Supplement

A Effective Reproductive Number

We derive the effective reproductive number R_e for the epidemic model using the next-generation method. The model has five infected host compartments:

$$x = \begin{bmatrix} E & E_m & I_1 & I_{1m} & I_{2m} \end{bmatrix}$$

Let $\mathcal{F}_i$ be the rate at which new infected individuals enter compartment i, and let $\mathcal{V}_i$ be the transfer of individuals into and out of compartment i. We define two matrices F and V, where $F_{ij} = \frac{\partial \mathcal{F}_i(x_0)}{\partial x_j}$, $V_{ij} = \frac{\partial \mathcal{V}_i(x_0)}{\partial x_j}$, and x_0 is the disease-free equilibrium. Using this notation, we have $\frac{\mathrm{d}x}{\mathrm{d}t} = (F - V)x$. For our model, F and V are as follows:

$$F = \begin{bmatrix} 0 & 0 & \beta_1 S & \beta_1 S(1-\eta_{I1}) & \beta_2 S(1-\eta_{I2}) \\ 0 & 0 & \beta_1 S_m(1-\eta_S) & \beta_1 S_m(1-\eta_S)(1-\eta_{I1}) & \beta_2 S_m(1-\eta_S)(1-\eta_{I2}) \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$V = \begin{bmatrix} w & 0 & 0 & 0 & 0 \\ 0 & w & 0 & 0 & 0 \\ -w & 0 & \gamma_1 + p & 0 & 0 \\ 0 & -w & 0 & \gamma_1 + p & 0 \\ 0 & 0 & -p & -p & \gamma_2 + \mu \end{bmatrix}$$

R_e is given by the largest eigenvalue of the next generation operator FV^{-1}, where the entry (i, j) represents the expected number of of secondary cases in compartment i caused by an individual in compartment j. FV^{-1} has only one non-zero eigenvalue. The effective reproductive number is given by:

$$R_e = \frac{S\Big(\beta_1(\gamma_2+\mu) + \beta_2(1-\eta_{I2})p\Big) + S_m(1-\eta_S)\Big(\beta_1(1-\eta_{I1})(\gamma_2+\mu) + \beta_2(1-\eta_{I2})p\Big)}{(\gamma_1+p)(\gamma_2+\mu)}$$

B Parameter Values Derivation

- d_2: Wu et al.[1] find that approximately 5% of COVID-19 cases are critical and 15% are severe. Furthermore, the average duration of a severe infection is estimated to be 6 days[2,3] and the average duration of a critical infection is 8 days.[4] The duration of a confirmed infection d_2 is the weighted average of the time spent in the confirmed infected state for severe and critical individuals:

  $$d_2 = \frac{0.05 \times (6+8) + 0.15 \times 6}{0.05 + 0.15} = 8$$

  In our natural history sensitivity analysis (Supplement D), we use a lower bound of $d_2 = 6.25$ and upper bound of $d_2 = 17.25$. These are derived in a similar fashion as explained above, using an interval of $(5, 14)$ for the average duration of a severe infection and $(5, 13)$ for the average duration of a critical infection.[5]

- f_1, f_2: Our model assumes that confirmed infected cases are symptomatic cases because they warrant testing, whereas mild cases are unlikely to be diagnosed. Therefore, to determine f_1 and f_2, we use the percentage of the population that is mild, severe, and critical found in Wu et al.,[1] instead of the confirmed and unconfirmed percentages. Specifically, f_2 is the sum of the percentage of severe and critical rounded to 20% and $f_1 = 1 - f_2 = 0.8$. We believe these numbers more accurately represent the progression of the disease rather than being dependent on testing capacity.

