5 Supplementary Material

5.1 Derivation of why the scaling method preserves the network activity statistics

The membrane voltage of neuron i is described by

$$\tau_m \frac{dV_i}{dt} = -V_i + RI_i(t), \tag{16}$$

$$\tau_s \frac{dI_i}{dt} = -I_i + \tau_s \sum_j w_{ij} \sum_{t_j^{\mathrm{sp}}} \delta\left(t - t_j^{\mathrm{sp}} - d\right), \tag{17}$$

where τ_m and τ_s are the membrane and synaptic conductances, R is the membrane resistance, I_i is the synaptic current, w_{ij} is the amplitude of the synaptic current in neuron i due to a spike in neuron j (synaptic weight), $\{t_j^{\mathrm{sp}}\}$ is the set of spike times of neuron j, and d is transmission delay. In these expressions, I_i and w_{ij} are in units of current but they can also be given in units of voltage by absorbing the membrane resistance in the definition of w (in such case, w would represent the amplitude of the postsynaptic potential). When V_i reaches the threshold θ, neuron i is assumed to emit a spike and V_i is reset to V_{reset} for a refractory period τ_{ref}.

Assuming that $w \ll \theta$, the synaptic current can be approximated by an average part plus a fluctuating part Brunel (2000)

$$I_i(t) = \mu_i(t) + \sigma_i \sqrt{\tau_m} \eta_i(t), \tag{18}$$

where $\eta_i(t)$ is a Gaussian white noise with zero mean and unit variance. The average and fluctuating parts can be further split into an internal part, relative to the recurrent connections received by neuron i from the network, and an external part,

$$\mu_i(t) = \mu_{i,\mathrm{int}}(t) + \mu_{i,\mathrm{ext}}, \tag{19}$$

$$\sigma_i^2(t) = \sigma_{i,\text{int}}^2(t) + \sigma_{i,\text{ext}}^2. \tag{20}$$

Assuming that the average firing rate of neurons from population j ($j \in$ L2/3e, L2/3i, L4e, L4i, $\ldots$) is given by $\langle f_j \rangle$ and the average synaptic weight of external connections is $\langle f_{\text{ext}} \rangle$, these expressions can be written as

$$\mu_i = \sum_j x_{ij} \langle w_{ij} \rangle \langle f_j \rangle \tau_m + X_{i,\text{ext}} \langle w_{i,\text{ext}} \rangle \langle f_{\text{ext}} \rangle \tau_m, \tag{21}$$

$$\sigma_i^2 = \sum_j x_{ij} \langle w_{ij} \rangle^2 \langle f_j \rangle \tau_m + X_{i,\text{ext}} \langle w_{i,\text{ext}} \rangle^2 \langle f_{\text{ext}} \rangle \tau_m, \tag{22}$$

where x_{ij} is the average number of connections neuron i receives from population j, $\langle w_{ij} \rangle$ is the average weight of synapses from neurons in population j to neuron i, $X_{i,\text{ext}}$ is the number of external synapses received by neuron i, and $\langle w_{i,\text{ext}} \rangle$ is the average synaptic weight of external connections to neuron i.

Assuming that $x_{ij} = p_{ij} N_j$ and all synapses have the same average weight w (one could introduce a different weight gw for inhibitory synapses without loss of generality), these expressions read:

$$\mu_i = \sum_j p_{ij} N_j w \langle f_j \rangle \tau_m + X_{i,\text{ext}} w \langle f_{\text{ext}} \rangle \tau_m, \tag{23}$$

$$\sigma_i^2 = \sum_j p_{ij} N_j w^2 \langle f_j \rangle \tau_m + X_{i,\text{ext}} w^2 \langle f_{\text{ext}} \rangle \tau_m. \tag{24}$$

