PNAS
www.pnas.org

Supplementary Information for

Conservative and liberal attitudes drive polarized neural responses to political content

Yuan Chang Leong, Janice Chen, Robb Willer, Jamil Zaki

Yuan Chang Leong
Email: ycleong@berkeley.edu

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www.pnas.org/cgi/doi/10.1073/pnas.2008530117

Supplementary text

Online pre-test

300 US-based participants (179 male, 120 female, 1 other; 21-72 years of age, mean age = 35.03 years) were recruited on the Amazon Mechanical Turk (AMT) online platform. Participants were first asked to indicate their political orientation on a 1-7 scale ($orientation_{score}$: 1 = extremely liberal, 4 = moderate, 7 = extremely conservative), and their support for each of the six immigration policies (1 = strongly not support, 7 = strongly support). 179 participants identified as liberal ($orientation_{score} < 4$), 49 participants identified as moderate ($orientation_{score} = 4$) and 72 participants as conservative ($orientation_{score} > 4$).

We tested if support for each policy differed between conservatives and liberals. Statistical significance was assessed using a Welch Two Sample t-test, and results are depicted graphically in Fig. S1. Relative to conservative participants, liberal participants were more likely to support allowing illegal/undocumented immigrants to work legally in the US ($M_{liberal} = 4.83$, $SE_{liberal} = 0.13$, $M_{conservative} = 2.12$, $SE_{conservative} = 0.19$, $t(142.1) = 11.5$, $p < 0.001$), allowing the use of federal funds to pay for emergency healthcare for undocumented/illegal immigrants ($M_{liberal} = 4.69$, $SE_{liberal} = 0.14$, $M_{conservative} = 2.58$, $SE_{conservative} = 0.23$, $t(121.6) = 7.81$, $p < 0.001$), and providing a pathway to citizenship for undocumented individuals brought into the U.S. illegally as children ($M_{liberal} = 6.08$, $SE_{liberal} = 0.10$, $M_{conservative} = 4.01$, $SE_{conservative} = 0.23$, $t(97.9) = 8.01$, $p < 0.001$).

Relative to liberal participants, conservative participants were more likely to support funding the construction of a wall along the U.S.-Mexico border to reduce illegal immigration ($M_{conservative} = 5.04$, $SE_{conservative} = 0.24$, $M_{liberal} = 1.73$, $SE_{liberal} = 0.10$, $t(98.7) = 12.7$, $p < 0.001$), banning refugees from Muslim-majority countries from entering the country ($M_{conservative} = 4.66$, $SE_{conservative} = 0.25$, $M_{liberal} = 2.11$, $SE_{liberal} = 0.11$, $t(104.2) = 9.33$, $p < 0.001$), cutting federal funding to sanctuary cities unless the cities agree to fully cooperate with the U.S. immigration and customs enforcement ($M_{conservative} = 5.29$, $SE_{conservative} = 0.25$, $M_{liberal} = 2.59$, $SE_{liberal} = 0.14$, $t(135.6) = 10.39$, $p < 0.001$).

Representational Similarity Analyses

Our main analyses relied on dividing participants into conservatives and liberals via a median split on their immigration attitude scores. This allowed us to identify voxels where the response was more similar within a group than between groups. An alternative approach is to treat immigration attitude score as a continuous measure and run a representational similarity analysis (RSA) by correlating the difference in attitude scores and the dissimilarity in neural responses for each pair of participants (1). We ran a whole-brain RSA to identify voxels where the difference in immigration attitudes correlated with the dissimilarity in neural responses.

We first calculated the difference in attitude scores between each pair of participants. For each voxel, we then calculated the dissimilarity in neural responses between each pair of participants as 1 – Pearson *r* between the timecourses of the two participants. Next, we calculated the correlation between the difference in attitude scores and the dissimilarity in neural responses across each unique pair of participants. A high correlation value would denote a voxel where a greater difference in immigration attitudes between two participants was associated with greater neural dissimilarity (i.e. a voxel where the response tracks scalar differences in immigration attitudes). Finally, we computed p-values by comparing the observed correlation against a null distribution generated by repeating the analysis 10,000 times with immigration scores randomly shuffled across participants.

This RSA yielded no significant clusters at FWE-corrected $p < 0.05$ with a cluster-forming threshold of $p < 0.001$. Examining the RSA map at less stringent thresholds show that the DMPFC cluster identified by the within-group > between-group ISC contrast overlaps with voxels with a strong RSA effect, indicating that the two analyses produce similar results, though the voxels identified using RSA did not survive correction for multiple comparisons (Fig. S8).

There are several reasons why an RSA is less sensitive than the one-to-group-average ISC approach we employed in the main text. RSA relies on pairwise correlations in neural responses between individual participants, and individual neural timecourses are relatively noisy compared to an average timecourse of multiple participants. Each correlation would thus be less reliable. In contrast, the one-to-group-average ISC approach benefits from assuming that there are two groups. Averaging the timecourses within each group prior to running the correlations increases the signal-to-noise for both within and between-group ISCs, resulting in greater statistical power.

