Web-based supporting materials for "Multi-armed Angle-based Direct learning for Estimating Optimal Individual Treatment Rules with Various Outcomes"

Zhengling Qi[1], Dacheng Liu[2], Haoda Fu[3], and Yufeng Liu[1*]

[1]Department of Statistics and Operations Research, Department of Genetics, Department of Biostatistics, Carolina Center for Genome Science, Lineberger Comprehensive Cancer Center, University of North Carolina, Chapel Hill
[2]Boehringer Ingelheim Pharmaceuticals, Inc., Ridgefield, CT, USA
[3]Eli Lilly and Company, Lilly Corporate Center, Indianapolis, IN, USA
[*]E-mail: yfliu@email.unc.edu

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1 Additional Simulation Studies

In this section, we include additional simulation results to further demonstrate the performance of our methods.

1.1 Continuous Outcome Studies

When the clinical outcome R is continuous, we report the simulation results with $n = 400, 800, p = 20$ and $n = 200, p = 20, 40$ for the three continuous simulation scenarios in the main paper.

Table 1: Results of average means (standard deviations) of empirical value functions and misclassification rates for four continuous-outcome simulations scenarios with 20 covariates. The best value functions and misclassification rates are in bold.

	$n = 400$		$n = 800$	
	Value	Misclassification	Value	Misclassification
		Scenario 1		
Pair-D	2.79(0.04)	0.45(0.02)	3.04(0.02)	0.29(0.02)
l_1-PLS	**3.12**(0.01)	**0.21**(0.01)	**3.16**(0.01)	**0.14**(0.01)
DL	2.67(0.03)	0.51(0.01)	2.78(0.02)	0.47(0.01)
ACWL-1	2.77(0.03)	0.43(0.01)	2.91(0.02)	0.37(0.01)
ACWL-2	2.91(0.02)	0.37(0.01)	3.04(0.01)	0.3(0.01)
VT	2.75(0.02)	0.48(0.01)	2.85(0.01)	0.43(0.01)
Group-AD	3.11(0.03)	**0.21**(0.02)	3.14(0.03)	0.16(0.02)
		Scenario 2		
Pair-D	2.82(0.11)	0.33(0.03)	2.92(0.1)	0.3(0.03)
l_1-PLS	2.93(0.11)	0.36(0.04)	2.99(0.1)	0.32(0.03)
DL	2.88(0.11)	0.34(0.04)	2.98(0.12)	0.28(0.04)
ACWL-1	2.78(0.11)	0.38(0.02)	2.96(0.1)	0.31(0.02)
ACWL-2	2.86(0.10)	0.37(0.02)	3.04(0.1)	**0.28**(0.03)
VT	**3.04**(0.09)	**0.30**(0.02)	**3.09**(0.1)	0.28(0.03)
Group-AD	2.91(0.1)	0.32(0.03)	2.9(0.11)	0.31(0.03)
		Scenario 3		
Pair-D	1.2(0.04)	0.75(0.03)	1.21(0.04)	0.75(0.03)
l_1-PLS	1.51(0.19)	0.54(0.15)	1.64(0.2)	0.41(0.18)
DL	1.43(0.1)	0.61(0.06)	1.52(0.07)	0.55(0.06)
ACWL-1	1.43(0.07)	0.61(0.05)	1.49(0.07)	0.56(0.05)
ACWL-2	1.47(0.07)	0.58(0.05)	1.63(0.06)	0.45(0.05)
VT	1.42(0.05)	0.62(0.03)	1.48(0.04)	0.57(0.03)
Group-D	**1.65**(0.11)	**0.43**(0.09)	**1.77**(0.04)	**0.29**(0.05)

Table 2: Results of average means (std) of empirical value functions and misclassification rates for four continuous-outcome simulation scenarios with $n = 200$. The best value functions and misclassification rates are in bold.

