Supporting Information for
"Inverse Probability Weighted Estimators of Vaccine Effects Accommodating Partial Interference and Censoring"
by

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Web Appendix A. Proposition Proof

From the definition of the IPCW estimator,

$$E\{\hat{F}_i(t,a,\alpha)\} = E\left\{\sum_{j=1}^{n_i} \frac{\pi(\mathbf{A}_{i,-j};\alpha)I(A_{ij}=a)I(\Delta_{ij}=1)I(X_{ij}\le t)}{S_C(X_{ij}|\mathbf{L}_i,\mathbf{A}_i)\Pr(\mathbf{A}_i|\mathbf{L}_i)n_i}\right\} \tag{1}$$

Noting $\Delta_{ij}=1$ if and only if $C_{ij} > T_{ij}(\mathbf{A}_i)$, by the law of total expectation and causal consistency the right side of (1) can be expressed as

$$E_{\mathbf{L}_i,\mathbf{A}_i,T_{ij}(\mathbf{A}_i)}E_{C_{ij}|\mathbf{L}_i,\mathbf{A}_i,T_{ij}(\mathbf{A}_i)}\left[\sum_{j=1}^{n_i} \frac{\pi(\mathbf{A}_{i,-j};\alpha)I(A_{ij}=a)I\{C_{ij}>T_{ij}(\mathbf{A})\}I\{T_{ij}(\mathbf{A}_i)\le t\}}{S_C\{T_{ij}(\mathbf{A})|\mathbf{L}_i,\mathbf{A}_i\}\Pr(\mathbf{A}_i|\mathbf{L}_i)n_i}\right]$$

Moving the inner expectation inside the summation and taking out terms that are constant with respect to that expectation, it follows that $E\{\hat{F}_i(t,a,\alpha)\}$ equals

$$E_{\mathbf{L}_i,\mathbf{A}_i,T_{ij}(\mathbf{A}_i)}\left[\sum_{j=1}^{n_i} \frac{\pi(\mathbf{A}_{i,-j};\alpha)I(A_{ij}=a)E\{I(C_{ij}>T_{ij}(\mathbf{A}_i))|\mathbf{L}_i,\mathbf{A}_i,T_{ij}(\mathbf{A}_i)\}I\{T_{ij}(\mathbf{A}_i)\le t\}}{S_C\{T_{ij}(\mathbf{A}_i)|\mathbf{L}_i,\mathbf{A}_i\}\Pr(\mathbf{A}_i|\mathbf{L}_i)n_i}\right]$$

Note by Assumption III that for any t

$$S_c(t|\mathbf{L}_i,\mathbf{A}_i) = \Pr\{C_{ij}>t|\mathbf{L}_i,\mathbf{A}_i,T_{ij}(\mathbf{A}_i)=t\} = E\{I(C_{ij}>t)|\mathbf{L}_i,\mathbf{A}_i,T_{ij}(\mathbf{A}_i)=t\}$$

Therefore

$$E\{\hat{F}_i(t,a,\alpha)\} = E_{\mathbf{A}_i,\mathbf{L}_i,T_{ij}(\mathbf{A}_i)}\left[\frac{\sum_{j=1}^{n_i}\pi(\mathbf{A}_{i,-j};\alpha)I(A_{ij}=a)I\{T_{ij}(\mathbf{A}_i)\le t\}}{\Pr(\mathbf{A}_i|\mathbf{L}_i)n_i}\right].$$

The remainder of the proof follows from the proof of Theorem 6 of Tchetgen Tchetgen and VanderWeele (2012).

Web Tables

Web Table 1

Simulation results as in Table 1 of the main text, but with $m = 400$ and $n_i = 10$ for all i, where: α denotes the allocation probability; μ_a denotes the value of the target parameters $\mu(100, a, \alpha)$ for $a = 0, 1$; Bias is the average of $\mu(100, a, \alpha) - \hat{\mu}(100, a, \alpha)$ for $a = 0, 1$; ESE is the empirical standard error; MSE is the median of the sandwich variance-based standard error estimates; EC denotes the empirical coverage of the 95% Wald confidence intervals based on the Normal distribution; and EC_t denotes empirical coverage of t-distribution-based Wald confidence intervals.

α	μ_0	Bias	ESE	MSE	EC	EC_t
0.1	0.39	-0.02	0.09	0.08	93%	93%
0.2	0.38	-0.01	0.04	0.04	96%	96%
0.3	0.38	0.00	0.03	0.03	96%	96%
0.4	0.37	0.00	0.03	0.03	94%	94%
0.5	0.37	0.00	0.03	0.02	94%	94%
0.6	0.36	0.00	0.03	0.02	92%	92%
0.7	0.35	0.00	0.03	0.03	95%	95%
0.8	0.35	0.00	0.03	0.03	94%	95%
0.9	0.34	0.00	0.05	0.05	94%	94%

α	μ_1	Bias	ESE	MSE	EC	EC_t
0.1	0.28	0.00	0.09	0.08	91%	91%
0.2	0.28	0.00	0.05	0.05	94%	94%
0.3	0.27	0.00	0.03	0.03	94%	94%
0.4	0.27	0.00	0.02	0.02	94%	94%
0.5	0.27	0.00	0.02	0.02	94%	94%
0.6	0.27	0.00	0.02	0.02	95%	95%
0.7	0.26	0.00	0.02	0.02	94%	94%
0.8	0.26	0.00	0.02	0.02	95%	95%
0.9	0.26	0.00	0.02	0.02	95%	95%

Web Table 2

Simulation results as in Table 1 of the main text, but with $m = 300$ and $n_i = 10$ for all i.

α	μ_0	Bias	ESE	MSE	EC	EC_t
0.1	0.39	-0.02	0.09	0.09	92%	93%
0.2	0.38	-0.01	0.05	0.05	95%	95%
0.3	0.38	-0.01	0.04	0.04	95%	95%
0.4	0.37	0.00	0.03	0.03	95%	95%
0.5	0.37	0.00	0.03	0.03	95%	95%
0.6	0.36	0.00	0.03	0.03	95%	95%
0.7	0.35	0.00	0.03	0.03	94%	94%
0.8	0.35	0.00	0.04	0.04	95%	95%
0.9	0.34	0.00	0.06	0.06	95%	95%

α	μ_1	Bias	ESE	MSE	EC	EC_t
0.1	0.28	-0.01	0.11	0.09	89%	89%
0.2	0.28	0.00	0.06	0.05	93%	93%
0.3	0.27	0.00	0.04	0.04	93%	93%
0.4	0.27	0.00	0.03	0.03	94%	94%
0.5	0.27	0.00	0.02	0.02	95%	95%
0.6	0.26	0.00	0.02	0.02	95%	95%
0.7	0.26	0.00	0.02	0.02	94%	94%
0.8	0.26	0.00	0.02	0.02	96%	96%
0.9	0.26	0.00	0.03	0.03	95%	95%

Web Table 3

Simulation results as in Table 1 of the main text, but with $m = 200$ and $n_i = 10$ for all i.

