_\rho + \sigma_\rho |\nabla \rho|} \right) \\
&\quad + r\left(1 + r_E \frac{E^m}{k_E^m + E^m}\right) n \left(1 - \frac{n}{n_0}\right) - \mu_n N_1 n \quad \text{in } \Omega_*,\ t > 0, \\
\frac{\partial N_1}{\partial t} &= \nabla \cdot \left(D_1 \nabla N_1 - \chi_1 N_1 \frac{\nabla C}{\delta_1 + \sigma_C |\nabla C|} \right) + \lambda_1 N_1 - \lambda_{12} G N_1 \quad \text{in } \Omega_*,\ t > 0, \\
\frac{\partial N_2}{\partial t} &= \nabla \cdot \left(D_2 \nabla N_2 - \chi_2 N_2 \frac{\nabla C}{\delta_2 + \sigma_C |\nabla C|} \right) + \lambda_{12} G N_1 + \lambda_2(G) N_2 \quad \text{in } \Omega_*,\ t > 0, \\
\frac{d\rho}{dt} &= -(\mu_{\rho 1} E + \mu_{\rho 2} P) n \quad \text{in } S,\ t > 0, \\
\frac{\partial C}{\partial t} &= D_C \Delta C + \lambda_C n - \mu_C C \quad \text{in } \Omega_*,\ t > 0, \\
\frac{\partial G}{\partial t} &= D_G \Delta G + \lambda_G n - \mu_G G - \mu_{AG} A G \quad \text{in } \Omega_*,\ t > 0, \\
\frac{\partial E}{\partial t} &= D_E \Delta E + \lambda_E N_2 - \mu_E E - \mu_{ED} \frac{E D^l}{K_D^l + D^l} \quad \text{in } \Omega_*,\ t > 0, \\
\frac{\partial P}{\partial t} &= D_P \Delta P + \lambda_P N_2 - \mu_{PM} \frac{P M^m}{K_M^m + M^m} - \mu_P P \quad \text{in } \Omega_*,\ t > 0, \\
\frac{\partial D}{\partial t} &= D_D \Delta D + \lambda_D I_{\Omega_I} - \mu_D D \quad \text{in } \Omega_*,\ t > 0, \\
\frac{\partial M}{\partial t} &= D_M \Delta M + \lambda_M - \mu_M M \quad \text{in } \Omega_*,\ t > 0, \\
\frac{\partial A}{\partial t} &= D_A \Delta A + \lambda_A - \mu_A A \quad \text{in } \Omega_*,\ t > 0.
\end{aligned}$$

We non-dimensionalize the variables and the parameters in the governing equations above as follows

$$\begin{aligned}
&\overline{t} = \frac{t}{T},\ \overline{x} = \frac{x}{L},\ \overline{n} = \frac{n}{n^*},\ \overline{N}_1 = \frac{N_1}{N_1^*},\ \overline{N}_2 = \frac{N_2}{N_2^*},\ \overline{\rho} = \frac{\rho}{\rho^*},\ \overline{C} = \frac{C}{C^*},\ \overline{G} = \frac{G}{G^*},\ \overline{E} = \frac{E}{E^*}, \overline{P} = \frac{P}{P^*}, \\
&\overline{D} = \frac{D}{D^*},\ \overline{M} = \frac{M}{M^*},\ \overline{A} = \frac{A}{A^*},\ \overline{D}_n = \frac{D_n}{D_\dagger},\ \overline{D}_1 = \frac{D_1}{D_\dagger},\ \overline{D}_2 = \frac{D_2}{D_\dagger},\ \overline{D}_C = \frac{D_C}{D_\dagger},\ \overline{D}_G = \frac{D_G}{D_\dagger}, \\
&\overline{D}_E = \frac{D_E}{D_\dagger},\ \overline{D}_P = \frac{D_P}{D_\dagger},\ \overline{D}_D = \frac{D_D}{D_\dagger},\ \overline{D}_M = \frac{D_M}{D_\dagger},\ \overline{D}_A = \frac{D_A}{D_\dagger},\ \overline{r} = rT,\ \overline{r}_E = r_E,\ \overline{k}_E = \frac{k_E}{E^*}, \\
&\overline{n}_0 = \frac{n_0}{n^*},\ \overline{\lambda}_1 = \lambda_1 T,\ \overline{\lambda}_{12} = \lambda_{12} T G^*,\ \overline{\lambda}_{12}^\dagger = \frac{\lambda_{12} T G^* N_1^*}{N_2^*},\ \overline{\lambda}_2 = \lambda_2 T,\ \overline{\lambda}_C = \frac{\lambda_C T n^*}{C^*},\ \overline{\lambda}_G = \frac{\lambda_G T n^*}{G^*}, \\
&\overline{\lambda}_E = \frac{\lambda_E T N_2^*}{E^*},\ \overline{\lambda}_P = \frac{\lambda_P T N_2^*}{P^*},\ \overline{\lambda}_D = \frac{\lambda_D T}{D^*},\ \overline{\lambda}_M = \frac{\lambda_M T}{M^*},\ \overline{\lambda}_A = \frac{\lambda_A T}{A^*},\ \overline{\mu}_n = \mu_n T N_1^*,
\end{aligned}$$

Table S1. Reference variables used in the model.

