

Supplementary Materials for

The use of differential privacy for census data and its impact on redistricting: The case of the 2020 U.S. Census

Christopher T. Kenny, Shiro Kuriwaki, Cory McCartan, Evan T. R. Rosenman, Tyler Simko, Kosuke Imai*

*Corresponding author. Email: imai@harvard.edu

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The supplementary materials contain the same type of analysis as the main text done with different versions of the DAS. See the correspondence table below.

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Simulations of MMDs in East Ramapo	-	Table S2.1	Table S4.2
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S1 Prediction of Individual Voters’ Race and Ethnicity: Accuracy Tables

Here, we give the full set of results for ethnicity classification on the North Carolina dataset. We assign each individual to the ethnic group with the highest posterior probability, using the Bayesian method described in the main text. Full results are given in Tables [S1.1](#), [S1.2](#), and [S1.3](#).

Ethnicity	Data	Census 2010	DAS-4.5	DAS-12.2
	Overall Error Rate	15.1%	15.6%	15.1%
White	False negative	7.5%	7.9%	7.6%
White	False positive	11.4%	11.7%	11.3%
Black	False negative	29.1%	29.7%	29.1%
Black	False positive	23.6%	24.2%	23.4%
Hispanic	False negative	30.3%	30.7%	27.7%
Hispanic	False positive	29.1%	29.9%	29.0%
Asian	False negative	36.2%	38.2%	34.9%
Asian	False positive	35.4%	35.1%	34.3%
Other	False negative	75.1%	76.0%	75.0%
Other	False positive	35.1%	50.1%	43.9%

Table S1.1: Overall classification error rate as well as false positive (Type I error) and false negative (Type II error) rates for White, Black, Hispanic, Asian, and Other voters using prediction based on geography and **last names only**. We classify each registered voter to the racial / ethnic category with the greatest predicted probability. The columns refer to the different datasets we use to set geographic ethnicity priors at the Census Block level.

Ethnicity	Data	Census 2010	DAS-4.5	DAS-12.2
	Overall Error Rate	12.1%	12.5%	12.0%
White	False negative	5.4%	5.8%	5.5%
White	False positive	9.3%	9.4%	9.1%
Black	False negative	23.4%	23.9%	23.3%
Black	False positive	17.3%	17.8%	17.1%
Hispanic	False negative	28.2%	28.1%	25.3%
Hispanic	False positive	25.1%	26.1%	24.9%
Asian	False negative	29.2%	31.2%	28.1%
Asian	False positive	30.6%	30.5%	29.5%
Other	False negative	69.8%	71.1%	69.5%
Other	False positive	36.6%	48.0%	43.5%

Table S1.2: Error rates using prediction based on geography as well as last and first names, for each geographic prior.

Ethnicity	Data	Census 2010	DAS-4.5	DAS-12.2
	Overall Error Rate	10.0%	10.3%	9.9%
White	False negative	3.9%	4.2%	4.0%
White	False positive	8.0%	8.1%	7.8%
Black	False negative	20.4%	20.9%	20.2%
Black	False positive	12.6%	12.9%	12.3%
Hispanic	False negative	25.2%	24.9%	22.1%
Hispanic	False positive	21.5%	22.6%	21.5%
Asian	False negative	22.8%	24.7%	21.8%
Asian	False positive	26.3%	26.4%	25.3%
Other	False negative	64.2%	66.0%	63.9%
Other	False positive	35.6%	45.1%	41.3%

Table S1.3: Error rates using prediction based on geography as well as last, first, and middle names, for each geographic prior.

S2 Majority Minority Districts from Block Level Simulations

The redistricting simulation findings in the East Ramapo Central School District suggest that, under the DAS-12.2 data, we estimate fewer majority minority districts than would have been estimated under the Census 2010 data.

Table S2.1: *East Ramapo MMDs under Census 2010 and DAS-12.2 data. The noise introduced in the DAS-12.2 leads us to undercount the number of majority minority districts in many plans, but never to overcount them.*

Census 2010	Number of MMDs from DAS-12.2				Plans
	0	1	2	3	
0	100%	0	0	0	50
1	10	90	0	0	3103
2	2	56	42	0	6774
3	0	77	22	0	73

Note: Percentages add to 100% by row.

S3 Undercounting Bias along Partisanship and with the DAS-4.5

In Figures 1 and 2 of the main text, we detail bias in total population counts along non-white and precinct diversity measures. In Figure S3.1 we demonstrate that this persists along partisan dimensions.

We next replicate three figures from the main paper using the DAS-4.5 data in place of the DAS-12.2 data. This comparison, along with our subsequent results for the DAS-19.61, demonstrates that the bias occurs across levels of the privacy budget. Figure S3.3 follows Figure 2, Figure S3.4 follows Figure S3.1, and Figure S3.2 follows Figure 1.