We note also that the bimodal distribution of attitude scores limits the gain in statistical power when treating attitude scores as a continuous measure in the RSA, relative to if the attitude scores were normally distributed (2). Other potential explanations for the lower sensitivity of the RSA include (i) a non-linear relationship between attitude scores and neural responses, (ii) additional dimension(s) (e.g., demographic variables) that moderate the effect of political attitudes on neural similarity and (iii) measurement noise in attitude scores. Teasing these explanations apart is not central to our claim that neural responses diverge between conservative-leaning and liberal-leaning participants when viewing political content, and the consequences this has on attitude polarization.

Finally, it is worth noting that the RSA and one-to-group-average ISC approaches are fundamentally different analyses that test different hypotheses about the data. The RSA tests whether two individuals with greater differences in political attitudes have greater dissimilarity in neural responses. For example, the RSA framework assumes that the neural response of a moderate liberal (attitude score = 12) and a moderate conservative (attitude score = 24) is as dissimilar as that between a moderate conservative (attitude score = 24) and extreme conservative (attitude score = 36). In contrast, the one-to-group-average ISC approach tests whether neural responses were different between two groups of individuals, and thus provides a more direct test of our central claim that neural responses to political content diverge between conservative-leaning and liberal-leaning participants.

ISFC-RSA. We took a similar RSA approach to the ISFC analyses. For each participant, we computed the neural dissimilarity between each voxel in that participant's brain and the DMPFC activity of every other participant. We then calculated the correlation between the difference in attitude scores and neural dissimilarity in each voxel across each pair of participants. Finally, we computed p-values by comparing the observed correlation against a null distribution generated by repeating the analysis 10,000 times with immigration scores randomly shuffled across participants. This analysis again yielded no significant clusters at FWE-corrected $p < 0.05$ with a cluster-forming threshold of 0.001.

Ridge Regression Analysis

Ridge regression is a regularized regression technique commonly used when there are many predictor variables relative to observations, and reduces the variability of model fits by imposing a penalty for large coefficients ("shrinkage") (3). We entered the percentage of words in each of the 50 semantic categories into the same ridge regression model to predict neural polarization in the DMPFC. The duration and number of words in each segment were again included as covariates of no interest.

As we were interested in individual coefficients, the regularization parameter λ was selected automatically using the method proposed in (4) rather than via cross-validation as it is unclear how coefficient estimates can be aggregated over cross-validation folds in a statistically appropriate manner. The model was fit using the *R* package lmridge with default settings (5). Briefly, the response variable was centered while the predictors were scaled to correlation form such that the correlation matrix has unit diagonal elements. Significance testing of the coefficients was then performed using non-exact t-tests (6) with effective degrees of freedom estimated following (7).

Simulation studies have shown that this procedure controls for the false positive rate when observations are independent (6, 8). We note, however, that we have multiple observations from the same video, thus violating the assumption of independence. Hence, the *p-values* obtained should be interpreted with caution. Nevertheless, this analysis complements the mass univariate test reported in the main text by providing coefficient estimates of each predictor adjusted for the influence of all other predictors.

The ridge regression analyses yielded similar results as the mass univariate test (Fig S5, Table S4). In particular, the percentage of risk-related words was the only variable that was significantly associated with neural polarization in the DMPFC after correcting for multiple comparisons (b = 0.034, 95% CI [0.011, 0.057], $t(5.2) = 3.83$, $p < 0.001$, Holm-Bonferroni adjusted $p = 0.013$) . The percentage of moral emotional words was the next strongest predictor, but similar to our other analysis, this relationship does not survive correction over 50 comparisons (b = 0.025, 95% CI [0.003, 0.046], $t(5.2) = 2.89$, $p = 0.005$, Holm-Bonferroni $p = 0.255$).

