Supplement to "Joint Modelling of Longitudinal and Survival Data in the Presence of Competing Risks with Applications to Prostate Cancer Data"

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S.1 Priors and Posterior Computation

In this section, we present the technical details of prior and MCMC sampling of the model parameters. We assume independent gamma priors for $\boldsymbol{\lambda}_k$ with $\lambda_{kg} \sim \text{Gamma}(a_{kg}, b_{kg})$, where $a_{kg} = 0.001$, $b_{kg} = 0.001$, for $g = 1, \ldots, G_k$ and $k = 1, 2$. Independent normal priors are assumed for the parameters $\boldsymbol{\gamma}, \boldsymbol{\theta}, \boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2$. Particularly, we have $\boldsymbol{\theta} \sim N(\boldsymbol{\mu}_\theta, \mathbf{V}_\theta)$, $\boldsymbol{\gamma} \sim N(\boldsymbol{\mu}_\gamma, \mathbf{V}_\gamma)$, $\boldsymbol{\alpha}_1^* \sim N(\boldsymbol{\mu}_{\alpha_1}, \mathbf{V}_{\alpha_1})$, $\boldsymbol{\alpha}_2^* \sim N(\boldsymbol{\mu}_{\alpha_2}, \mathbf{V}_{\alpha_2})$, $\boldsymbol{\beta_1} \sim N(\boldsymbol{\mu}_{\beta_1}, \mathbf{V}_{\beta_1})$, $\boldsymbol{\beta_2} \sim N(\boldsymbol{\mu}_{\beta_2}, \mathbf{V}_{\beta_2})$, considering the variance-covariance matrix of the prior distributions are positive definite. We specify an inverse gamma prior for the measurement error of the longitudinal data, i.e. $\sigma^2 \sim \text{IG}(a_0, b_0)$, where, $a_0 = 0.001, b_0 = 0.001$. For the variance covariance matrix of the random effects, we consider the prior distributions for the three components of the cholesky factor Γ. We assume $b_{11} \sim TN(0, \infty)$, $b_{22} \sim TN(0, \infty)$, and $b_{21} \sim N(0, 1)$. The prior distributions of the parameters are assumed to be independent to each other that will facilitate the computation of the Bayesian inference. The prespecified hyperparameters for the other parameters are given by $\boldsymbol{\mu}_\theta = \boldsymbol{\mu}_\gamma = \boldsymbol{\mu}_{\alpha_1} = \boldsymbol{\mu}_{\alpha_2} = \boldsymbol{\mu}_{\beta_1} = \boldsymbol{\mu}_{\beta_2} = \mathbf{0}, \mathbf{V}_\theta = 1000 I_2, \mathbf{V}_\gamma = 1000 I_p, \mathbf{V}_{\alpha_1} = 1000 I_2, \mathbf{V}_{\alpha_2} = 1000 I_2, \mathbf{V}_{\beta_1} = 1000 I_p$, and $\mathbf{V}_{\beta_2} = 1000 I_p$.

MCMC sampling of the model parameters is based on the augmented likelihood function. Within each step of MCMC sampling, we first impute the unknown cause of failure using equation (4.1) in Section 4. Then plugging in this censoring status information, we construct the augmented likelihood (discussed in Section 4). MCMC sampling of the model parameters is a combination of direct sampling from the full conditional distribution and Metropolis-Hastings (M-H) (Metropolis et al., 1953; Hastings, 1970) sampling or Adaptive rejection sampling (ARS) (Gilks and Wild, 1992). The details of MCMC sampling are summarized as follows.

1. Sample b_{11} from

$$b_{11}|\boldsymbol{\theta}, \boldsymbol{\theta}^R, b_{21}, b_{22}, \sigma^2, \mathrm{D_{obs}} \sim TN(\mu_{b_{11}}, \sigma^2_{b_{11}}, 0, \infty),$$

where $\mu_{b_{11}|\cdot} = \dfrac{\sum_{i=1}^n \theta_{i1}^R \sum_{j=1}^{m_i} \big(y_{ij} - (\theta_1 + \theta_2 a_{ij} + b_{21}\theta_{i1}^R a_{ij} + b_{22}\theta_{i2}^R a_{ij} + \mathbf{x}_i'\boldsymbol{\gamma})\big)/\sigma^2}{\sum_{i=1}^n m_i \theta_{i1}^{R2}/\sigma^2 + 1}$ and $\sigma^2_{b_{11}|\cdot} = \dfrac{1}{\sum_{i=1}^n m_i \theta_{i1}^{R2}/\sigma^2 + 1}$.

2. Sample b_{21} from

$$b_{21}|\boldsymbol{\theta}, \boldsymbol{\theta}^R, \sigma^2, b_{11}, b_{22}, \mathrm{D_{obs}} \sim N(\mu_{b_{21}}, \sigma^2_{b_{21}}),$$

where $\mu_{b_{21}|\cdot} = \dfrac{\sum_{i=1}^n \theta_{i1}^R \sum_{j=1}^{m_i} a_{ij}\big(y_{ij} - (\theta_1 + b_{11}\theta_{i1}^R + \theta_2 a_{ij} + b_{22}\theta_{i2}^R a_{ij} + \mathbf{x}_i'\boldsymbol{\gamma})\big)/\sigma^2}{\sum_{i=1}^n \sum_{j=1}^{m_i} \theta_{i1}^{R2} a_{ij}^2/\sigma^2 + 1}$ and $\sigma^2_{b_{21}|\cdot} = \dfrac{1}{\sum_{i=1}^n \sum_{j=1}^{m_i} \theta_{i1}^{R2} a_{ij}^2/\sigma^2 + 1}$.