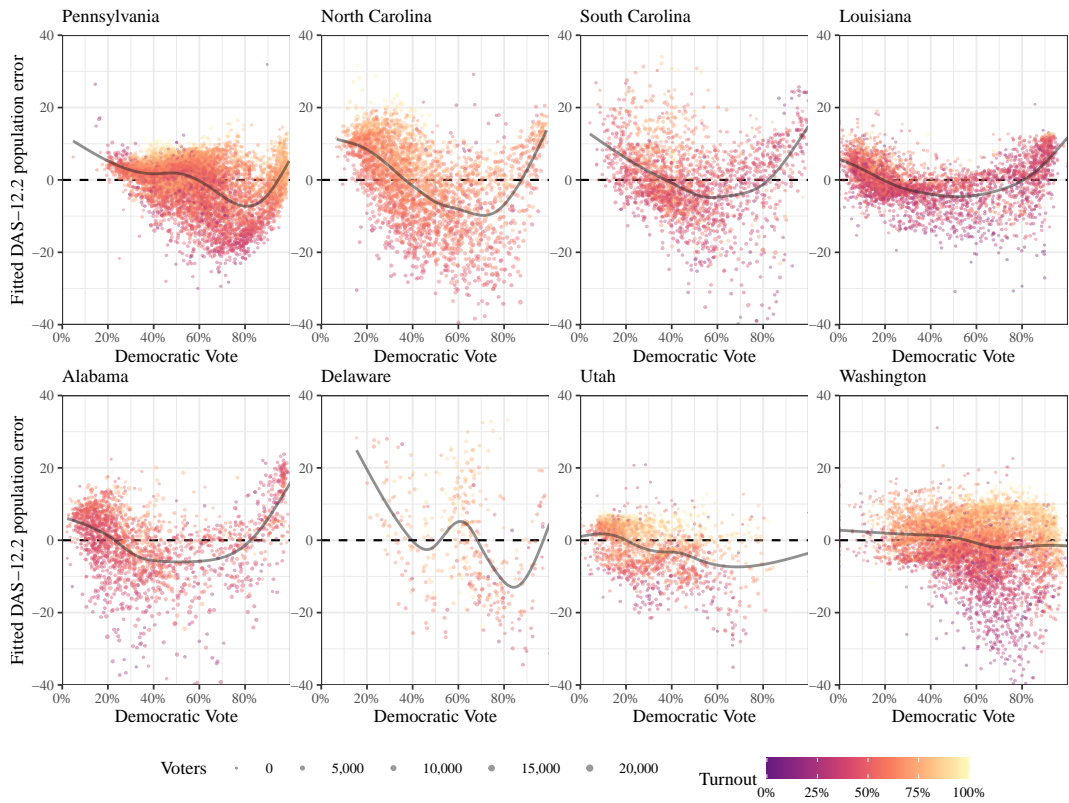


Figure S3.1: Model-smoothed error in precinct populations by Democratic two-party vote share, with color indicating turnout. A GAM smooth is overlaid to show the mean error by Democratic share.

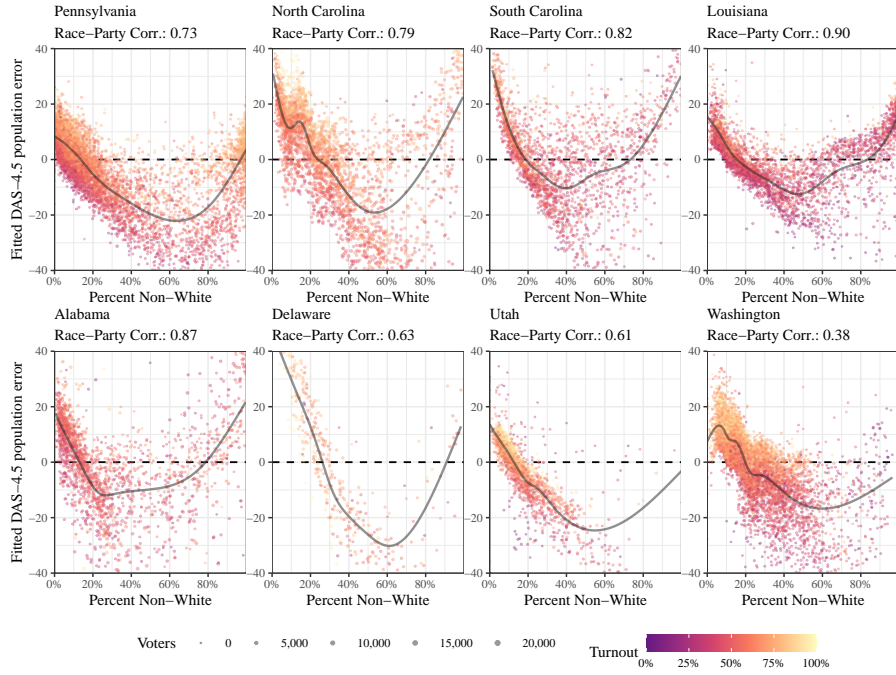


Figure S3.2: DAS-4.5 version of Figure 1.

