## Phase Velocity Estimation with Expanded Bandwidth in Viscoelastic Phantoms and Tissues – A Supplementary Material

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## I. TM PHANTOMS

Figure S1 presents dispersion curves for the TM phantoms corresponding to the results shown in Fig. 8, in the main manuscript [1]. Figure S1, in addition to the 2D-FT, EV, and GST-SFK methods shown in Fig. 8 in the main manuscript, includes results for the 2P-CWT technique [2]. Results for the 2P-CWT method are shown as an alternative approach based on wavelet transformation analysis.

The 2P-CWT method was implemented according to the theory described in [2]. The 2P-CWT approach considers the shear wave propagation measured only at two lateral locations, separated from each other by distance  $\Delta$ . For the continuous wavelet transform, the complex Morlet wavelet function as the mother wavelet, is used as a bank of filters to decompose the signals into a set of narrow-band details. Next, phase shift between the corresponding details is computed. The 2P-CWT technique uses reduced data (by means of using only two tracking locations) as opposed to a more complete data set (i.e. shear wave propagation is measured at multiple lateral locations) in the 2D-FT, EV, and GST-SFK methods [2].

First, multiple dispersion curves, for the 2P-CWT approach, were calculated for selected first signal positions (7 locations) and selected distances between two measurement signals (50 locations). The first signal position varied from 6-7.1 mm measured from the focal depth location, and the distance between two measurement signals varied from 3 to 10.7 mm. Then, the mean and standard deviation from the obtained set of dispersion curves were calculated. Results are shown as a shaded blue region in Fig. S1.

The phase velocity curves for the 2P-CWT method coincide with the 2D-FT, EV, and GST-SFK methods for frequencies up to around 950 Hz, 800 Hz, 800 Hz, and 1100 Hz, for

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Fig. S1: Phase velocity curves calculated for the 2P-CWT (blue shaded area), 2D-FT (black dots), Eigenvector (green triangles), and GST-SFK (red diamonds) methods. Results were computed for the experimental, custom-made TM viscoelastic phantoms, for (a) Phantom A, (b) Phantom B, (c) Phantom C, and (d) Phantom D.

Phantoms A, B, C, and D, respectively. Above that, dispersion curves flatten and have a downward trend, and their standard deviation widens for 2P-CWT. The 2D-FT and EV approaches provide scattered results, and the proposed GST-SFK method delivers stable phase velocity estimates for higher frequencies.

## REFERENCES

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This work was supported by the Ministry of Science and Higher Education of Poland under agreement no. 0177/E-356/STYP/13/2018. The second author would like to acknowledge financial support from grant R01DK092255, from the National Institutes of Health. The content is solely the responsibility of authors and does not necessarily represent the official views of the National Institute of Diabetes and Digestive and Kidney Diseases or the National Institutes of Health. (*Corresponding author: Piotr Kijanka.*)