Sense2Stop MRT Analysis

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This supplementary appendix describes the analysis method used to address the primary aim of the Sense2Stop micro-randomized trial. The method is based on a generalization of Robins' multiplicative structural nested log-linear model [6] for use with data arising from a micro-randomized trial. It is an extension of the approaches described in [1, 2, 4] with the use of a log-link function to accommodate the primary outcome in this study, which is a vector of trichotomous outcomes.

1 Notation and Study Set-up

- Decision Point: Each minute within a user's 12-hour day is a decision point. Over the course of 10 days $t = 1, \ldots, T$ where $T = 720 \times 10 = 7200$.
- Primary Outcome: $Y_t = 1$ if at minute t, the user is within a *probably stressed* episode; $Y_t = 2$ if at minute t, the user is within a *physically active* episode; $Y_t = 3$ if at minute t, the user is within a *probably not stressed* episode. We will assess the causal effect of the intervention prompt at a decision point, t on the primary outcome in the subsequent 120 minutes, $Y_{t+1}, \ldots, Y_{t+120}$.
- Missing Data Indicator: M_t is set to 1 if at minute t, the primary outcome, Y_t is observed and $M_t = 0$ if Y_t is missing.
- Availability: $I_t = 1$ if user is available at time t and is 0 otherwise. At each of the 720 decision points per day, a user is considered available if they: *(i)* have not received a random EMA in the last 10 minutes; *(ii)* have not received a smoking EMA in the last 10 minutes; *(iii)* have not received an EMI in the last hour; *(iv)* are at the peak[1] of either a probably stressed or probably not stressed episode; *(v)* satisfy other conditions:

[1] The peak of an episode is determined by the MACD method (See [7] for details.)

data quality in the last 5 minutes is good[2], they are not driving, phone battery level is greater than 10%, not currently physically active[3], e.g., walking or moving. An available decision point is a decision point when $I_t = 1$. If $M_t = 0$ then $I_t = 0$.

- Intervention prompt randomization: $A_t = 1$ if randomized to receive an intervention and is 0 otherwise. Delivery of intervention vs. no intervention is randomized only if the user is available, $I_t = 1$.

- History (covariate data): H_t is all of the data we have observed on the user, up to and including time t, including M_t, I_t and current stress classification, Y_t, but excluding randomizations, A_t.

- Covariates: The analysis will include three sets of covariates - X_t, L_t and Z_t - all of which are features (summaries) of the history H_t; $X_t \subset Z_t \subset L_t$.

- Moderators: The analysis will include one moderator. For decision point t, $X_t = 1$ if $Y_t = 1$ and $X_t = 0$ if $Y_t = 3$. Note that for a decision point to be available, the current detection is either *probably stressed* ($Y_t = 1$) or *probably not stressed* ($Y_t = 3$).

- L_t is a vector of control covariates to reduce noise. To test the primary hypothesis, L_t will include two covariates: episode-ind, an indicator of stress episode type at the available decision point (probably stressed vs. probably not stressed) and intervention-ind, an indicator for the randomization (intervention vs. no intervention). In Section 6, we provide the plan for choosing additional covariates in L_t.

- Z_t is a vector of covariates for the missing at random assumption to hold. Z_t is a subset of H_t such that Y_{t+j} is independent of M_{t+j}, conditional on $Z_t, I_t = 1$, $j = 1, \ldots, 120$. We include X_t in Z_t and we include Z_t in L_t. So we always have $X_t \subset Z_t \subset L_t \subset H_t$ by definition. In Section 6, we provide a plan for choosing Z_t.

2 The Causal Effects

At every mth minute following an available decision point (i.e., at every mth minute following minute t at which $I_t = 1$ and the user is randomized at minute t), there are two potential intervention assignments: (1) intervention at time t and no other interventions for the next $m - 1$ minutes and (2) no intervention at time t and no other interventions for the next $m - 1$ minutes. The potential outcomes corresponding to these two assignments are denoted

[2]Good quality data corresponds to not having more than 33% of a minute's corresponding data missing due to sensor detachment or sensor off the body, low phone or sensor battery, momentary wireless data loss or software crash.