Applying the scaling transformations,

$$w' = \frac{w}{\sqrt{k}}; \quad (p_{ij} N_j)' = k\,(p_{ij} N_j)\,; \quad X'_{i,\text{ext}} = k X_{i,\text{ext}}, \tag{25}$$

to eqns. (23 and (24) we get

$$\mu_i' = \sqrt{k}\left(\sum_j p_{ij}N_j w\langle f_j\rangle\tau_m + X_{i,\mathrm{ext}} w\langle f_{\mathrm{ext}}\rangle\tau_m\right) = \sqrt{k}\mu_i, \tag{26}$$

$$\sigma_i'^2 = \sum_j p_{ij}N_j w^2\langle f_j\rangle\tau_m + X_{i,\mathrm{ext}} w^2\langle f_{\mathrm{ext}}\rangle\tau_m = \sigma_i^2. \tag{27}$$

So, the variance of the synaptic current is preserved but not the mean. Now add to neuron i a DC current input given by

$$I_{\mathrm{DC}} = (1-\sqrt{k})\left(\sum_j p_{ij}N_j w\langle f_j\rangle\tau_m + X_{i,\mathrm{ext}} w\langle f_{\mathrm{ext}}\rangle\tau_m\right) = (1-\sqrt{k})\mu_i. \tag{28}$$

With this, the average part of the synaptic current is also preserved.

5.2 Supplementary Tables

Population Scaling	L2/3e	L2/3i	L4e	L4i	L5e	L5i	L6e	L6i
100%	0.90	2.80	4.39	5.70	6.80	8.22	1.14	7.60
80%	0.82 (9%)	2.81 (0%)	4.45 (1%)	5.74 (1%)	7.10 (4%)	8.24 (0%)	1.17 (3%)	7.64 (1%)
60%	0.85 (6%)	2.79 (0%)	4.42 (1%)	5.74 (1%)	6.25 (8%)	8.22 (0%)	1.09 (4%)	7.57 (0%)
50%	0.75 (17%)	2.87 (3%)	4.57 (4%)	5.84 (2%)	7.35 (8%)	8.31 (1%)	1.17 (3%)	7.70 (1%)
40%	0.70 (22%)	2.95 (5%)	4.62 (5%)	5.90 (4%)	7.90 (16%)	8.35 (2%)	1.22 (7%)	7.79 (3%)
30%	**0.69 (23%)**	3.06 (9%)	4.71 (7%)	5.98 (5%)	8.34 (23%)	8.45 (3%)	1.18 (4%)	7.92 (4%)
20%	0.78 (13%)	2.99 (7%)	4.57 (4%)	5.97 (5%)	6.17 (9%)	8.54 (4%)	1.06 (7%)	7.80 (3%)
10%	0.75 (17%)	3.28 (17%)	4.76 (8%)	6.20 (9%)	6.55 (4%)	**8.97 (9%)**	1.10 (4%)	8.06 (6%)
5%	**0.69 (23%)**	3.88 (39%)	4.74 (8%)	**6.33 (11%)**	**9.66 (42%)**	8.83 (7%)	1.09 (4%)	**8.60 (13%)**
2 %	**0.69 (23%)**	3.8 (36%)	4.15 (5%)	5.90 (4%)	7.08 (4%)	8.70 (6%)	1.02 (11%)	7.96 (5%)
1%	0.71 (21%)	**4.08 (46%)**	**3.71 (15%)**	5.46 (4%)	8.33 (23%)	8.92 (9%)	**0.93 (18%)**	7.43 (2%)
NEST trial	0.86	-	4.45	-	7.59	-	1.09	-

Table S1: Population firing rates (Hz) for rescaled NetPyNE versions of the original PDCM model with Poisson external input. All results calculated from 60 s simulations and all neurons. Relative deviations in relation to the full-scale NetPyNE version $\left(\left|f_{\mathrm{X}\%} - f_{100\%}\right| / f_{100\%}\right)$, are shown within parentheses, and their maxima in bold. For comparison, the last row shows the mean firing rates of the NEST implementation (see (Potjans and Diesmann, 2014a) for details).