3. Sample b_{22} from

$$b_{22}|\boldsymbol{\theta}, \boldsymbol{\theta}^R, \sigma^2, b_{11}, b_{21}, \mathrm{D_{obs}} \sim TN(\mu_{b_{22}}, \sigma^2_{b_{22}}, 0, \infty),$$

where $\mu_{b_{22}|\cdot} = \dfrac{\sum_{i=1}^n \theta_{i2}^R \sum_{j=1}^{m_i} a_{ij}\big(y_{ij} - (\theta_1 + b_{11}\theta_{i1}^R + \theta_2 a_{ij} + b_{21}\theta_{i1}^R a_{ij} + \mathbf{x}_i'\boldsymbol{\gamma})\big)/\sigma^2}{\sum_{i=1}^n \sum_{j=1}^{m_i} \theta_{i2}^{R2} a_{ij}^2/\sigma^2 + 1}$ and $\sigma^2_{b_{22}|\cdot} = \dfrac{1}{\sum_{i=1}^n \sum_{j=1}^{m_i} \theta_{i2}^{R2} a_{ij}^2/\sigma^2 + 1}$.

4. Sample $\boldsymbol{\gamma}$ from

$$\boldsymbol{\gamma}|\sigma^2, \boldsymbol{\theta}^R, \mathrm{D_{obs}} \sim N(\Sigma_\gamma\{\frac{1}{\sigma^2}\sum_{i=1}^n \mathbf{x}_i(\mathbf{y}_i - \mathbf{g}(a_{ij})'\Gamma\boldsymbol{\theta}_i^R)\}, \Sigma_\gamma),$$

where $\Sigma_\gamma = (\frac{1}{\sigma^2}\sum_{i=1}^n \mathbf{x}_i\mathbf{x}_i' + V_\gamma^{-1})^{-1}$.

5. Sample $\boldsymbol{\theta}$ from

$$\boldsymbol{\theta}|\Gamma, \boldsymbol{\theta}^R, \mathrm{D_{obs}} \sim N(\Sigma_\theta\{\frac{1}{\sigma^2}\sum_{i=1}^n \mathbf{g}(a_{ij})(\mathbf{y}_i - \mathbf{g}(a_{ij})'\Gamma\boldsymbol{\theta}_i^R)\}, \Sigma_\theta),$$

where $\Sigma_\theta = (\frac{1}{\sigma^2}\sum_{i=1}^n \mathbf{g}(a_{ij})\mathbf{g}(a_{ij})' + V_\theta^{-1})^{-1}$.

6. Sample σ^2 from

$$\sigma^2|\boldsymbol{\gamma}, \boldsymbol{\theta}^R, \mathrm{D_{obs}} \sim IG\Big(a_0 + \sum_{i=1}^n m_i/2, b_0 + \frac{1}{2}\sum_{i=1}^n \big(\mathbf{y}_i - \mathbf{g}(a_{ij})'(\boldsymbol{\theta} + \Gamma\boldsymbol{\theta}_i^R) - \mathbf{x}_i'\boldsymbol{\gamma}\big)'\big(\mathbf{y}_i - \mathbf{g}(a_{ij})'(\boldsymbol{\theta} + \Gamma\boldsymbol{\theta}_i^R) - \mathbf{x}_i'\boldsymbol{\gamma}\big)\Big).$$

7. Sample $\lambda_{kg}, k = 1, 2, g = 1, \ldots, G_k$ from

$$\lambda_{kg}|\boldsymbol{\alpha}_1^*, \boldsymbol{\alpha}_2^*, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \boldsymbol{\theta}^R, \boldsymbol{\delta}^*, D_{\mathrm{obs}} \sim \mathrm{Gamma}(a_{kg} + d_{kg}, b_{kg} + \tau_{kg}),\ k = 1, 2,\ g = 1, \ldots, G_k$$

where $d_{kg} = \sum_{i=1}^n \delta_i^* I(s_{k,g-1} < t_i \le s_{kg})$ and $\tau_{kg} = \sum_{i=1}^n \exp(\mathbf{z}_i'\boldsymbol{\beta}_k)H_{ik}(t_i)$.

8. M-H algorithm is used to sample from the conditional distribution of $\boldsymbol{\alpha}_k$ given by

$$\pi(\boldsymbol{\alpha}_k^*|\boldsymbol{\delta}^*, \boldsymbol{\lambda}_k, \boldsymbol{\beta}_k, \boldsymbol{\theta}^R, \mathrm{D}_{\mathrm{obs}}) \propto \exp\Big[\sum_{i=1}^n I(\delta_i^* = k)\Big(\boldsymbol{\theta}_i^{R'}\boldsymbol{\alpha}_k^* + \mathbf{z}_i'\boldsymbol{\beta}_k\Big)$$
$$- \sum_{i=1}^n \exp(\boldsymbol{\theta}_i^{R'}\boldsymbol{\alpha}_k^* + \mathbf{z}_i'\boldsymbol{\beta}_k)H_{ik}(t_i) - \frac{1}{2}(\boldsymbol{\alpha}_k^* - \boldsymbol{\mu}_{\alpha_k})'\mathbf{V}_{\alpha_k}^{-1}(\boldsymbol{\alpha}_k^* - \boldsymbol{\mu}_{\alpha_k})\Big], k = 1, 2,$$

where $H_{ik}(t_i) = \sum_{g=1}^{j-1} \lambda_{kg}(s_{k,g} - s_{k,g-1}) + \lambda_{kj}(t_i - s_{kg})$.