  In our natural history sensitivity analysis (Supplement D), we derive an upper bound of the fraction of infections that remain unconfirmed $f_1 = 0.94$ using Anand et al.[6] In particular, the study estimates the SARS-CoV-2 seroprevalence in New York state to be 33.6% in their sample population in July 2020. We then computed the lower bound for the fraction of infection that are confirmed f_2 as the total confirmed cases in New York state in mid-July divided by the New York state population multiplied by the seroprevalence estimate. That is, $f_2 = 409476/(19.45 \times 10^6 \times 0.336) \approx 0.06$ and therefore $f_1 = 1 - f_2 \approx 0.94$.

- ξ: We estimate the fatality ratio for confirmed cases (and therefore the rate at which individuals with confirmed infection die, or recover and become immune) based on data from New York state. We calculate this ratio as the total number of estimated deaths divided by the number of estimated infections as of July 15, 2020.[7]

C Model Parameter Calculations

The rate at which an individual leaves the unconfirmed infected compartment is $\frac{1}{d_1}$. Given that only a fraction f_1 of individuals remain unconfirmed and recover, the transition rate from unconfirmed infected to recovered (γ_1) is simply the product of f_1 and $\frac{1}{d_1}$. Additionally, a fraction $1 - f_1$ of individuals becomes confirmed infected, and thus the transition rate from unconfirmed infected to confirmed infected (p) is equal to $\frac{1-f_1}{d_1}$. We similarly compute the transition rates γ_2 and μ. Using this reasoning, we obtain:

$$\gamma_1 = \frac{f_1}{d_1}$$
$$p = \frac{1 - f_1}{d_1}$$
$$\mu = \frac{1}{d_2} \times \frac{\xi}{f_2}$$
$$\gamma_2 = \frac{1}{d_2} - \mu$$

D Supplemental Analyses

D.1 Threshold Analysis of R_e

D.1.1 Initial Outbreak

Figure D.1 shows two-way sensitivity analyses on R_e as a function of reduction in susceptibility and infectivity. The figure assumes that mask effectiveness in reducing transmission is the same for all infected individuals ($\eta_{I1} = \eta_{I2} = \eta_I$). Each row shows three different times of intervention (mask wearing begins 0, 20, or 50 days after the onset of the epidemic), and the columns show two coverage levels (80% and 100%). The red line is the contour line at which $R_e = 1$ for each pair (η_S, η_I). The white diagonal line represents the points at which $\eta_I = \eta_S$. Only the lower right triangle is relevant since the reduction in infectivity is greater than the reduction in susceptibility;[8,9] however, for completeness, we show the contour plot for the entire range of η_I and η_S.

We find that only 100% coverage of masks with high effectiveness can reduce R_e below 1. This trend holds across the natural history sensitivity analyses (Supplemental Table D.2). For both 80% and 100% coverage, the threshold at which $R_e = 1$ is similar when the time of intervention is between 0 and 20 days. This occurs because there are few new daily cases and the growth of the epidemic is not yet exponential. However, if the time of intervention is 50 days after the start of the epidemic, then the epidemic is closer to the peak and the proportion of the population that remains susceptible is smaller, so R_e is smaller. Hence, the longer people wait to wear masks, the lower the needed effectiveness to stop the outbreak, but at the cost of more infections in the beginning of the epidemic.

If mask coverage is only 80%, then masks with higher effectiveness are necessary to decrease R_e below 1 compared to 100% coverage. Additionally, the impact of η_I and η_S on R_e is asymmetric: to maintain $R_e = 1$, if η_I decreases slightly, η_S must increase more than η_I if η_S instead decreases slightly. We can explain this asymmetry by looking at the expression for R_e. Since we assume for this analysis that $\eta_{I1} = \eta_{I2} = \eta_I$, we have:

$$R_e = \frac{S\Big(\beta_1(\gamma_2+\mu) + \beta_2(1-\eta_I)p\Big) + S_m(1-\eta_S)(1-\eta_I)\Big(\beta_1(\gamma_2+\mu)+\beta_2 p\Big)}{(\gamma_1+p)(\gamma_2+\mu)}$$

The reduction in infectivity, η_I, modifies both the first and second terms of the numerator, whereas the reduction in susceptibility, η_S, only modifies the second term. Intuitively, because we assume that individuals with confirmed cases always wear masks, it is more advantageous to prioritize masks with higher reduced infectivity (η_I) than masks with higher reduced susceptibility (η_S). In other words, prioritizing masks that reduce the risk of an infected individual from spreading the infection rather than the risk of a susceptible individual from getting infected yields the greatest benefit.