Population Scaling	L2/3e	L2/3i	L4e	L4i	L5e	L5i	L6e	L6i
100%	1.02	2.89	4.32	5.60	7.02	8.20	0.90	7.46
80%	0.85 (17%)	2.83 (2%)	4.40 (2%)	5.64 (1%)	7.55 (8%)	8.20 (0%)	1.02 (13%)	7.49 (1%)
60%	0.89 (13%)	2.77 (4%)	4.32 (0%)	5.59 (0%)	6.38 (9%)	8.10 (1%)	0.94 (4%)	7.37 (1%)
50%	0.75 (26%)	2.82 (2%)	4.46 (3%)	5.66 (1%)	7.46 (6%)	8.16 (0%)	1.03 (14%)	7.47 (0%)
40%	**0.66 (35%)**	2.86 (1%)	4.49 (4%)	5.70 (2%)	8.17 (16%)	8.16 (0%)	**1.12 (24%)**	7.54 (1%)
30%	**0.66 (35%)**	2.93 (1%)	**4.53 (5%)**	**5.71 (2%)**	**8.46 (21%)**	8.21 (0%)	1.05 (17%)	7.50 (2%)
20%	0.76 (25%)	2.77 (4%)	4.29 (1%)	5.58 (0%)	6.08 (13%)	**8.11 (1%)**	0.89 (1%)	7.30 (2%)
10%	0.87 (15%)	**3.06 (6%)**	4.25 (2%)	5.46 (2%)	6.62 (6%)	8.17 (0%)	0.80 (11%)	**7.17 (4%)**
NEST trial	0.86	-	4.45	-	7.59	-	1.09	-

Table S2: Population firing rates (Hz) for rescaled NetPyNE versions of the original PDCM model with DC current input. All results calculated from 60-seconds simulations and all neurons. Relative deviations in relation to the full-scale NetPyNE version $\left(\left|f_{X\%} - f_{100\%}\right| / f_{100\%}\right)$ are shown within parentheses, and their maxima in bold. For comparison, the last row shows sampled mean firing rates of the NEST implementation (see (Potjans and Diesmann, 2014a) for details).

Population Scaling	L2/3e	L2/3i	L4e	L4i	L5e	L5i	L6e	L6i
100%	0.938	0.916	0.891	0.873	0.847	0.809	0.924	0.819
80%	**0.934 (0.45%)**	0.916 (0.06%)	0.890 (0.17%)	0.873 (0.07%)	0.842 (0.63%)	0.808 (0.15%)	0.927 (0.29%)	0.818 (0.19%)
60%	**0.934 (0.45%)**	0.917 (0.04%)	0.891 (0.02%)	0.874 (0.12%)	0.858 (1.25%)	0.813 (0.48%)	0.930 (0.63%)	0.822 (0.27%)
50%	**0.934 (0.39%)**	0.917 (0.06%)	0.889 (0.19%)	0.873 (0.06%)	0.838 (1.12%)	0.808 (0.24%)	0.930 (0.68%)	0.816 (0.38%)
40%	0.937 (0.08%)	0.914 (0.31%)	0.887 (0.44%)	0.872 (0.12%)	0.828 (2.30%)	0.807 (0.36%)	0.932 (0.88%)	0.816 (0.40%)
30%	**0.931 (0.72%)**	0.913 (0.37%)	0.887 (0.45%)	0.873 (0.06%)	0.822 (2.95%)	0.807 (0.32%)	0.931 (0.72%)	0.815 (0.60%)
20%	0.936 (0.15%)	0.921 (0.50%)	0.887 (0.41%)	0.871 (0.28%)	0.858 (1.22%)	0.806 (0.41%)	0.929 (0.55%)	0.814 (0.64%)
10%	0.937 (0.05%)	0.919 (0.31%)	0.883 (0.87%)	0.869 (0.51%)	0.853 (0.62%)	0.800 (1.23%)	**0.938 (1.55%)**	0.810 (1.11%)
5%	**0.926 (1.21%)**	0.899 (1.90%)	0.884 (0.82%)	0.867 (0.76%)	**0.797 (5.99%)**	0.802 (0.97%)	0.934 (1.03%)	**0.803 (1.97%)**
2%	0.935 (0.32%)	**0.895 (2.34%)**	0.891 (0.03%)	**0.863 (1.21%)**	0.837 (1.23%)	0.811 (0.15%)	0.932 (0.81%)	0.817 (0.24%)
1%	**0.929 (0.92%)**	0.895 (2.30%)	**0.901 (1.12%)**	0.884 (1.16%)	0.823 (2.90%)	**0.785 (2.98%)**	0.932 (0.82%)	0.814 (0.66%)
NetPyNE-10 trial	0.936 ± 0.146	0.916 ± 0.106	0.890 ± 0.107	0.875 ± 0.101	0.847 ± 0.107	0.809 ± 0.118	0.926 ± 0.142	0.822 ± 0.125