9. ARS algorithm is used to sample $\boldsymbol{\beta}_k$ from

$$\pi(\boldsymbol{\beta}_k|\boldsymbol{\delta}^*, \boldsymbol{\lambda}_k, \boldsymbol{\alpha}_k^*, \boldsymbol{\theta}^R, \mathrm{D}_{\mathrm{obs}}) \propto \exp\Big[\sum_{i=1}^n I(\delta_i^* = k)\Big(\boldsymbol{\theta}_i^{R'}\boldsymbol{\alpha}_k^* + \mathbf{z}_i'\boldsymbol{\beta}_k\Big)$$
$$- \sum_{i=1}^n \exp(\boldsymbol{\theta}_i^{R'}\boldsymbol{\alpha}_k^* + \mathbf{z}_i'\boldsymbol{\beta}_k)H_{ik}(t_i) - \frac{1}{2}(\boldsymbol{\beta}_{\boldsymbol{k}} - \boldsymbol{\mu}_{\beta_k})'\mathbf{V}_{\beta_k}^{-1}(\boldsymbol{\beta}_{\boldsymbol{k}} - \boldsymbol{\mu}_{\beta_k})\Big], k = 1, 2$$

10. M-H algorithm is used to sample $\boldsymbol{\theta}_i^R$ from

$$\pi(\boldsymbol{\theta}_i^R|\boldsymbol{\theta}, \sigma^2, \Gamma, \boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2, \boldsymbol{\alpha}_1^*, \boldsymbol{\alpha}_2^*, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2, D_{\mathrm{obs}}) \propto \exp\Big[\sum_{k=1}^2 I(\delta_i^* = k)(\boldsymbol{\theta}_i^{R'}\boldsymbol{\alpha}_k^* + \mathbf{z}_i'\boldsymbol{\beta}_k)$$
$$- \exp(\boldsymbol{\theta}_i^{R'}\boldsymbol{\alpha}_1^* + \mathbf{z}_i'\boldsymbol{\beta}_1)H_{i1}(t_i) - \exp(\boldsymbol{\theta}_i^{R'}\boldsymbol{\alpha}_2^* + \mathbf{z}_i'\boldsymbol{\beta}_2)H_{i2}(t_i)$$
$$- \frac{1}{2\sigma^2}\sum_{i=1}^n \big(\mathbf{y}_i - \mathbf{g}(a_{ij})'(\boldsymbol{\theta} + \Gamma\boldsymbol{\theta}_i^R) - \mathbf{x}_i'\boldsymbol{\gamma}\big)'\big(\mathbf{y}_i - \mathbf{g}(a_{ij})'(\boldsymbol{\theta} + \Gamma\boldsymbol{\theta}_i^R) - \mathbf{x}_i'\boldsymbol{\gamma}\big) - \frac{1}{2}\boldsymbol{\theta}_i^{R'}\boldsymbol{\theta}_i^R\Big].$$

The computer codes for the proposed method are written in FORTRAN 95 with IMSL subroutines. The software in the form of FORTRAN 95 codes can be provided upon request. Tables S.1 and S.2 present the simulation results under the proposed joint model and the survival data only model, respectively, based on the simulated data from True-Surv scenario defined in Section 5.1. The trace plots of the MCMC chains of the parameters under the proposed joint model for the SELECT data are presented in Figures S.1–S.7.

References

Gilks, W. R. and Wild, P. (1992). Adaptive rejection sampling for Gibbs sampling. *Journal of the Royal Statistical Society: Series C (Applied Statistics)*, 41(2):337–348.

Hastings, W. K. (1970). Monte carlo sampling methods using Markov chains and their applications. *Biometrika*, 57:97–109.

Metropolis, N., Rosenbluth, A. W., Rosenbluth, M. N., Teller, A. H., and Teller, E. (1953). Equation of state calculations by fast computing machines. *The Journal of Chemical Physics*, 21(6):1087–1092.

S.2 Additional Tables and Figures

Table S.1: Posterior estimates under the joint model for True-Surv in Section 5.1

Parameter	True	Est.	Bias	SE	SD	Coverage
γ_1	0.15	0.150	0.000	0.011	0.011	0.960
γ_2	0.30	0.298	0.002	0.011	0.012	0.930
σ^2	0.50	0.500	0.000	0.002	0.002	0.935
b_{11}	1.00	1.001	-0.001	0.009	0.009	0.940
b_{21}	0.50	0.502	-0.002	0.012	0.011	0.955
b_{22}	0.80	0.800	-0.000	0.008	0.007	0.975
θ_1	0.50	0.501	-0.001	0.013	0.014	0.945
θ_2	1.00	1.001	-0.001	0.012	0.013	0.945
α^*_{11}	0.00	0.001	-0.001	0.020	0.020	0.940
α^*_{12}	0.00	0.002	-0.002	0.020	0.020	0.945
α^*_{21}	0.00	-0.002	0.002	0.022	0.022	0.960
α^*_{22}	0.00	-0.003	0.003	0.022	0.022	0.925
β_{11}	0.50	0.495	0.005	0.020	0.018	0.975
β_{12}	0.60	0.588	0.012	0.022	0.021	0.915
β_{21}	0.20	0.204	-0.004	0.022	0.020	0.970
β_{22}	-0.50	-0.479	-0.021	0.024	0.022	0.860
λ_1	0.10	0.100	-0.000	0.002	0.002	0.965
λ_2	0.08	0.081	-0.001	0.002	0.002	0.895

Table S.2: Posterior estimates under the survival data only model for True-Surv in Section 5.1