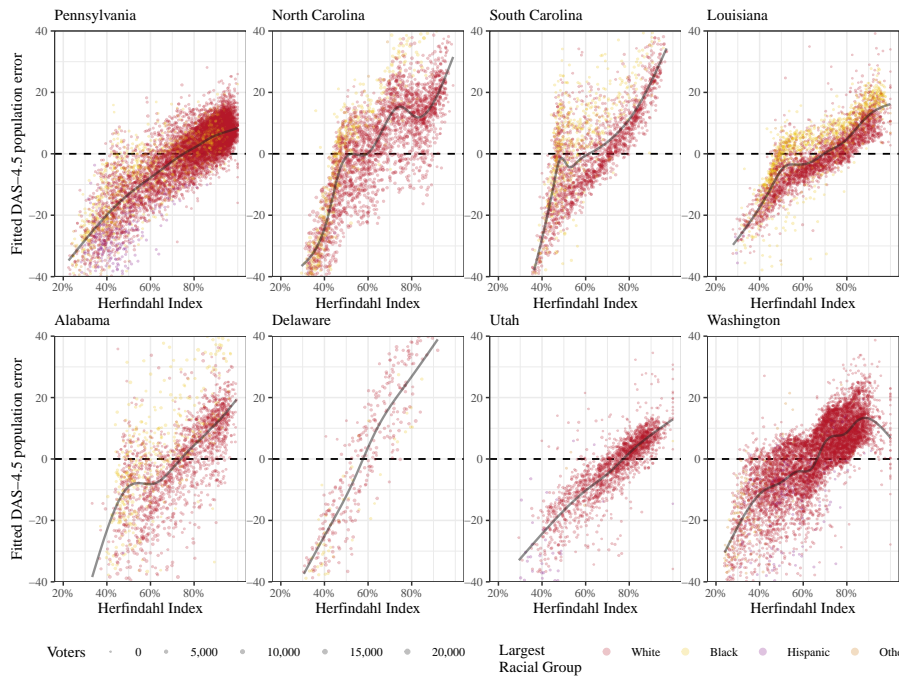


Figure S3.3: DAS-4.5 version of Figure 2.

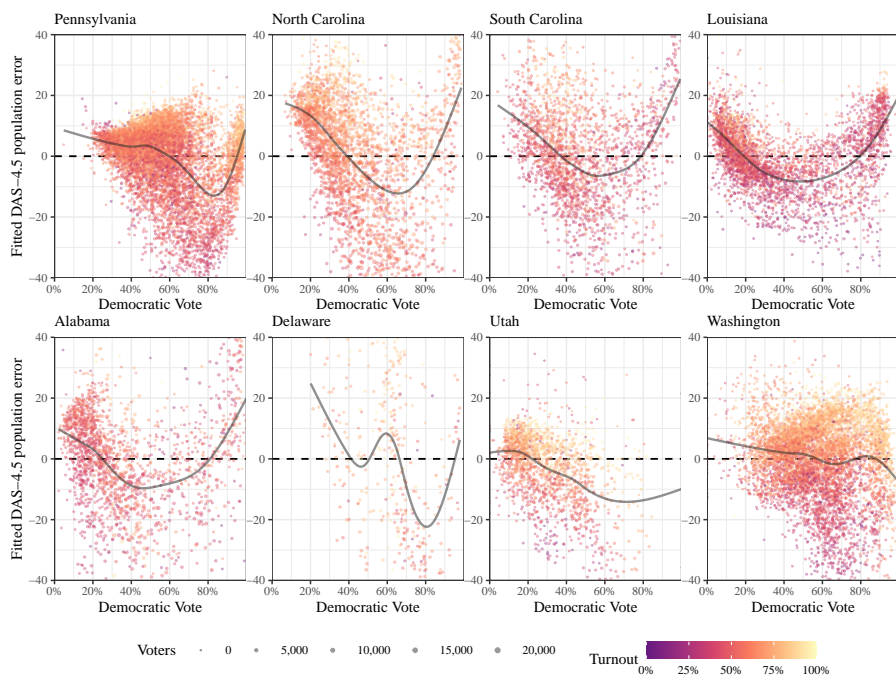


Figure S3.4: DAS-4.5 version of Figure S3.1.

