Supplementary Information
Embryonic tissues as active foams

Sangwoo Kim, Marie Pochitaloff, Georgina A. Stooke-Vaughan & Otger Campàs

Captions of supplementary movies

Supplementary movie 1

Simulations of equilibrium configurations showing the effect of decreasing the relative adhesion quasi-statically from $W/T_0 = 0.6$ to $W/T_0 = 0$, both for low cell density ($\rho = 0.81$) and high cell density ($\rho = 1.21$).

Supplementary movie 2

Simulations of the system with an imposed strain step in the absence of tension fluctuations ($\Delta T/T_0 = 0$) for both non-confluent ($\rho = 1$, $W/T_0 = 0.2$) and confluent ($\rho = 1$, $W/T_0 = 1$) regimes. Cells that undergo topological transitions are color-coded in red.

Supplementary movie 3

Simulations of the system dynamics in the non-confluent regime and in the presence of active tension fluctuations of small magnitude ($\rho = 1$, $W/T_0 = 0.2$, and $\Delta T/T_0 = 0.5$) and large magnitude ($\rho = 1$, $W/T_0 = 0.2$, and $\Delta T/T_0 = 1.5$). Trajectories of four cells are shown in different colors and cells that undergo topological transitions are color-coded in red after $t/\tau_R = 150$.

Supplementary movie 4

Simulations of the system dynamics in the confluent regime and in the presence of active tension fluctuations of small magnitude ($\rho = 1$, $W/T_0 = 1$, and $\Delta T/T_0 = 0.5$) and large magnitude ($\rho = 1$, $W/T_0 = 1$, and $\Delta T/T_0 = 1.5$). Trajectories of four cells are shown in different colors and cells that undergo topological transitions are color-coded in red after $t/\tau_R = 150$.

Supplementary movie 5

Simulations of the system with an imposed strain step in the presence of tension fluctuations ($\Delta T/T_0 = 1$) for both non-confluent ($\rho = 1$, $W/T_0 = 0.2$) and confluent ($\rho = 1$, $W/T_0 = 1$) regimes. Cells that undergo topological transitions are color-coded in red.

Supplementary movie 6

Simulations of the system dynamics showing the spatiotemporal tension fluctuations for small magnitude of tension fluctuations ($\rho = 1$, $W/T_0 = 1$ and $\Delta T/T_0 = 0.5$) and for large magnitude ($\rho = 1$, $W/T_0 = 1$ and $\Delta T/T_0 = 1.5$): identical samples of Supplementary video 3. A tension gradient color scheme is rainbow color, with high tension as red and low tension as purple.

1 Stability condition for small triangular extracellular spaces

For small triangular extracellular spaces, the contribution from normal forces becomes negligible due to vanishing junctional lengths, and force balance of tensions at the vertices determines whether these extracellular spaces open up or close down (Fig. S1). In the absence of tension fluctuations, tensions have fixed values that correspond to the average tensions: $2T_0 - W$ for cell-cell contacts and T_0 for free cell boundaries. Assuming that the shape of very small extracellular spaces is close to regular triangle, these open up (close down) if the cell-cell contact tension is larger (smaller) than the sum of two free cell boundaries tension projected in the direction of the cell-cell contact (Fig. S1), or $2T_0 - W > 2T_0 \cos\frac{\pi}{6}$ ($2T_0 - W < 2T_0 \cos\frac{\pi}{6}$). Hence, small triangular spaces can only be stabilized under a critical relative adhesion strength at cell-cell contacts, namely

$$\frac{W}{T_0} < 2 - 2\cos\frac{\pi}{6} = 2 - \sqrt{3} \approx 0.23\,. \tag{1}$$

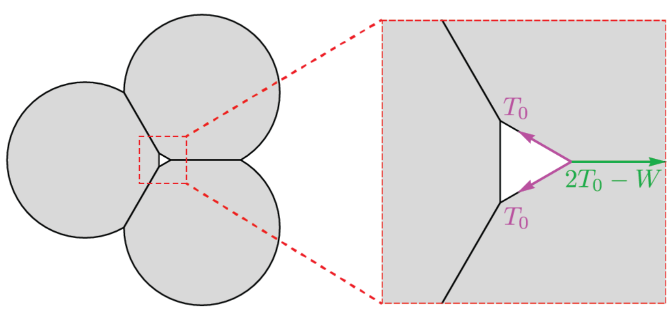

Figure S1: Schematics of small triangular extracellular spaces and tension force balance at vertices.