α	μ_0	Bias	ESE	MSE	EC	EC_t
0.1	0.39	-0.02	0.12	0.10	90%	90%
0.2	0.38	-0.01	0.06	0.06	94%	94%
0.3	0.38	-0.01	0.05	0.04	92%	93%
0.4	0.37	0.00	0.04	0.04	93%	93%
0.5	0.37	0.00	0.04	0.03	94%	94%
0.6	0.36	0.00	0.03	0.03	95%	95%
0.7	0.35	0.00	0.04	0.04	93%	94%
0.8	0.35	0.00	0.05	0.05	93%	94%
0.9	0.34	0.00	0.08	0.07	91%	91%

α	μ_1	Bias	ESE	MSE	EC	EC_t
0.1	0.39	-0.02	0.12	0.10	90%	90%
0.2	0.38	-0.01	0.06	0.06	94%	94%
0.3	0.38	-0.01	0.05	0.04	92%	93%
0.4	0.37	0.00	0.04	0.04	93%	93%
0.5	0.37	0.00	0.04	0.03	94%	94%
0.6	0.36	0.00	0.03	0.03	95%	95%
0.7	0.35	0.00	0.04	0.04	93%	94%
0.8	0.35	0.00	0.05	0.05	93%	94%
0.9	0.34	0.00	0.08	0.07	91%	91%

Web Table 4

Simulation results as in Table 1 of the main text, but with $m = 100$ and $n_i = 10$ for all i.

α	μ_0	Bias	ESE	MSE	EC	EC$_t$
0.1	0.39	-0.01	0.17	0.13	84%	84%
0.2	0.38	-0.01	0.09	0.08	92%	93%
0.3	0.38	-0.01	0.07	0.06	93%	94%
0.4	0.37	0.00	0.06	0.05	93%	93%
0.5	0.36	0.00	0.05	0.05	93%	93%
0.6	0.36	0.01	0.05	0.04	93%	93%
0.7	0.35	0.01	0.05	0.05	93%	93%
0.8	0.35	0.01	0.07	0.06	92%	92%
0.9	0.34	0.00	0.11	0.10	89%	90%

α	μ_1	Bias	ESE	MSE	EC	EC$_t$
0.1	0.39	-0.01	0.17	0.13	84%	84%
0.2	0.38	-0.01	0.09	0.08	92%	93%
0.3	0.38	-0.01	0.07	0.06	93%	94%
0.4	0.37	0.00	0.06	0.05	93%	93%
0.5	0.36	0.00	0.05	0.05	93%	93%
0.6	0.36	0.01	0.05	0.04	93%	93%
0.7	0.35	0.01	0.05	0.05	93%	93%
0.8	0.35	0.01	0.07	0.06	92%	92%
0.9	0.34	0.00	0.11	0.10	89%	90%

Web Table 5

Simulation results as in Table 1 of the main text, but with $m = 50$ and $n_i = 10$ for all i.

α	μ_0	Bias	ESE	MSE	EC	EC_t
0.1	0.39	-0.01	0.27	0.13	74%	75%
0.2	0.38	-0.01	0.15	0.10	88%	88%
0.3	0.38	-0.01	0.11	0.08	90%	91%
0.4	0.37	0.00	0.09	0.07	90%	91%
0.5	0.37	0.00	0.08	0.06	91%	92%
0.6	0.36	0.01	0.07	0.06	91%	92%
0.7	0.35	0.01	0.08	0.07	91%	92%
0.8	0.35	0.00	0.10	0.09	89%	90%
0.9	0.34	0.00	0.16	0.13	84%	85%

α	μ_1	Bias	ESE	MSE	EC	EC_t
0.1	0.28	-0.01	0.27	0.12	68%	68%
0.2	0.28	-0.01	0.15	0.11	82%	83%
0.3	0.28	0.00	0.10	0.08	88%	89%
0.4	0.27	0.00	0.08	0.06	90%	90%
0.5	0.27	0.00	0.06	0.05	91%	92%
0.6	0.27	0.00	0.05	0.04	91%	91%
0.7	0.26	0.00	0.05	0.04	90%	90%
0.8	0.26	0.00	0.05	0.05	90%	91%
0.9	0.26	0.00	0.07	0.06	89%	90%

Web Table 6

Simulation results as in Table 1 of the main text, but with $m = 10$ and $n_i = 10$ for all i (53 of the 1000 simulated data sets where the fitted mixed model estimated covariance matrices were singular are excluded).

α	μ_0	Bias	ESE	MSE	EC	EC_t
0.1	0.39	0.11	0.40	0.07	44%	69%
0.2	0.38	0.04	0.33	0.11	62%	87%
0.3	0.38	0.01	0.28	0.12	67%	94%
0.4	0.37	0.02	0.22	0.11	73%	98%
0.5	0.37	0.03	0.18	0.10	73%	99%
0.6	0.36	0.03	0.17	0.10	73%	99%
0.7	0.35	0.03	0.19	0.11	75%	98%
0.8	0.35	0.00	0.25	0.14	72%	95%
0.9	0.34	-0.02	0.42	0.14	60%	82%

α	μ_1	Bias	ESE	MSE	EC	EC_t
0.1	0.28	-0.01	0.67	0.02	34%	53%
0.2	0.28	0.00	0.43	0.08	48%	75%
0.3	0.28	0.01	0.28	0.09	57%	88%
0.4	0.27	0.02	0.19	0.08	67%	96%
0.5	0.27	0.02	0.15	0.08	70%	98%
0.6	0.27	0.02	0.13	0.07	72%	99%
0.7	0.26	0.01	0.13	0.07	75%	100%
0.8	0.26	0.01	0.14	0.08	74%	99%
0.9	0.26	0.02	0.17	0.08	67%	94%

Web Table 7

Simulation results as in Table 1 of the main text, but with $m = 500$ and $n_i = 30$ for all i.

α	μ_0	Bias	ESE	MSE	EC	EC_t
0.1	0.39	-0.01	0.11	0.10	92%	92%
0.2	0.38	-0.01	0.06	0.05	93%	93%
0.3	0.38	0.00	0.04	0.04	94%	94%
0.4	0.37	0.00	0.04	0.03	93%	93%
0.5	0.36	0.00	0.03	0.03	93%	93%
0.6	0.36	0.00	0.03	0.03	94%	94%
0.7	0.35	0.00	0.03	0.03	94%	94%
0.8	0.34	0.00	0.04	0.03	95%	95%
0.9	0.34	-0.01	0.05	0.05	95%	95%

α	μ_1	Bias	ESE	MSE	EC	EC_t
0.1	0.28	0.00	0.10	0.09	89%	89%
0.2	0.28	0.00	0.06	0.05	92%	92%
0.3	0.27	0.00	0.04	0.03	93%	93%
0.4	0.27	0.00	0.03	0.03	93%	94%
0.5	0.27	0.00	0.02	0.02	94%	94%
0.6	0.26	0.00	0.02	0.02	94%	94%
0.7	0.26	0.00	0.02	0.02	93%	93%
0.8	0.26	0.00	0.02	0.02	95%	95%
0.9	0.25	0.00	0.02	0.02	96%	96%

Web Table 8

Simulation results as in Table 1 of the main text, but with $m = 500$ and $n_i = 50$ for all i.

