Supporting Information

Hydrogel-based diffractive optical elements (hDOEs) using rapid digital photopatterning

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SI-1 Setup to test the optical performance of hDOEs.

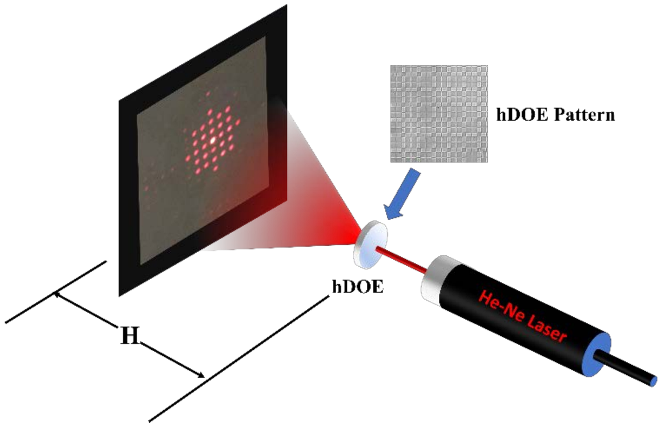

Figure S1 Setup to test the optical performance of hDOEs using He-Ne laser. The diffractive optical element was directly illuminated by coherent laser source. An imaging screen was used to capture the diffraction patterns.

SI-2 Setup to test imaging performance of hDOEs.

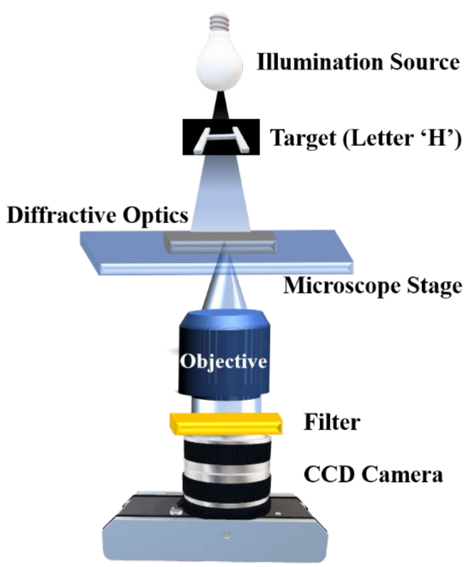

Figure S2 Setup to test imaging performance of hDOEs under bright-field microscope. The target, letter 'H', was illuminated by bright-field illumination source and imaged by diffractive optics, Fresnel Zone Plate. The image was captured by a CCD camera with objective. A filter was used to block non-visible spectrum.

SI-3 Theoretical model to generate digital mask designs using computer generated holography

(A) Diffraction grating

In order to obtain the digital mask of diffraction grating, we used the computer-generated holography method programmed in MATLAB. The details are shown in Fig. 1 and the supporting theoretical model is explained as follows.

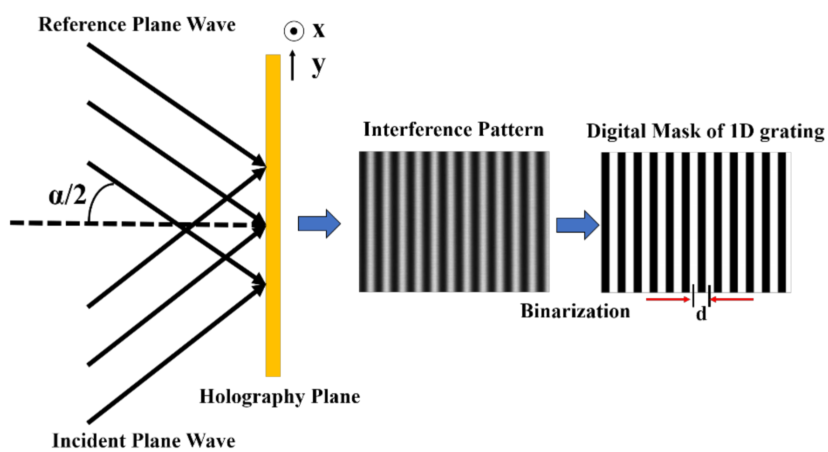

Figure S3 Schematic of 1D grating mask design based on two beams holographic interference.