Supplementary Figures

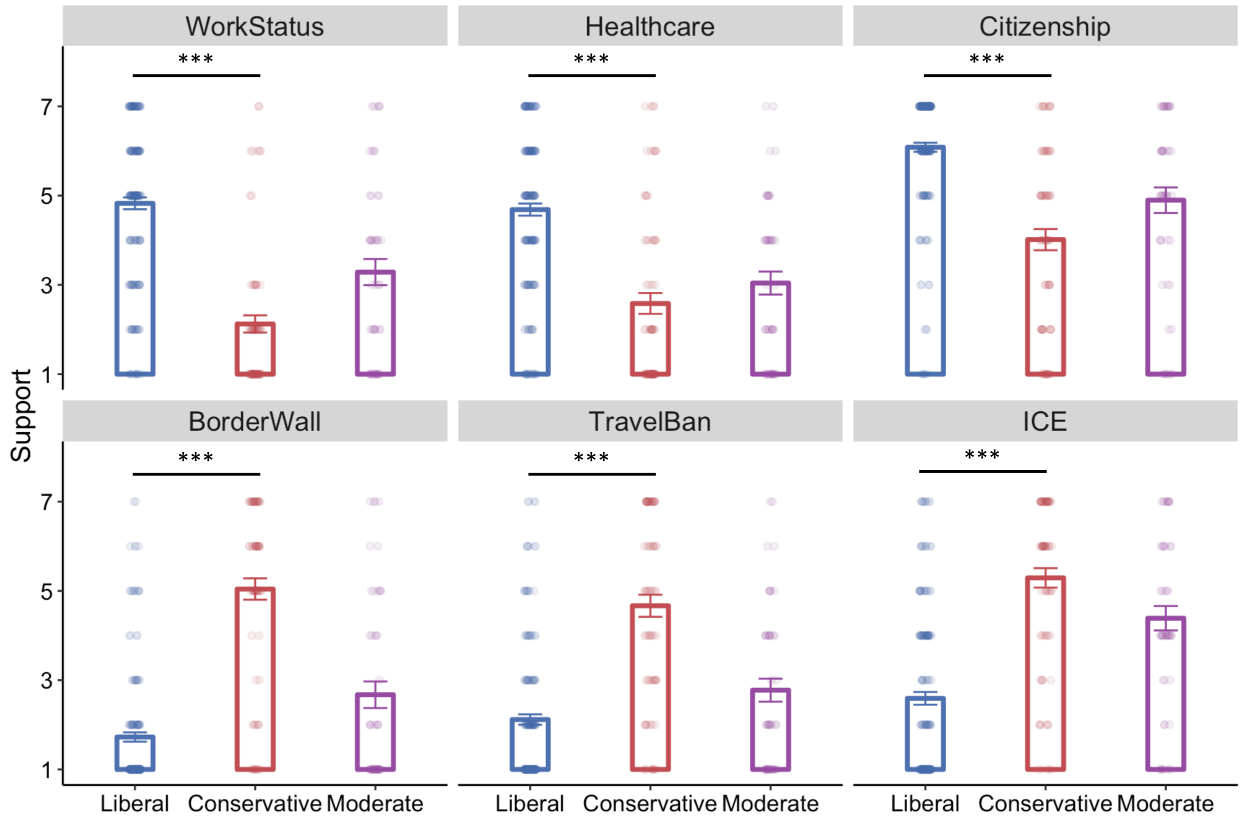

Figure S1. Liberals and conservatives differ significantly on their support for each of the six policies. See Fig. 1B and Supplemental Results for full description of each policy. Data points indicate individual participants' support for the policy with horizontal jitter added for clearer visualization. Support by participants identifying as moderate are shown in purple for comparison. Error bars indicate SEM. *** p < 0.001

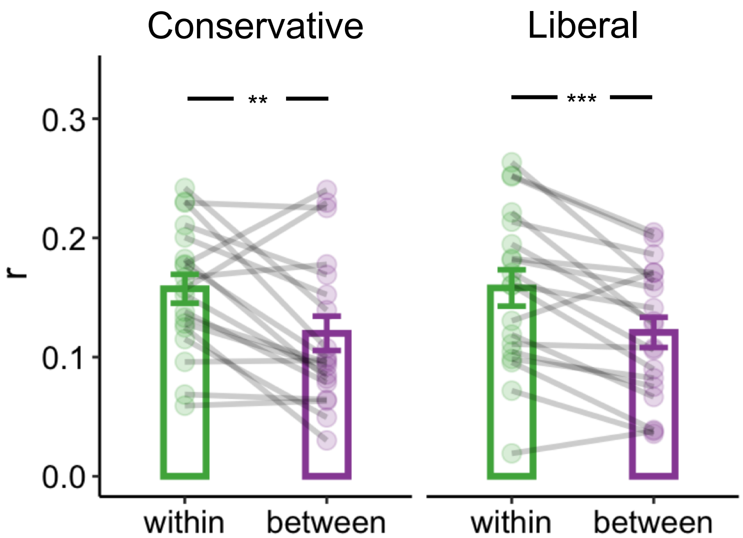

Figure S2. Within-group ISC was higher than between-group ISC in both conservative (*t(18)* = 3.01, p = 0.007) and liberal participants (*t(18)* = 4.57, *p* < 0.001). Furthermore, political orientation did not moderate the within vs. between group ISC difference (*t(36)* = 0.007, *p* = 0.994) . ** p < 0.01, *** p < 0.001.

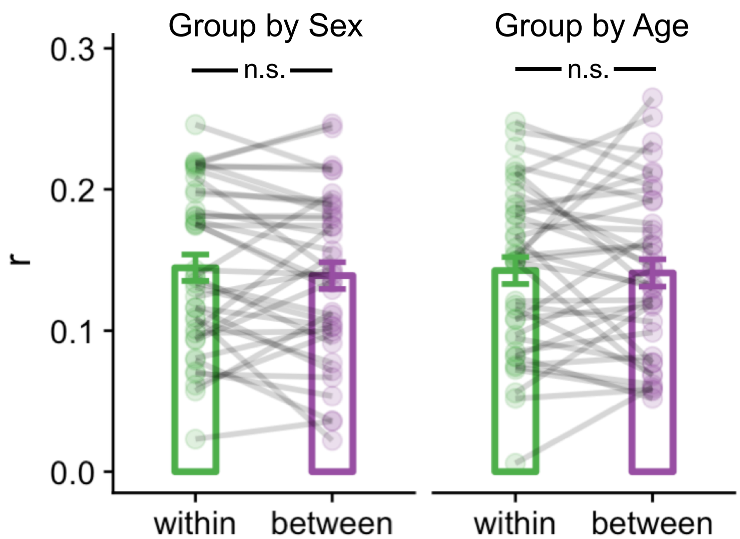

Figure S3. Within-group and between-group ISC in the DMPFC was not significantly different when participants were grouped by sex ($t(37) = 0.95$, $p = 0.348$) or a median split by age (t(37) = 0.23, p = 0.819).