Parameter	True	Est.	Bias	SE	SD	Coverage
β_{11}	0.50	0.495	0.005	0.020	0.018	0.970
β_{12}	0.60	0.587	0.013	0.022	0.021	0.925
β_{21}	0.20	0.204	-0.004	0.022	0.020	0.970
β_{22}	-0.50	-0.478	-0.022	0.024	0.022	0.860
λ_1	0.10	0.100	-0.000	0.002	0.002	0.965
λ_2	0.08	0.081	-0.001	0.002	0.002	0.885

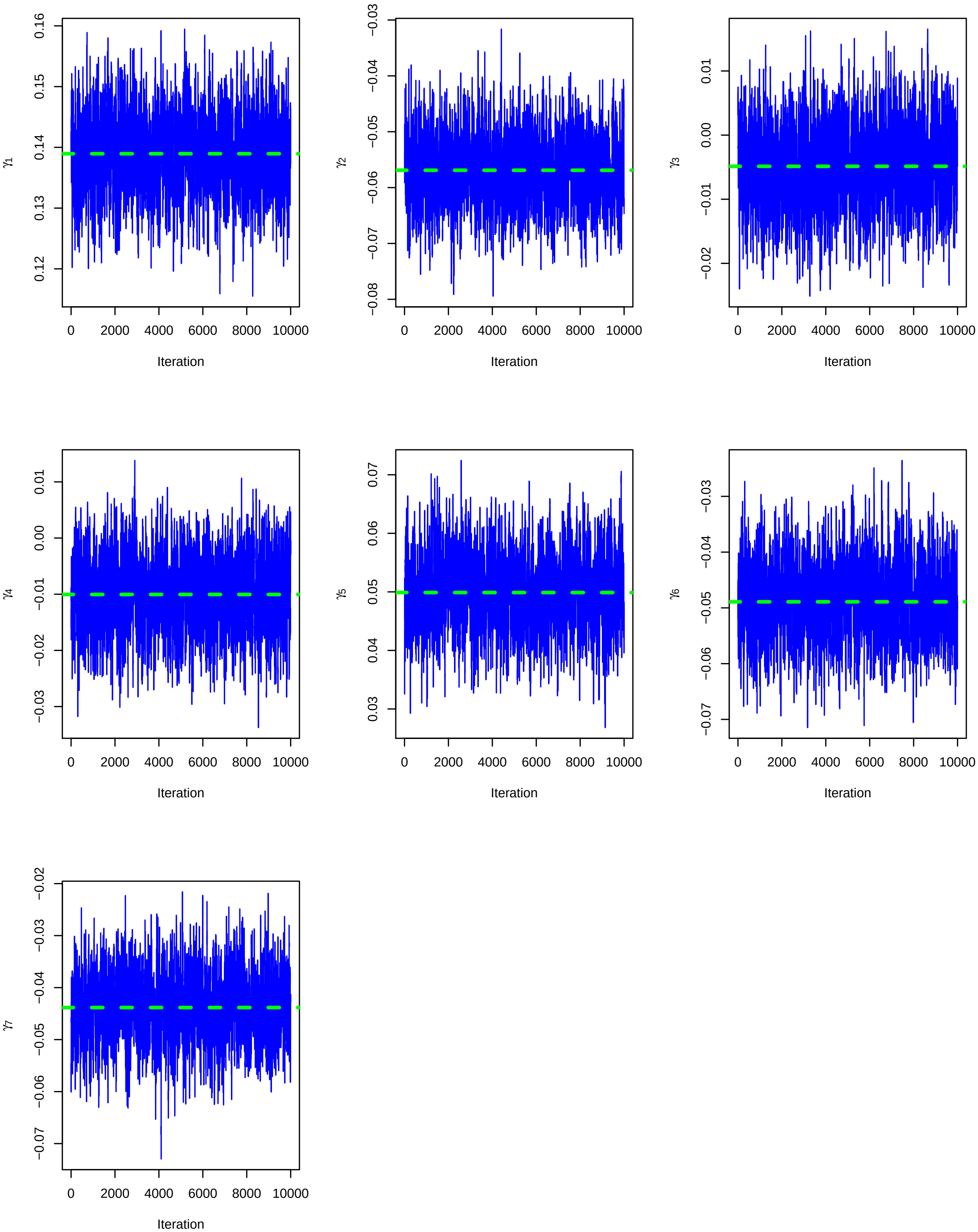

Figure S.1: Trace plots of MCMC samples of $\boldsymbol{\gamma}$

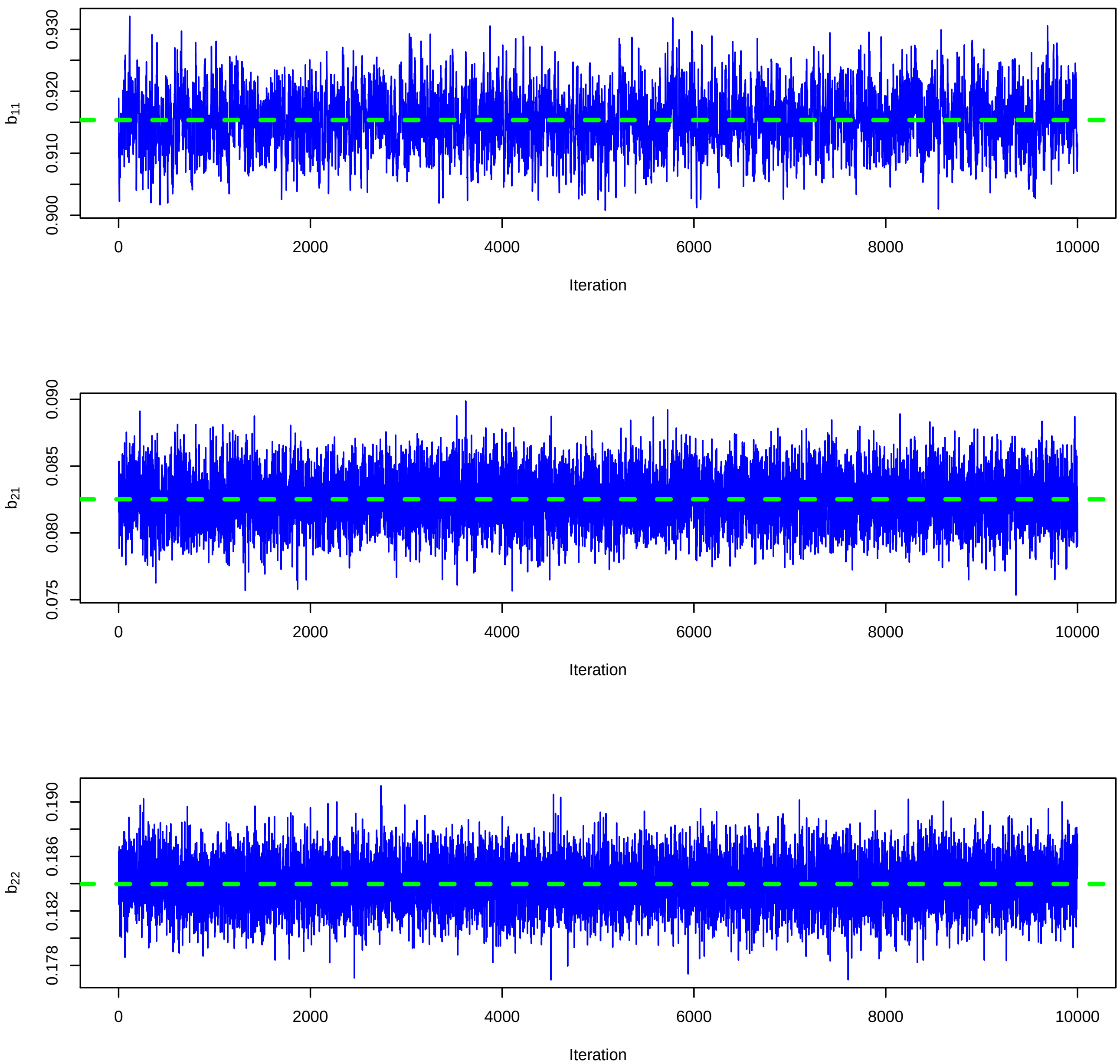