	$p = 20$		$p = 40$	
	Value	Misclassification	Value	Misclassification
		Scenario 1		
Pair-D	2.54(0.06)	0.56(0.02)	2.36(0.05)	0.63(0.02)
l_1-PLS	3.04(0.02)	0.29(0.01)	2.94(0.03)	0.36(0.02)
DL	2.51(0.04)	0.58(0.02)	2.46(0.04)	0.6(0.01)
ACWL-1	2.64(0.04)	0.5(0.02)	2.28(0.05)	0.65(0.02)
ACWL-2	2.7(0.03)	0.48(0.01)	2.3(0.05)	0.64(0.02)
VT	2.64(0.03)	0.53(0.01)	2.57(0.03)	0.55(0.01)
Group-AD	**3.05**(0.03)	**0.28**(0.02)	**3.02**(0.03)	**0.31**(0.02)
		Scenario 2		
Pair-D	2.71(0.13)	0.37(0.04)	2.73(0.14)	0.36(0.05)
l_1-PLS	2.81(0.12)	0.41(0.04)	2.83(0.12)	0.41(0.05)
DL	2.72(0.12)	0.39(0.04)	2.71(0.13)	0.4(0.04)
ACWL-1	2.57(0.12)	0.43(0.03)	2.05(0.13)	0.57(0.03)
ACWL-2	2.63(0.12)	0.43(0.03)	2.1(0.13)	0.57(0.03)
VT	**2.97**(0.1)	**0.33**(0.02)	**3.01**(0.09)	**0.33**(0.02)
Group-AD	2.86(0.11)	0.34(0.03)	2.9(0.11)	0.34(0.02)
		Scenario 3		
Pair-D	1.2(0.03)	0.75(0.03)	1.2(0.03)	0.75(0.03)
l_1-PLS	1.37(0.15)	0.65(0.1)	1.31(0.11)	0.69(0.08)
DL	1.34(0.09)	0.67(0.06)	1.29(0.09)	0.7(0.05)
ACWL-1	1.27(0.08)	0.71(0.05)	1.2(0.04)	0.74(0.02)
ACWL-2	1.27(0.08)	0.71(0.05)	1.2(0.05)	0.75(0.03)
VT	1.35(0.07)	0.66(0.04)	**1.33**(0.06)	**0.68**(0.04)
Group-D	**1.38**(0.16)	**0.64**(0.11)	1.31(0.13)	**0.68**(0.09)

1.2 Binary Outcome Studies

For the binary outcome R, we report the simulation results with $n = 400, 800, p = 20$ and $n = 200, p = 20, 40$ in addition to the results in the main paper.

Table 3: Results of average means (standard deviations) of empirical value functions and misclassification rates for two binary-outcome simulation scenarios with 20 covariates. The best value functions and misclassification rates are in bold.

	$n = 400$		$n = 800$	
	Value	Misclassification	Value	Misclassification
		Scenario 1		
l_1-PLR	0.89(0.01)	0.53(0.03)	**0.92**(0)	0.4(0.02)
DL	0.86(0.01)	0.64(0.02)	0.88(0.01)	0.58(0.02)
VT	0.85(0.01)	0.66(0.02)	0.85(0)	0.67(0.02)
Binary-AD	**0.91**(0.01)	**0.41**(0.03)	**0.92**(0)	**0.31**(0.03)
		Scenario 2		
l_1-PLR	0.84(0.01)	0.63(0.02)	**0.87**(0)	0.56(0.03)
DL	0.82(0.01)	0.53(0.04)	0.85(0.01)	0.45(0.04)
VT	0.84(0.01)	0.42(0.05)	0.83(0.01)	0.5(0.05)
Binary-AD	**0.86**(0.01)	**0.44**(0.03)	**0.87**(0.01)	**0.42**(0.02)

Table 4: Results of average means (standard deviation) of empirical value functions and misclassification rates for two binary-outcome simulation scenarios with $n = 200$. The best value functions and misclassification rates are in bold.