Table S3: Irregularity of single-unit spike trains for rescaled NetPyNE versions of the original PDCM model with Poisson external input. All results calculated from 60-seconds simulations and approximately 1000 neurons per population. Relative deviations in relation to the full-scale NetPyNE version $\left(\left|f_{X\%} - f_{100\%}\right| / f_{100\%}\right)$ are shown within parentheses, and their maxima in bold.

Population Scaling	L2/3e	L2/3i	L4e	L4i	L5e	L5i	L6e	L6i
100%	0.936	0.909	0.875	0.862	0.820	0.775	0.923	0.788
80%	0.934 (0.21%)	0.912 (0.33%)	0.874 (0.11%)	0.858 (0.46%)	0.807 (1.59%)	0.772 (0.39%)	0.921 (0.22%)	0.786 (0.25%)
60%	0.935 (0.11%)	0.911 (0.22%)	0.875 (0.00%)	0.858 (0.46%)	0.831 (1.34%)	0.776 (0.13%)	0.922 (0.11%)	0.787 (0.13%)
50%	0.933 (0.32%)	0.905 (0.44%)	0.872 (0.34%)	**0.855 (0.81%)**	0.807 (1.59%)	0.773 (0.26%)	**0.920 (0.33%)**	0.785 (0.38%)
40%	0.931 (0.53%)	0.910 (0.11%)	0.871 (0.46%)	0.856 (0.70%)	0.792 (3.41%)	0.775 (0.00%)	0.921 (0.22%)	0.783 (0.63%)
30%	**0.929 (0.75%)**	0.906 (0.33%)	0.867 (0.91%)	0.857 (0.58%)	**0.784 (4.39%)**	0.777 (0.26%)	0.921 (0.22%)	0.777 (1.40%)
20%	**0.929 (0.75%)**	0.908 (0.11%)	0.870 (0.57%)	0.857 (0.58%)	0.829 (1.10%)	0.768 (0.90%)	0.921 (0.22%)	0.782 (0.76%)
10%	0.937 (0.11%)	**0.896 (1.43%)**	**0.866 (1.03%)**	**0.855 (0.81%)**	0.817 (0.37%)	**0.766 (1.16%)**	**0.920 (0.33%)**	**0.771 (2.16%)**

Table S4: Irregularity of single-unit spike trains for rescaled NetPyNE versions of the original PDCM model with DC current input. All results calculated from 60-seconds simulations and approximately 1000 neurons per population. Relative deviations in relation to the full-scale NetPyNE version $\left(\left|f_{\mathrm{X}\%} - f_{100\%}\right| / f_{100\%}\right)$ are shown within parentheses, and their maxima in bold.

Layer Scaling	L2/3e	L2/3i	L4e	L4i	L5e	L5i	L6e	L6i
100%	2.9	1.8	3.0	1.7	4.4	1.1	1.2	1.0
80%	2.3 (21%)	1.6 (11%)	2.4 (20%)	1.4 (18%)	4.0 (9%)	1.0 (9%)	1.2 (0%)	0.9 (10%)
60%	2.1 (28%)	1.5 (17%)	2.2 (27%)	1.4 (18%)	3.6 (18%)	**0.9 (18%)**	1.2 (0%)	0.9 (10%)
50%	**1.8 (38%)**	1.5 (17%)	2.0 (33%)	1.3 (24%)	4.3 (2%)	**0.9 (18%)**	1.2 (0%)	0.9 (10%)
40%	**1.8 (38%)**	1.6 (11%)	2.1 (30%)	1.4 (18%)	5.0 (14%)	1.0 (9%)	1.3 (8%)	**0.8 (20%)**
30%	1.9 (34%)	1.7 (6%)	2.1 (30%)	1.4 (18%)	5.9 (34%)	1.1 (0%)	1.3 (8%)	0.9 (10%)
20%	2.0 (31%)	1.9 (6%)	2.4 (20%)	1.7 (0%)	5.4 (23%)	1.0 (9%)	1.4 (17%)	0.9 (10%)
10%	2.6 (10%)	2.1 (17%)	3.1 (3%)	1.9 (12%)	4.5 (2%)	1.1 (0%)	1.7 (42%)	1.0 (0%)
5%	3.4 (17%)	**2.4 (33%)**	4.1 (37%)	**2.1 (24%)**	5.3 (20%)	1.2 (9%)	**1.8 (50%)**	1.1 (10%)
2 %	2.5 (14%)	1.9 (6%)	2.7 (10%)	1.8 (6%)	2.4 (45%)	1.1 (0%)	1.4 (17%)	1.1 (10%)
1%	1.9 (34%)	1.6 (11%)	**1.7 (43%)**	1.4 (18%)	**2.0 (55%)**	1.1 (0%)	1.2 (0%)	1.0 (0%)