Incident plane wave

$$\mathrm{O(x,y)} = \mathrm{O_0}\exp(i\phi_o(x,y))\ ;\ \text{where}\ \phi_o(x,y) = \frac{2\pi}{\lambda}\sin(\frac{\alpha}{2})x \tag{1}$$

Reference plane wave

$$\mathrm{R(x,y)} = R_0\exp(i\phi_R(x,y));\ \text{where}\ \phi_R(x,y) = \frac{2\pi}{\lambda}\sin(\frac{\alpha}{2})x \tag{2}$$

Where O(x,y) is the wave equation of the incident light, R(x,y) is the wave equation of the reference light, ϕ represents their phase which is determined by their incident angle α.

For the holographic interference, the intensity distribution I(x,y) of the interference pattern is calculated using the equation

$$I(x,y)=(O+R)(O^*+R^*)=R_0^2+O_0^2+2R_0O_0\cos(\phi_R-\phi_O) \quad (3)$$

The pattern is determined by the phase of the incident and reference lights

$$\phi_R-\phi_O=\frac{2\pi}{\lambda}2\sin\left(\frac{\alpha}{2}\right)x=2m\pi,\ \text{where m=1, 2 …… m} \quad (4)$$

Therefore, the period *d* of the interference pattern is calculated as follows

$$d=\frac{\lambda}{2\sin(\frac{\alpha}{2})} \quad (5)$$

2D grating can be designed by simply adding two 1D gratings as shown in **Figure S4**. In the Figure, color red and black represents 1 (corresponds to phase π), while white color represents 0 (0 corresponds to phase 0). After their addition, their transmissions are determined by the conditions shown below in **Figure S4.**

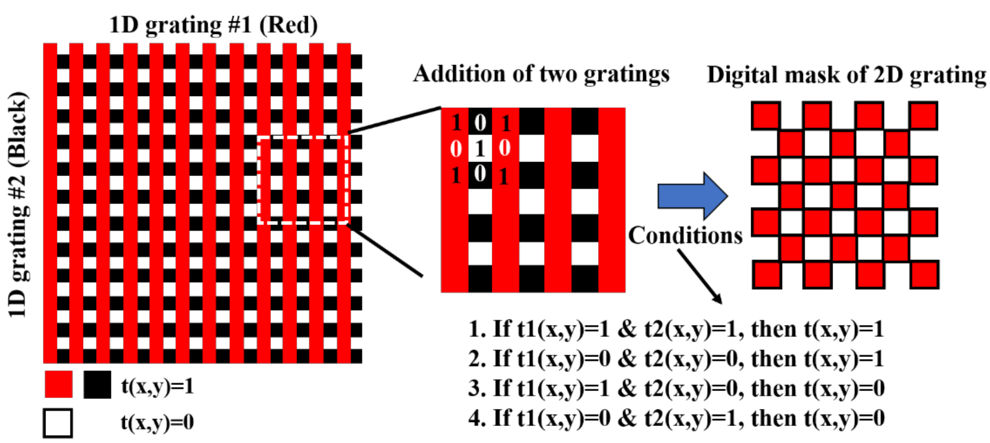

Figure S4 Schematic of 2D grating design.

(B) Dammann Grating (Beam Splitter)

Figure S5 shows the general shape of a 1D DG. For simplicity we set the period of the grating to unity and require the grating period to be symmetric with respect to the origin. The transmission function has only two values 1 and -1, corresponding to phase value 0 and π respectively. Thus, the grating is characterized by its N transition points, x_1, x_2, ……x_N, where the phase jump occurs. The transition points are the variable parameters that are determined by the design procedure such as the diffraction order.

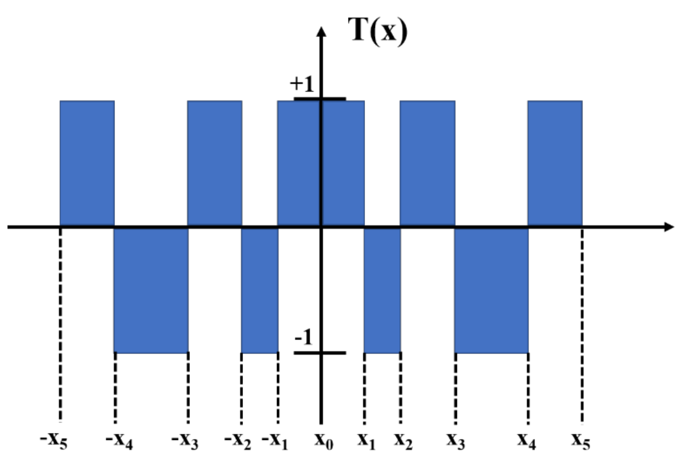

Figure S5 One period of transmission function of 1D binary DG.