	$p = 20$		$p = 40$	
	Value	Misclassification	Value	Misclassification
		Scenario 1		
l_1-PLR	0.86(0.01)	0.62(0.02)	0.85(0.01)	0.66(0.02)
DL	0.85(0.01)	0.68(0.01)	0.84(0.01)	0.7(0.01)
VT	0.84(0.01)	0.66(0.01)	0.83(0.01)	0.68(0.01)
Binary-AD	**0.88**(0.01)	**0.54**(0.02)	**0.87**(0.01)	**0.57**(0.02)
		Scenario 2		
l_1-PLR	0.81(0.01)	0.68(0.05)	0.79(0.01)	0.7(0.05)
DL	0.78(0.01)	0.59(0.01)	0.76(0.01)	0.61(0.01)
VT	**0.84**(0.01)	**0.38**(0.01)	**0.83**(0.01)	**0.36**(0.01)
Binary-AD	0.83(0.01)	0.49(0.04)	0.82(0.01)	0.52(0.04)

1.3 Survival Outcome Studies

For the survival outcome R, we report the simulation results with $n = 400, 800, p = 20$ and $n = 200, p = 20, 40$ in addition to the results in the main paper.

Table 5: Results of average means (standard deviations) of empirical value functions and misclassification rates for two survival-outcome simulation scenarios with 20 covariates. The best value functions and misclassification rates are in bold.

	$n = 400$		$n = 800$	
	Value	Misclassification	Value	Misclassification
		Scenario 1		
l_1-CPH	43.18(2.02)	0.26(0.04)	45.70(1.02)	**0.17**(0.01)
Surv-AD	**44.32**(1.27)	**0.23**(0.02)	**45.81**(1.06)	**0.17**(0.01)
		Scenario 2		
l_1-CPH	**22.48**(0.69)	0.53(0.04)	**23.52**(0.57)	0.47(0.04)
Surv-AD	22.28(0.61)	**0.45**(0.02)	22.77(0.49)	**0.43**(0.02)

Table 6: Results of average means (standard deviation) of empirical value functions and misclassification rates for two survival-outcome simulation scenarios with $n = 200$. The best value functions and misclassification rates are in bold.

	$p = 20$		$p = 40$	
	Value	Misclassification	Value	Misclassification
		Scenario 1		
l_1-CPH	36.27(3.25)	0.44(0.06)	32.19(3.47)	0.52(0.06)
Surv-AD	**40.41**(1.85)	**0.35**(0.03)	**39.46**(2.03)	**0.38**(0.03)
		Scenario 2		
l_1-CPH	20.98(0.92)	0.6(0.03)	19.82(1.08)	0.63(0.03)
Surv-AD	**21.14**(0.95)	**0.49**(0.04)	**20.63**(0.95)	**0.51**(0.04)

1.4 Low Rank Simulation Studies

When the clinical outcome R is continuous, we generate our data from the following model with

$$R_i = \mu(\mathbf{X}_i) + \sum_{k=1}^{K} (\mathbf{X}_i^T \beta_k)\mathbb{I}(A = k) + \epsilon_i,$$

where $i = 1, \cdots, n$, each covariate is generated by the uniform distribution from -1 to 1, and ϵ_i follows from the standard normal distribution. Let the coefficient matrix $\Gamma = (\beta_1, \cdots, \beta_k)$. We consider the following two simulation scenarios:

1. $\mu(\mathbf{X}) = 1 + X_1 + X_2$ and the coefficient matrix $\Gamma = \mathbf{U}\mathbf{V}^T$, where $\mathbf{U} \in \mathbb{R}^{p\times 2}$ and $\mathbf{V} \in \mathbb{R}^{p\times 2}$. Each element of $\mathbf{U}$ and $\mathbf{V}$ is generated by the uniform distribution from -1 to 1;

2. $\mu(\mathbf{X}) = 1 + X_1^2 + X_2^2$ and the coefficient matrix Γ is the same as Scenario 1.

The difference in these two scenarios lies in the linear and nonlinear main effect functions. For each simulation scenario, we compare the following methods:

(1) l_1-PLS proposed by Qian and Murphy (2011) with basis $(1, \mathbf{X}, \mathbf{X}A)$;

(2) Pairwise D-learning;

(3) AD-learning with the group sparsity penalty;

(4) AD-learning with the nuclear norm penalty.

All the tuning parameters are selected via 10-fold cross-validation. We report the value functions and misclassification errors for both $p = 20$ and $p = 40$ on 10000 independently generated test data in the following tables. We can see that our AD-learning has some advantages over l_1-PLS and pairwise D-learning.

Table 7: Results of average means (std) of empirical value functions and misclassification rates for four continuous-outcome simulation scenarios with 20 covariates. The best value functions and misclassification rates are in bold.