S4 Empirical Results with the DAS-19.61

Here we provide re-analyses of the main figures in the text with the DAS-19.61 data released August 12, 2021.

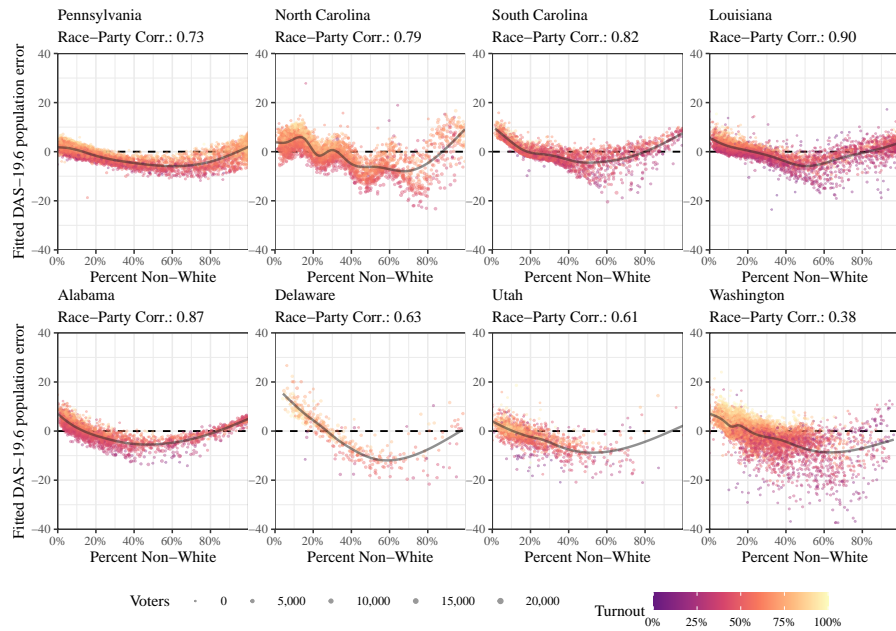


Figure S4.1: DAS-19.61 version of Figure 1.

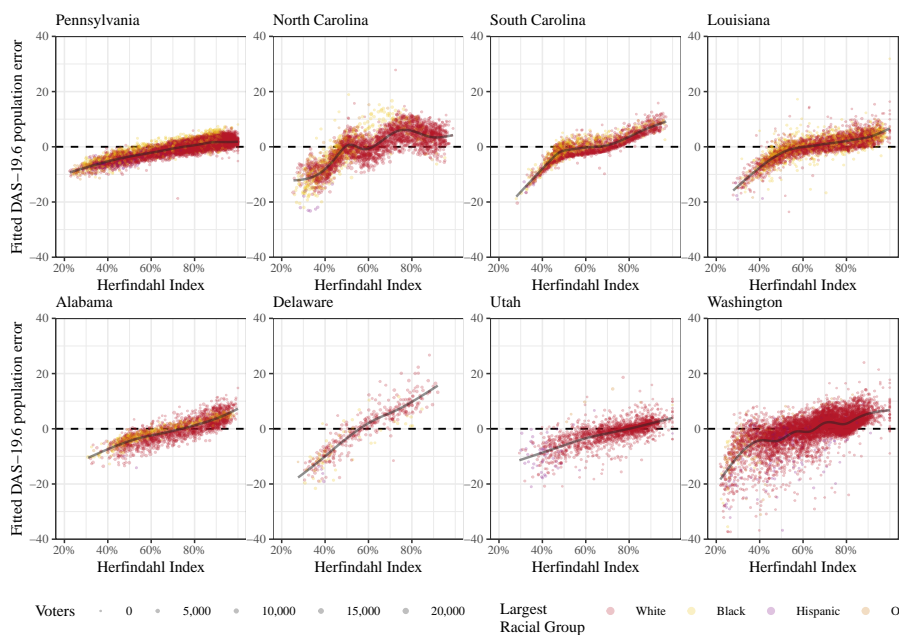


Figure S4.2: DAS-19.61 version of Figure 2.

