# *Event-based update of synapses in voltage-based learning rules: Supplementary Material*

| A: Simulation parameters | | |
|---|---|---|
| **Symbol** | **Value** | **Description** |
| $f_{\text{pair}}$ | $[10,\ 11,\ldots,50]$ Hz | frequency of occurrence of spike pairs |
| $\Delta t$ | $\pm 10\,\text{ms}$ | time shift of spike pair |
| $w_{\text{init}}$ [mV] | $0.5\ \text{mV}$ or $\text{pA}/\text{ms}$ | initial weight (unit is $\text{mV}$ for aeif and $\text{pA}/\text{ms}$ for hh neuron) |
| **B: Parameters of `aeif_psc_delta_clopath`** | | |
| **Symbol** | **Value** | **Description** |
| $E_L$ | $-54.402\,\text{mV}$ | leak reversal potential |
| $E_{\text{Na}}$ | $50.0\,\text{mV}$ | sodium reversal potential |
| $E_{\text{K}}$ | $-77.0\,\text{mV}$ | potassium reversal potential |
| $g_L$ | $30.0\,\text{nS}$ | leak conductance |
| $g_{\text{Na}}$ | $12 \cdot 10^3\,\text{nS}$ | sodium peak conductance |
| $g_{\text{K}}$ | $3.6 \cdot 10^3\,\text{nS}$ | potassium peak conductance |
| $C_m$ | $100\,\text{pF}$ | membrane capacitance |
| $\tau_{\text{ex}}$ | $0.2\,\text{ms}$ | rise time of the exc. synaptic alpha funct. |
| $\tau_{\text{in}}$ | $2.0\,\text{ms}$ | rise time of the inh. synaptic alpha funct. |
| $\theta_-$ | $-64.9\,\text{mV}$ | threshold |
| $\theta_+$ | $-35\,\text{mV}$ | threshold |
| $A_{\text{LTD}}$ | $1 \cdot 10^{-4}\,1/\text{mV}$ | amplitude of LTD |
| $A_{\text{LTP}}$ | $12 \cdot 10^{-4}\,1/\text{mV}^2$ | amplitude of LTP |
| $\tau_-$ | $10\,\text{ms}$ | time constant of $\bar{u}_-$ |
| $\tau_+$ | $114\,\text{ms}$ | time constant of $\bar{u}_+$ |
| $\tau_s$ | $15\,\text{ms}$ | time constant of $s_j^*$ |
| $d_s$ | $5\,\text{ms}$ | delay of $\bar{u}_\pm$ |
| **C: Parameters of `hh_psc_alpha_clopath`** | | |
| **Symbol** | **Value** | **Description** |
| $E_L$ | $-70.6\,\text{mV}$ | leak reversal potential |
| $g_L$ | $30\,\text{nS}$ | leak conductance |
| $C_m$ | $281\,\text{pF}$ | membrane capacitance |
| $V_{\text{reset}}$ | $-60\,\text{mV}$ | reset value of membr. pot. after spike |
| $V_{\text{peak}}$ | $33\,\text{mV}$ | spike detection threshold |
| $\Delta_T$ | $2\,\text{mV}$ | slope factor |
| $\tau_w$ | $144\,\text{ms}$ | spike adaptation time constant |
| $\tau_z$ | $40\,\text{ms}$ | spike adaptation time constant |
| $V_{\text{th,max}}$ | $30.4\,\text{mV}$ | threshold potential after spike |
| $\tau_{V,\text{th}}$ | $50\,\text{ms}$ | threshold potential time constant |
| $a$ | $4\,\text{nS}$ | subthreshold adaptation |
| $b$ | $0.0805\,\text{pA}$ | spike triggered adaptation |
| $\theta_-$ | $-70.6\,\text{mV}$ | threshold of $\bar{u}_-$ |
| $\theta_+$ | $-45.3\,\text{mV}$ | threshold of $\bar{u}_+$ |
| $A_{\text{LTD}}$ | $14 \cdot 10^{-5}\,1/\text{mV}$ | amplitude of LTD |
| $A_{\text{LTP}}$ | $8 \cdot 10^{-5}\,1/\text{mV}^2$ | amplitude of LTP |
| $\tau_-$ | $10\,\text{ms}$ | time constant of $\bar{u}_-$ |
| $\tau_+$ | $7\,\text{ms}$ | time constant of $\bar{u}_+$ |
| $\tau_s$ | $15\,\text{ms}$ | time constant of $s_j^*$ |
| $d_s$ | $4\,\text{ms}$ | delay of $\bar{u}_\pm$ |

**Table S1. Parameters of the spike pairing experiment using the Clopath rule.** The values for the aeif model are taken from (**Clopath et al.**, 2010, Tab. 1 and appendix) and those for the hh model are extracted from the reference implementation by B. Torben-Nielson on ModelDB (**Hines et al.**, 2004).