Due to the symmetry requirements, the positive and negative diffraction orders have the same intensity. DG with N transition points will have 2N+1 diffraction orders which may be specified in the design. The transmission function T(x) of the grating can be written as

$$\mathrm{T(x)} = \sum_{n=0}^{N}(-1)^n rect(\frac{x-(x_{n+1}+x_n)/2}{x_{n+1}-x_n}) \quad (6)$$

The negative part from -N to 0 is chosen symmetrically. By Fourier-transformation of Equation (6), the amplitudes of the diffraction orders can be calculated to be

$$t_m = \frac{1}{\pi m}\sum_{n=0}^{N}(-1)^n\,[\sin(2\pi m x_{n+1}) - \sin(2\pi m x_n)] \quad (7)$$

The intensity of the diffraction order is |tm|2. For a perfect design of a DG, we require the intensity uniformity of the diffraction orders (m≤N) to be smaller than 1%. In this work, N is 2. The iteration Fourier transformation is run using Gercheberg-Saxtion(GS) algorithm which is used to obtain the local optimized result. As we set the period to be unity, the transition points x_0, x_1, x_2, x_3 and x_4 are 0, 0.034837, 0.3869454, 0.6516349 and 1. Therefore, the ratio of width between each transition point is 1:9:7:9 as shown in **Figure S6**. Similarly, 2D DG is designed by the addition of two 1D DGs as shown in Figure S6. In the Figure, color red and black represents 1, while white color represents 0 (where 1 and 0 correspond to phase π and 0 respectively). Their transmissions (t) are determined by the conditions shown in Figure S6**.**

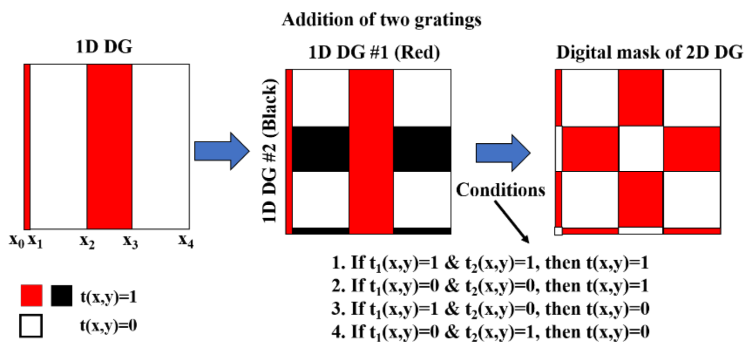

Figure S6 Schematic of mask design of 2D DG.

(C) Fresnel Zone Plate (FZP)

In order to generate the digital mask of FZP, we used the computer-generated holography technique. The FZP is obtained through the holographic interferences between incident spherical and reference plane waves. The details are shown in **Figure S7** and theoretical model is explained as follows.

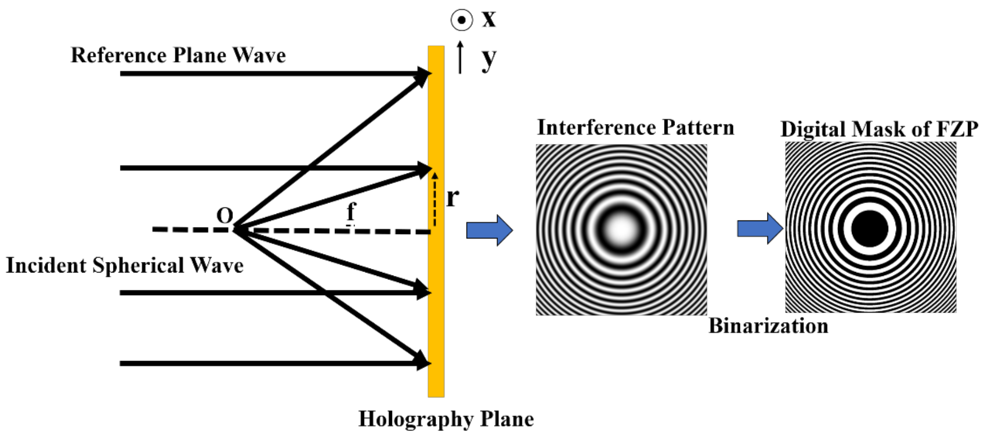

Figure S7 Schematic of mask design of FZP based on the interference between spherical and plane waves.