α	μ_0	Bias	ESE	MSE	EC	EC_t
0.1	0.39	-0.01	0.15	0.12	89%	89%
0.2	0.38	0.00	0.08	0.07	92%	93%
0.3	0.38	0.00	0.06	0.05	93%	93%
0.4	0.37	0.00	0.05	0.04	92%	92%
0.5	0.36	0.00	0.04	0.04	92%	92%
0.6	0.36	0.00	0.04	0.03	93%	93%
0.7	0.35	0.00	0.04	0.03	92%	92%
0.8	0.34	0.00	0.04	0.04	93%	94%
0.9	0.34	0.00	0.06	0.05	93%	93%

α	μ_1	Bias	ESE	MSE	EC	EC_t
0.1	0.28	-0.01	0.14	0.10	86%	86%
0.2	0.28	-0.01	0.07	0.06	92%	92%
0.3	0.27	0.00	0.05	0.04	92%	92%
0.4	0.27	0.00	0.04	0.03	91%	91%
0.5	0.27	0.00	0.03	0.03	92%	92%
0.6	0.26	0.00	0.03	0.02	91%	91%
0.7	0.26	0.00	0.03	0.02	93%	93%
0.8	0.26	0.00	0.03	0.02	93%	93%
0.9	0.25	0.00	0.03	0.03	94%	94%

Web Table 9

Simulation results as in Table 1 of the main text, but with $m = 500$ and $n_i = 75$ for all i.

α	μ_0	Bias	ESE	MSE	EC	EC_t
0.1	0.39	-0.01	0.18	0.14	87%	87%
0.2	0.38	0.00	0.10	0.08	91%	91%
0.3	0.38	0.00	0.08	0.07	91%	91%
0.4	0.37	0.00	0.11	0.06	92%	92%
0.5	0.36	0.00	0.08	0.05	90%	90%
0.6	0.36	0.00	0.14	0.04	91%	91%
0.7	0.35	0.00	0.07	0.04	90%	90%
0.8	0.34	0.00	0.06	0.04	92%	92%
0.9	0.34	0.00	0.06	0.06	93%	93%

α	μ_1	Bias	ESE	MSE	EC	EC_t
0.1	0.28	-0.01	0.15	0.11	84%	84%
0.2	0.28	-0.01	0.08	0.07	91%	91%
0.3	0.27	0.00	0.07	0.05	89%	89%
0.4	0.27	0.00	0.08	0.04	90%	90%
0.5	0.27	0.00	0.06	0.04	90%	90%
0.6	0.26	0.00	0.10	0.03	90%	90%
0.7	0.26	0.00	0.05	0.03	91%	91%
0.8	0.26	0.00	0.04	0.03	92%	92%
0.9	0.25	0.00	0.04	0.04	94%	94%

Web Table 10

Simulation results as in Table 1 of main text, but with $m = 500$ and $n_i = 100$ for all i.

α	μ_0	Bias	ESE	MSE	EC	EC_t
0.1	0.39	0.01	0.21	0.14	80%	80%
0.2	0.38	0.00	0.13	0.10	86%	87%
0.3	0.38	0.00	0.12	0.08	88%	88%
0.4	0.37	0.00	0.12	0.07	87%	88%
0.5	0.36	0.01	0.08	0.06	88%	88%
0.6	0.36	0.01	0.07	0.05	86%	86%
0.7	0.35	0.00	0.07	0.05	89%	89%
0.8	0.34	0.00	0.07	0.05	91%	91%
0.9	0.34	0.00	0.07	0.06	92%	92%

α	μ_1	Bias	ESE	MSE	EC	EC_t
0.1	0.28	0.00	0.17	0.11	79%	80%
0.2	0.28	0.00	0.11	0.07	85%	85%
0.3	0.27	0.00	0.09	0.06	87%	87%
0.4	0.27	0.00	0.08	0.05	88%	88%
0.5	0.27	0.00	0.06	0.04	87%	87%
0.6	0.26	0.00	0.05	0.04	87%	87%
0.7	0.26	0.00	0.05	0.04	89%	89%
0.8	0.26	0.00	0.05	0.04	91%	91%
0.9	0.25	0.00	0.04	0.04	93%	93%

Web Table 11

Simulation results as in Table 1 of main text, but with $m = 500$ and $n_i = 200$ for all i.

α	μ_0	Bias	ESE	MSE	EC	EC_t
0.1	0.39	-0.03	1.17	0.17	71%	71%
0.2	0.38	0.01	0.24	0.13	76%	76%
0.3	0.38	0.00	0.27	0.11	75%	76%
0.4	0.37	0.01	0.22	0.10	76%	76%
0.5	0.36	0.00	0.25	0.09	76%	76%
0.6	0.36	0.01	0.22	0.08	74%	74%
0.7	0.35	-0.01	0.19	0.08	82%	82%
0.8	0.34	0.00	0.14	0.08	84%	84%
0.9	0.34	0.00	0.10	0.08	87%	87%

α	μ_1	Bias	ESE	MSE	EC	EC_t
0.1	0.28	-0.01	0.50	0.12	71%	71%
0.2	0.28	0.01	0.19	0.09	75%	75%
0.3	0.27	0.00	0.22	0.08	75%	75%
0.4	0.27	0.00	0.17	0.07	75%	76%
0.5	0.27	0.01	0.16	0.06	76%	76%
0.6	0.26	0.00	0.18	0.06	75%	75%
0.7	0.26	-0.01	0.15	0.06	82%	82%
0.8	0.26	0.00	0.10	0.06	85%	85%
0.9	0.25	0.00	0.08	0.06	88%	88%

Web Table 12

Simulation results as in Table 1 of main text, but censoring times C_{ij} randomly sampled from an Exponential distribution with mean $1/\lambda_0$ where $\lambda_0 = 0.0025\exp(0.002L_{1ij} + 0.015L_{2ij})e_i$, resulting in an average of 33% of individuals being censored.

