Reaction-diffusion in a Growing 3D Domain of Skin Scales Generates a Discrete Cellular Automaton

SUPPLEMENTARY INFORMATION

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Supplementary Figure 1 | **Sensitivity analysis** – The prism colour homogeneity error, which measures, at steady state, the deviation from homogeneously-coloured prisms, decreases with decreasing h_p/h ratio. When the inter-prism thickness reaches 30% of the prism thickness ($h_p/h = 0.3$), the chains of black scales tend to exhibit a width of 1 to 2 scales and the prism colour homogeneity error (averaged across all prisms) is below 4%. An error of zero would correspond to each prism having all its simulation grid points of identical colour; see Methods). Green line: simulation starting with random assignment of prism colours: either black (u,v,w)=(1.2, 6.6, 2.3) or green (u,v,w)=(5.3, 0.92, 4)). Red line: simulation starting with all prisms assigned the uniform steady-state colour (u,v,w)=(3.5, 3.0, 3.4) ±1% noise. Inset images show steady-state top views for $h_p/h = 0.3$, 0.6 and 0.9.



Supplementary Figure 2 I Changing initial conditions in two-component RD models. (A) Steady state obtained with the Gray-Scott RD model (ϵ =0.1, F=0.042, k=0.063, Du = 0.15 and Dv=0.075) with initial condition as in¹: the entire field is first placed in the trivial state (u,v)=(1,0); second the 100 x 100 mesh point area in the centre of the simulation domain is set to (u,v)=(0.5, 0.25); and third, ±1% random noise is added to break the symmetry of the square. (B) Steady state obtained with the Schnakenberg model (a=0.1, b=0.9, γ=1, d=40) with homogeneous initial condition (u,v)=(1,0.9) perturbed with ±1% random noise. In both (A) and (B), the domain size is 200 x 173.

Reference 1: Pearson, J. E. Complex patterns in a simple system. Science 261, 189-192 (1993).



Supplementary Figure 3 | Geometry of large skin patches – (A) 3D geometry bounded form below and above by two smooth 2D functions $z_1(x,y)$ and $z_2(x,y)$. (B) Elementary volume V centred in (x,y) of width and depth ε , thickness $z = z_2(x,y) - z_1(x,y)$ and bounded by the surfaces S_{x-} and S_{x+} in the x direction and S_{y-} and S_{y+} in the y direction.



Supplementary Figure 4 | Pattern length scale quantification - (A) 2D image of the dorsal skin patch of a 45 week-old ocellated lizard. (B) Grayscale version of the image in A. (C) Histogram of grayscale intensities after blurring the image (inset) with a Gaussian filter. A 2-means clustering algorithm separates (red vertical line) the light and dark pixels. (D) Binary image resulting from thresholding based on the histogram in C. (E) Red lines indicate borders of green stripes detected on the original image in A. (F) Pixels belonging to green stripes are coloured by their normalised distance (measured in the number of skin scale diameters) to the closest border. Scale bars: 2 mm.



Supplementary Figure 5 I Growth affects patterning dynamics – Time evolution of average Menger curvature of the pattern border in a real animal and in a quasi-static growth simulation. Curvature ($1/r_M$) is computed, for points (white dots in inset) sampled every r_{avg} (*i.e.*, the average skin scale radius), along the border contour using neighbouring points (blue) that are 5. r_{avg} away from the point where the curvature is computed (purple). *I.e.*, curvature at the purple point is computed as the reciprocal of the radius (purple) of the circle (yellow) intercepting the purple and blue points. The curvature is normalised by r_{avg} .



Supplementary Figure 6 I Patterns within tail scales — (**A**) Ocellated lizards tend to exhibit black and green patterns *within* tail scales as illustrated by an individual at the juvenile (32 week-old) and adult (106 week-old) stages. (**B**) Accurate 3D geometry of a 4x5 scales patch of tail skin (top panel) reconstructed with high-resolution episcopic microscopy sections (one section is displayed in the bottom panel). (**C**) Using the same RD model as in Figure 3E and 3F, simulations in 3D using the geometry reconstructed in (B) generates a pattern *within* scales both in a domain of size (ε =0.6; top panel) equivalent to a 32 week-old animal (initial condition = uniform steady state ±1% random noise), as well as after growth (using a quasi-static approach) to reach a size (ε =0.96; bottom panel) equivalent to a 106 week-old animal, *i.e.*, 60% larger than the 32 week-old juvenile. Yellow and red dotted lines in (C) indicate the position of transversal (red) and parasagittal (yellow) sections. Scale bars: 2 mm in (A) and 300 μ m in (B).