

Martingales and the characteristic functions of absorption time on bipartite graphs

Travis Monk and André van Schaik

Article citation details

R. Soc. open sci. **8**: 210657.

<http://dx.doi.org/10.1098/rsos.210657>

Review timeline

Original submission: 15 April 2021
Revised submission: 13 September 2021
Final acceptance: 15 September 2021

Note: Reports are unedited and appear as submitted by the referee. The review history appears in chronological order.

Note: This manuscript was transferred from another Royal Society journal without peer review.

Review History

RSOS-210657.R0 (Original submission)

Review form: Reviewer 1

Is the manuscript scientifically sound in its present form?

Yes

Are the interpretations and conclusions justified by the results?

Yes

Is the language acceptable?

Yes

Do you have any ethical concerns with this paper?

No

Have you any concerns about statistical analyses in this paper?

No

Recommendation?

Accept as is

Comments to the Author(s)

This is a well written manuscript that develops a theory of conditional characteristic functions and applies it to get formulas for the number of times that the mutant population size changes before going extinct or fixing in a bipartite graphs.

The paper can be accepted as is but the authors may consider adding some comparisons relating their results to the mean fixation or absorption times (such as in the form of figures for one or more bipartite graphs with mutant fitness on x axis and different times on the y axis, perhaps after an appropriate transformation)

Review form: Reviewer 2

Is the manuscript scientifically sound in its present form?

Yes

Are the interpretations and conclusions justified by the results?

Yes

Is the language acceptable?

Yes

Do you have any ethical concerns with this paper?

No

Have you any concerns about statistical analyses in this paper?

No

Recommendation?

Accept with minor revision (please list in comments)

Comments to the Author(s)

Please see attached report file (Appendix A).

Decision letter (RSOS-210657.R0)

We hope you are keeping well at this difficult and unusual time. We continue to value your support of the journal in these challenging circumstances. If Royal Society Open Science can assist you at all, please don't hesitate to let us know at the email address below.

Dear Dr Monk

On behalf of the Editors, we are pleased to inform you that your Manuscript RSOS-210657 "Martingales and the characteristic functions of absorption time on bipartite graphs" has been accepted for publication in Royal Society Open Science subject to minor revision in accordance

with the referees' reports. Please find the referees' comments along with any feedback from the Editors below my signature.

We invite you to respond to the comments and revise your manuscript. Below the referees' and Editors' comments (where applicable) we provide additional requirements. Final acceptance of your manuscript is dependent on these requirements being met. We provide guidance below to help you prepare your revision.

Please submit your revised manuscript and required files (see below) no later than 7 days from today's (ie 07-Sep-2021) date. Note: the ScholarOne system will 'lock' if submission of the revision is attempted 7 or more days after the deadline. If you do not think you will be able to meet this deadline please contact the editorial office immediately.

Please note article processing charges apply to papers accepted for publication in Royal Society Open Science (<https://royalsocietypublishing.org/rsos/charges>). Charges will also apply to papers transferred to the journal from other Royal Society Publishing journals, as well as papers submitted as part of our collaboration with the Royal Society of Chemistry (<https://royalsocietypublishing.org/rsos/chemistry>). Fee waivers are available but must be requested when you submit your revision (<https://royalsocietypublishing.org/rsos/waivers>).

Thank you for submitting your manuscript to Royal Society Open Science and we look forward to receiving your revision. If you have any questions at all, please do not hesitate to get in touch.

Kind regards,
Royal Society Open Science Editorial Office
Royal Society Open Science
openscience@royalsociety.org

on behalf of Professor Andreas Kyprianou (Associate Editor) and Mark Chaplain (Subject Editor)
openscience@royalsociety.org

Associate Editor Comments to Author (Professor Andreas Kyprianou):

Associate Editor: 1

Comments to the Author:

The two referees were both positive about the contribution. There are some minor things that need thinking about that both referee's suggest. I don't see a need to go through another round of refereeing, so I would simply ask that you take your time in making the corrections before submitting the final files for publication.