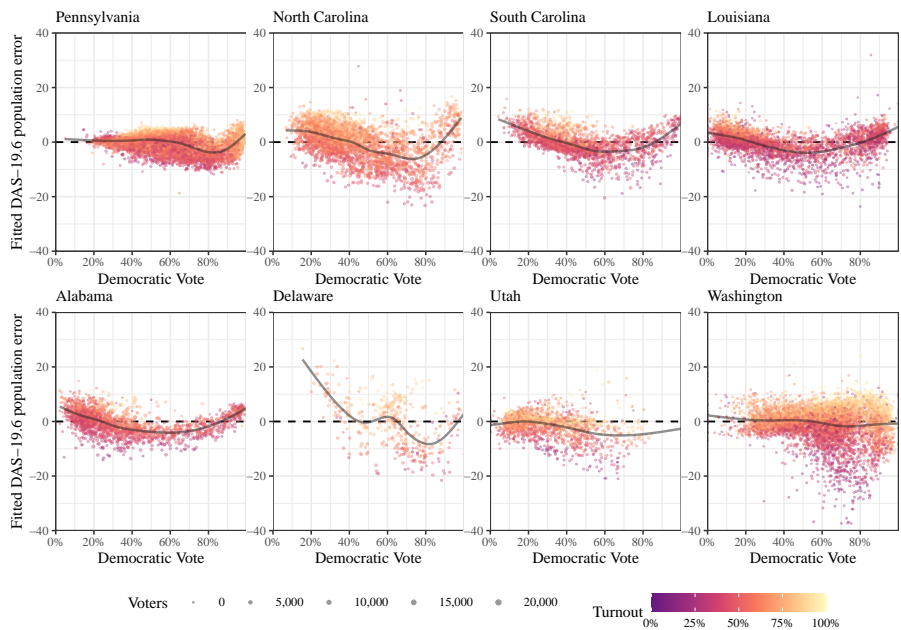


Figure S4.3: DAS-19.61 version of Figure S3.1.

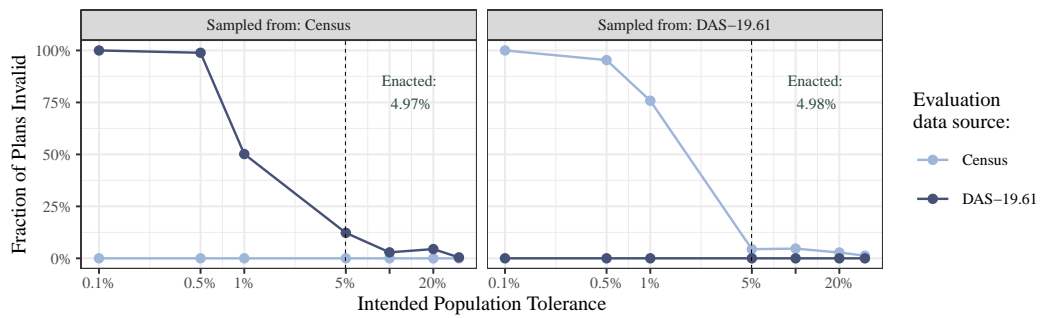


Figure S4.4: DAS-19.61 version of Figure 4. Fraction of Louisiana State Senate plans simulated under one data source with a population parity constraint which are invalid when measured under another. The horizontal axis shows the tolerance constraint for the original simulation on the log10 scale. The vertical axis shows the percent of plans that exceed the intended tolerance according to the evaluation data. The dashed line shows the maximum deviation from parity of the enacted 2010 map.