Figure S.2: Trace plots of MCMC samples of Γ parameters

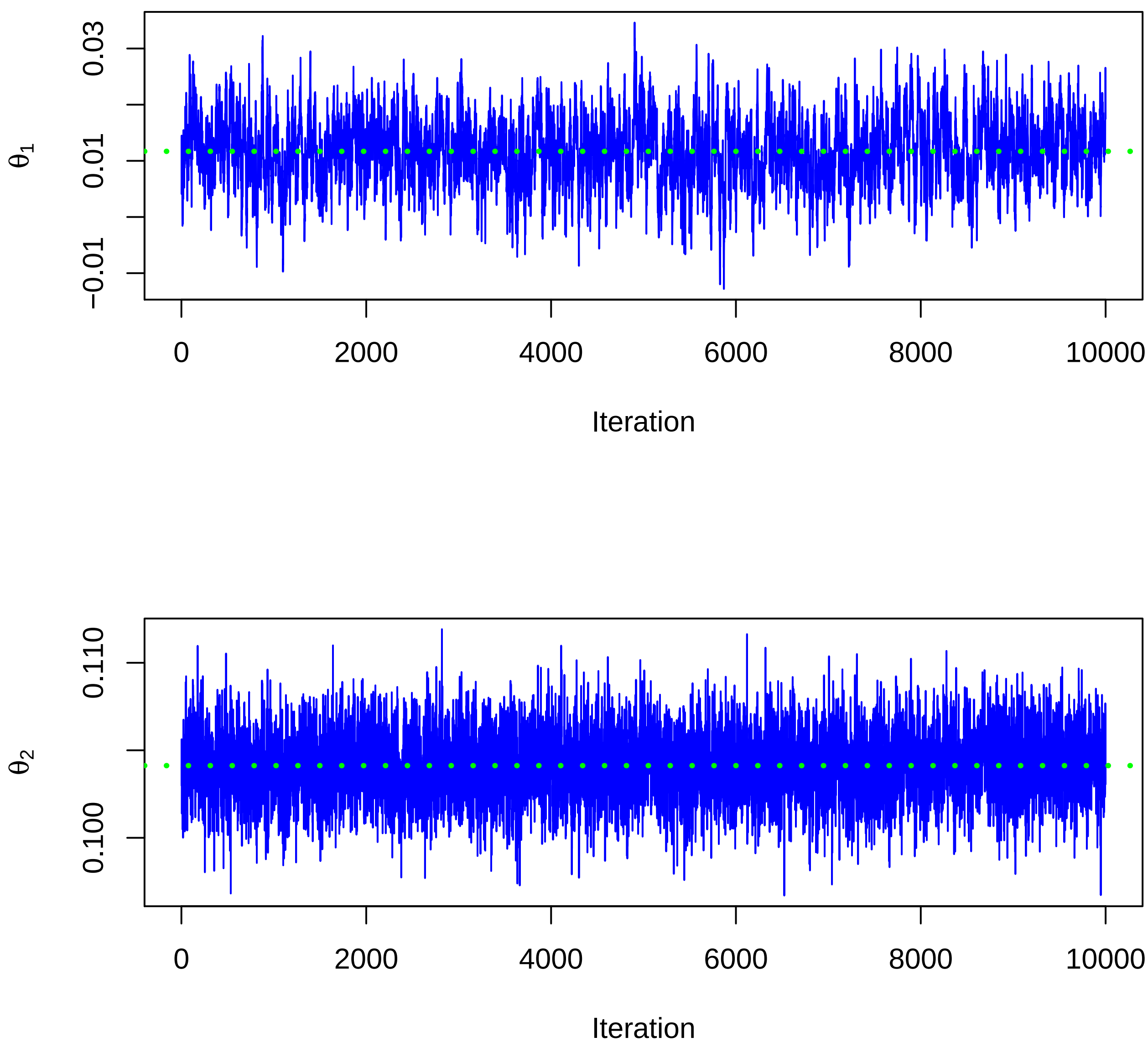

Figure S.3: Trace plots of MCMC samples of $\boldsymbol{\theta}$ parameters

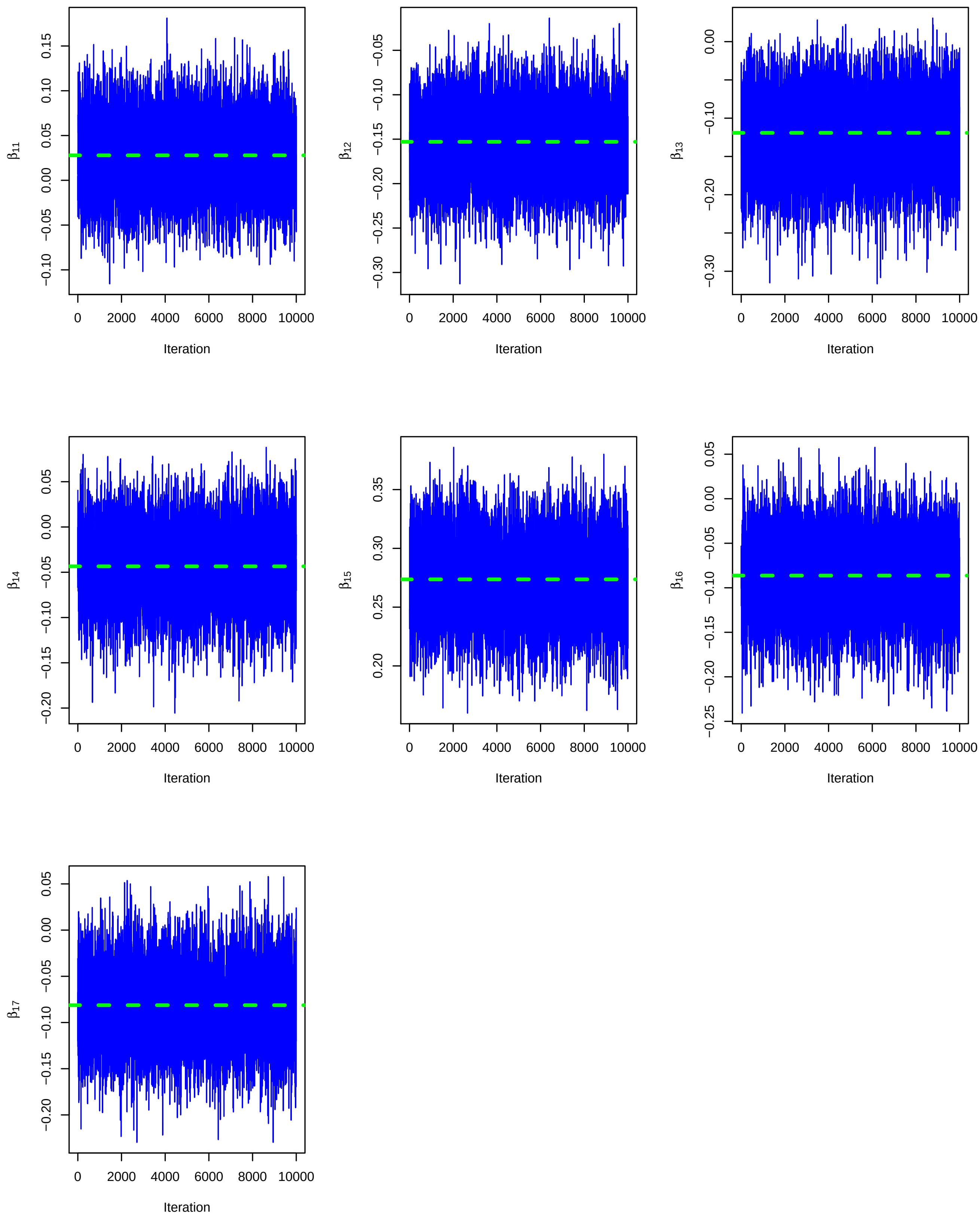