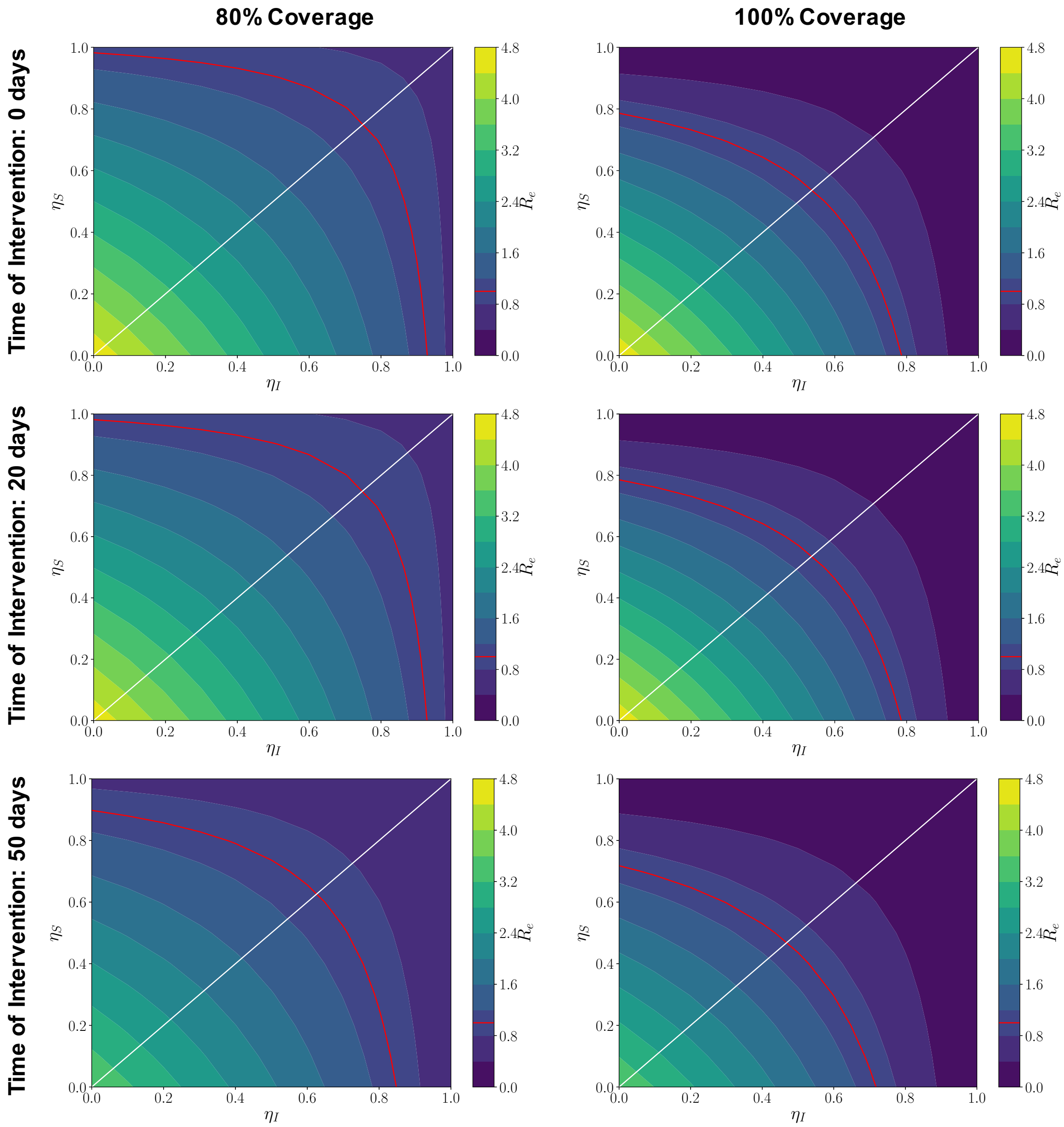
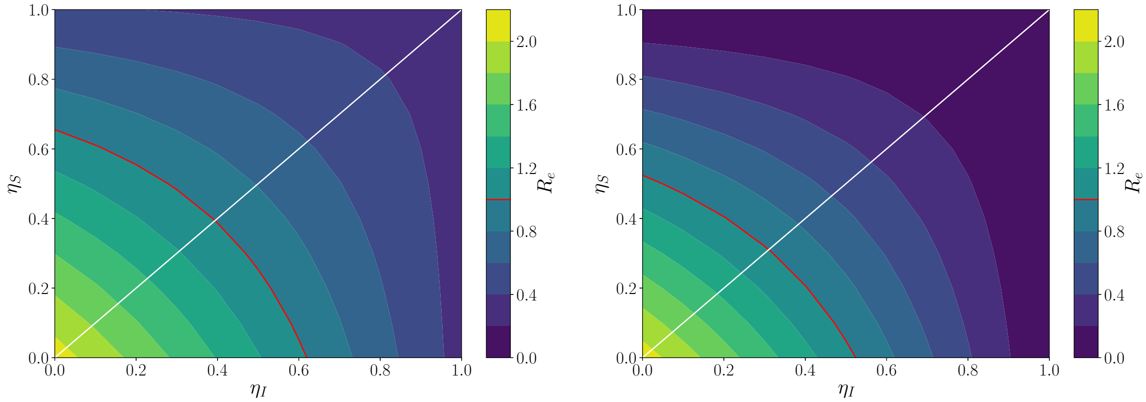