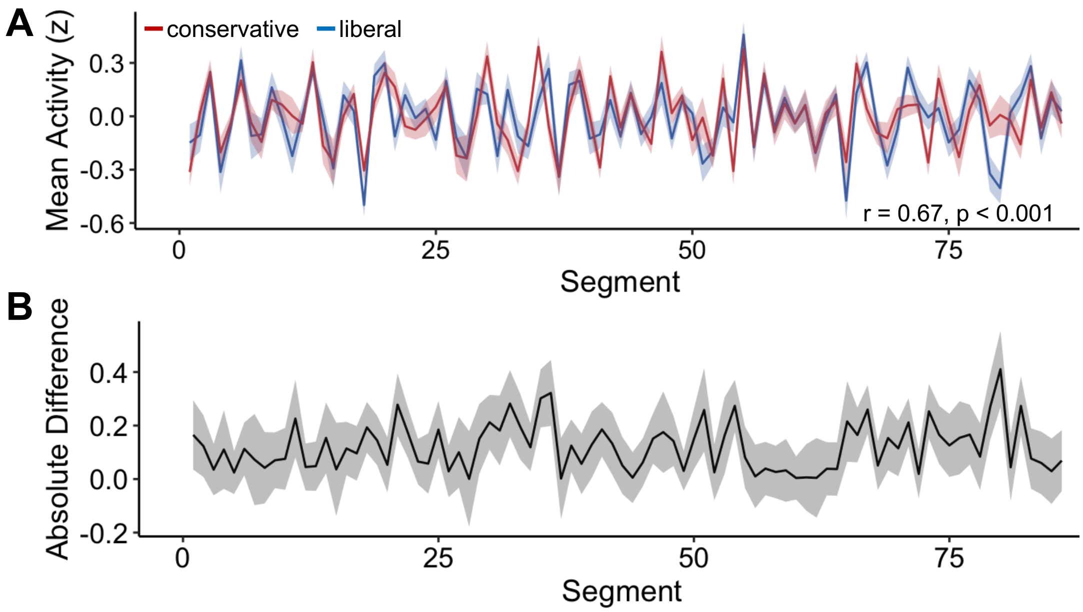

Figure S4. Neural polarization in the DMPFC. **A.** Average DMPFC timecourse of conservative (red) and liberal (blue) participants over the 86 segments. The two timecourses were moderately correlated (r = 0.66, $p < 0.001$). **B.** Absolute difference between average conservative and average liberal DMPFC activity. Shaded errors indicate SEM.

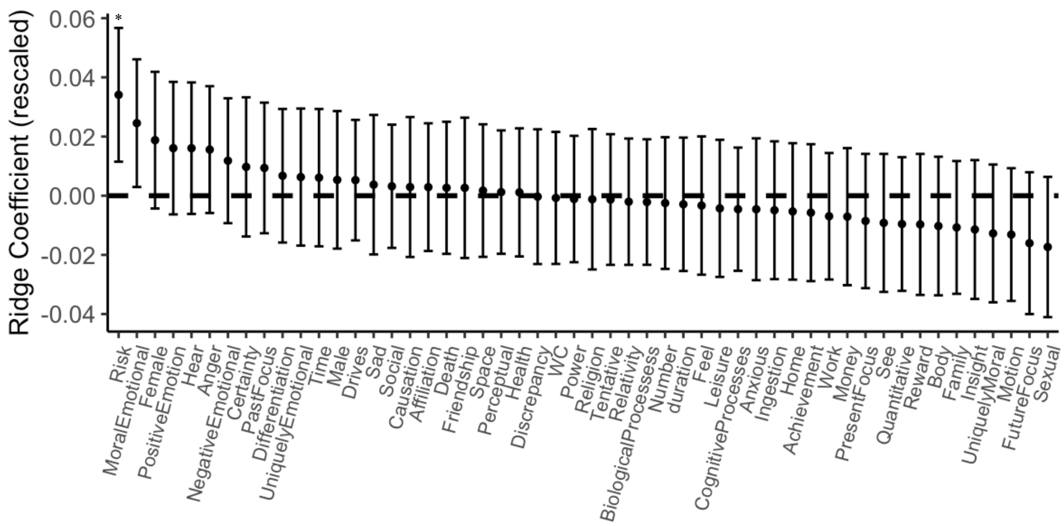

Figure S5. Regression coefficients estimated from ridge regression model. Error bars indicate 95% confidence intervals. Holm-Bonferroni adjusted *p* * < 0.05.

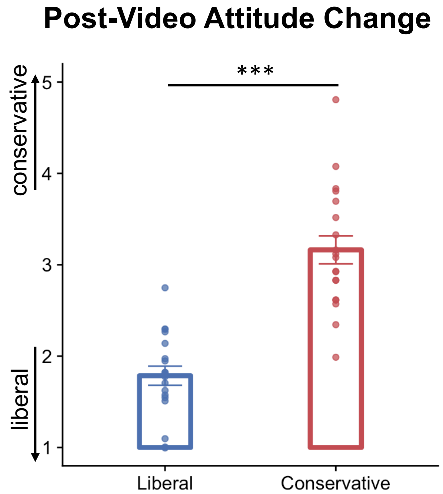

Figure S6. Ratings of post-video attitude changes averaged across videos separately for liberal-leaning (blue) and conservative-leaning (red) participants. Higher ratings denote attitude change towards the conservative position while lower ratings denote attitude change towards the liberal position. Datapoints indicate average rating for individual participants. *** $p < 0.001$