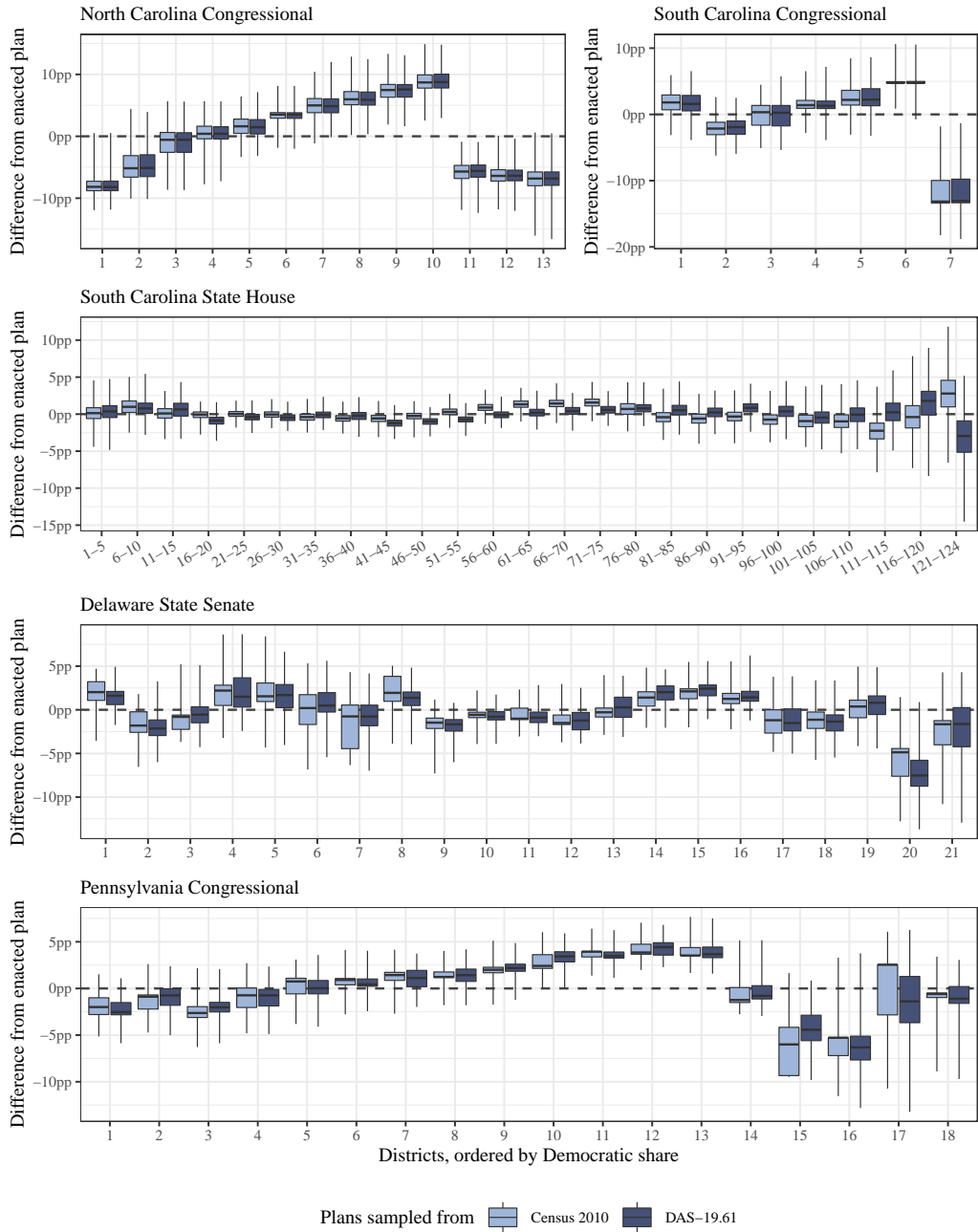


Figure S4.5: DAS-19.61 version of Figure 5.