[3]Physical activity was determined by using activity recognition algorithms that automatically analyze data from the AutoSense-based accelerometer to classify participants' current activity.

as $Y_{t+m}(\bar{A}_{t-1}, 1, \bar{0}_{m-1})$ and $Y_{t+m}(\bar{A}_{t-1}, 0, \bar{0}_{m-1})$, respectively. Recall that Y_{t+m} is a trichotomous outcome (i.e., with category 1 denoting a minute in a *probably stressed* episode, category 2 denoting a minute in a *physically active* episode and category 3 denoting a minute in a *probably not stressed* episode).

The causal effect we are interested in is, for $k = 1, 2$ and for $m = 1, 2, \ldots, 120$,

$$\frac{P\left\{Y_{t+m}(\bar{A}_{t-1}, 1, \bar{0}_{m-1}) = k \mid I_t = 1, X_t\right\}}{P\left\{Y_{t+m}(\bar{A}_{t-1}, 0, \bar{0}_{m-1}) = k \mid I_t = 1, X_t\right\}}. \tag{1}$$

For a given $k \in \{1, 2\}$, display (1) is the relative risk between the probabilities of $Y_{t+m} = k$ for two potential intervention assignments. The relative risk in (1) is conditional on the individual being available at decision point t because we are only interested in the causal effect at available moments. The relative risk is conditional on whether the individual is currently within a probably stressed or probably not stressed episode (X_t), because we are interested in how the causal effect differs depending on the individual's current stress level. The causal effect in (1) is on the relative risk scale; a value greater than 1 indicates that delivering an intervention increases the probability that the proximal outcome belongs to category k in the m-th minute following the decision point.

In the analysis, we will model the causal relative risk in (1) using a log-linear model

$$\frac{P\left\{Y_{t+m}(\bar{A}_{t-1}, 1, \bar{0}_{m-1}) = k \mid I_t = 1, X_t\right\}}{P\left\{Y_{t+m}(\bar{A}_{t-1}, 0, \bar{0}_{m-1}) = k \mid I_t = 1, X_t\right\}} = e^{\beta_{k1} X_t + \beta_{k0}(1 - X_t)}. \tag{2}$$

Here, $\exp(\beta_{11})$, for which the first subscript corresponds $k = 1$ and the second subscript to $X_t = 1$, captures the causal relative risk between the probabilities of an individual being in a probably stressed episode at time $t + m$ under the aforementioned two intervention assignments, conditional on the individual being within a probably stressed episode at decision point t. This model (2), assumes that the causal relative risks, β_{kj}'s ($k \in \{1, 2, 3\}$, $j \in \{0, 1\}$), are the same for all $1 \leq t \leq T$ and $1 \leq m \leq 120$. It is possible that the true causal relative risk is different at different t and for different m; in such cases the estimated β_{kx}'s from the primary analysis serves as an average over $1 \leq t \leq T$ and $1 \leq m \leq 120$. See Section 5 below.

We further assume a log linear model on the missingness data indicator:

$$E\left\{M_{t+m}(\bar{A}_{t-1}, a_t, \bar{0}_{m-1}) \mid I_t = 1, H_t\right\} = e^{Z_t^T \xi + a_t Z_t^T \eta}. \tag{3}$$

This model is auxiliary in the sense that the parameters ξ and η are not of interest for our primary analysis, and its purpose is solely to facilitate estimation of the parameters of interest, β_{kx}.

3 Randomization Probabilities

In this section we review how the randomization probabilities in the Sense2Stop trial will be determined[4]. These randomization probabilities will be used in the estimation of the causal effects in Section 4.

Each day, the goal is to deliver interventions uniformly over all the available decision points. One of the difficulties in Sense2Stop is that the total number of available decision points is not known ahead of time, because availability is affected by many time-varying variables (see Section 1). To overcome this difficulty and deliver interventions uniformly over all the available decision points, the algorithm that determines the randomization probabilities makes use of and generalizes a sequential algorithm [5] from the EMA literature.