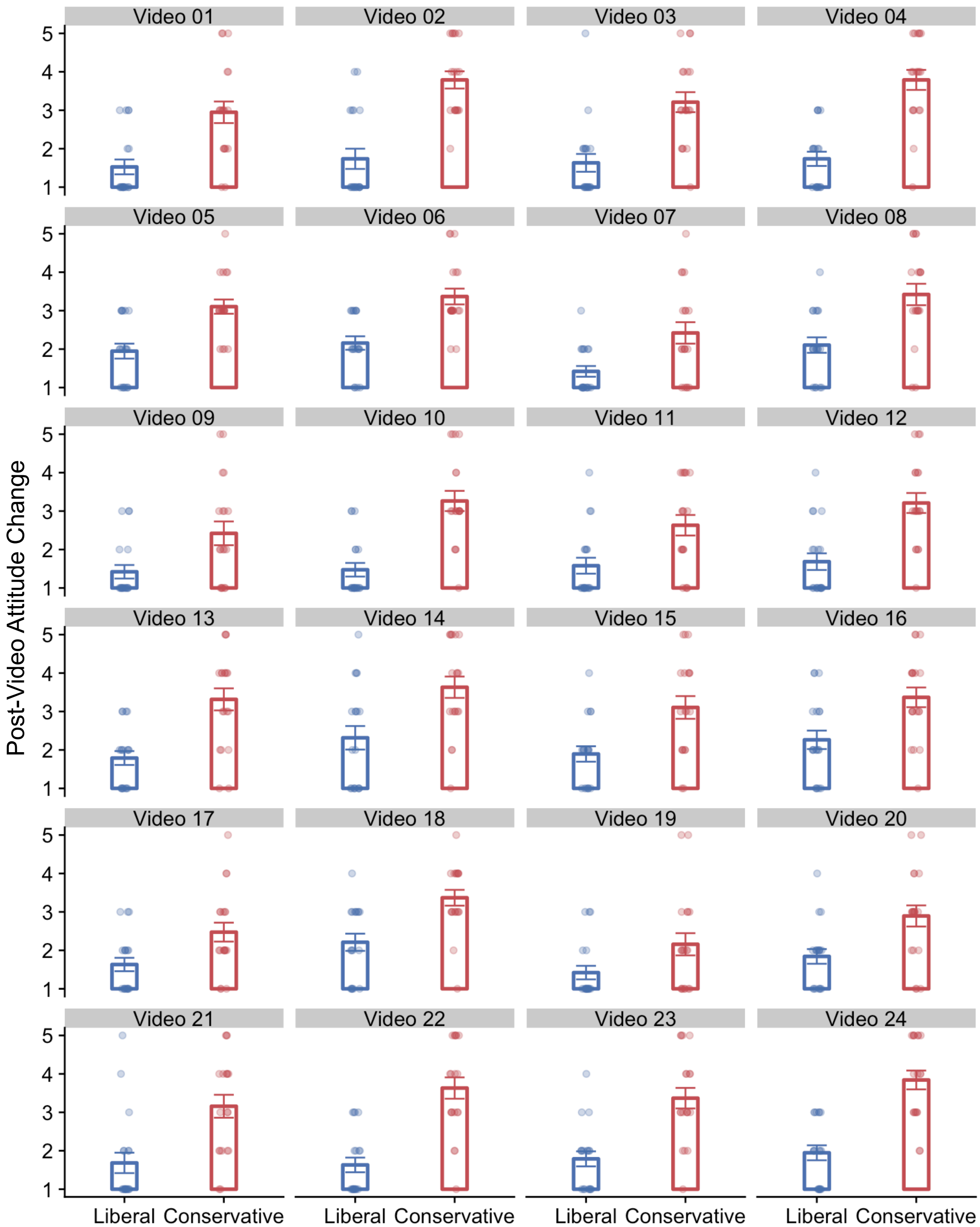

Figure S7. Post-video attitude change for each video separately for liberal-leaning (blue) and conservative-leaning (red) participants. Higher ratings denote attitude change towards the conservative position while lower ratings denote attitude change towards the liberal position. Datapoints indicate rating for each participant with horizontal jitter added for clearer visualization.