Table S5: Synchrony of population spikes for rescaled NetPyNE versions of the original PDCM model with Poisson external input. All results calculated from 60-seconds simulations and approximately 1000 neurons per population. Relative deviations in relation to the full-scale NetPyNE version $\left(\left|f_{\mathrm{X}\%} - f_{100\%}\right| / f_{100\%}\right)$ are shown within parentheses, and their maxima in bold.

Layer Scaling	L2/3e	L2/3i	L4e	L4i	L5e	L5i	L6e	L6i
100%	5.1	3.3	5.5	2.7	8.0	2.0	1.5	1.3
80%	3.6 (29%)	2.5 (24%)	4.1 (25%)	2.1 (22%)	6.6 (18%)	1.5 (25%)	1.4 (7%)	1.2 (8 %)
60%	3.4 (33%)	2.5 (24%)	3.6 (35%)	2.1 (22%)	5.6 (30%)	1.4 (30%)	1.4 (7%)	1.1 (15 %)
50%	2.6 (49%)	2.1 (36%)	3.0 (45%)	1.8 (33%)	5.8 (28%)	**1.2 (40%)**	1.4 (7%)	1.0 (23%)
40%	2.4 (53%)	2.2 (33%)	2.8 (49%)	1.8 (33%)	6.5 (19%)	**1.2 (40%)**	1.4 (7%)	1.0 (23%)
30%	2.5 (51%)	2.5 (24%)	2.8 (49%)	2.0 (26%)	7.8 (3%)	1.3 (35%)	1.5 (0%)	1.1 (15%)
20%	3.3 (35%)	3.6 (9%)	3.9 (29%)	2.9 (7%)	7.3 (9%)	1.3 (35%)	1.6 (7%)	1.2 (8%)
10%	**10.7 (110%)**	**10.3 (212%)**	**15.6 (184%)**	**8.5 (215%)**	**11.4 (43%)**	2.4 (20%)	**3.3 (120%)**	**2.3 (77%)**

Table S6: Synchrony of population spikes for rescaled NetPyNE versions of the original PDCM model with DC current input. All results calculated from 60-seconds simulations and approximately 1000 neurons per population. Relative deviations in relation to the full-scale NetPyNE version $\left(\left|f_{\mathrm{X}\%} - f_{100\%}\right| / f_{100\%}\right)$ are shown within parentheses, and their maxima in bold.