We first provide an intuitive account of how the randomization probability is determined. In the Sense2Stop study, every day comprises 12-hours (or 720 minutes) within which each minute is a candidate for an available decision point. At each available decision point (i.e. at minute t where $I_t = 1$), the randomization algorithm calculates the difference between: *(i)* the number of interventions to be delivered in the entire 720-minute day; and *(ii)* the number of interventions that have been delivered so far in the day; and divides this difference by a forecast of the number of available decision points left in the day. This ratio is the randomization probability at decision point t.

We provide a more detailed explanation in the following. This is a special case of the method developed in [3], where instead of splitting the day into k time blocks, we instead consider each 12-hour day to be one block of time, i.e., k=1. We also provide the values of the tuning parameters when applying the method in [3] to the Sense2Stop trial. Note that the randomization probabilities will be calculated separately for available decision points during a probably stressed episode ($X_t = 1$) and available decision points during a probably not stressed episode ($X_t = 0$). The tuning parameters used as inputs of the randomization algorithm are the following:

- N_x is the average number of interventions to be delivered during the day for episode type x ($x = 0, 1$). $x = 1$ refers to a probably stressed episode and $x = 0$ refers to a probably not stressed episode.

- $\lambda \in (0, 1)$ is a tuning parameter that controls the variability in sampling available decision points.

- $g(x, r)$ is a forecast of the remaining number of available decision points in the current day with $X_t = x$, where r denotes the remaining time (in minutes) in the current day.

[4] For a more general description on the algorithm used to assign randomization probabilities in the Sense2Stop trial, see [3]. The open source production code can be found at https://github.com/MD2Korg/mCerebrum-Configuration/tree/master/1.0/Northwestern/STU00201566/mCerebrum/org.md2k.ema_scheduler.

$x = 1$ refers to a probably stressed episode and $x = 0$ refers to a probably not stressed episode. For a discussion on the limitations of forecasting in this setting, see [3].

Furthermore, let s index the episodes that will involve randomization to intervention vs. no intervention. That is, s indexes episodes that include decision point t where $I_t = 1$. In the Sense2Stop trial, we plan to only conduct randomization to intervention vs. no intervention one minute following the peak (i.e., $t + 1$) of either a probably stressed or probably not stressed episode given that the individual is available (i.e., $I_t = 1$). T_s is the minute count occurring one minute following the peak of episode s. For example, suppose the peak of the sth episode occurs during minute 150 of an individual's 720 minute day, then $T_s = 151$.

The randomization probability in Sense2Stop for delivering an intervention prompt vs. no prompt at an available decision point is

$$p_t(H_t) = \frac{N_{X_t} - \sum_{\tau=1}^{t-1} [\lambda_\tau A_\tau + (1-\lambda_\tau) p_\tau(H_\tau)] \mathbb{1}_{\{X_\tau = X_t\}}}{1 + g(X_t, 720 - T_s)}, \quad \text{for} \quad X_t \in \{0, 1\},$$

where $\lambda_\tau = \lambda^{T_s - T_\tau}$. Note that when $s = 1$ the sum in the numerator is defined as 0, e.g., in this case, $p_t(H_t)$ is equal to $N_{X_t}/(1 + g(X_t, 720 - T_1))$. In addition, the randomization probability is restricted within the interval $[0.05, 0.95]$ for probably stressed episodes and $[0, 1]$ for probably not stressed episodes.

The values of the tuning parameters are $\lambda = 0.4$,

$$N_0 = \begin{cases} 1.6, & \text{if during pre-lapse} \\ 1.65, & \text{if during post-lapse} \end{cases}, \quad N_1 = \begin{cases} 2.25, & \text{if during pre-lapse} \\ 3, & \text{if during post-lapse} \end{cases},$$

and the function g depends on another tuning parameter η_x which takes on values

$$\eta_0 = \begin{cases} 1.1, & \text{if during pre-lapse} \\ 1.2, & \text{if during post-lapse} \end{cases}, \quad \eta_1 = 0.5.$$

4 Estimation

We describe a three-step procedure for estimating parameters in the causal effect of interest, β_{kx} for $k \in \{1, 2\}$ and $x \in \{0, 1\}$.