Figure D.1: Initial outbreak. Sensitivity analysis for R_e varying η_S and η_I with a red contour line at $R_e = 1$ and a white line at which $\eta_S = \eta_I$, for 80% and 100% coverage.

D.1.2 Resurgence

Figure D.2 shows a threshold analysis on R_e for two coverage levels (80% and 100%) with immediate intervention. Unlike in the initial outbreak, masks with high ($\eta_S = 0.6$, $\eta_I = 0.7$) and medium ($\eta_S = 0.4$, $\eta_I = 0.5$) effectiveness at 80% coverage can decrease R_e below 1. Because the starting R_e at the time of resurgence is lower, the reductions in susceptibility and infectivity (η_S, η_I) needed to reduce R_e below 1 are smaller than during the initial outbreak.

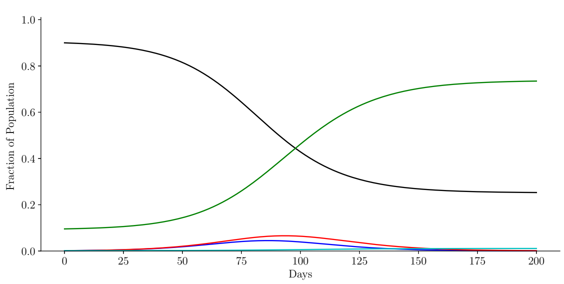
Figure D.3 compares the trajectory of the epidemic under three scenarios: only



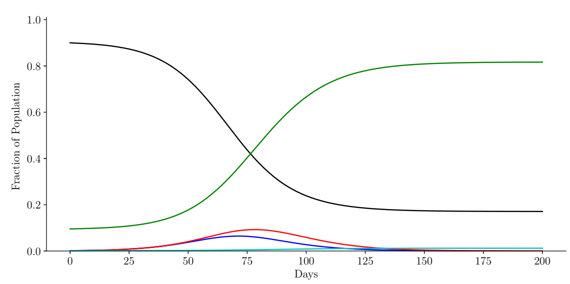
(a) Coverage level: 80% (b) Coverage level: 100%