	$n = 800$		$n = 1600$	
	Value	Misclassification	Value	Misclassification
Scenario 1				
l_1-PLS	1.64(0.41)	0.52(0.29)	1.7(0.38)	0.47(0.27)
Pair-D	2(0.05)	0.32(0.06)	2.06(0.04)	0.25(0.05)
Group-AD	1.97(0.06)	0.35(0.07)	2.05(0.04)	0.26(0.05)
Low rank-AD	**2.05(0.04)**	**0.22(0.05)**	**2.09(0.04)**	**0.17(0.03)**
Scenario 2				
l_1-PLS	2.37(0.36)	0.47(0.24)	2.52(0.35)	0.35(0.23)
Pair-D	2.6(0.1)	0.38(0.1)	2.69(0.06)	0.29(0.08)
Group-AD	2.6(0.07)	0.38(0.07)	2.68(0.05)	0.3(0.06)
Low rank-AD	**2.7(0.06)**	**0.24(0.06)**	**2.75(0.04)**	**0.19(0.03)**

Table 8: Results of average means (std) of empirical value functions and misclassification rates for four continuous-outcome simulation scenarios with 40 covariates. The best value functions and misclassification rates are in bold.

	$n = 800$		$n = 1600$	
	Value	Misclassification	Value	Misclassification
	Scenario 1			
l_1-PLS	1.64(0.41)	0.52(0.29)	1.7(0.38)	0.47(0.27)
Pair-D	2(0.05)	0.32(0.06)	2.06(0.04)	0.25(0.05)
Group-AD	1.97(0.06)	0.35(0.07)	2.05(0.04)	0.26(0.05)
Low rank-AD	**2.05(0.04)**	**0.22(0.05)**	**2.09(0.04)**	**0.17(0.03)**
	Scenario 2			
l_1-PLS	2.37(0.36)	0.47(0.24)	2.52(0.35)	0.35(0.23)
Pair-D	2.6(0.1)	0.38(0.1)	2.69(0.06)	0.29(0.08)
Group-AD	2.6(0.07)	0.38(0.07)	2.68(0.05)	0.3(0.06)
Low rank-AD	**2.7(0.06)**	**0.24(0.06)**	**2.75(0.04)**	**0.19(0.03)**

1.5 Further Comparison with l_1-PLS

In this section, we compare our proposed AD-learning with l_1-PLS when the main effect functions $\mu(\mathbf{X}) = 1 + X_1^2 + X_2^2$ in the first scenario of continuous outcome settings with the non-zero coefficients are generated by uniform distribution from -1 to 1. Table 9 demonstrates the advantage of our proposed method over l_1-PLS by avoiding modeling main effect functions.

Table 9: Results of average means (std) of empirical value functions and misclassification rates for two simulation scenarios with 20 covariates. The best value functions and misclassification rates are in bold.

	$n = 400$		$n = 800$	
	Value	Misclassification	Value	Misclassification
	Scenario 1			
l_1-PLS	1.64(0.41)	0.52(0.29)	1.7(0.38)	0.47(0.27)
Group-AD	**1.97**(0.06)	**0.35**(0.07)	**2.05**(0.04)	**0.26**(0.05)

1.6 Comparison between Group l_1-PLS and l_1-PLS

In this section, we compare the performance of group l_1-PLS and l_1-PLS using the first simulation scenario of the continuous outcome study. Table 10 demonstrates the performance of l_1-PLS in our simulation scenarios is similar to group l_1-PLS.

Table 10: Results of average means (std) of empirical value functions and misclassification rates for one continuous-outcome simulation scenarios with 20 covariates. The best value functions and misclassification rates are in bold.

	$n = 400$		$n = 800$	
	Value	Misclassification	Value	Misclassification
		Scenario 1		
Group l_1-PLS	3.12(0.07)	0.21(0.05)	3.16(0.04)	0.14(0.03)
l_1-PLS	**3.12**(0.06)	**0.2**(0.05)	**3.17** (0.04)	**0.14**(0.03)

References

M. Qian and S. A. Murphy. Performance guarantees for individualized treatment rules. *Annals of Statistics*, 39(2):1180, 2011.