Figure S.4: Trace plots of MCMC samples of $\boldsymbol{\beta}_1$ parameters

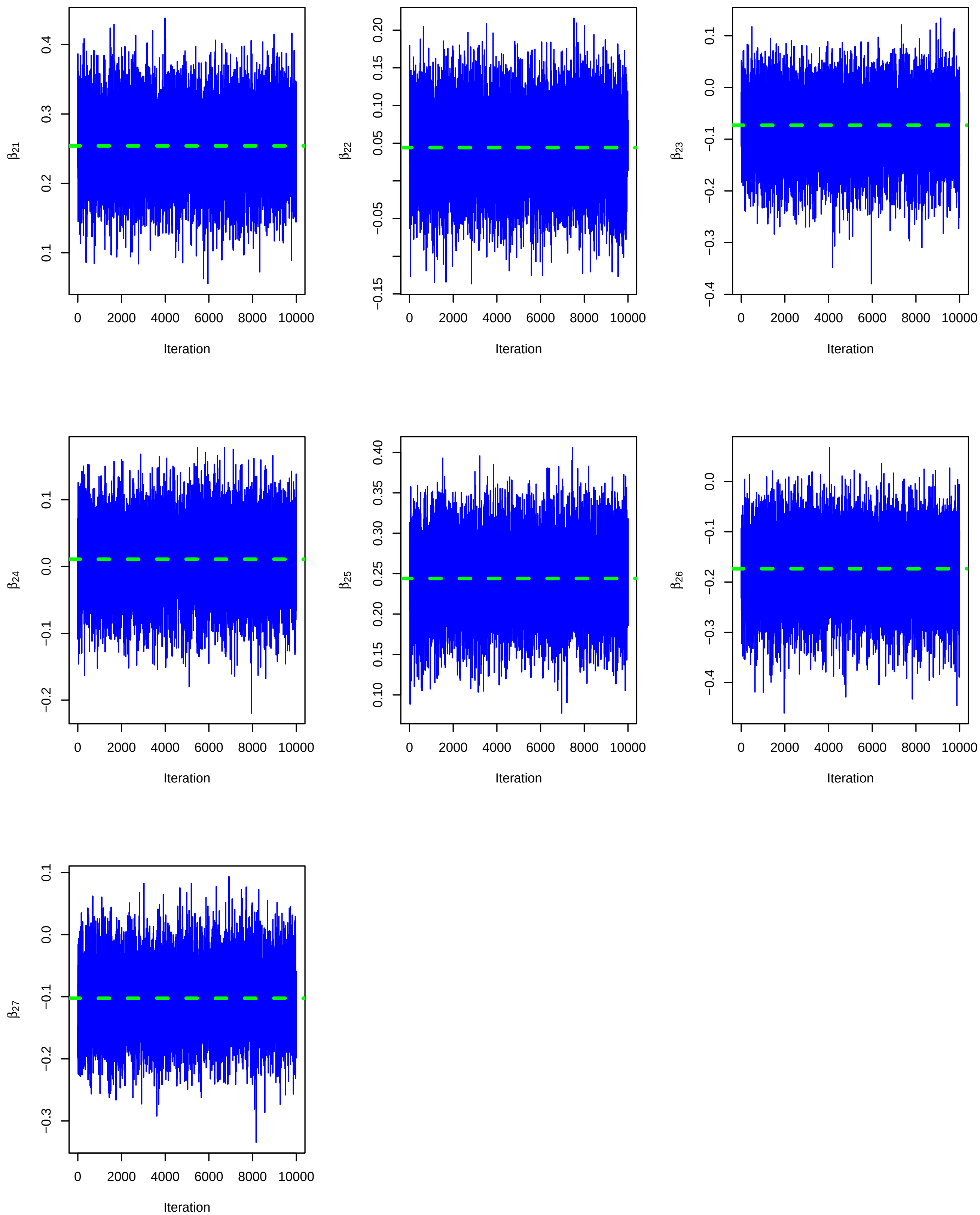

Figure S.5: Trace plots of MCMC samples of $\boldsymbol{\beta}_2$ parameters