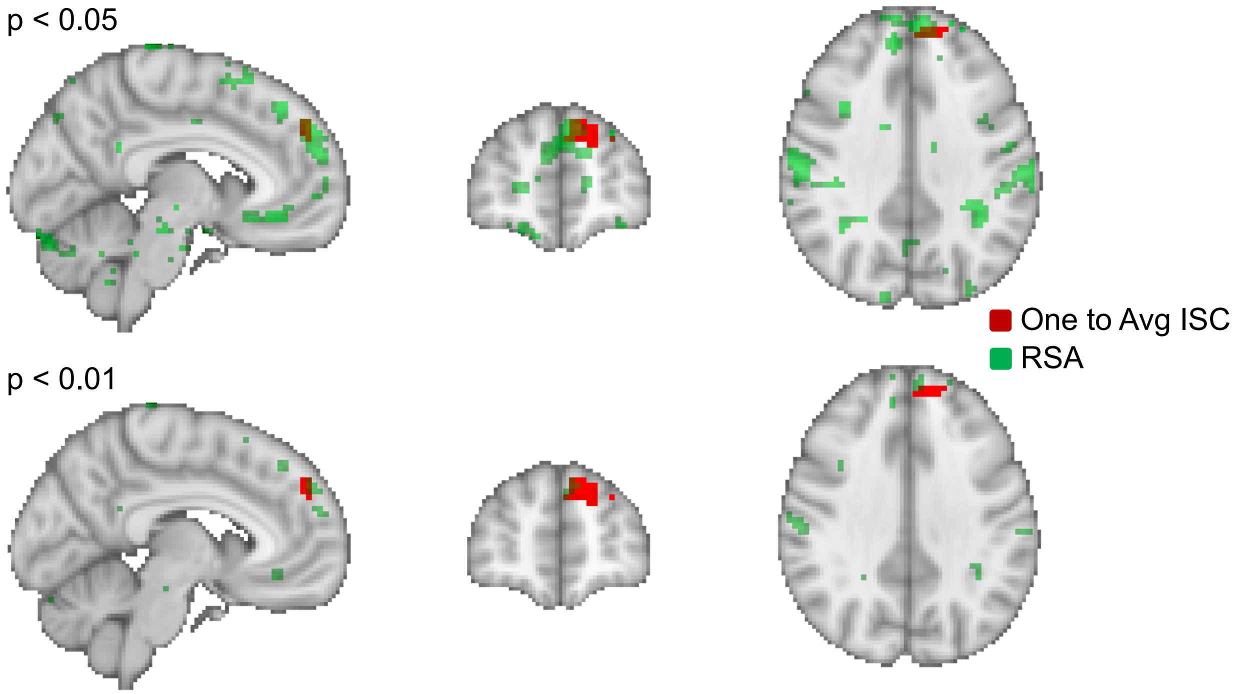

Figure S8. DMPFC cluster overlaid on RSA map thresholded at different *p-values*. Red: Cluster where within-group ISC was greater the between-group ISC as identified using a one-to-group-average ISC approach, FWE-corrected $p < 0.05$ with cluster-forming threshold of $p < 0.001$. Green: RSA map thresholded at $p < 0.05$ (top) and $p < 0.01$ (bottom).

Supplementary Tables

Table S1. Immigration attitude was not related to potentially confounding variables. Liberal-leaning and conservative-leaning participants did not differ significantly on age, sex, income, education and head-motion in the scanner (FD: framewise displacement). A paired t-test was used to test for differences between groups for all variables except sex, where a chi-square test was used instead. We also computed the Spearman correlation coefficient to test for continuous relationships between immigration attitude and each variable.[1]chi-square statistic.

			Categorical		Continuous	
	Liberal	**Conservative**	***t***	***p***	**Spearman *rho***	***p***
Mean Age	30.1 (2.1)	33.0 (3.1)	-0.78	0.44	0.16	0.34
Sex	9F 10M	8F 11M	0.11[1]	0.74	-	-
Median Income	\$80,000-\$89,000	\$80,000-\$89,000	-0.38	0.70	0.09	0.56
Median Education	4-year college degree	4-year college degree	1.02	0.31	-0.01	0.97
Race	14 White (1 Hispanic) 2 Black 3 Asian	15 White (1 Hispanic) 4 Asian				
Head Motion: Mean FD (SE)	0.34 (0.19)	0.27 (0.11)	1.46	0.16	-0.01	0.95

Table S2. Description, length and URL of videos included in the study. Parenthesis indicates segment clipped from video. YouTube playlist:
https://www.youtube.com/playlist?list=PLSMjBX9VG4dvFxQ3TQ9MoD_4ab31jxgte

Video #	Issue	Length	Description	URL
1	Border Wall	1:10	CNN interview with Senator Dick Durbin (D) on federal funding for a wall on the US-Mexico border	https://www.youtube.com/watch?v=G0Plr4-A_6c (0:00 to 1:10)
2	Border Wall	0:57	BBC news showing an existing barrier between California and Mexico, followed by an interview with a Border Patrol Agent who supports the construction of a wall along the US-Mexico border	https://www.youtube.com/watch?v=EmLLlt76YqQ (0:00 to 0:57)
3	Border Wall	1:14	Animated public service announcement on why a wall along the US-Mexico border is unlikely to reduce illegal immigration	https://www.youtube.com/watch?v=jrB0otm2VRo (0:00 to 1:14)
4	Border Wall	1:47	Fox Business interview with a researcher from the Center for Immigration Studies on why a wall along the US-Mexico border would reduce illegal immigration	https://www.youtube.com/watch?v=c5VmX3p5kKE (0:17 to 2:04)
5	Work Authorization	1:00	UC Davis Economics professor explaining how immigrants push Americans towards better paying jobs	https://www.youtube.com/watch?v=OTrZTaz9jAE (0:00 to 0:12, 0:37 to 1:23)
6	Work Authorization	1:04	CBS News segment with panelist explaining how immigrants suppress the wages of American workers	https://www.youtube.com/watch?v=v8xTJ8WCGNU (0:00 to 1:04)
7	Work Authorization	1:30	Video clip from President Obama's November 2014 remarks on why he will sign an executive order allowing work authorization of undocumented immigrants	https://www.youtube.com/watch?v=6Q_Xk66gsRU (4:49 to 6:19)
8	Work Authorization	1:43	Animated public service announcement on how illegal immigration results in unemployment and underemployment of American workers	https://www.youtube.com/watch?v=H8lLU7XjcWc (0:14 to 1:57)
9	Refugee Ban	1:55	Vox video clip with information on the Syrian refugee crisis, followed by criticism of President Trump's executive order to temporarily suspend immigrants and refugees from 7 Muslim-majority countries	Video no longer available at time of writing
10	Refugee Ban	1:26	Local news segment (Buffalo, NY) showing clips from protests against President Trump's executive order to temporarily suspend immigrants and refugees from 7 Muslim-majority countries, followed by clips of individuals voicing their support for the order.	https://www.youtube.com/watch?v=CxeCkCc0mS0 (0:06 to 1:32)
11	Refugee Ban	1:48	CNN journalist Fareed Zakaria providing statistics suggesting that there is "no rational basis" for President Trump's executive order to temporarily suspend immigrants and refugees from 7 Muslim-majority countries	https://www.youtube.com/watch?v=Ywsz4ozvo48 (0:00 to 1:48)
12	Refugee Ban	1:57	Local news segment (Oakland, CA) showing a gathering of Americans who support President Trump's executive order to temporarily suspend	https://www.youtube.com/watch?v=W5fNVdU1Y4g (0:00 to 1:57)