Reviewer comments to Author:

Reviewer: 1

Comments to the Author(s)

This is a well written manuscript that develops a theory of conditional characteristic functions and applies it to get formulas for the number of times that the mutant population size changes before going extinct or fixing in a bipartite graphs.

The paper can be accepted as is but the authors may consider adding some comparisons relating their results to the mean fixation or absorption times (such as in the form of figures for one or more bipartite graphs with mutant fitness on x axis and different times on the y axis, perhaps after an appropriate transformation)

Reviewer: 2

Comments to the Author(s)

Please see attached report file.

===PREPARING YOUR MANUSCRIPT===

Your revised paper should include the changes requested by the referees and Editors of your manuscript. You should provide two versions of this manuscript and both versions must be provided in an editable format:

one version identifying all the changes that have been made (for instance, in coloured highlight, in bold text, or tracked changes);

a 'clean' version of the new manuscript that incorporates the changes made, but does not highlight them. This version will be used for typesetting.

Please ensure that any equations included in the paper are editable text and not embedded images.

Please ensure that you include an acknowledgements' section before your reference list/bibliography. This should acknowledge anyone who assisted with your work, but does not qualify as an author per the guidelines at <https://royalsociety.org/journals/ethics-policies/openness/>.

While not essential, it will speed up the preparation of your manuscript proof if you format your references/bibliography in Vancouver style (please see <https://royalsociety.org/journals/authors/author-guidelines/#formatting>). You should include DOIs for as many of the references as possible.

If you have been asked to revise the written English in your submission as a condition of publication, you must do so, and you are expected to provide evidence that you have received language editing support. The journal would prefer that you use a professional language editing service and provide a certificate of editing, but a signed letter from a colleague who is a native speaker of English is acceptable. Note the journal has arranged a number of discounts for authors using professional language editing services (<https://royalsociety.org/journals/authors/benefits/language-editing/>).

===PREPARING YOUR REVISION IN SCHOLARONE===

To revise your manuscript, log into <https://mc.manuscriptcentral.com/rsos> and enter your Author Centre - this may be accessed by clicking on "Author" in the dark toolbar at the top of the page (just below the journal name). You will find your manuscript listed under "Manuscripts with Decisions". Under "Actions", click on "Create a Revision".

Attach your point-by-point response to referees and Editors at Step 1 'View and respond to decision letter'. This document should be uploaded in an editable file type (.doc or .docx are preferred). This is essential.

Please ensure that you include a summary of your paper at Step 2 'Type, Title, & Abstract'. This should be no more than 100 words to explain to a non-scientific audience the key findings of your research. This will be included in a weekly highlights email circulated by the Royal Society press office to national UK, international, and scientific news outlets to promote your work.

At Step 3 'File upload' you should include the following files:

-- Your revised manuscript in editable file format (.doc, .docx, or .tex preferred). You should upload two versions:

- 1) One version identifying all the changes that have been made (for instance, in coloured highlight, in bold text, or tracked changes);
- 2) A 'clean' version of the new manuscript that incorporates the changes made, but does not highlight them.

-- An individual file of each figure (EPS or print-quality PDF preferred [either format should be produced directly from original creation package], or original software format).

-- An editable file of each table (.doc, .docx, .xls, .xlsx, or .csv).

-- An editable file of all figure and table captions.

Note: you may upload the figure, table, and caption files in a single Zip folder.

-- Any electronic supplementary material (ESM).

-- If you are requesting a discretionary waiver for the article processing charge, the waiver form must be included at this step.

-- If you are providing image files for potential cover images, please upload these at this step, and inform the editorial office you have done so. You must hold the copyright to any image provided.

-- A copy of your point-by-point response to referees and Editors. This will expedite the preparation of your proof.

At Step 6 'Details & comments', you should review and respond to the queries on the electronic submission form. In particular, we would ask that you do the following:

-- Ensure that your data access statement meets the requirements at

<https://royalsociety.org/journals/authors/author-guidelines/#data>. You should ensure that you cite the dataset in your reference list. If you have deposited data etc in the Dryad repository, please only include the 'For publication' link at this stage. You should remove the 'For review' link.