$$\mathrm{O}(\mathrm{x},\mathrm{y}) = \mathrm{O}_0\exp(i\phi_o(x,y)) \text{ ; where } \phi_o(x,y) = \frac{2\pi}{\lambda}\frac{r^2}{2f} \text{ and } r = \sqrt{x^2+y^2} \qquad (8)$$

$$\mathrm{R}(\mathrm{x},\mathrm{y}) = R_0\exp(i\phi_R(x,y)); \text{ where } \phi_R(x,y) = 0 \qquad (9)$$

Where O(x,y) is the wave equation of the incident light, R(x,y) is the wave equation of the reference light, ϕ represents their phases which are determined by the position r in holographic plane.

For the holographic interference, the intensity distribution I(x,y) of interference pattern is calculated as

$$\mathrm{I(x,y)=(O+R)(O^*+R^*)=R_0{}^2+ O_0{}^2+2R_0O_0\cos(\phi_R-\phi_O)} \qquad (10)$$

The pattern is determined by the phase of incident light and reference light

$$\phi_R - \phi_O = \frac{2\pi}{\lambda}\frac{r^2}{2f} = m\pi, \text{ where m=1, 2 …… m} \qquad (11)$$

Therefore, the radius of zone rings of FZP is

$$r^2 = fm\lambda \qquad (12)$$

Where f is the distance between dot source and holographic plane, the primary focusing length of FZP; m is the interference pattern order.

(D) Fork Grating (FG)

In order to obtain the digital mask of FG, we used the computer-generated holography technique in MATLAB. The FG is generated via holographic interferences between incident, spiral, and reference plane waves. The details are shown in **Figure S8** and theoretical model is explained as follows.

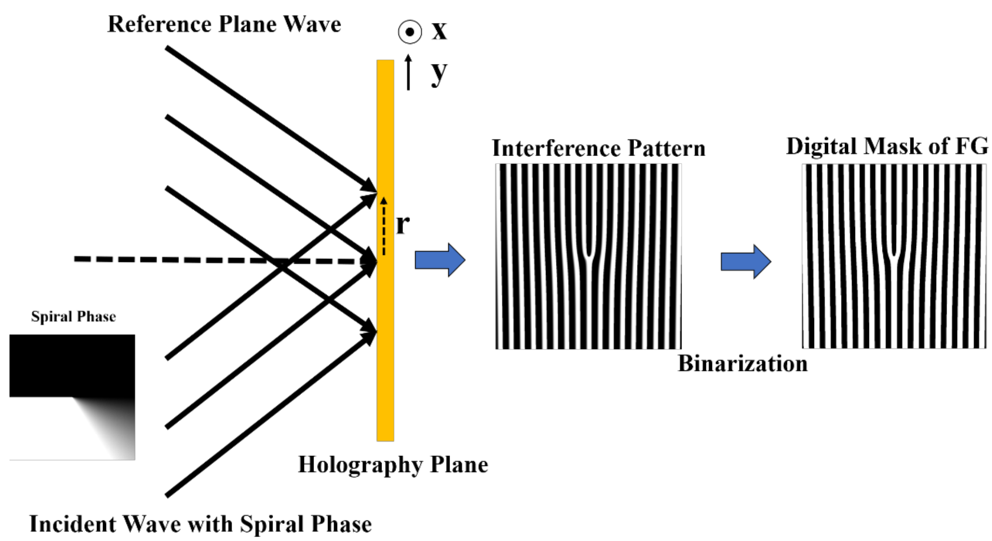

Figure S8 Schematic of mask design of FG based on the interferences between wave with spiral phase and plane waves.

$$\mathrm{O(x,y)} = \mathrm{O_0}\exp(i\phi_o(x,y))\text{ ; where }\phi_o(x,y) = \varphi, \mathrm{x} = \mathrm{r}\cos(\varphi)\text{ and }\mathrm{y} = \mathrm{r}\sin(\varphi) \quad (13)$$

$$\mathrm{R(x,y)} = R_0\exp(i\phi_R(x,y))\text{; where }\phi_R(x,y) = \frac{2\pi}{\lambda}\sin(\alpha)x \quad (14)$$

Where the O(x,y) is the wave equation of the incident light, R(x,y) is the wave equation of the reference light and ϕ represents their phase, $\boldsymbol{\varphi}$ is the azimuth angle in the cylindrical coordinate, r is the radial distance in the cylindrical coordinate and α is the incident angle of plane wave.