	Description	Dimensional Value	Refs.
T	Time	$1\ h$	
L	Length	$1.0\ mm$	
$D_\dagger$	Diffusion coefficients ($= L^2/T$)	$2.78 \times 10^{-6}\ cm^2 s^{-1}$	
n^*	Tumor density	$2.5{\times}10^4\ cells/cm^3$	[1,2]
N_1^*	N1 neutrophil density	$1.0{\times}10^5\ cells/cm^3$	[1,3,4]
N_2^*	N2 neutrophil density	$= N_1^*$	[1,3]
ρ^*	ECM density	$5.0{\times}10^{-4} g/cm^3$	[5–10]
C^*	CXCL8 (IL-8) concentration	$1{\times}10^{-12}\ g/cm^3$	[11]
G^*	TGF-β concentration	$1.1\ {\times}10^{-8}\ g/cm^3$	[2,12,13]
E^*	Neutrophil elastase concentration	$6.46\ {\times}10^{-9}\ g/cm^3$	[14,15]
P^*	MMP concentration	$1.6{\times}10^{-9}\ g/cm^3$	[16]
D^*	NE inhibitor (DNase I) concentration	$3.2{\times}10^{-9}\ g/cm^3$	[17–19]
M^*	TIMP concentration	$4.6385\ {\times}10^{-8}\ g/cm^3$	[20]
A^*	Concentration of anti-body of TGF-β	$0.172\ \mu M$	[21]

$$
\begin{aligned}
&\overline{\mu}_{\rho 1} = \frac{\mu_{\rho 1} T E^* n^*}{\rho^*},\ \overline{\mu}_{\rho 2} = \frac{\mu_{\rho 2} T P^* n^*}{\rho^*},\ \overline{\mu}_C = \mu_C T,\ \overline{\mu}_G = \mu_G T,\ \overline{\mu}_E = \mu_E T,\ \overline{\mu}_{ED} = \mu_{ED} T,\\
&\overline{K}_D = \frac{K_D}{D^*},\ \overline{\mu}_P = \mu_P T,\ \overline{\mu}_D = \mu_D T,\ \overline{K}_M = \frac{K_M}{M^*},\ \overline{\mu}_M = \mu_M T,\ \overline{\mu}_{PM} = \mu_{PM} T,\ \overline{\mu}_A = \mu_A T,\\
&\overline{\mu}_{AG} = \mu_{AG} T A^*,\ \overline{\chi}_E = \frac{\chi_E T}{L},\ \overline{\delta}_E = \frac{\delta_E L}{E^*},\ \overline{\sigma}_E = \sigma_E,\ \overline{\chi}_\rho = \frac{\chi_\rho T}{L},\ \overline{\delta}_\rho = \frac{\delta_\rho L}{\rho^*},\ \overline{\sigma}_\rho = \sigma_\rho,\\
&\overline{\chi}_1 = \frac{\chi_1 T}{L},\ \overline{\delta}_1 = \frac{\delta_1 L}{C^*},\ \overline{\sigma}_C = \sigma_C,\ \overline{\chi}_2 = \frac{\chi_2 T}{L},\ \overline{\delta}_2 = \frac{\delta_2 L}{C^*}.
\end{aligned}
$$

where reference values $T, L, D, n^*, N_1^*, N_2^*, \rho^*, C^*, G^*, E^*, P^*, D^*, M^*, A^*$ are given in Table S1. Here, $D_\dagger = L^2/T$ is the characteristic diffusion coefficient for characteristic length L and time T in Table S1.