2 Active stress estimation

In the presence of active tension fluctuations, the unperturbed system (no applied macroscopic strains) will show global shear stress fluctuations around zero, with larger amplitudes for increasing magnitudes of active tension fluctuations (Fig. S2**a**). We obtain the magnitude of active, global shear stresses, σ_A, by computing the standard deviation of shear stress fluctuations for each simulation. Then we obtain the ensemble averaged σ_A from multiple simulations with the same values of parameters (Fig. S2**b**). Regardless of relative adhesion strength, the level of active shear stress fluctuations increases monotonically as tension fluctuations increase. In non-confluent regimes, the level of active shear stress at any given magnitude of tension fluctuations increases as the relative adhesion strength increases, due to the higher volume fraction in the system. In the confluent regime, the level of active shear stress is independent of relative adhesion strength (Fig. S2**b**).

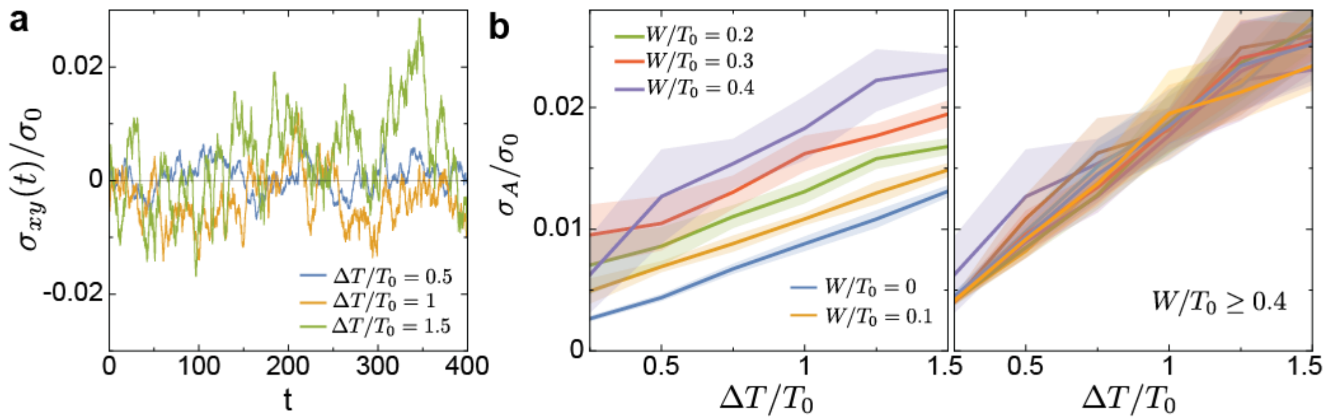

Figure S2: Active, global shear stress fluctuations. **a**, Temporal evolution of the global shear stress displaying fluctuations induced by active tension fluctuations at the cell scale. **b**, Level of active shear stress σ_A in terms of $\Delta T/T_0$ for different values of the relative adhesion strength W/T_0.

3 Dependence of volume fraction on tension fluctuations

The system bistability for $W/T_0 \geq 0.23$ makes its volume fraction ϕ depend on the level of tension fluctuations. Higher tension fluctuations move the structural transition between non-confluent to confluent states to larger relative adhesion values, an effect that is specially strong at low cell densities ($\rho < 1$) because bistability is stronger in this regime. However, as the cell density ρ increases, the effect of tension fluctuations $\Delta T/T_0$ on the volume fraction ϕ diminishes. At $\rho = 1.1025$, tension fluctuations have only a mild effect on the volume fraction for magnitudes of tension fluctuations in the range $0.3 < \Delta T/T_0 < 0.6$, which is in contrast with the substantially larger effect observed at $\rho = 1$. As the cell density is increased, the volume fraction becomes essentially independent of tension fluctuations (at $\rho \simeq 1.21$), and it is solely determined by the relative adhesion strength W/T_0 and cell density ρ (Fig. S3).