The intensity distribution I(x,y) of interference pattern is calculated as

$$I(x,y)=(O+R)(O^*+R^*)=R_0^2+O_0^2+2R_0O_0\cos(\phi_R-\phi_O) \quad (15)$$

The pattern is determined by the phase of incident light and reference light

$$\phi_R - \phi_O = \frac{2\pi}{\lambda}\sin(\alpha)\, r\cos(\varphi) - \varphi = m\pi, \text{ where m=1, 2 …… m} \quad (16)$$

(E) Computer-generated hologram of user-defined object

In order to obtain the digital mask of CGH, we used the computer-generated holography technique. The CGH is obtained through the holographic interference between the Fourier pattern of user-defined object with reference plane wave. The details are shown in **Figure S9** and theoretical model is explained as follows.

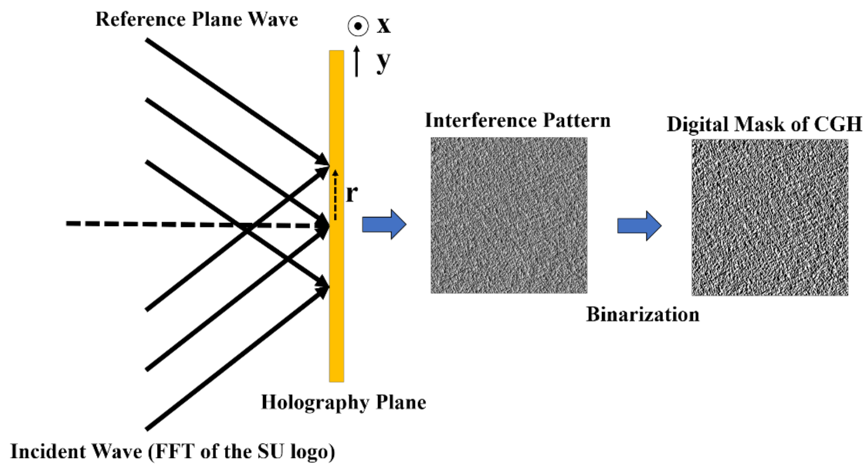

Figure S9 Schematic of mask design of CGH based on the interference between Fourier pattern of object and plane wave.

Incident wave

$$\mathrm{O(x, y)} = FFT\big(S(x,y)\big) = \mathrm{O}_0\exp(i\phi_o(x,y)) \quad (13)$$

Reference plane wave

$$\mathrm{R(x, y)} = R_0\exp(i\phi_R(x,y)); \text{ where } \phi_R(x,y) = \frac{2\pi}{\lambda}\sin(\alpha)x \quad (14)$$

Where O(x,y) is the wave equation of the incident light which is the Fourier transformation pattern of S(x,y), S(x,y) is the optical field distribution of a user-defined object, R(x,y) is the wave equation of the reference light and α is the incident angle of plane wave.