Proposed IPCW estimator

α	μ_0	Bias	ESE	MSE	EC	EC_t
0.1	0.39	-0.02	0.06	0.06	94%	94%
0.2	0.38	-0.01	0.03	0.03	96%	96%
0.3	0.38	-0.01	0.02	0.02	96%	96%
0.4	0.37	0.00	0.02	0.02	94%	94%
0.5	0.37	0.00	0.02	0.02	94%	94%
0.6	0.36	0.00	0.02	0.02	95%	95%
0.7	0.35	0.00	0.02	0.02	95%	95%
0.8	0.35	0.00	0.02	0.02	95%	95%
0.9	0.34	0.00	0.04	0.04	94%	94%

α	μ_1	Bias	ESE	MSE	EC	EC_t
0.1	0.28	-0.01	0.07	0.06	94%	94%
0.2	0.28	-0.01	0.04	0.04	94%	94%
0.3	0.27	0.00	0.02	0.02	95%	95%
0.4	0.27	0.00	0.02	0.02	94%	94%
0.5	0.27	0.00	0.01	0.01	94%	94%
0.6	0.26	0.00	0.01	0.01	94%	94%
0.7	0.26	0.00	0.01	0.01	95%	95%
0.8	0.26	-0.00	0.01	0.01	96%	96%
0.9	0.26	0.00	0.02	0.02	96%	96%

Tchetgen Tchetgen and VanderWeele (2012) estimator

α	μ_0	Bias	ESE	MSE	EC	EC_t
0.1	0.39	-0.15	0.08	0.06	40%	40%
0.2	0.38	-0.14	0.03	0.03	12%	12%
0.3	0.38	-0.13	0.03	0.02	8%	8%
0.4	0.37	-0.13	0.02	0.02	3%	3%
0.5	0.37	-0.12	0.02	0.02	5%	5%
0.6	0.36	-0.12	0.02	0.02	7%	7%
0.7	0.35	-0.12	0.02	0.02	7%	7%
0.8	0.35	-0.13	0.03	0.02	6%	6%
0.9	0.34	-0.13	0.04	0.03	6%	6%

α	μ_1	Bias	ESE	MSE	EC	EC_t
0.1	0.28	-0.15	0.07	0.06	32%	32%
0.2	0.28	-0.15	0.04	0.03	6%	6%
0.3	0.27	-0.15	0.03	0.02	3%	3%
0.4	0.27	-0.14	0.02	0.02	4%	4%
0.5	0.27	-0.14	0.02	0.01	5%	5%
0.6	0.26	-0.14	0.02	0.01	5%	5%
0.7	0.26	-0.14	0.02	0.01	3%	3%
0.8	0.26	-0.15	0.02	0.01	2%	2%
0.9	0.26	-0.16	0.02	0.02	7%	7%

Web Table 13

Simulation results as in Table 1 of main text, but with the censoring times generated in step (vi) from an Exponential distribution with mean $1/\lambda_0$ where $\lambda_0 = 0.005\exp(0.002L_{1ij} + 0.015L_{2ij})e_i$, resulting in an average of 45% of individuals being censored.

Proposed IPCW estimator

α	μ_0	Bias	ESE	MSE	EC	EC_t
0.1	0.39	-0.02	0.06	0.06	96%	96%
0.2	0.38	-0.01	0.03	0.03	96%	96%
0.3	0.38	-0.01	0.02	0.02	95%	95%
0.4	0.37	0.00	0.02	0.02	96%	96%
0.5	0.37	0.00	0.02	0.02	96%	96%
0.6	0.36	0.00	0.02	0.02	95%	95%
0.7	0.35	0.00	0.02	0.02	94%	94%
0.8	0.35	0.00	0.03	0.03	94%	95%
0.9	0.34	0.00	0.04	0.04	94%	94%

α	μ_1	Bias	ESE	MSE	EC	EC_t
0.1	0.28	-0.01	0.07	0.06	92%	92%
0.2	0.28	-0.00	0.04	0.04	94%	94%
0.3	0.27	-0.00	0.03	0.02	94%	94%
0.4	0.27	0.00	0.02	0.02	94%	94%
0.5	0.27	0.00	0.01	0.01	94%	94%
0.6	0.27	0.00	0.01	0.01	94%	94%
0.7	0.26	0.00	0.01	0.01	94%	94%
0.8	0.26	-0.00	0.01	0.01	95%	95%
0.9	0.26	-0.01	0.02	0.02	96%	97%

Tchetgen Tchetgen and VanderWeele (2012) estimator

α	μ_0	Bias	ESE	MSE	EC	EC_t
0.1	0.39	-0.23	0.09	0.05	13%	13%
0.2	0.38	-0.22	0.05	0.03	6%	6%
0.3	0.38	-0.21	0.03	0.02	4%	4%
0.4	0.37	-0.21	0.03	0.02	2%	2%
0.5	0.37	-0.21	0.02	0.02	3%	3%
0.6	0.36	-0.20	0.02	0.02	3%	3%
0.7	0.35	-0.21	0.02	0.02	3%	3%
0.8	0.35	-0.21	0.03	0.02	3%	3%
0.9	0.34	-0.21	0.04	0.03	1%	1%

α	μ_1	Bias	ESE	MSE	EC	EC_t
0.1	0.28	-0.24	0.08	0.05	6%	6%
0.2	0.28	-0.24	0.05	0.03	2%	2%
0.3	0.27	-0.24	0.03	0.02	1%	1%
0.4	0.27	-0.23	0.02	0.02	2%	2%
0.5	0.27	-0.23	0.02	0.02	3%	3%
0.6	0.27	-0.23	0.02	0.02	2%	2%
0.7	0.26	-0.24	0.02	0.02	1%	1%
0.8	0.26	-0.24	0.02	0.02	1%	1%
0.9	0.26	-0.25	0.03	0.02	4%	4%

Web Table 14

Simulation results as in Table 1 of main text, but censoring times C_{ij} randomly sampled from an Exponential distribution with mean $1/\lambda_0$ where $\lambda_0 = 0.015\exp(0.002L_{1ij} + 0.015L_{2ij})e_i$, resulting in an average of 65% of individuals being censored.

Proposed IPCW estimator

α	μ_0	Bias	ESE	MSE	EC	EC_t
0.1	0.39	-0.02	0.07	0.07	95%	95%
0.2	0.38	-0.01	0.04	0.04	95%	95%
0.3	0.38	-0.01	0.03	0.03	95%	95%
0.4	0.37	-0.00	0.03	0.02	95%	95%
0.5	0.37	0.00	0.03	0.02	96%	96%
0.6	0.36	0.00	0.02	0.02	94%	95%
0.7	0.35	0.01	0.02	0.02	95%	95%
0.8	0.35	0.00	0.03	0.03	96%	96%
0.9	0.34	0.00	0.05	0.05	95%	95%

α	μ_1	Bias	ESE	MSE	EC	EC_t
0.1	0.28	-0.01	0.08	0.08	92%	92%
0.2	0.28	-0.00	0.04	0.04	95%	95%
0.3	0.27	-0.00	0.03	0.03	94%	94%
0.4	0.27	0.00	0.02	0.02	94%	94%
0.5	0.27	0.00	0.02	0.02	94%	94%
0.6	0.27	0.00	0.02	0.02	94%	94%
0.7	0.26	0.00	0.01	0.01	96%	96%
0.8	0.26	-0.00	0.02	0.02	96%	96%
0.9	0.26	-0.00	0.02	0.02	96%	96%