Figure D.2: Resurgence. Sensitivity analysis for R_e varying η_S and η_I with a red contour line at $R_e = 1$ and a white line at which $\eta_S = \eta_I$ assuming two coverage levels (80% and 100%) and a 50% reduction in the transmission rates (β_1, β_2) due to social distancing.

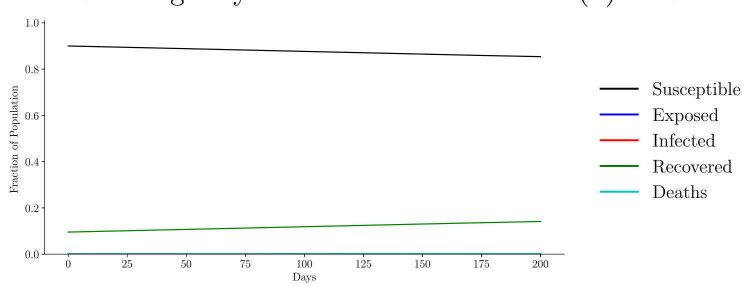
social distancing (Figure D.3a); only masks with low effectiveness, immediate intervention, and 100% coverage (Figure D.3b); both intervention strategies combined (Figure D.3c). The percentage of the population that becomes infected during the resurgence within 500 days is 65%, 73%, and 8%, respectively, for the three scenarios, emphasizing the synergy between social distancing and masks.



(a) Social distancing only



(b) Masks only



(c) Both social distancing and masks

Figure D.3: Resurgence. Trajectory of the epidemic assuming only social distancing, only masks (low effectiveness, immediate intervention, 100% coverage), and both intervention strategies combined.

D.2 Sensitivity Analysis on Disease Natural History

Table D.1: Lower and upper bounds for one-way sensitivity analysis on disease natural history

Parameter	*Description*	*Lower Bound*	*Base Case*	*Upper Bound*	*Source*
d_2	Average duration of confirmed infection (days)	6.25	8	17.25	[5]
f_1	Fraction of infections that remain unconfirmed	0.67	0.8	0.94	[6,10]
w	Daily rate of progression from exposed to infected	1/9	1/5	1/4	[11,12]

Table D.2: Disease natural history sensitivity analysis: values of R_e assuming immediate intervention and masks with high and medium effectiveness

			Scenario						
			Base Case	$d_2 = 6.25$	$d_2 = 17.25$	$f_1 = 0.67$	$f_1 = 0.94$	$w = 1/9$	$w = 1/4$
High effectiveness mask	*Initial outbreak*	100% Coverage	0.56	0.55	0.74	0.67	0.49	0.58	0.56
		80% Coverage	1.23	1.23	1.48	1.40	1.15	1.29	1.23
	Resurgence	100% Coverage	0.25	0.25	0.33	0.30	0.22	0.26	0.25
		80% Coverage	0.55	0.55	0.67	0.63	0.52	0.58	0.55
	Intervention fatigue	100% Coverage	0.38	0.37	0.50	0.45	0.33	0.39	0.38
		80% Coverage	0.83	0.83	1.00	0.94	0.78	0.87	0.83
Medium effectiveness mask	*Initial outbreak*	100% Coverage	1.27	1.27	1.55	1.46	1.17	1.34	1.27
		80% Coverage	1.80	1.81	2.14	2.02	1.70	1.88	1.80
	Resurgence	100% Coverage	0.57	0.57	0.70	0.66	0.53	0.60	0.57
		80% Coverage	0.81	0.81	0.96	0.91	0.77	0.85	0.81
	Intervention fatigue	100% Coverage	0.86	0.86	1.05	0.98	0.79	0.90	0.86
		80% Coverage	1.22	1.22	1.44	1.37	1.15	1.27	1.22