Layer Scaling	L2/3e	L2/3i	L4e	L4i	L5e	L5i	L6e	L6i
100%	0.036	0.053	0.056	0.057	0.057	0.058	0.039	0.057
80%	0.034 (5 %)	0.054 (1 %)	0.056 (1 %)	0.057 (0 %)	0.057 (0 %)	0.058 (0 %)	0.039 (1 %)	0.057 (0 %)
60%	0.035 (3 %)	0.055 (2 %)	0.056 (2 %)	0.057 (1 %)	0.057 (0 %)	0.059 (1 %)	0.039 (1 %)	0.058 (1 %)
50%	0.033 (8 %)	0.055 (3 %)	0.057 (2 %)	0.057 (1 %)	0.058 (1 %)	0.059 (2 %)	0.040 (4 %)	0.058 (1 %)
40%	**0.032 (12 %)**	0.055 (4 %)	0.057 (2 %)	0.058 (2 %)	0.058 (1 %)	0.060 (3 %)	0.040 (5 %)	0.058 (1 %)
30%	**0.032 (12 %)**	0.056 (5 %)	0.057 (3 %)	0.058 (2 %)	0.058 (1 %)	0.060 (4 %)	0.040 (5 %)	0.059 (2 %)
20%	0.034 (7 %)	0.056 (6 %)	0.057 (3 %)	0.058 (3 %)	0.058 (2 %)	0.062 (6 %)	0.039 (1 %)	0.059 (3 %)
10%	0.034 (7 %)	0.057 (7 %)	0.057 (3 %)	0.059 (5 %)	0.059 (3 %)	0.067 (15 %)	0.040 (4 %)	0.061 (6 %)
5%	0.032 (11 %)	0.058 (10 %)	0.058 (4 %)	0.061 (7 %)	0.061 (7 %)	0.071 (22 %)	0.040 (4 %)	0.063 (10 %)
2 %	0.036 (0 %)	0.060 (12 %)	**0.059 (6 %)**	0.062 (10 %)	0.066 (15 %)	0.079 (36 %)	**0.043 (11 %)**	**0.066 (15 %)**
1%	**0.041 (12 %)**	**0.063 (18 %)**	0.060 (0 %)	**0.063 (11 %)**	**0.076 (34 %)**	**0.106 (82 %)**	0.042 (8 %)	0.063 (10 %)

Table S7: Average cross correlation between pairs of neurons per population for rescaled NetPyNE versions of the original PDCM model with Poisson input. All results calculated from 5-seconds simulations and all pair of neurons combination per population. Relative deviations in relation to the full-scale NetPyNE version $\left(\left|f_{\mathrm{X}\%} - f_{100\%}\right| / f_{100\%}\right)$ are shown within parentheses, and their maxima in bold.

Layer Scaling	L2/3e	L2/3i	L4e	L4i	L5e	L5i	L6e	L6i
100%	0.039	0.055	0.057	0.057	0.058	0.059	0.036	0.058
80%	0.035 (9 %)	0.055 (0 %)	0.057 (0 %)	0.057 (0 %)	0.058 (0 %)	0.059 (1 %)	0.038 (4 %)	0.058 (0 %)
60%	0.036 (6 %)	0.054 (1 %)	0.057 (0 %)	0.058 (1 %)	0.058 (0 %)	0.060 (1 %)	0.037 (2 %)	0.059 (1 %)
50%	0.033 (14 %)	0.055 (1 %)	0.057 (0 %)	0.058 (1 %)	0.058 (0 %)	0.061 (3 %)	0.038 (5 %)	0.059 (1 %)
40%	0.032 (17 %)	0.055 (0 %)	0.057 (0 %)	0.058 (1 %)	0.058 (1 %)	0.061 (3 %)	**0.039 (8 %)**	0.059 (2 %)
30%	**0.031(18 %)**	0.055 (0 %)	0.057 (0 %)	0.058 (1 %)	0.059 (2 %)	0.062 (5 %)	0.038 (5 %)	0.060 (3 %)
20%	0.033 (13 %)	0.055 (1 %)	0.057 (0 %)	0.059 (2 %)	0.058 (1 %)	0.064 (8 %)	0.036 (1 %)	0.061 (5 %)
10%	0.039 (2 %)	**0.057 (3 %)**	**0.058 (2 %)**	**0.060 (5 %)**	**0.060 (4 %)**	**0.070 (18 %)**	0.035 (2 %)	**0.063 (9 %)**

Table S8: Average cross correlation between pairs of neurons per population for rescaled NetPyNE versions of the original PDCM model with DC input. All results calculated from 5-seconds simulations and all pair of neurons combination per population. Relative deviations in relation to the full-scale NetPyNE version $\left(\left|f_{\mathrm{x}\%} - f_{100\%}\right| / f_{100\%}\right)$ are shown within parentheses, and their maxima in bold.