Tchetgen Tchetgen and VanderWeele (2012) estimator

α	μ_0	Bias	ESE	MSE	EC	EC_t
0.1	0.39	-0.39	0.10	0.05	6%	6%
0.2	0.38	-0.38	0.05	0.03	5%	5%
0.3	0.38	-0.37	0.04	0.03	2%	2%
0.4	0.37	-0.36	0.03	0.03	1%	1%
0.5	0.37	-0.36	0.03	0.03	2%	2%
0.6	0.36	-0.36	0.02	0.03	2%	2%
0.7	0.35	-0.36	0.02	0.03	2%	2%
0.8	0.35	-0.37	0.03	0.03	2%	2%
0.9	0.34	-0.38	0.05	0.04	1%	1%

α	μ_1	Bias	ESE	MSE	EC	EC_t
0.1	0.28	-0.43	0.08	0.05	3%	3%
0.2	0.28	-0.42	0.05	0.04	1%	1%
0.3	0.27	-0.42	0.04	0.03	0%	0%
0.4	0.27	-0.41	0.03	0.03	1%	1%
0.5	0.27	-0.41	0.02	0.03	1%	1%
0.6	0.27	-0.42	0.02	0.03	1%	1%
0.7	0.26	-0.42	0.02	0.03	1%	1%
0.8	0.26	-0.43	0.02	0.03	1%	1%
0.9	0.26	-0.44	0.03	0.03	2%	2%

Web Table 15

Simulation results as in Table 1 of the main text, but with the distribution of the random effect b_i in the treatment model misspecified. Data were generated in step (ii) such that b_i was from a logistic distribution with mean 0 and variance 1.0859. However, the mixed effect logistic treatment model was fit assuming a Normal random effect distribution.

α	μ_0	Bias	ESE	MSE	EC	EC_t
0.1	0.39	-0.05	0.09	0.08	95%	95%
0.2	0.38	-0.01	0.04	0.04	97%	97%
0.3	0.38	0.00	0.03	0.03	95%	95%
0.4	0.37	0.00	0.02	0.02	94%	94%
0.5	0.37	0.00	0.02	0.02	95%	95%
0.6	0.36	0.00	0.02	0.02	96%	96%
0.7	0.35	-0.00	0.02	0.02	96%	96%
0.8	0.35	0.00	0.03	0.03	94%	94%
0.9	0.34	0.00	0.05	0.05	93%	93%

α	μ_1	Bias	ESE	MSE	EC	EC_t
0.1	0.28	0.00	0.08	0.08	89%	89%
0.2	0.28	0.01	0.05	0.04	92%	92%
0.3	0.27	0.00	0.03	0.03	93%	93%
0.4	0.27	0.00	0.02	0.02	94%	94%
0.5	0.27	0.00	0.02	0.02	95%	95%
0.6	0.27	-0.00	0.02	0.02	95%	95%
0.7	0.26	0.00	0.01	0.01	96%	96%
0.8	0.26	0.00	0.02	0.02	96%	96%
0.9	0.26	-0.00	0.02	0.02	97%	97%

Web Table 16

Simulation results as in Table 1 of the main text, but with the distribution of the random effect b_i in the treatment model misspecified. Data were generated in step (ii) such that b_i was from a Gumbel distribution with mean 0 and variance 1.0859. However, the mixed effect logistic treatment model was fit assuming a Normal random effect distribution.

α	μ_0	Bias	ESE	MSE	EC	EC_t
0.1	0.39	0.09	0.07	0.06	64%	64%
0.2	0.38	0.03	0.04	0.04	85%	85%
0.3	0.38	-0.01	0.03	0.03	96%	96%
0.4	0.37	-0.02	0.02	0.02	90%	90%
0.5	0.36	-0.02	0.02	0.02	89%	89%
0.6	0.36	-0.01	0.02	0.02	95%	95%
0.7	0.35	0.01	0.02	0.02	89%	89%
0.8	0.35	0.03	0.03	0.03	81%	81%
0.9	0.34	0.03	0.05	0.05	84%	85%

α	μ_1	Bias	ESE	MSE	EC	EC_t
0.1	0.28	0.01	0.07	0.07	88%	89%
0.2	0.28	-0.01	0.04	0.04	96%	96%
0.3	0.27	-0.02	0.03	0.03	93%	93%
0.4	0.27	-0.01	0.02	0.02	93%	93%
0.5	0.27	-0.00	0.02	0.02	96%	96%
0.6	0.27	0.01	0.02	0.01	92%	92%
0.7	0.26	0.01	0.01	0.01	82%	82%
0.8	0.26	0.01	0.02	0.02	93%	94%
0.9	0.26	-0.03	0.03	0.03	88%	88%

Web Table 17

Simulation results as in Table 1 of the main text, but with the distribution of the random effect b_i in the treatment model misspecified. Data were generated in step (ii) such that b_i was from a t distribution with 10 degrees of freedom, mean 0 and variance 1.25. However, the mixed effect logistic treatment model was fit assuming a Normal random effect distribution.

α	μ_0	Bias	ESE	MSE	EC	EC_t
0.1	0.39	-0.04	0.08	0.07	95%	95%
0.2	0.38	-0.01	0.04	0.04	96%	96%
0.3	0.38	0.00	0.03	0.03	96%	96%
0.4	0.37	0.00	0.02	0.02	95%	95%
0.5	0.37	0.00	0.02	0.02	94%	94%
0.6	0.36	0.00	0.02	0.02	93%	94%
0.7	0.35	0.00	0.02	0.02	94%	94%
0.8	0.35	0.00	0.03	0.03	95%	95%
0.9	0.34	0.00	0.05	0.05	94%	94%

α	μ_1	Bias	ESE	MSE	EC	EC_t
0.1	0.28	-0.00	0.08	0.07	92%	92%
0.2	0.28	0.00	0.04	0.04	94%	94%
0.3	0.27	0.00	0.03	0.03	94%	94%
0.4	0.27	0.00	0.02	0.02	96%	96%
0.5	0.27	0.00	0.02	0.02	95%	95%
0.6	0.27	0.00	0.02	0.02	95%	95%
0.7	0.26	0.00	0.02	0.01	95%	95%
0.8	0.26	0.00	0.02	0.02	95%	95%
0.9	0.26	-0.00	0.02	0.02	96%	96%

Web Table 18

Simulation results as in Table 1 of the main text, but with the distribution of the random effect b_i in the treatment model misspecified. Data were generated in step (ii) such that b_i was from a 50:50 mixture of two Normal distributions with means -1 and 1, and standard deviations equal to 0.3, so b_i had mean 0 and variance approximately 1.09. However, the mixed effect logistic treatment model was fit assuming a Normal random effect distribution.