Table D.3: Disease natural history sensitivity analysis: fraction of infections averted (%) during the initial outbreak

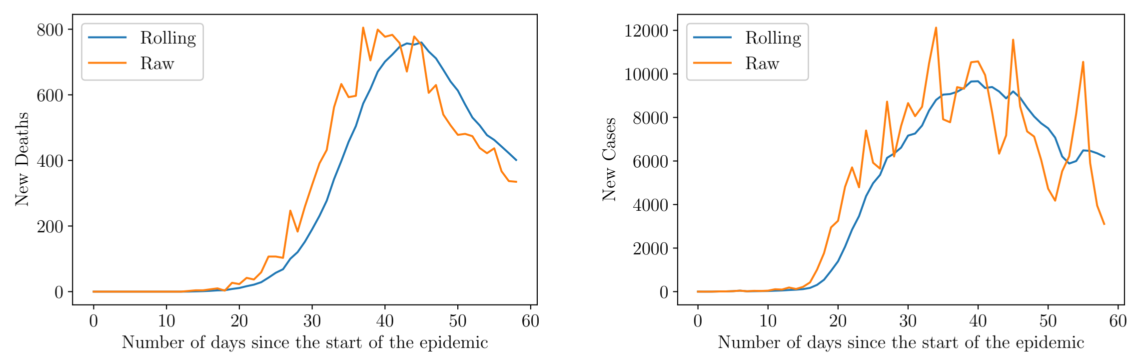
		Scenario						
		Base Case	$d_2 = 6.25$	$d_2 = 17.25$	$f_1 = 0.67$	$f_1 = 0.94$	$w = 1/9$	$w = 1/4$
Varying mask effectiveness:	Low	12.4	12.2	7.6	8.9	14.4	11.0	12.4
immediate intervention, 100%	Medium	72.8	73.7	38.9	45.5	90.2	66.4	71.6
coverage, $T = 365$	High	100	100	100	100	100	100	100
Varying mask effectiveness:	Low	3.9	3.8	2.4	2.8	4.4	3.4	3.9
immediate intervention, 50%	Medium	11.6	11.5	7.9	8.9	13.0	10.5	11.6
coverage, $T = 365$	High	25.7	25.5	19.5	21.3	27.9	23.9	25.7
Varying mask effectiveness:	Low	8.8	8.8	4.6	6.5	9.4	8.8	8.1
intervention after 50 days, 100%	Medium	28.1	28.4	14.8	22.5	28.1	32.2	24.7
coverage, $T = 365$	High	49.5	50.2	29.6	44.8	46.4	60.2	43.1
Varying mask effectiveness:	Low	3.1	3.0	1.7	2.3	3.3	3.0	3.0
intervention after 50 days, 50%	Medium	8.3	8.3	4.7	6.4	8.7	8.5	7.7
coverage, $T = 365$	High	16.7	16.7	9.9	13.9	16.7	18.1	15.2
Varying coverage level:	60 to 70%	41.6	41.9	41.2	41.3	42.5	42.5	41.4
immediate intervention, high	70 to 80%	67.7	68.7	49.4	51.4	67.5	66.9	67.4
effectiveness, $T = 365$								
Varying intervention timing:	0 days	100	100	100	100	100	100	100
high effectiveness, 100%	30 days	96.3	96.8	91.4	96.0	96.1	95.0	96.6
coverage, $T = 365$	50 days	49.5	50.2	29.6	44.8	46.4	60.2	43.1
Varying intervention timing: low	0 days	59.0	58.6	35.3	46.5	60.2	67.0	53.8
effectiveness, 100% coverage,	30 days	20.0	19.3	11.8	14.2	21.1	27.6	17.5
$T = 100$	50 days	9.8	9.6	5.3	7.2	10.1	13.0	8.6