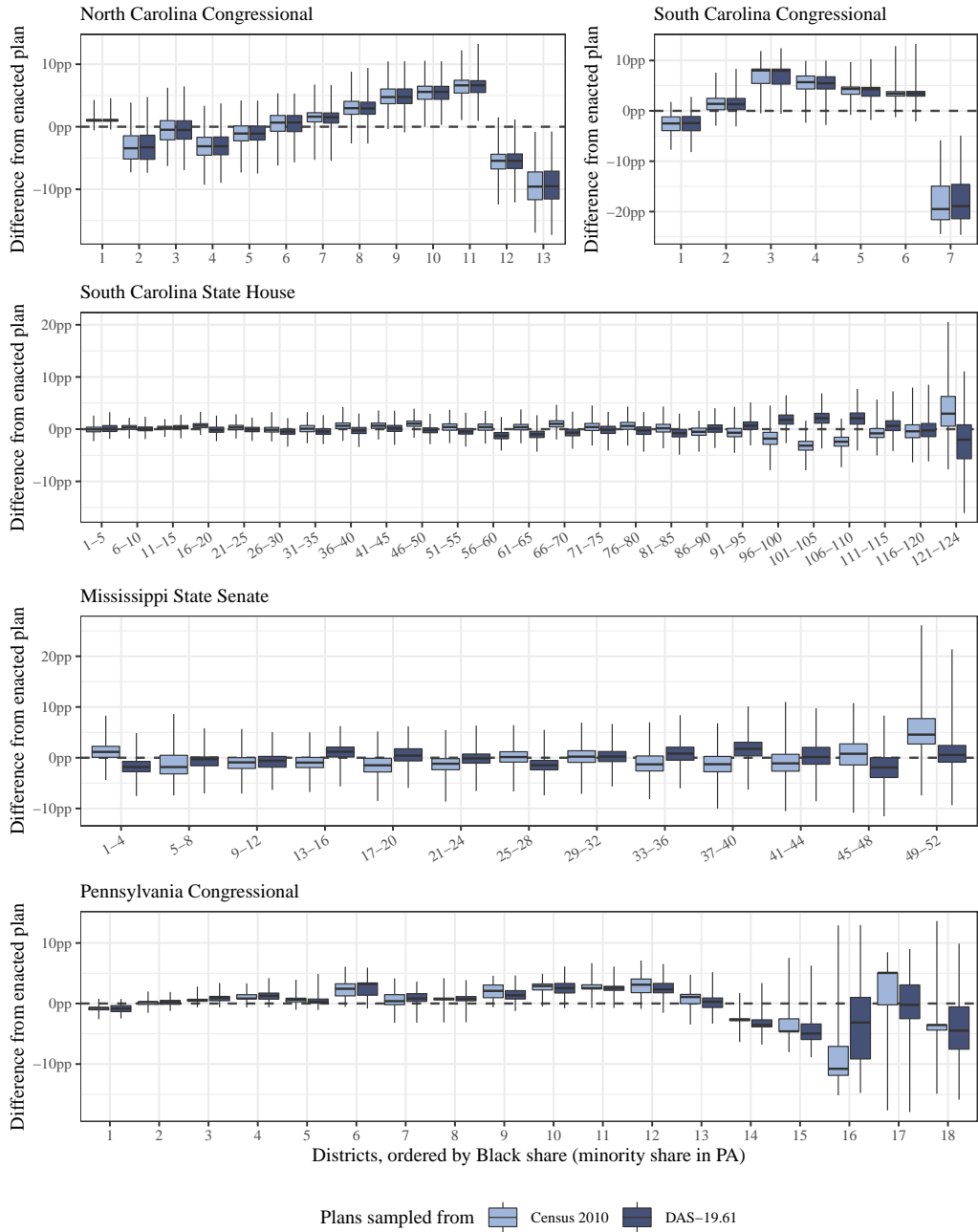


Figure S4.6: DAS-19.61 version of Figure 6.

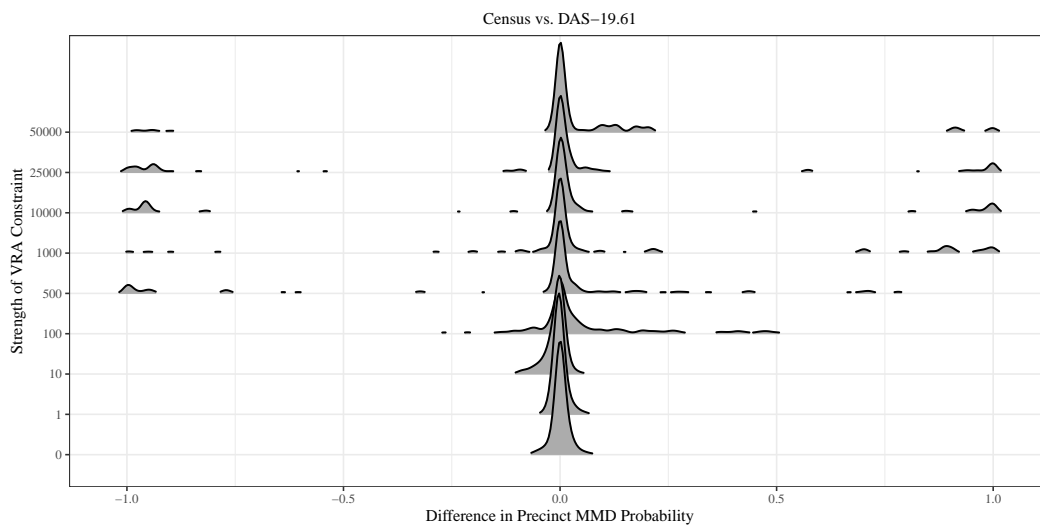


Figure S4.7: DAS-19.61 version of Figure 7. The calculated probability of being assigned to a majority-minority district can be much higher or lower for individual precincts, and these differences grow as a constraint encouraging the formation of MMDs is strengthened.

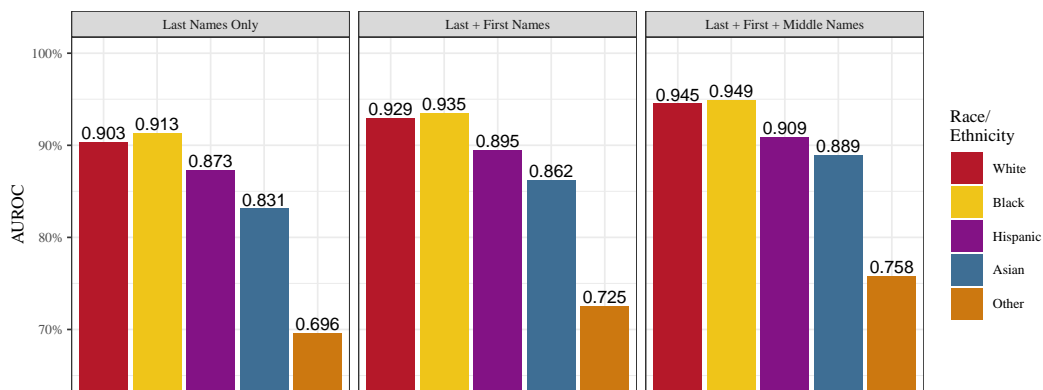


Figure S4.8: DAS-19.61 version of Figure 8. Area under the Receiver Operating Characteristic Curve (AUROC) percentage values for the prediction of each individual voter's race and ethnicity using the North Carolina voter file, with geographic priors given by the DAS-19.61 dataset.