α	μ_0	Bias	ESE	MSE	EC	EC_t
0.1	0.39	0.10	0.05	0.04	46%	46%
0.2	0.38	-0.00	0.04	0.03	95%	95%
0.3	0.38	-0.05	0.03	0.03	72%	72%
0.4	0.37	-0.04	0.03	0.03	70%	70%
0.5	0.37	-0.01	0.03	0.02	93%	94%
0.6	0.36	0.02	0.02	0.02	76%	76%
0.7	0.35	0.04	0.02	0.02	63%	63%
0.8	0.35	0.02	0.03	0.03	89%	89%
0.9	0.34	-0.03	0.05	0.05	95%	95%

α	μ_1	Bias	ESE	MSE	EC	EC_t
0.1	0.28	-0.03	0.07	0.07	95%	95%
0.2	0.28	-0.05	0.05	0.04	86%	86%
0.3	0.27	-0.04	0.03	0.03	83%	83%
0.4	0.27	-0.01	0.03	0.02	94%	94%
0.5	0.27	0.02	0.02	0.02	79%	80%
0.6	0.27	0.02	0.02	0.01	62%	62%
0.7	0.26	0.01	0.02	0.01	83%	83%
0.8	0.26	-0.01	0.02	0.02	92%	92%
0.9	0.26	-0.02	0.02	0.02	91%	91%

Web Table 19

Simulation results as in Table 1 of the main text, but with the distribution of the random effect b_i in the treatment model misspecified. Data were generated in step (ii) such that $b_i = X - E[X]$ where X was from an Exponential distribution with mean 1.0421, such that b_i had mean 0, variance approximately 1.09, and skewness 2. However, the mixed effect logistic treatment model was fit assuming a Normal random effect distribution.

α	μ_0	Bias	ESE	MSE	EC	EC_t
0.1	0.39	0.17	0.07	0.05	28%	28%
0.2	0.38	0.08	0.04	0.04	46%	46%
0.3	0.38	0.01	0.03	0.03	92%	92%
0.4	0.37	-0.02	0.02	0.02	87%	87%
0.5	0.37	-0.03	0.02	0.02	75%	75%
0.6	0.36	-0.01	0.02	0.02	95%	95%
0.7	0.35	0.02	0.02	0.02	88%	88%
0.8	0.35	0.04	0.03	0.03	64%	65%
0.9	0.34	0.05	0.05	0.05	76%	76%

α	μ_0	Bias	ESE	MSE	EC	EC_t
0.1	0.28	0.04	0.07	0.06	76%	76%
0.2	0.28	-0.00	0.04	0.04	94%	94%
0.3	0.27	-0.03	0.03	0.03	90%	90%
0.4	0.27	-0.02	0.02	0.02	85%	86%
0.5	0.27	-0.01	0.02	0.02	94%	95%
0.6	0.27	0.01	0.01	0.01	86%	86%
0.7	0.26	0.02	0.01	0.01	64%	64%
0.8	0.26	0.00	0.02	0.02	92%	92%
0.9	0.26	-0.06	0.03	0.03	64%	65%

Web Table 20

Simulation results as in Table 1 of the main text, but with the distribution of the random effect b_i in the treatment model misspecified. Data were generated in step (ii) such that $b_i = X - E[X]$ where $X = 2.8B$ and B was from a Beta distribution with shape parameters 0.2 and 0.4, such that the mean of b_i was 0, variance was approximately 1.09, and skewness was approximately 0.69. However, the mixed effect logistic treatment model was fit assuming a Normal random effect distribution.

α	μ_0	Bias	ESE	MSE	EC	EC_t
0.1	0.39	0.09	0.24	0.09	55%	55%
0.2	0.38	0.04	0.11	0.08	74%	74%
0.3	0.38	0.01	0.05	0.05	89%	89%
0.4	0.37	-0.00	0.03	0.03	95%	95%
0.5	0.37	-0.00	0.02	0.02	95%	95%
0.6	0.36	-0.00	0.02	0.02	94%	94%
0.7	0.35	0.00	0.02	0.02	94%	94%
0.8	0.35	0.01	0.03	0.03	92%	93%
0.9	0.34	0.00	0.06	0.05	92%	92%

α	μ_1	Bias	ESE	MSE	EC	EC_t
0.1	0.28	0.02	0.15	0.08	68%	68%
0.2	0.28	0.00	0.07	0.06	88%	88%
0.3	0.27	-0.00	0.04	0.04	92%	92%
0.4	0.27	-0.00	0.02	0.02	93%	93%
0.5	0.27	-0.00	0.02	0.02	95%	95%
0.6	0.27	0.00	0.01	0.01	95%	95%
0.7	0.26	0.00	0.02	0.01	95%	95%
0.8	0.26	-0.01	0.02	0.02	94%	94%
0.9	0.26	-0.02	0.04	0.04	94%	94%

Web Table 21

Simulation results as in Table 1 of the main text, but with the distribution of the random effect b_i in the treatment model misspecified. Data were generated in step (ii) such that $b_i = X - E[X]$ where $X = 6.8B$ and B was from a Beta distribution with shape parameters 2 and 4, such that the distribution of b_i was bimodal with mean of 0, variance approximately 1.14, and skewness approximately 0.47. However, the mixed effect logistic treatment model was fit assuming a Normal random effect distribution.

α	μ_0	Bias	ESE	MSE	EC	EC_t
0.1	0.39	0.07	0.06	0.06	69%	69%
0.2	0.38	0.01	0.03	0.04	93%	93%
0.3	0.38	-0.02	0.03	0.03	94%	94%
0.4	0.37	-0.02	0.03	0.02	87%	87%
0.5	0.37	-0.01	0.02	0.02	93%	93%
0.6	0.36	0.00	0.02	0.02	95%	95%
0.7	0.35	0.02	0.02	0.02	89%	89%
0.8	0.35	0.02	0.03	0.03	86%	86%
0.9	0.34	0.02	0.05	0.05	92%	92%

α	μ_1	Bias	ESE	MSE	EC	EC_t
0.1	0.28	-0.01	0.07	0.07	92%	92%
0.2	0.28	-0.02	0.04	0.04	94%	94%
0.3	0.27	-0.02	0.03	0.03	92%	92%
0.4	0.27	-0.01	0.02	0.02	94%	94%
0.5	0.27	0.00	0.02	0.02	94%	94%
0.6	0.27	0.01	0.02	0.02	86%	86%
0.7	0.26	0.01	0.02	0.01	84%	84%
0.8	0.26	0.00	0.02	0.02	93%	93%
0.9	0.26	-0.02	0.02	0.02	94%	94%

Web Table 22

Simulation results as in Table 1 of the main text, but with the distribution of the random effect b_i in the treatment model misspecified. Data were generated in step (ii) such that b_i Uniform$(-1.8, 1.8)$, with mean 0 and variance 1.08. However, the mixed effect logistic treatment model was fit assuming a Normal random effect distribution.