| **A: Model summary** | |
|---|---|
| **Populations** | Three: excitatory, inhibitory, external input |
| **Connectivity** | all-to-all, fixed out-degree, fixed in-degree |
| **Neuron model** | adaptive exponential integrate-and-fire (aeif, Clopath) |
| **Plasticity** | Clopath synapse/ |
| **Input** | independent homogeneous Poisson spike trains |
| **Measurements** | synapse weight |

| **B: Populations** | | |
|---|---|---|
| **Name** | **Elements** | **Population size** |
| E | aeif/two-comp. | $N_E = 10$ |
| I | aeif/two-comp. | $N_I = 3$ |
| $E_{ext}$ | Poisson generator | $N_p = 500$ |

| **C: Connectivity** | | | |
|---|---|---|---|
| **Name** | **Source** | **Target** | **Pattern** |
| ExcExc | E | E | all-to-all (no autapses) |
| ExcInh | E | I | fixed in-degree $C_E = 8$ |
| InhExc | I | E | fixed out-degree $C_I = 6$ |
| ExtExc | $E_{Ext}$ | E | all-to-all |
| ExtInh | $E_{Ext}$ | I | all-to-all |

| **D: Neurons** | |
|---|---|
| **Name** | `aeif_psc_delta_clopath` |
| **Type** | adaptive exponential integrate-and-fire |
| **Details** | see **Clopath et al.** (2010) |
| **Parameters** | see Table S3 |

| **E: Synapses** | | | |
|---|---|---|---|
| **Name** | **Model** | **Initial weight [mV]** | **Max. weight [mV]** |
| ExcExc | `clopath_synapse` | 0.25 | 0.75 |
| ExcInh | `static_synapse` | 1.0 | – |
| InhExc | `static_synapse` | 1.0 | – |
| ExtExc | `clopath_synapse` | random uniform from $[0.5, 1.5]$ | 3.0 |
| ExtInh | `static_synapse` | random uniform from $[0.0, 0.5]$ | – |

| **F: Input** | |
|---|---|
| **Type** | Poisson generator |
| **Description** | homogeneous Poisson spike trains, independent for each neuron, modulated rate |
| **Parameters** | see Table S3 |

**Table S2. Model description of a small excitatory-inhibitory network after Nordlie et al.** (2009). This network reproduces the emergence of strong bidirectional couplings using the Clopath rule shown in Figure 7 . The values of the parameters are shown in Table S3.

| A: Parameters of `aeif_psc_delta_clopath` | | |
|---|---|---|
| **Symbol** | **Value** | **Description** |
| $E_L$ | $-70.6\,\mathrm{mV}$ | leak reversal potential |
| $g_L$ | $30\,\mathrm{nS}$ | leak conductance |
| $C_m$ | $281\,\mathrm{pF}$ | membrane capacitance |
| $V_{\mathrm{reset}}$ | $-60\,\mathrm{mV}$ | reset value of membr. pot. after spike |
| $V_{\mathrm{peak}}$ | $20\,\mathrm{mV}$ | spike detection threshold |
| $\Delta_T$ | $2\,\mathrm{mV}$ | slope factor |
| $\tau_w$ | $144\,\mathrm{ms}$ | spike adaptation time constant |
| $\tau_z$ | $40\,\mathrm{ms}$ | spike adaptation time constant |
| $V_{\mathrm{th,max}}$ | $30.4\,\mathrm{mV}$ | threshold potential after spike |
| $\tau_{V,\mathrm{th}}$ | $50\,\mathrm{ms}$ | threshold potential time constant |
| $a$ | $4\,\mathrm{nS}$ | subthreshold adaptation |
| $b$ | $0.0805\,\mathrm{pA}$ | spike triggered adaptation |
| $\theta_-$ | $-70.6\,\mathrm{mV}$ | threshold of $\bar{u}_-$ |
| $\theta_+$ | $-45.3\,\mathrm{mV}$ | threshold of $\bar{u}_+$ |
| $A_{\mathrm{LTD}}$ | $14 \cdot 10^{-5}\,1/\mathrm{mV}$ | amplitude of LTD |
| $u_{\mathrm{ref}}$ | $60$ | reference value for $\bar{\bar{u}}$ |
| $\bar{\bar{\tau}}$ | $1.5 \cdot 10^{3}\,\mathrm{ms}$ | time constant of $\bar{\bar{u}}$ |
| $A_{\mathrm{LTP}}$ | $8 \cdot 10^{-5}\,1/\mathrm{mV}^2$ | amplitude of LTP |
| $\tau_-$ | $10\,\mathrm{ms}$ | time constant of $\bar{u}_-$ |
| $\tau_+$ | $7\,\mathrm{ms}$ | time constant of $\bar{u}_+$ |
| $\tau_s$ | $15\,\mathrm{ms}$ | time constant of $s_j^*$ |
| $d_s$ | $4\,\mathrm{ms}$ | delay of $\bar{u}_\pm$ |
| **B: Input parameters** | | |
| **Symbol** | **Value** | **Description** |
| $A_p$ | 66 [Hz] | amplitude of Gaussian rate profile |
| $\sigma_p$ | 10 | width of Gaussian rate profile |
| $c_p$ | 0.48 [Hz] | offset of Gaussian rate profile |
| $s_p$ | $[25, 75, \ldots, 475]$ | set of possible values for the center of the Gaussian $\mu_p$ |
| $t_\mu$ | 100 [ms] | interval after which a new value for $\mu_p$ is drawn |
| $N_\mu$ | 100 | number of intervals $t_\mu$ |

