Dynamic Conditional Correlation Estimation

The Dynamic Conditional Correlation (DCC) method (Engle, 2002) is an example of a model-based multivariate volatility method, popular in finance, and recently introduced to neuroimaging (Lindquist et al., 2014). It provides a way to estimate conditional correlations between time series.

We begin by introducing generalized autoregressive conditional heteroscedastic (GARCH) models (Engle, 1982; Bollerslev, 1986), often used to model the volatility in univariate time series. These models express the conditional variance of a time series at time *t* as a linear combination of past values of the conditional variance and of the squared process itself. To illustrate, assume that we are observing a univariate process

$$y_t = \sigma_t \varepsilon_t$$

where ε_t is a *N* (0, 1) random variable and σ_t represents the time-varying variance term we want to model. A GARCH(1,1) process models the conditional variance as follows:

$$\sigma_t^2 = \omega + \alpha y_{t-1}^2 + \beta \sigma_{t-1}^2$$

where $\omega > 0$, $\alpha, \beta \geq 0$ and $\alpha + \beta < 1$. Here α controls the influence of past values of the time series on the variance and β the influence of past values of the conditional variance on its present value.

In the DCC approach, we assume $\mathbf{y}_t = \varepsilon_t$ is a bivariate mean zero time series with conditional covariance matrix Σ_t. The first order form of DCC can be expressed as follows:

$$\sigma_{i,t}^2 = \omega_i + \alpha_i y_{i,t-1}^2 + \beta_i \sigma_{i,t-1}^2 \text{ for } i = 1, 2 \quad (1)$$

$$\mathbf{D}_t = diag\{\sigma_{1,t}, \sigma_{2,t}\} \quad (2)$$

$$\varepsilon_t = \mathbf{D}_t^{-1} \mathbf{e}_t \quad (3)$$

$$\mathbf{Q}_t = (1 - \theta_1 - \theta_2)\overline{\mathbf{Q}} + \theta_1 \varepsilon_{t-1} \varepsilon_{t-1}^{'} + \theta_2 \mathbf{Q}_{t-1} \quad (4)$$

$$\mathbf{R}_t = diag\{\mathbf{Q}_t\}^{-1/2} \mathbf{Q}_t \, diag\{\mathbf{Q}_t\}^{-1/2} \quad (5)$$

$$\sum\nolimits_t = \mathbf{D}_t \mathbf{R}_t \mathbf{D}_t \quad (6)$$

In Eqs. 1-3, univariate GARCH(1,1) models are fit (Eq. 1) to each of the two univariate time series that make up $\mathbf{y}_t$, and used to compute standardized residuals (Eq. 3). In Eq. 4, an exponentially weighted moving average is applied to the standardized residuals and used to compute a non-normalized version of the time-varying correlation matrix. Here $\bar{\mathbf{Q}}$ represents the unconditional covariance matrix of ε_t and (θ_1, θ_2) are non-negative scalars satisfying $0 < \theta_1+\theta_2 < 1$. Eq. 5 is a rescaling that ensures a proper correlation matrix is created, while Eq. 6 computes the time-varying covariance matrix. Model parameters (ω_1, α_1, β_1, ω_2, α_2, β_2, θ_1, θ_2) can be estimated using a two-step approach. This approach has been shown (Engle and Sheppard, 2001; Engle, 2002) to provide estimates that are consistent and asymptotically normal with a variance that can be computed using the generalized method of moments approach.

Figure S1. Network heritability estimates in each session for the DCC variance, DCC mean, and static connectivity measures. VN = visual network, SMN = sensorimotor network, VAN = ventral attention network, DAN = dorsal attention network, DMN = default mode network, FPN = frontoparietal network