-- If you are requesting an article processing charge waiver, you must select the relevant waiver option (if requesting a discretionary waiver, the form should have been uploaded at Step 3 'File upload' above).

-- If you have uploaded ESM files, please ensure you follow the guidance at

<https://royalsociety.org/journals/authors/author-guidelines/#supplementary-material> to include a suitable title and informative caption. An example of appropriate titling and captioning may be found at https://figshare.com/articles/Table_S2_from_Is_there_a_trade-off_between_peak_performance_and_performance_breadth_across_temperatures_for_aerobic_scope_in_teleost_fishes_/3843624.

At Step 7 'Review & submit', you must view the PDF proof of the manuscript before you will be able to submit the revision. Note: if any parts of the electronic submission form have not been completed, these will be noted by red message boxes.

Author's Response to Decision Letter for (RSOS-210657.R0)

See Appendix B.

Decision letter (RSOS-210657.R1)

We hope you are keeping well at this difficult and unusual time. We continue to value your support of the journal in these challenging circumstances. If Royal Society Open Science can assist you at all, please don't hesitate to let us know at the email address below.

Dear Dr Monk,

I am pleased to inform you that your manuscript entitled "Martingales and the characteristic functions of absorption time on bipartite graphs" is now accepted for publication in Royal Society Open Science.

If you have not already done so, please remember to make any data sets or code libraries 'live' prior to publication, and update any links as needed when you receive a proof to check - for instance, from a private 'for review' URL to a publicly accessible 'for publication' URL. It is good practice to also add data sets, code and other digital materials to your reference list.

You can expect to receive a proof of your article in the near future. Please contact the editorial office (openscience@royalsociety.org) and the production office (openscience_proofs@royalsociety.org) to let us know if you are likely to be away from e-mail contact -- if you are going to be away, please nominate a co-author (if available) to manage the proofing process, and ensure they are copied into your email to the journal. Due to rapid publication and an extremely tight schedule, if comments are not received, your paper may experience a delay in publication.

Please see the Royal Society Publishing guidance on how you may share your accepted author manuscript at <https://royalsociety.org/journals/ethics-policies/media-embargo/>. After publication, some additional ways to effectively promote your article can also be found here <https://royalsociety.org/blog/2020/07/promoting-your-latest-paper-and-tracking-your-results/>.

On behalf of the Editors of Royal Society Open Science, thank you for your support of the journal and we look forward to your continued contributions to Royal Society Open Science.

Kind regards,
Royal Society Open Science Editorial Office
Royal Society Open Science
openscience@royalsociety.org

on behalf of Professor Andreas Kyprianou (Associate Editor) and Mark Chaplain (Subject Editor)
openscience@royalsociety.org

Follow Royal Society Publishing on Twitter: @RSocPublishing
Follow Royal Society Publishing on Facebook:
<https://www.facebook.com/RoyalSocietyPublishing.FanPage/>
Read Royal Society Publishing's blog:
<https://royalsociety.org/blog/blogsearchpage/?category=Publishing>

Appendix A

REFEREE REPORT ON RSOS-210657
“Martingales and the characteristic functions of
absorption time on bipartite graphs”
by T. Monk & A. van Schaik

Date of report August 17, 2021

1. PAPER SUMMARY

The paper investigates the Moran model on bipartite graphs and develops a method to explicitly compute the conditional characteristic functions

$$\mathbb{E}[\exp(i\lambda C_T)|\text{fixation at } T], \quad \mathbb{E}[\exp(i\lambda C_T)|\text{extinction at } T],$$

where T is the time at which the mutants become either extinct or comprise the whole population, and where $C_t, t = 1, 2, \dots$ counts the number of instants up to time t at which the size of the mutant population changes. The resulting characteristic functions are then discussed for several parameter values and compared to numerically obtained values for the characteristic function of T .