**Table S3. Neuron and input parameters for simulation of network producing bidirectional connections using the Clopath rule.** The values are taken from (**Clopath et al.**, 2010, Tab. 1 and appendix). The same values are used for the performance measurements shown in Figures 9 and 10.

| **A: Simulation parameters** | | |
|---|---|---|
| **Symbol** | **Value** | **Description** |
| $T_p$ | $1000\,\mathrm{ms}$ | pattern duration |
| $N_{\mathrm{rep}}$ | $100$ | number of pattern repetitions |
| $N_{\mathrm{p}}$ | $200$ | number of input spike trains |
| $f_{\mathrm{p}}$ | $10\,\mathrm{Hz}$ | input firing rate |
| $w_{g_E}$ | $(18\,\sin{(2\pi t)}+4.8)\;\mathrm{nS}$ | weights to generate periodic excitatory conductance |
| $w_{g_I}$ | $3\,\mathrm{nS}$ | weights to generate constant inhibitory conductance |
| **B: Parameters of `pp_cond_exp_mc_urbanczik (soma)`** | | |
| **Symbol** | **Value** | **Description** |
| $C_m$ | $300\,\mathrm{pF}$ | membrane capacitance |
| $E_L$ | $-70\,\mathrm{mV}$ | leak reversal potential |
| $g_L$ | $30.0\,\mathrm{nS}$ | leak conductance |
| $E_{\mathrm{ex}}$ | $0.0\,\mathrm{mV}$ | exc. reversal potential |
| $E_{\mathrm{in}}$ | $-75.0\,\mathrm{mV}$ | inh. reversal potential |
| $\tau_{\mathrm{ex}}$ | $3.0\,\mathrm{ms}$ | rise time of the exc. synaptic alpha funct. |
| $\tau_{\mathrm{in}}$ | $3.0\,\mathrm{ms}$ | rise time of the inh. synaptic alpha funct. |
| $t_{\mathrm{ref}}$ | $3.0\,\mathrm{ms}$ | refractory time |
| **C: Parameters of `pp_cond_exp_mc_urbanczik (dendrite)`** | | |
| **Symbol** | **Value** | **Description** |
| $C_m$ | $300\,\mathrm{pF}$ | membrane capacitance |
| $E_L$ | $-70\,\mathrm{mV}$ | leak reversal potential |
| $g_L$ | $30.0\,\mathrm{nS}$ | leak conductance |
| $\tau_{\mathrm{ex}}$ | $3.0\,\mathrm{ms}$ | rise time of the exc. synaptic alpha funct. |
| $\tau_{\mathrm{in}}$ | $3.0\,\mathrm{ms}$ | rise time of the inh. synaptic alpha funct. |
| $\phi\,(U)$ | $\frac{0.15\,\mathrm{kHz}}{1+\frac{1}{2}\exp\left(\frac{-55\,\mathrm{mV}-U}{3\,\mathrm{mV}}\right)}$ | rate function |
| $g_{\mathrm{sp}}$ | $600.0\,\mathrm{nS}$ | coupling dendrite to soma |

**Table S4. Parameters of the simulation of the learning experiment using the Urbanczik-Senn rule.** The same values of the neuron parameters are used for the performance measurements shown in Figures 11 and 12.