Then, the governing equations in a dimensionless form are

$$
\begin{aligned}
\frac{\partial \overline{n}}{\partial \overline{t}} &= \nabla \cdot (\overline{D}_n \nabla \overline{n}) - \nabla \cdot \left(\overline{\chi}_E \; \overline{n} \; \frac{\nabla \overline{E}}{\overline{\delta}_E + \overline{\sigma}_E |\nabla \overline{E}|} \right) - \nabla \cdot \left(\overline{\chi}_\rho \; I_{\overline{S}} \overline{n} \; \frac{\nabla \overline{\rho}}{\overline{\delta}_\rho + \overline{\sigma}_\rho |\nabla \overline{\rho}|} \right) \\
&\quad + \overline{r} \left(1 + \overline{r}_E \frac{\overline{E}^m}{\overline{k}_E^m + \overline{E}^m} \right) \overline{n} \left(1 - \frac{\overline{n}}{\overline{n}_0} \right) - \overline{\mu}_n \overline{N}_1 \overline{n} \quad \text{in } \overline{\Omega}_*, \; \overline{t} > 0, \\
\frac{\partial \overline{N}_1}{\partial \overline{t}} &= \nabla \cdot \left(\overline{D}_1 \nabla \overline{N}_1 - \overline{\chi}_1 \overline{N}_1 \frac{\nabla \overline{C}}{\overline{\delta}_1 + \overline{\sigma}_C |\nabla \overline{C}|} \right) + \overline{\lambda}_1 \overline{N}_1 - \overline{\lambda}_{12} \overline{G} \overline{N}_1 \quad \text{in } \overline{\Omega}_*, \; \overline{t} > 0, \\
\frac{\partial \overline{N}_2}{\partial \overline{t}} &= \nabla \cdot \left(\overline{D}_2 \nabla \overline{N}_2 - \overline{\chi}_2 \overline{N}_2 \frac{\nabla \overline{C}}{\overline{\delta}_2 + \overline{\sigma}_C |\nabla \overline{C}|} \right) + \overline{\lambda}_{12} \overline{G} \overline{N}_1 + \overline{\lambda}_2(\overline{G}) \overline{N}_2 \quad \text{in } \overline{\Omega}_*, \; \overline{t} > 0, \\
\frac{d\overline{\rho}}{d\overline{t}} &= -(\overline{\mu}_{\rho 1} \overline{E} + \overline{\mu}_{\rho 2} \overline{P}) \overline{n} \qquad \text{in } \overline{S}, \; t > 0, \\
\frac{\partial \overline{C}}{\partial \overline{t}} &= \overline{D}_C \overline{\Delta C} + \overline{\lambda}_C \overline{n} - \overline{\mu}_C \overline{C} \qquad \text{in } \overline{\Omega}_*, \; \overline{t} > 0, \\
\frac{\partial \overline{G}}{\partial \overline{t}} &= \overline{D}_G \overline{\Delta G} + \overline{\lambda}_G \overline{n} - \overline{\mu}_G \overline{G} - \overline{\mu}_{AG} \overline{AG} \qquad \text{in } \overline{\Omega}_*, \; \overline{t} > 0, \\
\frac{\partial \overline{E}}{\partial \overline{t}} &= \overline{D}_E \overline{\Delta E} + \overline{\lambda}_E \overline{N}_2 - \overline{\mu}_E \overline{E} - \overline{\mu}_{ED} \frac{\overline{E} \; \overline{D}^l}{\overline{K}_D^l + \overline{D}^l} \qquad \text{in } \overline{\Omega}_*, \; \overline{t} > 0, \\
\frac{\partial \overline{P}}{\partial \overline{t}} &= \overline{D}_P \overline{\Delta P} + \overline{\lambda}_P \overline{N}_2 - \overline{\mu}_{PM} \frac{\overline{P} \; \overline{M}^m}{\overline{K}_M^m + \overline{M}^m} - \overline{\mu}_P \overline{P} \qquad \text{in } \overline{\Omega}_*, \; \overline{t} > 0, \\
\frac{\partial \overline{D}}{\partial \overline{t}} &= \overline{D}_D \overline{\Delta D} + \overline{\lambda}_D I_{\Omega_I} - \overline{\mu}_D \overline{D} \qquad \text{in } \overline{\Omega}_*, \; \overline{t} > 0, \\
\frac{\partial \overline{M}}{\partial \overline{t}} &= \overline{D}_M \overline{\Delta M} + \overline{\lambda}_M - \overline{\mu}_M \overline{M} \qquad \text{in } \overline{\Omega}_*, \; \overline{t} > 0, \\
\frac{\partial \overline{A}}{\partial \overline{t}} &= \overline{D}_A \overline{\Delta A} + \overline{\lambda}_A - \overline{\mu}_A \overline{A} \qquad \text{in } \overline{\Omega}_*, \; \overline{t} > 0.
\end{aligned}
$$

Note that $\overline{\lambda}_{12}^\dagger = \overline{\lambda}_{12}$ under the assumption of $N_1^* = N_2^*$; otherwise $\overline{\lambda}_{12}^\dagger$ is different from $\overline{\lambda}_{12}$.

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