Ethnicity	Data	Last	Last, First	Last, First, Middle
Overall Error Rate		15.5%	12.4%	10.2%
White	False negative	7.9%	5.8%	4.2%
White	False positive	11.5%	9.3%	7.9%
Black	False negative	29.5%	23.7%	20.7%
Black	False positive	24.2%	17.7%	12.8%
Hispanic	False negative	30.2%	27.4%	24.1%
Hispanic	False positive	29.1%	25.1%	21.8%
Asian	False negative	37.1%	30.2%	23.8%
Asian	False positive	34.5%	29.7%	25.7%
Other	False negative	74.7%	69.5%	64.0%
Other	False positive	48.8%	47.6%	45.0%

Table S4.1: DAS-19.61 version of Tables S1.1, S1.2, S1.3. Overall classification error rate as well as false positive (Type I error) and false negative (Type II error) rates for White, Black, Hispanic, Asian, and Other voters using prediction based on geographic priors derived from the DAS-19.61 data.

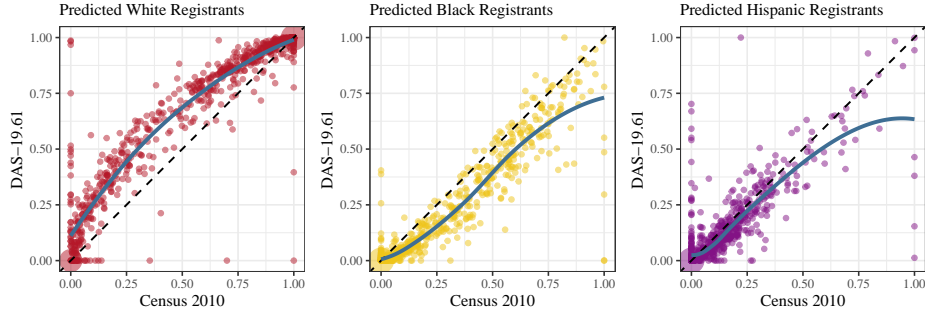


Figure S4.9: DAS-19.61 version of Figure 9. Imputed Racial Registrants by Census Blocks. The x-axis represents the percent of a group, as measured by the most likely race from racial imputation using the Census 2010 data. The y-axis represents the corresponding imputation using the DAS-19.61 data.

Table S4.2: DAS-19.61 version of Table S2.1. East Ramapo MMDs under Census 2010 and DAS-19.61 data. The noise introduced in the DAS-19.61 leads us to undercount the number of majority minority districts in many plans, but never to overcount them.

Census 2010	Number of MMDs from DAS-19.61				Plans
	0	1	2	3	
0	100%	0	0	0	4
1	26	74	0	0	3,435
2	66	14	20	0	6,462
3	0	42	58	0	99

Note: Percentages add to 100% by row.

Table S4.3: *Difference in the population of existing Congressional Districts as computed by three DAS populations, relative to the enacted Congressional District Map. For each table and each state we show the minimum, median, maximum, and standard deviation of the difference in population. All numbers are rounded to whole numbers.*

DAS-4.5

State	Min	Median	Mean	Max	SD	CDs
Alabama	-597	-2	0	865	444	7
Delaware	0	0	0	0		1
Louisiana	-1221	35	0	669	660	6
Mississippi	-262	12	0	237	210	4
North Carolina	-1301	-9	0	1659	657	13
Pennsylvania	-3996	0	0	3805	1453	18
South Carolina	-467	-173	0	1318	633	7
Utah	-1303	166	0	971	1065	4
Washington	-3128	54	0	2784	1617	10

DAS-12.2

State	Min	Median	Mean	Max	SD	CDs
Alabama	-405	-2	0	483	270	7
Delaware	0	0	0	0		1
Louisiana	-471	30	0	314	259	6
Mississippi	-210	27	0	156	159	4
North Carolina	-601	-18	0	680	277	13
Pennsylvania	-2153	1	0	2164	765	18
South Carolina	-287	-6	0	485	256	7
Utah	-328	-18	0	363	291	4
Washington	-2099	98	0	1413	1075	10

DAS-19.61

State	Min	Median	Mean	Max	SD	CDs
Alabama	-163	17	0	161	102	7
Delaware	0	0	0	0		1
Louisiana	-196	56	0	96	115	6
Mississippi	-150	36	0	79	104	4
North Carolina	-216	10	0	319	133	13
Pennsylvania	-122	-12	0	121	61	18
South Carolina	-100	8	0	89	72	7
Utah	-42	6	0	29	30	4
Washington	-143	13	0	200	92	10