2. GENERAL REMARKS

The paper is original and mathematically sound. It extends previous works by the authors on how to exploit martingale techniques in a clever way to study fixation probabilities and fixation times in the Moran process. Their method is not only mathematically appealing but also of practical value, as it provides an analytical understanding of quantities which have hitherto only been numerically studied. The only drawback of the approach is that it relies heavily on symmetries in the specific example considered and it is dubious whether the method can be adapted for graphs which are not either bipartite or complete. The presentation of the paper could be a bit more polished in some places, I have provided a list of pointers below.

My recommendation is that the paper be accepted for publication after minor corrections.

3. SOME COMMENTS ON THE TEXT

- p.1, l.50 The first sentence is somewhat misleading, as it is not the (sole) purpose of stochastic processes to “model the spread of some novelty in a population”. How about *The spread of some novelty in a population can be modelled by stochastic processes* ?
- p.2, l.39/40 Instead of “the Moran process on evolutionary graphs” write *the Moran process on **more general** evolutionary graphs* or even just *the Moran process on **more general** graphs*. The latter suggestion applies to many instances throughout the paper, the graphs discussed in evolutionary graph theory are not evolutionary per se,

- but become “evolutionary graphs” by being used in the context of evolution models.
- p.2, last line Replace “complete” by something like *more detailed*.
- p.3, l. 41 It is not the graph which can be considered as a bivariate random walk, but the process on it.
- p.3, l. 56 Writing $\Pr(T|\cdot)$ for the conditional distribution clashes with the use of $\Pr(\cdot)$ as shorthand for ‘probability of’. It would be better to use e.g. $(\Pr(T = t|\cdot)_{t=0}^\infty)$.
- p.4, eq (21) The variable h is throughout a complex number, it would help if that was mentioned at its introduction. E.g. use *free complex variable* instead of just “free variable”.
- p.4, l.21 Calling (21) a martingale is semantically problematic. Either call it *product martingale property* or *product martingale relation*, or say that $h, f(h), g(h)$ satisfying (21) define a product martingale.
- p.5, l.10 “Assume that f and g are convex such that they have two possible complex values in the neighbourhood about $\tau = 0$.” This sentence is very hard to understand, the general reference [26] you mention does not help very much either. It becomes somewhat clearer what you mean here only in the next section, when given h you solve for $f(h), g(h)$. It seems you are referring to $x = f(h), y = g(h)$ as valid solutions to the martingale equations for given h ; f and g are then the implicit solution curves, but the connection to convexity of f, g in h is not apparent to me.
- p.8, Figure 3 In the top right panel the legend reads “ $\alpha \ 0.889 \approx 0.889$ ”.

Appendix B



Dr. Travis Monk,
International Centre for Neuromorphic Systems
The MARCS Institute
Western Sydney University
Sydney, Australia

internet www.westernsydney.edu.au/icns
e-mail
travis.monk@westernsydney.edu.au
telephone +61 2 4736 0668 (ext. 2668)
mailing address. Dr. Travis Monk
Western Sydney University

Bag 1797

Penrith, NSW, Australia 2751

Locked

Dear Prof. Kyprianou,

We thank you and the referees for your careful reading of our manuscript. We are pleased that our submission is of sufficient quality and interest to be published in *RSOS*. Please find below our responses to referees, and how we revised the manuscript.

Referee 1

Thank you for complimenting our manuscript's writing.

Comment 1: The authors may consider relating their results to the mean absorption times (such as in the form of figures for one or more bipartite graphs with mutant fitness on x axis and different times on the y axis).

Response: We added the requested figure as Fig. 9. We compare our results to mean absorption times for two bipartite graphs with mutant fitness on the x-axis and times on the y-axis. The final two paragraphs of the Results section discuss the new figure.

Referee 2

Thank you for noting the originality and mathematical rigour of our submission.

Comment 1: Referee 2 suggested 9 minor edits of the text to improve our presentation.

Response: We agree with all 9 suggested edits. We implemented them to improve our presentation. We highlighted them in red in our submitted document that marks all changes.

Again, we sincerely thank you and the referees for your time and expertise. We believe our manuscript has been improved by your diligence.

Sincerely,
Travis Monk and André van Schaik