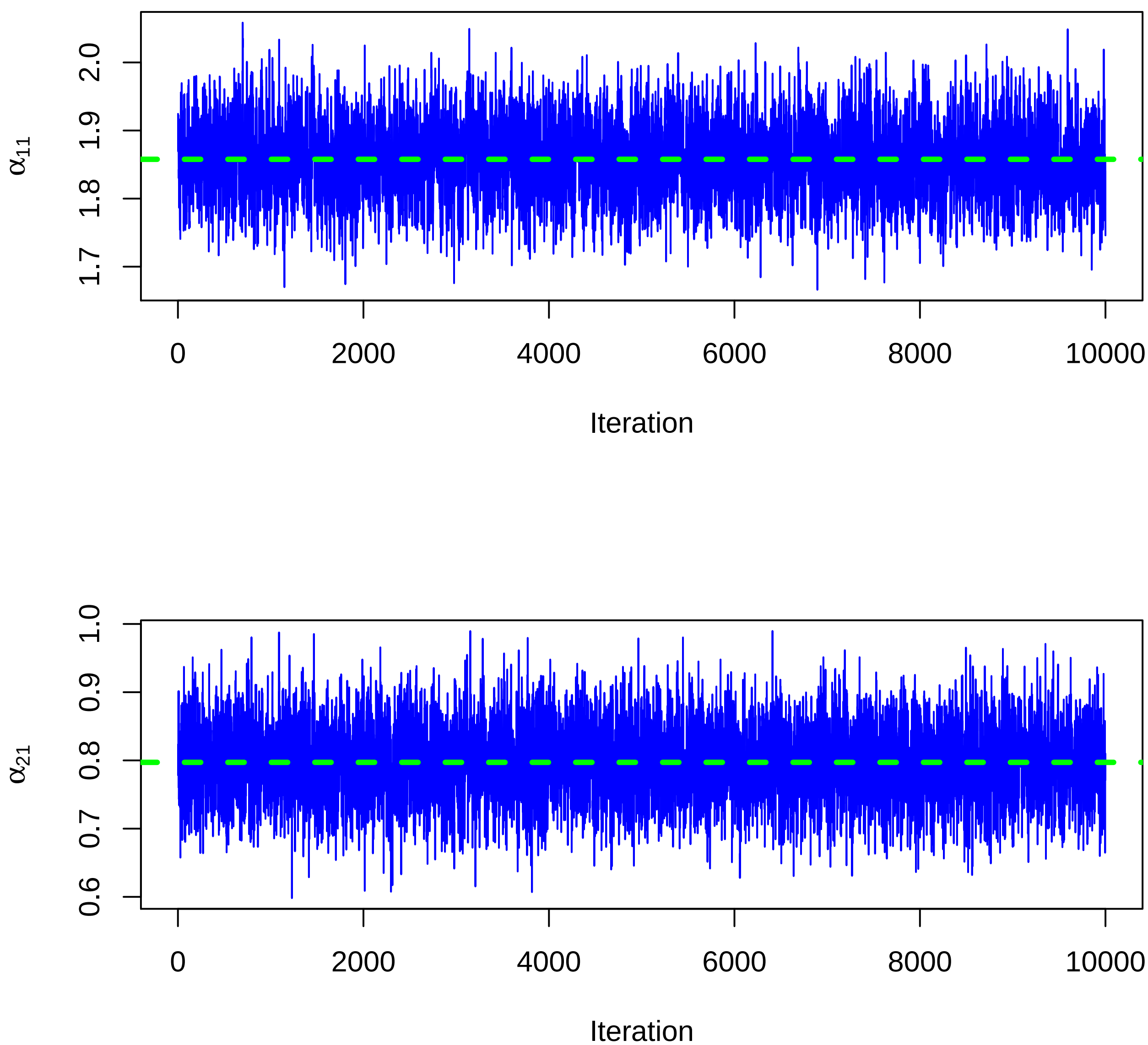

Figure S.6: Trace plots of MCMC samples of $\boldsymbol{\alpha}_1$ parameters

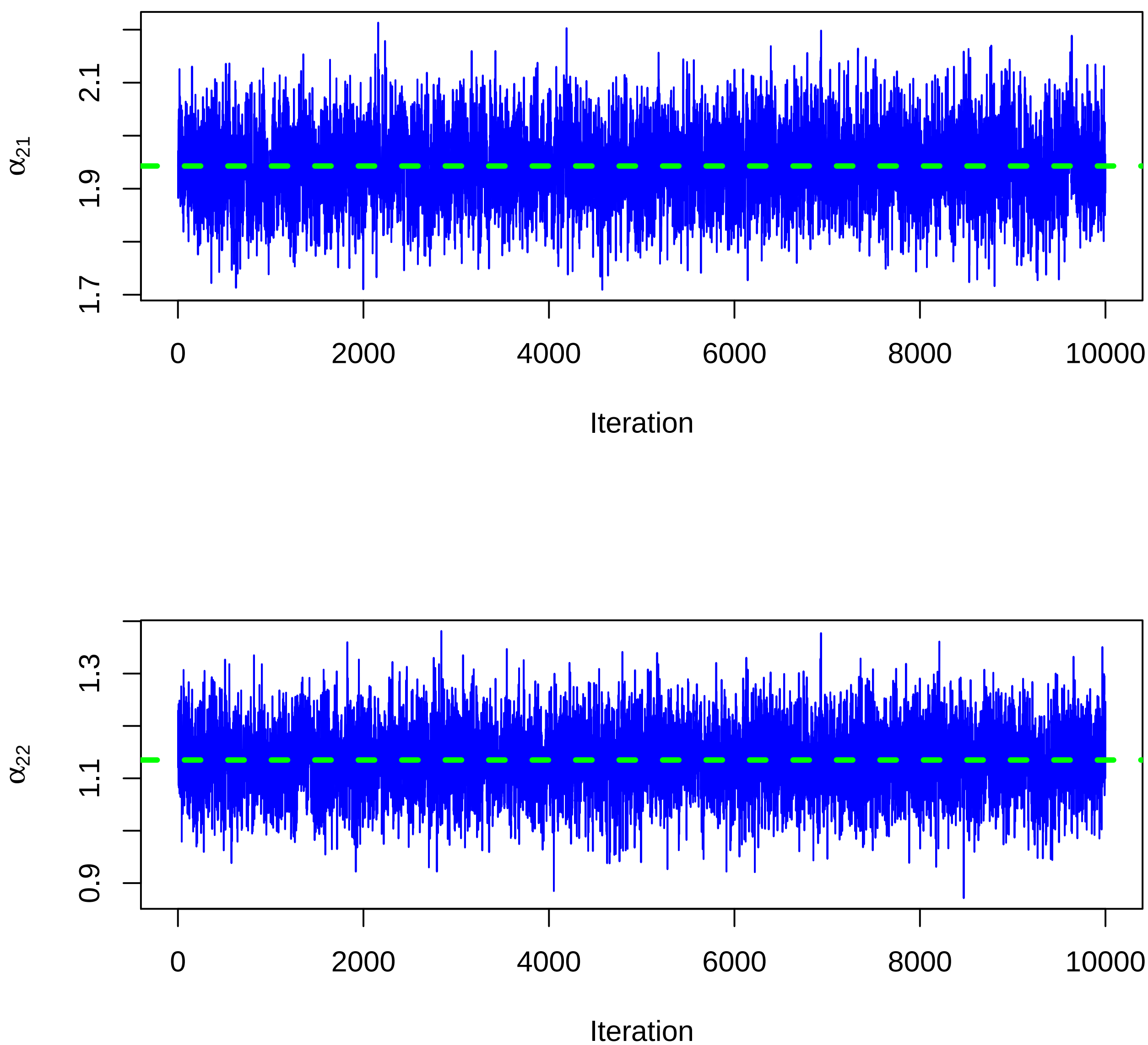

Figure S.7: Trace plots of MCMC samples of $\boldsymbol{\alpha}_2$ parameters