D.3 Effectiveness of Social Distancing Measures During a Resurgence

Table D.4: Sensitivity analysis on effectiveness of social distancing measures during a resurgence: values of R_e assuming immediate intervention

		Reduction in Transmission			
		25%	50%	75%	90%
Masks with high effectiveness	100% Coverage	0.38	0.25	0.13	0.05
	80% Coverage	0.83	0.55	0.28	0.11
Masks with medium effectiveness	100% Coverage	0.86	0.57	0.29	0.11
	80% Coverage	1.22	0.81	0.41	0.16
Masks with low effectiveness	100% Coverage	1.53	1.02	0.51	0.20
	80% Coverage	1.76	1.17	0.59	0.23

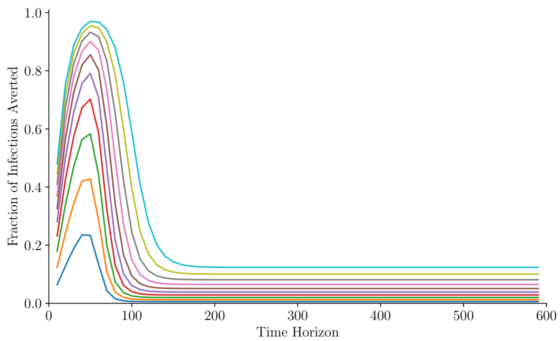
E Supplemental Figures



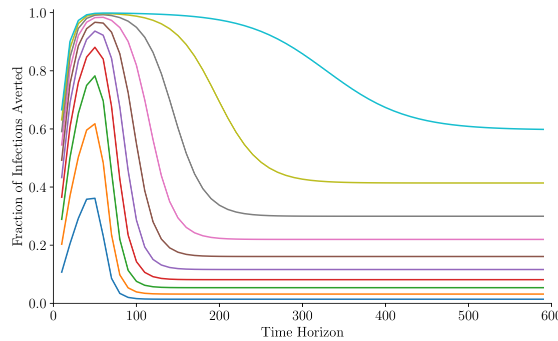
(a) New daily confirmed deaths

(b) New daily confirmed cases

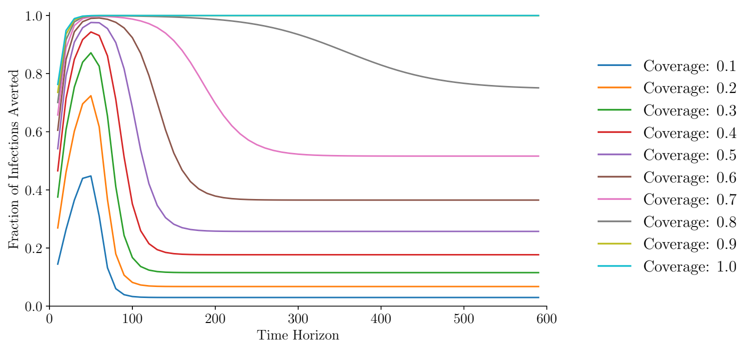
Figure E.1: New daily confirmed COVID-19 deaths and cases in New York state beginning from March 1: raw numbers and 7-day rolling average



(a) Scenario 1



(b) Scenario 2



(c) Scenario 3

Figure E.2: Initial outbreak. Fraction of infections averted for different time horizons (ranging from 10 to 600 days in 10 day increments) as a function of coverage level, assuming immediate intervention. Scenario 1: $\eta_S = 0.2$, $\eta_{I1} = 0.3$; Scenario 2: $\eta_S = 0.4$, $\eta_{I1} = 0.5$; Scenario 3: $\eta_S = 0.6$, $\eta_{I1} = 0.7$, where η_S = reduction in susceptibility for an uninfected person wearing a mask, η_{I1} = reduction in infectivity for an unconfirmed infected person wearing a mask.

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