

Choosing a model for laser speckle contrast imaging: supplement

CHANG LIU,^{1,2}  KIVILCIM KILIÇ,³ SEFIK EVREN ERDENER,^{3,4}
DAVID A. BOAS,^{1,3} AND DMITRY D. POSTNOV^{3,5,*} 

¹Department of Biomedical Engineering, Boston University, Massachusetts 02215, USA

²Department of Bioengineering, Northeastern University, Massachusetts 02115, USA

³Neurophotonics Center, Boston University, Massachusetts 02215, USA

⁴Institute of Neurological Sciences and Psychiatry, Hacettepe University, Ankara, Turkey

⁵Department of Biomedical Sciences, Copenhagen University, Copenhagen, Denmark

*dpostnov@sund.ku.dk

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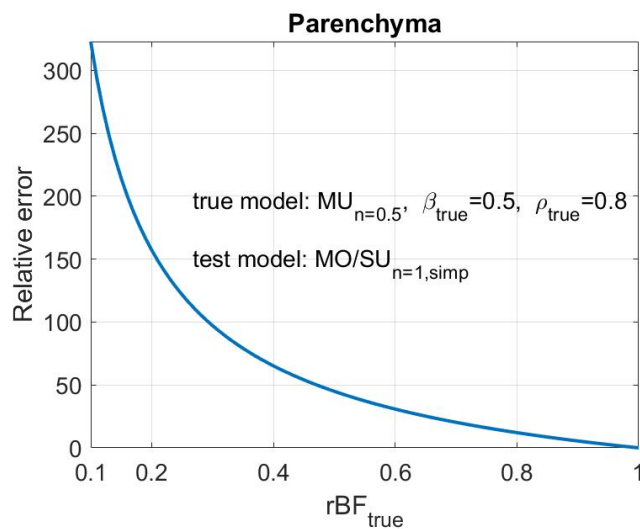


Fig. S1. Relative error caused by a typical combination of wrong assumptions when measuring blood flow in the parenchyma. Ground truth model was set to $MU_{n=0.5}$ with $\tau_c = 1ms$, $T = 5ms$, $\beta = 0.5$, $\rho = 0.8$, while the conventionally used $MO/SU_{n=1,simp}$ was tested.

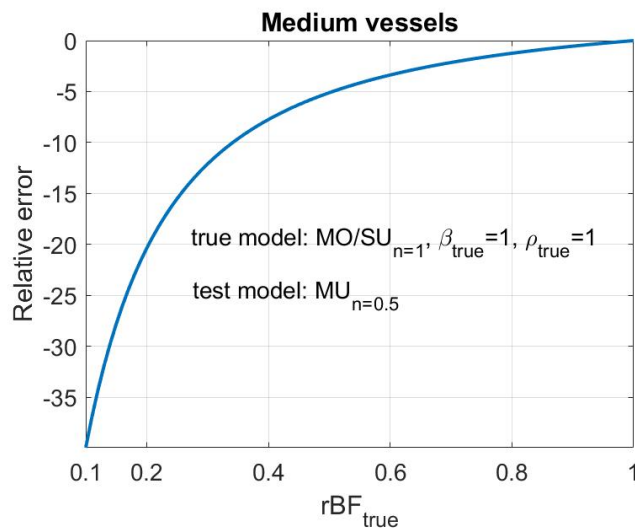


Fig. S2. Relative error caused by using $MU_{n=0.5}$ model in medium-sized vessels. Ground truth model was set to $MO/SU_{n=1}$, with $T = 5ms$, $\tau_c = 0.2ms$, $\beta = 1$, $\rho = 1$.

Derivation of Eq. 12

Field autocorrelation function for multiple scattering unordered motion regime is given by:

$$g_1(\tau) = \exp\left(-\sqrt{\tau/\tau_c}\right) \quad (S1)$$

With Eq. 10, g_2 becomes:

$$g_2 = 1 + \beta \left[\rho^2 \exp(-\sqrt{\tau/\tau_c}) + 2\rho(1-\rho) |\exp(-\sqrt{\tau/\tau_c})| + (1-\rho^2) \right] \quad (S2)$$

Substitute the new g_2 into Eq. 3 to compute the autocovariance $C_t(\tau)$:

$$C_t(\tau) = \beta \left[\rho^2 \exp(-\sqrt{\tau/\tau_c})^2 + 2\rho(1-\rho) \exp(-\sqrt{\tau/\tau_c}) + (1-\rho^2) \right] \langle I \rangle_t^2 \quad (S3)$$

From Eq. 1 the contrast is derived as:

$$K_s^2 = \frac{2\beta}{T} \int_0^T \left(1 - \frac{\tau}{T}\right) \left\{ \rho^2 \exp(-\sqrt{\tau/\tau_c})^2 + 2\rho(1-\rho) \exp(-\sqrt{\tau/\tau_c}) + (1-\rho^2) \right\} d\tau \quad (S4)$$

Compute the definite integral to obtain Eq.12 .

Derivation of Eq. 13

For single scattering and ordered motion regime, the proper model of autocorrelation function is

$$g_1(\tau) = \exp\left(-(\tau/\tau_c)^2\right) \quad (S5)$$

Substitute Eq. S5 in Eq. 10, g_2 becomes:

$$g_2 = 1 + \beta \left\{ \rho^2 \exp\left(-(\tau/\tau_c)^2\right)^2 + 2\rho(1-\rho) \exp\left(-(\tau/\tau_c)^2\right) + (1-\rho^2) \right\} \quad (S6)$$

The autocovariance function $C_t(\tau)$ in this case becomes:

$$C_t(\tau) = \beta \left\{ \rho^2 \exp\left(-(\tau/\tau_c)^2\right)^2 + 2\rho(1-\rho) \exp\left(-(\tau/\tau_c)^2\right) + (1-\rho^2) \right\} \langle I_t \rangle^2 \quad (S7)$$

Substitute Eq. S7 into Eq. 1, the contrast equation is:

$$K_s^2 = \frac{2\beta}{T} \int_0^T \left(1 - \frac{\tau}{T}\right) \left\{ \rho^2 \exp\left(-(\tau/\tau_c)^2\right)^2 + 2\rho(1-\rho) \exp\left(-(\tau/\tau_c)^2\right) + (1-\rho^2) \right\} d\tau \quad (S8)$$

Compute the definite integral to obtain Eq.13.