The intensity distribution I(x,y) of interference pattern is calculated

$$I(x,y)=(O+R)(O^*+R^*)=R_0^2+O_0^2+2R_0O_0\cos(\phi_R-\phi_O) \quad (15)$$

The pattern is determined by the phase of incident light and reference light

$$\phi_R-\phi_O=m\pi\text{, where m=1, 2 …… m} \quad (16)$$

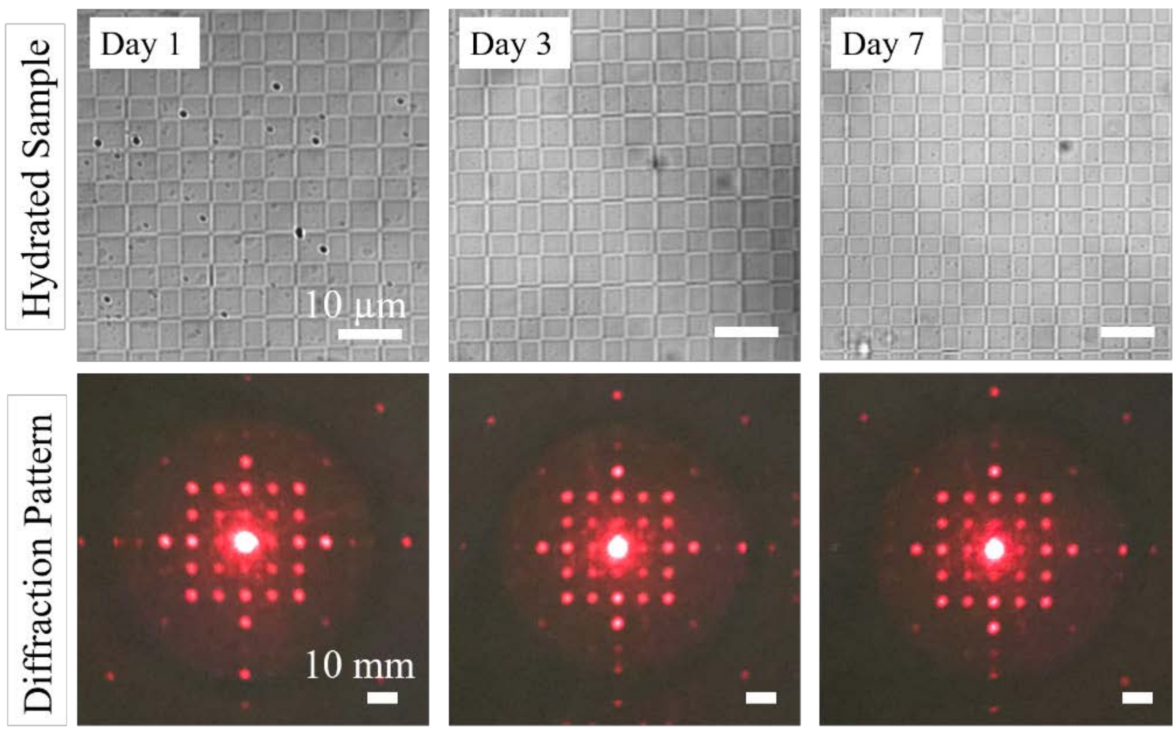

Figure S10 Testing the effect of water evaporation on 'as printed' Damman grating DOEs