4 Fraction of cell edges with vanishing tension

The fluctuation-induced transition is qualitatively distinct from the density-independent rigidity transition, which requires vanishing effective tension at cell-cell contacts. In our simulations, up to $W/T_0 \leq 1.2$ and for

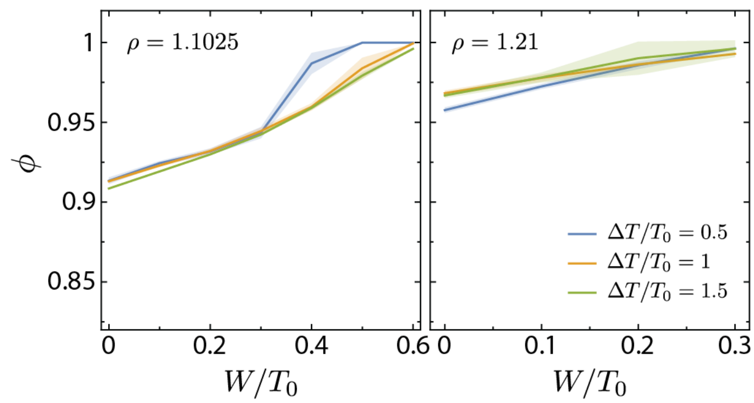

Figure S3: Cellular volume fraction for varying relative adhesion strength W/T_0 and different magnitudes of tension fluctuations $\Delta T/T_0$. As cell density increases, the volume fraction of the system becomes independent of tension fluctuations.

$\Delta T/T_0 \leq 1.5$, the mean effective tension is finite (Fig. S4**a**) and the system contains only a few or none zero tension edges (Fig. S4**b**). Even for the largest tension fluctuations that we explored, $\Delta T/T_0 = 1.5$, the system shows no zero tension edges up to $W/T_0 \approx 0.75$. For $W/T_0 > 0.75$, the system starts to show a very small fraction of zero tension edges, but it is less than 1% of the edges even for large adhesion values, $W/T_0 = 1.2$. The density-independent rigidity transition requires percolating zero tension edges in the system, so the fraction of zero tension edges is far larger than what we observe in our simulations. Hence, it is the spatial and temporal variations in tension that induce the rigidity transition in our description, as the fraction of zero tension edges is very small for essentially all the explored parameter space, including extreme cases.

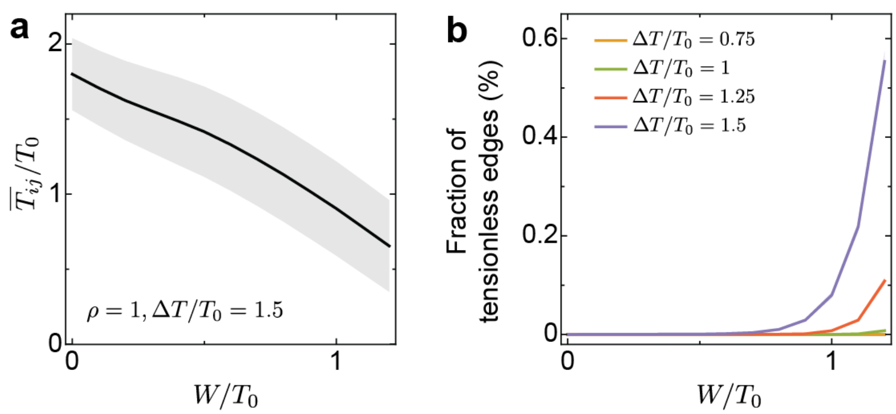

Figure S4: Characterization of tensionless edges. **a**, the mean edge tension changes with the relative adhesion strength. The shaded region corresponds to the standard deviation of the tension distribution. **b**, a fraction of zero tension edges is zero for small relative adhesion and stays less than 1% for large relative adhesion and large tension fluctuations.

5 Initial stress jump induced by a sudden step in strain

A large step strain is generated by imposing an affine deformation, with all vertex positions being adjusted according to a simple shear deformation. The instantaneous adjustment of vertex positions makes cell shapes anisotropic, leading to a sudden increase in shear stress. For all cell density values ρ, the initial stress jump is maximum when relative adhesion strength is zero, $W/T_0 = 0$, due to the maximum mean effective tension at cell-cell contacts. In the confluent regime, the initial stress jump is independent of cell density values because it is solely determined by cell shape and mean effective tensions (Fig. S5) .

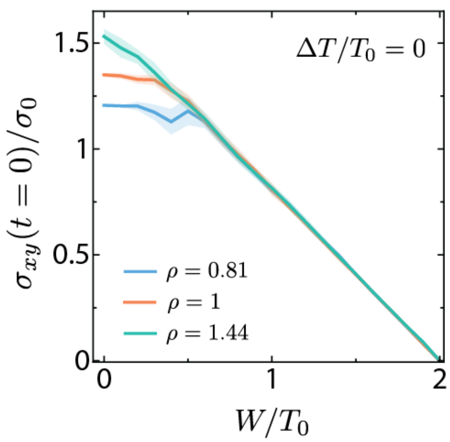

Figure S5: Initial stress jump level as a function of relative adhesion strength in the absence of tension fluctuations.