			immigrants and refugees from 7 Muslim-majority countries, followed by clips of protests against the order at airports.	
13	Healthcare Provision	1:34	Animated public service announcement describing how undocumented immigrants pay substantial taxes and should be entitled to healthcare benefits	https://www.youtube.com/watch?v=7v6Pj2_OFiU (0:00 to 1:34)
14	Healthcare Provision	1:31	Fox Business segment with former Arizona governor Jan Brewer criticizing California's petition to expand the Affordable Care Act to include undocumented immigrants, followed by clips showing arrests made by border patrol agents	https://www.youtube.com/watch?v=Lh2-3TS0tqA (0:34 to 2:05)
15	Healthcare Provision	1:55	MSNBC News segment with panelist describing how healthcare is a human right and should be provided regardless of legal status in the country	https://www.youtube.com/watch?v=jd0WvJh_L24 (0:00 to 1:55)
16	Healthcare Provision	1:32	Fox News segment indicating that $2 billion of Medicaid funds is spent on providing emergency care to illegal immigrants, which is a practice that is in violation of existing laws.	Video no longer available at time of writing
17	Dream Act	0:56	CNN interview with Rep. Carlos Curbelo (R) on a Republican-led bill to provide a pathway to legal status for Dreamers	https://www.youtube.com/watch?v=b127AdJ-McU (0:00 to 0:56)
18	Dream Act	0:57	Segment from The Young Turks with panelist describing how the Dream Act encourages dangerous border crossings and puts children's lives at risk	https://www.youtube.com/watch?v=yCsldlO7198 (3:45 to 3:58, 6:14 to 6:58)
19	Dream Act	1:42	Remarks by Senator Dick Durbin (D) describing and advocating for the Dream Act	https://www.youtube.com/watch?v=RGAV7UIN3hQ (0:00 to 1:42)
20	Dream Act	1:28	Animated public service announcement on why the Dream Act is bad policy	https://www.youtube.com/watch?v=N6bZQoFdb1c (0:00 to 1:28)
21	Sanctuary Cities	1:50	AJ+ news segment describing sanctuary policies, including clips of gatherings in support of sanctuary cities and interviews with activists and undocumented immigrants	https://www.youtube.com/watch?v=s4-9PaRd6xc (0:00 to 1:50)
22	Sanctuary Cities	1:30	Fox News segment with panelist explaining why sanctuary policies are dangerous and illegal	Video no longer available at time of writing
23	Sanctuary Cities	1:36	Vice news segment showing clips of arrests made by Immigration and Customs Enforcement, discussing issues surrounding sanctuary policies, including how it lowers crimes.	Video no longer available at time of writing
24	Sanctuary Cities	1:20	Animated public service announcement describing the history of sanctuary cities and how the policies result in the release of 8000 convicted illegal immigrants.	https://www.youtube.com/watch?v=BMfGpOYyKFc (0:00 to 1:20)

Table S3. Regression coefficients from linear mixed effects models predicting neural polarization in the DMPFC from semantic categories. *p*-values were corrected for 50 comparisons using the Holm-Bonferroni procedure.