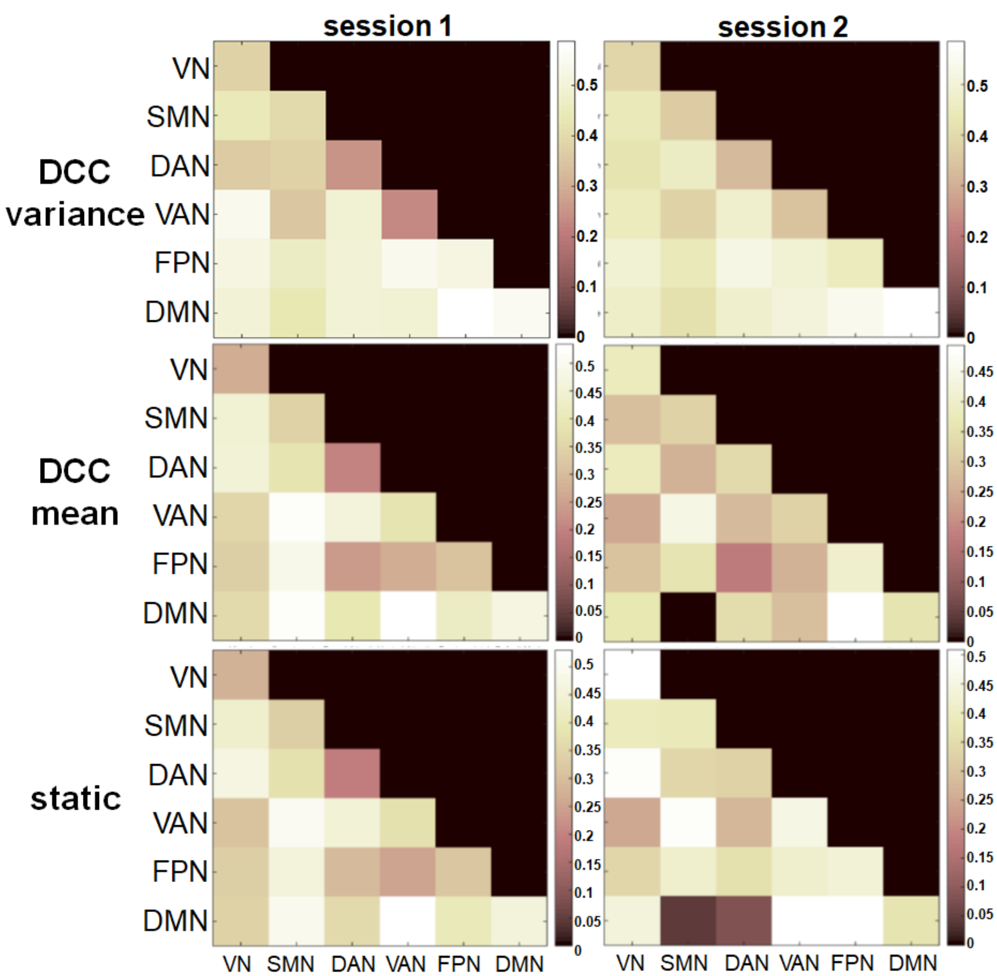

Table S1. Network heritability estimates in each session for the DCC variance, DCC mean, and static connectivity measures computed at different ICA dimensionalities. VN = visual network, SMN = sensorimotor network, VAN = ventral attention network, DAN = dorsal attention network, DMN = default mode network, FPN = frontoparietal network

metric	dimensionality	session	h^2 min	h^2 max	h^2 mean
DCC variance	25	1	0.28	0.50	0.41
		2	0.24	0.49	0.36
	50	1	0.27	0.53	0.43
		2	0.27	0.53	0.41
	100	1	0.27	0.60	0.44
		2	0.27	0.57	0.45
	200	1	0.33	0.58	0.46
		2	0.33	0.55	0.44
	300	1	0.24	0.59	0.45
		2	0.32	0.59	0.45
DCC mean	25	1	0.00	0.56	0.30
		2	0.04	0.45	0.27
	50	1	0.00	0.53	0.32
		2	0.00	0.47	0.32
	100	1	0.04	0.53	0.39
		2	0.00	0.64	0.34
	200	1	0.13	0.50	0.34
		2	0.01	0.46	0.26
	300	1	0.21	0.53	0.39
		2	0.00	0.49	0.31
static	25	1	0.00	0.56	0.30
		2	0.16	0.49	0.32
	50	1	0.00	0.52	0.31
		2	0.23	0.55	0.36
	100	1	0.03	0.54	0.37
		2	0.00	0.59	0.36
	200	1	0.15	0.47	0.34
		2	0.00	0.51	0.31
	300	1	0.19	0.53	0.38
		2	0.05	0.51	0.37

Table S2. Individual connection heritability estimates in each session for the DCC variance, DCC mean, and static connectivity measures computed at the ICA dimensionality of 300 components.

metric	dimensionality	session	h^2 min	h^2 max	h^2 mean
DCC variance	300	1	0.00	0.62	0.33
		2	0.00	0.55	0.31
DCC mean	300	1	0.00	0.59	0.26
		2	0.00	0.63	0.22
static	300	1	0.00	0.62	0.27
		2	0.00	0.65	0.27

References

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