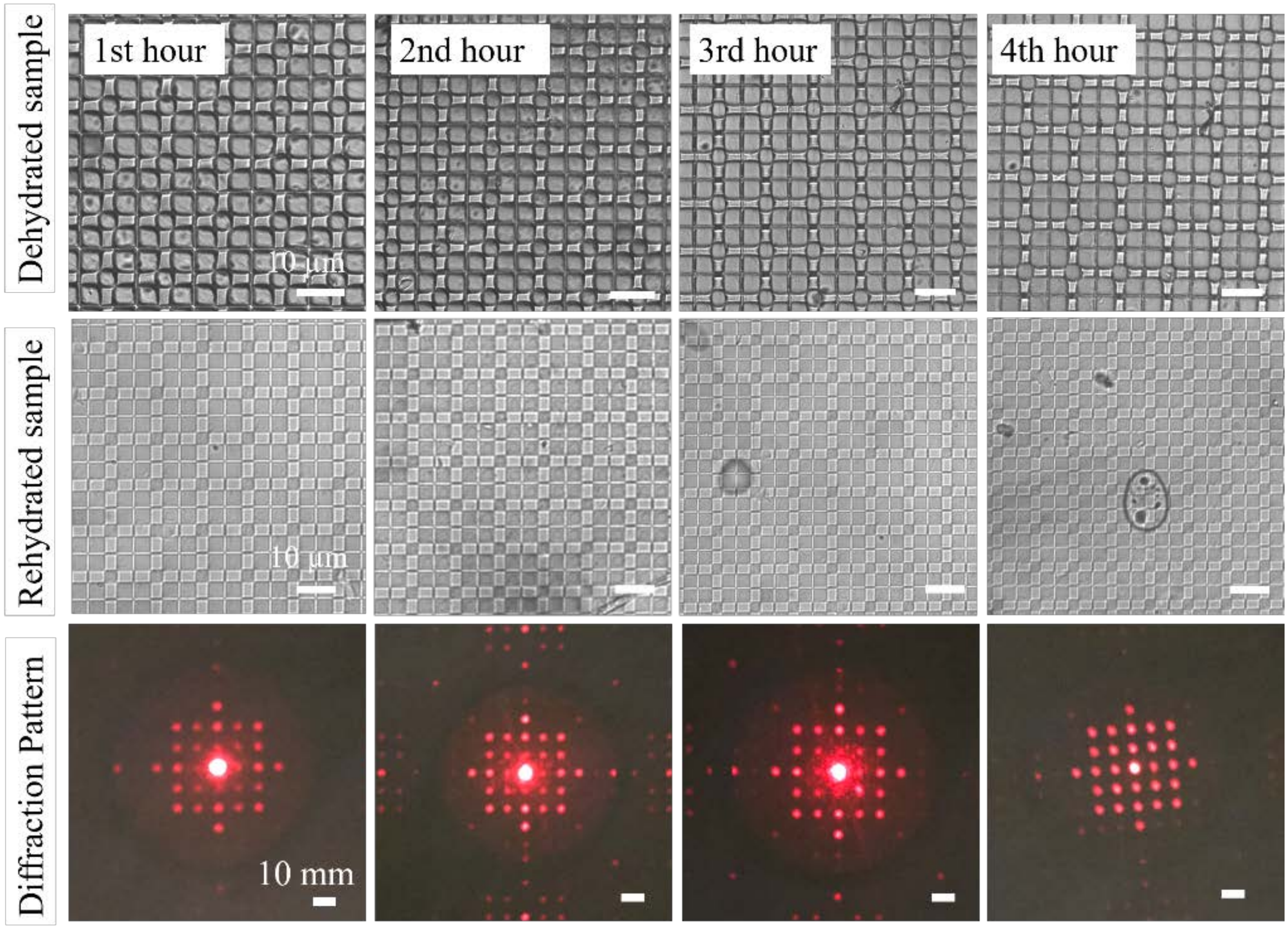

Figure S11 Testing the robustness of the Dammann grating DOEs under repeated dehydration and rehydration processes

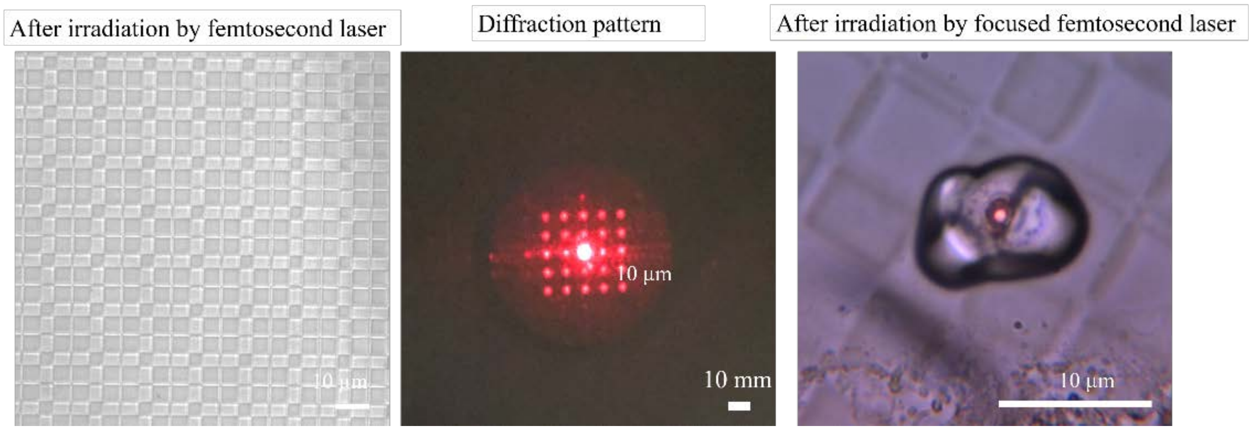

Figure S12 Testing the durability of the Dammann grating DOEs under femtosecond laser irradiation (Wavelength: 800nm, Pulse duration: 140fs, Repetition rate: 80MHz, Power: 1W).