α	μ_0	Bias	ESE	MSE	EC	EC$_t$
0.1	0.39	0.05	0.06	0.06	79%	79%
0.2	0.38	-0.01	0.04	0.04	95%	95%
0.3	0.38	-0.03	0.03	0.03	90%	90%
0.4	0.37	-0.02	0.03	0.02	90%	90%
0.5	0.37	-0.00	0.02	0.02	95%	95%
0.6	0.36	0.01	0.02	0.02	89%	89%
0.7	0.35	0.02	0.02	0.02	85%	85%
0.8	0.35	0.01	0.03	0.03	92%	92%
0.9	0.34	-0.01	0.05	0.05	94%	94%

α	μ_1	Bias	ESE	MSE	EC	EC$_t$
0.1	0.28	-0.02	0.07	0.07	95%	95%
0.2	0.28	-0.03	0.04	0.04	94%	94%
0.3	0.27	-0.02	0.03	0.03	95%	95%
0.4	0.27	-0.00	0.02	0.02	95%	95%
0.5	0.27	0.01	0.02	0.02	90%	90%
0.6	0.27	0.01	0.01	0.02	86%	86%
0.7	0.26	0.01	0.01	0.01	92%	92%
0.8	0.26	-0.00	0.02	0.02	96%	96%
0.9	0.26	-0.01	0.02	0.02	94%	95%

Web Table 23

Simulation results as in Table 1 of the main text, but with the frailty censoring model mis-specified; the covariate L_{2ij} was incorrectly omitted from the model, despite L_{2ij} being a component of the data generation process for censoring times C_{ij}.

α	μ_0	Bias	ESE	MSE	EC	EC_t
0.1	0.39	-0.02	0.07	0.07	94%	94%
0.2	0.38	-0.01	0.04	0.04	95%	95%
0.3	0.38	-0.00	0.03	0.03	96%	96%
0.4	0.37	0.00	0.03	0.02	96%	96%
0.5	0.36	0.00	0.02	0.02	94%	94%
0.6	0.36	0.01	0.02	0.02	94%	94%
0.7	0.35	0.01	0.02	0.02	93%	93%
0.8	0.35	0.00	0.03	0.03	95%	95%
0.9	0.34	0.00	0.05	0.05	93%	93%

α	μ_1	Bias	ESE	MSE	EC	EC_t
0.1	0.28	-0.01	0.08	0.08	91%	91%
0.2	0.28	-0.00	0.05	0.04	93%	93%
0.3	0.27	0.00	0.03	0.03	93%	93%
0.4	0.27	0.00	0.02	0.02	95%	95%
0.5	0.27	0.00	0.02	0.02	95%	95%
0.6	0.27	0.00	0.02	0.02	95%	95%
0.7	0.26	0.00	0.02	0.01	95%	95%
0.8	0.26	-0.00	0.02	0.02	96%	96%
0.9	0.26	-0.00	0.02	0.02	96%	96%

Web Table 24

Simulation results as in Table 1 of the main text, but with the censoring times generated in step (vi) such that $\lambda_0 = 0.015\exp(0.002L_{1ij} + 0.015L_{2ij} + 0.01A_{ij})$. *The IPCW estimator frailty censoring model was correctly specified by including terms for each of the three covariates* L_{1ij}, L_{2ij}, *and* A_{ij}.

α	μ_0	Bias	ESE	MSE	EC	EC_t
0.1	0.39	-0.02	0.08	0.07	95%	95%
0.2	0.38	-0.01	0.04	0.04	96%	96%
0.3	0.38	-0.00	0.03	0.03	96%	96%
0.4	0.37	0.00	0.02	0.02	95%	95%
0.5	0.36	0.00	0.02	0.02	94%	95%
0.6	0.36	0.00	0.02	0.02	94%	94%
0.7	0.35	0.00	0.02	0.02	94%	94%
0.8	0.35	0.00	0.03	0.03	95%	95%
0.9	0.34	-0.00	0.05	0.05	95%	95%

α	μ_1	Bias	ESE	MSE	EC	EC_t
0.1	0.28	-0.00	0.08	0.07	90%	90%
0.2	0.28	-0.00	0.04	0.04	93%	93%
0.3	0.27	0.00	0.03	0.03	93%	93%
0.4	0.27	0.00	0.02	0.02	93%	93%
0.5	0.27	0.00	0.02	0.02	93%	93%
0.6	0.27	0.00	0.02	0.02	94%	94%
0.7	0.26	0.00	0.01	0.01	95%	95%
0.8	0.26	-0.00	0.02	0.02	95%	95%
0.9	0.26	-0.00	0.02	0.02	95%	95%

Web Table 25

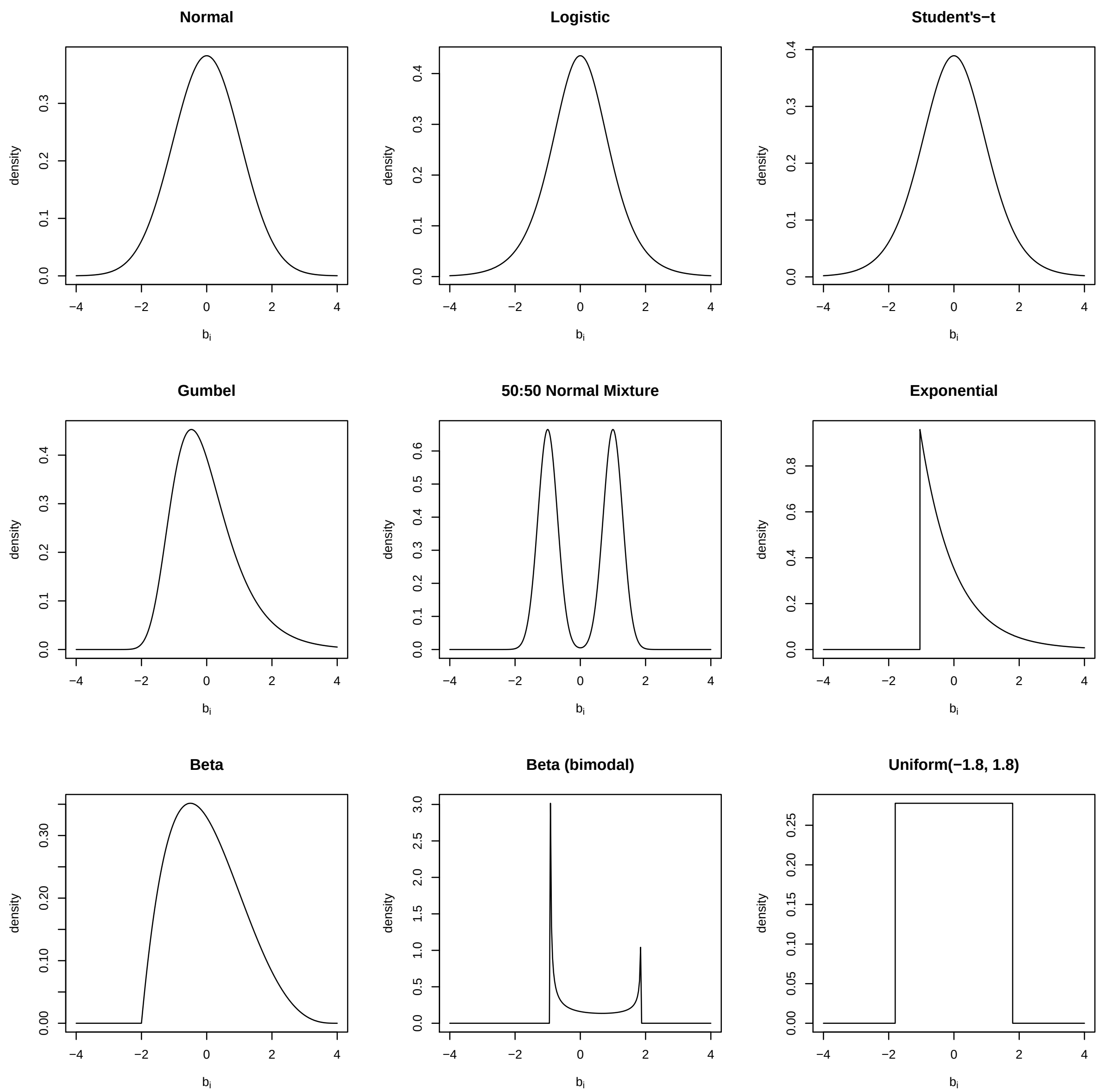
Parameter estimates and estimated standard errors (SE)
for treatment and censoring model