Regressor	Coefficient	SE	*t*	*p*	Corrected *p*
Risk	0.038	0.009	4.202	< 0.001	0.003
Moral Emotional	0.029	0.009	3.132	0.002	0.12
Anger	0.019	0.01	1.935	0.057	1
Female	0.018	0.01	1.806	0.075	1
Positive Emotional	0.017	0.01	1.697	0.094	1
Hear	0.016	0.01	1.592	0.115	1
Negative Emotional	0.015	0.01	1.508	0.135	1
Past Focus	0.011	0.01	1.046	0.299	1
Differentiation	0.009	0.01	0.924	0.358	1
Certainty	0.007	0.01	0.759	0.45	1
Causation	0.006	0.01	0.648	0.519	1
Death	0.006	0.01	0.648	0.519	1
Number	0.005	0.01	0.525	0.601	1
Perceptual	0.005	0.01	0.528	0.599	1
Social	0.005	0.011	0.500	0.618	1
Uniquely Emotional	0.005	0.01	0.485	0.629	1
Time	0.005	0.01	0.48	0.633	1
Affiliation	0.005	0.01	0.474	0.636	1
Drives	0.004	0.01	0.403	0.688	1
Space	0.002	0.01	0.246	0.807	1
Sad	0.002	0.01	0.212	0.833	1
Feel	0.001	0.01	0.125	0.901	1
Friendship	0.001	0.01	0.093	0.926	1
Male	0	0.01	0.024	0.981	1
Health	0	0.01	-0.047	0.963	1
Religion	-0.001	0.01	-0.054	0.957	1
Home	-0.001	0.01	-0.088	0.93	1
Discrepancy	-0.001	0.01	-0.097	0.923	1
Tentative	-0.002	0.01	-0.235	0.815	1
Cognitive Processes	-0.003	0.01	-0.25	0.803	1
Relativity	-0.003	0.01	-0.313	0.755	1
Anxious	-0.005	0.01	-0.482	0.631	1
Leisure	-0.005	0.01	-0.525	0.601	1
Achievement	-0.006	0.01	-0.595	0.553	1
See	-0.007	0.01	-0.667	0.507	1
Biological Processes	-0.007	0.01	-0.647	0.519	1
Money	-0.007	0.011	-0.609	0.545	1
Power	-0.007	0.01	-0.672	0.504	1
Family	-0.007	0.01	-0.723	0.472	1
Work	-0.008	0.01	-0.807	0.422	1
Ingestion	-0.009	0.01	-0.961	0.34	1
Insight	-0.011	0.01	-1.091	0.278	1
Quantitative	-0.011	0.01	-1.149	0.254	1
Present Focus	-0.011	0.01	-1.127	0.263	1
Body	-0.012	0.009	-1.243	0.218	1
Reward	-0.012	0.01	-1.258	0.212	1
Motion	-0.015	0.01	-1.483	0.142	1
Future Focus	-0.015	0.01	-1.585	0.117	1
Sexual	-0.016	0.009	-1.648	0.103	1
Uniquely Moral	-0.018	0.01	-1.742	0.085	1

Table S4. Regression coefficients from ridge regression model predicting neural polarization in the DMPFC from semantic categories. *p*-values were corrected for 50 comparisons using the Holm-Bonferroni procedure.

Regressor	Coefficient	SE	t	p	Corrected p
Risk	0.034	0.009	3.831	0	0.013
Moral Emotional	0.025	0.008	2.887	0.005	0.255
Female	0.019	0.009	2.064	0.042	1
Positive Emotional	0.016	0.009	1.822	0.072	1
Hear	0.016	0.009	1.834	0.07	1
Anger	0.016	0.008	1.846	0.069	1
Negative Emotional	0.012	0.008	1.423	0.159	1
Certainty	0.01	0.009	1.051	0.297	1
Past Focus	0.009	0.009	1.08	0.283	1
Differentiation	0.007	0.009	0.759	0.45	1
Uniquely Emotional	0.006	0.009	0.691	0.492	1
Time	0.006	0.009	0.667	0.507	1
Male	0.005	0.009	0.585	0.56	1
Drives	0.005	0.008	0.656	0.514	1
Sad	0.004	0.009	0.402	0.689	1
Social	0.003	0.008	0.388	0.699	1
Causation	0.003	0.009	0.315	0.754	1
Affiliation	0.003	0.008	0.341	0.734	1
Death	0.003	0.009	0.304	0.762	1
Friendship	0.003	0.009	0.284	0.777	1
Space	0.002	0.009	0.196	0.845	1
Perceptual	0.001	0.008	0.149	0.882	1
Health	0.001	0.009	0.132	0.895	1
Discrepancy	0	0.009	-0.036	0.971	1
WC	-0.001	0.009	-0.085	0.933	1
Power	-0.001	0.008	-0.133	0.894	1
Religion	-0.001	0.009	-0.129	0.898	1
Tentative	-0.001	0.009	-0.148	0.883	1
Relativity	-0.002	0.008	-0.241	0.81	1
Biological Processes	-0.002	0.008	-0.256	0.799	1
Number	-0.002	0.009	-0.284	0.777	1
duration	-0.003	0.009	-0.326	0.745	1
Feel	-0.003	0.009	-0.361	0.719	1
Leisure	-0.004	0.009	-0.469	0.64	1
Cognitive Processes	-0.005	0.008	-0.556	0.58	1
Anxious	-0.005	0.009	-0.485	0.629	1
Ingestion	-0.005	0.009	-0.535	0.594	1
Home	-0.005	0.009	-0.585	0.56	1
Achievement	-0.006	0.009	-0.629	0.531	1
Work	-0.007	0.008	-0.824	0.412	1
Money	-0.007	0.009	-0.775	0.44	1
Present Focus	-0.009	0.009	-0.957	0.342	1
See	-0.009	0.009	-1	0.32	1
Quantitative	-0.01	0.009	-1.075	0.285	1
Reward	-0.01	0.009	-1.034	0.304	1
Body	-0.01	0.009	-1.11	0.27	1
Family	-0.011	0.009	-1.213	0.229	1
Insight	-0.011	0.009	-1.236	0.22	1
Uniquely Moral	-0.013	0.009	-1.389	0.169	1
Motion	-0.013	0.009	-1.486	0.141	1

Supplementary References

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