| **A: Model summary** | |
|---|---|
| **Populations** | Three: excitatory, inhibitory, external input |
| **Connectivity** | random with fixed indegree |
| **Neuron model** | adaptive exponential integrate-and-fire (aeif, Clopath)/<br>two-compartment Poisson (two-comp., Urbanczik-Senn) |
| **Plasticity** | Clopath synapse/<br>Urbanczik-Senn synapse |
| **Input** | independent homogeneous Poisson spike trains |
| **Measurements** | — |

| **B: Populations** | | |
|---|---|---|
| **Name** | **Elements** | **Population size** |
| E | aeif/two-comp. | $N_E = 4N_I$ |
| I | aeif/two-comp. | $N_I$ |
| $\text{E}_\text{ext}$ | Poisson generator | 1 |

| **C: Connectivity** | | | | |
|---|---|---|---|---|
| **Name** | **Source** | **Target** | **Pattern** | **Weight** |
| Exc | E | E+I | fixed in-degree $C_E$ | $J_\text{ex}$ |
| Inh | I | E+I | fixed in-degree $C_I$ | $J_\text{in}$ |
| Ext | $\text{E}_\text{Ext}$ | E+I | all-to-all | $J$ |

| **D: Neurons** | |
|---|---|
| **Name** | `aeif_psc_delta_clopath` |
| **Type** | adaptive exponential integrate-and-fire |
| **Details** | see **Clopath et al.** (2010) |
| **Parameters** | see Table S3 |
| **Name** | `pp_cond_exp_mc_urbanczik` |
| **Type** | two-compartment neuron with spike generation via inhomogeneous Poisson process |
| **Details** | see **Urbanczik and Senn** (2014) |
| **Parameters** | see Table S4 |

| **E: Synapses** | |
|---|---|
| **Name** | **Model** |
| Exc | `clopath/urbanczik_synapse` |
| Inh | `clopath/urbanczik_synapse` |
| Ext | `static_synapse` |

| **F: Input** | |
|---|---|
| **Type** | **Description** |
| Poisson generator | homogeneous Poisson spike trains, independent for each neuron, rate $f_\text{ext} = \nu_\text{ext} C_E$ |

**Table S5. Model description of Brunel network after Nordlie et al.** (2009). This network is used to produce the performance measurement shown in Figures 9, 10, 11, and 12. The values of the parameters are shown in Table S6.

| A: Global simulation parameters | | |
|---|---|---|
| **Symbol** | **Value** | **Description** |
| $T$ | $2 \cdot 10^3$ ms | biological time |
| $h$ | $0.1$ ms | resolution |
| **B: Network sizes** | | |
| **Symbol** | **Value** | **Description** |
| $N = N_E + N_I$ | $1.92 \cdot 10^6$ | number of neurons in Clopath simulation with small indegree $K = 100$ |
| $N$ | $1.54 \cdot 10^5$ | number of neurons in Clopath simulation with large indegree $K = 5000$ |
| $N$ | $3.84 \cdot 10^5$ | number of neurons in Urbanczik simulation with small indegree $K = 100$ |
| $N$ | $3.84 \cdot 10^4$ | number of neurons in Urbanczik simulation with large indegree $K = 5000$ |
| **C: Connectivity** | | |
| **Symbol** | **Value** | **Description** |
| $g$ | 7.0 | ratio inh./exc. weight |
| $J$ | 0.1 | postsynaptic amplitude The unit depends on the neuron model. In case of the aeif model and the Clopath rule it is $[\mathrm{mV}]$ and for the Urbanczik-Senn rule it is $[\mathrm{pA}]$ |
| $J_{\mathrm{ex}}$ | $J$ | amplitude of exc. postsyn. potential |
| $J_{\mathrm{in}}$ | $-gJ_{\mathrm{ex}}$ | amplitude of inh. postsyn. potential |
| $K = C_E + C_I$ | 100 or 5000 | total number of excitatory synapses per neuron |
| $C_E$ | $0.8K$ | number of excitatory synapses per neuron |
| $C_I$ | $0.2K$ | number of inhibitory synapses per neuron |
| $\eta$ | 0.0 | learning rate |
| **D: External input** | | |
| **Symbol** | **Value** | **Description** |
| $\nu_{\mathrm{ext}}$ | $6.75 \cdot 10^{-3}$ Hz | factor in rate of external Poisson input $f_{\mathrm{ext}} = \nu_{\mathrm{ext}} C_E$ |

**Table S6. Parameters of the Brunel network**.

## REFERENCES


Clopath, C., Büsing, L., Vasilaki, E., and Gerstner, W. (2010), Connectivity reflects coding: a model of voltage-based STDP with homeostasis, *Nat. Neurosci.*, 13, 344–352

Hines, M. L., Morse, T., Migliore, M., Carnevale, N. T., and Shepherd, G. M. (2004), ModelDB: A database to support computational neuroscience, *J. Comput. Neurosci.*, 17, 1, 7–11, doi:10.1023/B:JCNS.0000023869.22017.2e

Nordlie, E., Gewaltig, M.-O., and Plesser, H. E. (2009), Towards reproducible descriptions of neuronal network models, *PLOS Comput. Biol.*, 5, 8, e1000456, doi:10.1371/journal.pcbi.1000456

Urbanczik, R. and Senn, W. (2014), Learning by the dendritic prediction of somatic spiking, *Neuron*, 81, 3, 521–528