Treatment model		
	Estimate	SE
Intercept	0.28	0.075
Age	-0.06	0.004
Distance	-0.09	0.069
Squared Age	0.03	0.002
Squared Distance	0.08	0.015

Censoring model		
	Estimate	SE
θ_r	0.159	0.025
θ_h	0.039	0.001
Age	0.015	0.009

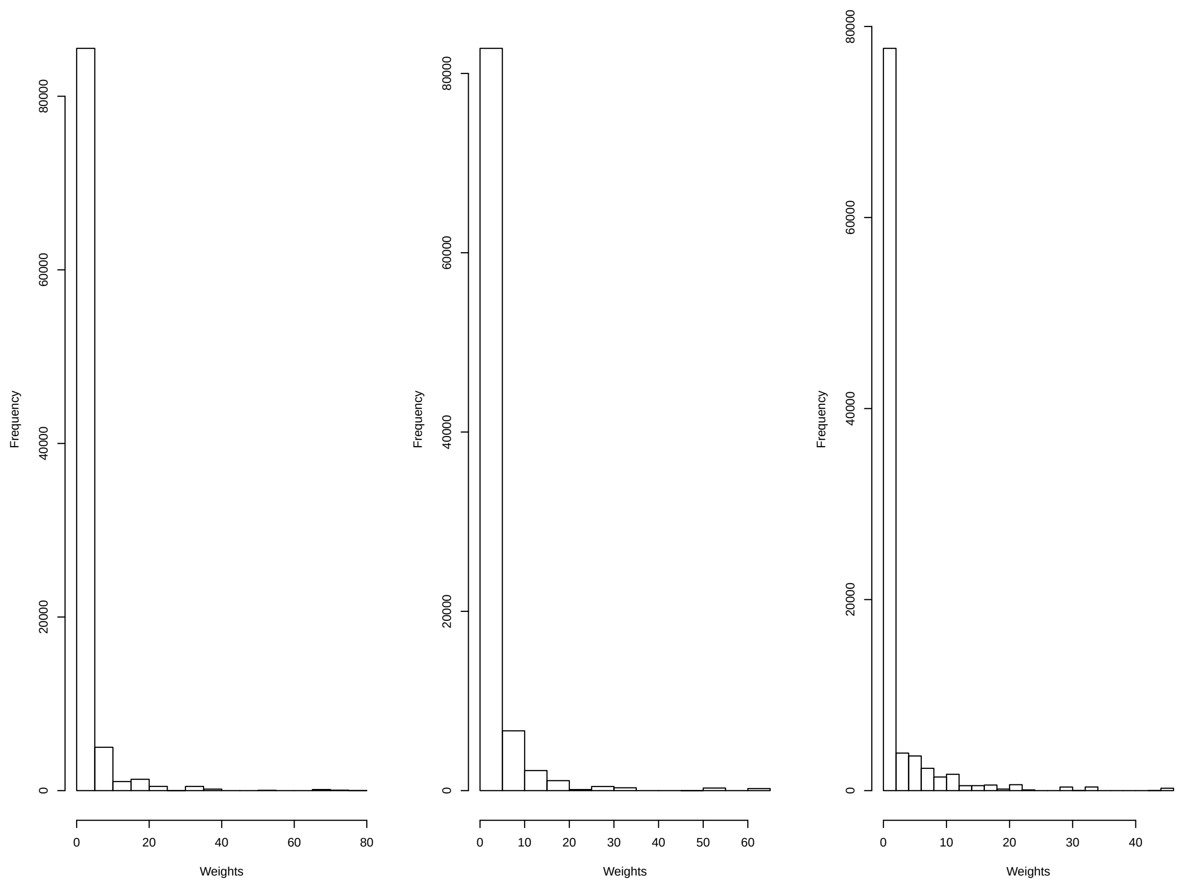
Web Figure 1

Generating distributions of random effect b_i in simulation studies corresponding to Table 1 (correct specification) and Web Tables 15–22 (mis-specification).



Web Figure 2

Histograms of weights $w_{ij} = \pi(\mathbf{A}_{i,-j};\alpha)/\{\Pr(\mathbf{A}_i|\mathbf{L}_i,\hat{\boldsymbol{\beta}})S_C(X_{ij}|\mathbf{L}_i,\hat{\boldsymbol{\gamma}})\}$ *for* $\alpha = 0.30$ *(left),* $\alpha = 0.45$ *(center) and* $\alpha = 0.60$ *(right)*



Web Figure 3

Histograms of weights $\tilde{w}_{ij} = \pi(\mathbf{A}_i; \alpha)/\{\Pr(\mathbf{A}_i|\mathbf{L}_i, \hat{\boldsymbol{\beta}})S_C(X_{ij}|\mathbf{L}_i, \hat{\boldsymbol{\gamma}})\}$ *for* $\alpha = 0.30$ *(left),* $\alpha = 0.45$ *(center) and* $\alpha = 0.60$ *(right)*