4.1 Step 1: Form the weights

The method uses a weighted regression approach. For $x \in \{0, 1\}$, we calculate the numerator probabilities as follows. For $x = 0, 1$, let

$$\hat{p}(x) = \frac{\mathbb{P} \sum_{t=1}^{T} I_t 1(X_t = x) p_t(H_t)}{\mathbb{P} \sum_{t=1}^{T} I_t 1(X_t = x)}. \tag{4}$$

The weight is

$$\hat{W}_t = \left\{\frac{\hat{p}(x)}{p_t(H_t)}\right\}^{A_t}\left\{\frac{1-\hat{p}(x)}{1-p_t(H_t)}\right\}^{1-A_t} \times \prod_{m=1}^{120}\frac{1(A_{t+m}=0)}{1-p_{t+m}(H_{t+m})}.$$

By design, all users who receive an intervention at an available decision point (t when $I_t = 1$) will not receive another intervention in the next hour. However, this is not true of users who do not receive an intervention at an available decision point. Intuitively, the weights $\hat{W}_t$ subsets and reweights data from users so that only data from users who do not receive an intervention in the following 120 minutes following an available decision point is used in estimation (which is consistent with the causal estimand in display (1)).

4.2 Step 2: Estimating the Missingness Mechanism

We compute $(\hat{\xi}, \hat{\eta})$ that solves the following estimating equation:

$$\mathbb{P}\sum_{t=1}^{T}\sum_{m=1}^{120} I_t\hat{W}_t\left\{e^{-A_tZ_t^T\eta}M_{t+m} - e^{Z_t^T\xi}\right\}\begin{bmatrix} Z_t \\ \{A_t-\hat{p}(X_t)\}Z_t\end{bmatrix} = 0. \tag{5}$$

4.3 Step 3: Estimating β, the Parameter of Interest

Let $e^{L_t^T\alpha_k}$ be a working model for

$$P\{Y_{t+m}(\bar{A}_{t-1}, 0, \bar{0}_{m-1}) = k \mid H_t, I_t = 1\},$$

where L_t are the control variables. This model does not need to be correct for the resulting estimator for β to be consistent; rather, this model is used to reduce estimation variance.

Define the residuals for $k = 1, 2$, $1 \le t \le T$, $1 \le m \le 120$ as follows:

$$R_{ktm}(\alpha,\beta) = e^{-A_t\{X_t\beta_{k1}+(1-X_t)\beta_{k0}\}}1(Y_{t+m}=k) - e^{L_t^T\alpha_k}. \tag{6}$$

The estimators for $(\alpha_1, \alpha_2, \beta_{10}, \beta_{11}, \beta_{20}, \beta_{21})$ solve the following equation:

$$\mathbb{P}\sum_{t=1}^{T}\sum_{m=1}^{120} I_t\hat{W}_t e^{-Z_t^T\hat{\xi}-A_tZ_t^T\hat{\eta}}M_{t+m}\begin{bmatrix} R_{1tm}(\alpha,\beta)L_t \\ R_{2tm}(\alpha,\beta)L_t \\ R_{1tm}(\alpha,\beta)\{A_t-\hat{p}(X_t)\}X_t \\ R_{1tm}(\alpha,\beta)\{A_t-\hat{p}(X_t)\}(1-X_t) \\ R_{2tm}(\alpha,\beta)\{A_t-\hat{p}(X_t)\}X_t \\ R_{2tm}(\alpha,\beta)\{A_t-\hat{p}(X_t)\}(1-X_t)\end{bmatrix} = 0, \tag{7}$$

where $\mathbb{P}$ indicates that one should evaluate the formula with each user's data and then average the results over all users. The estimators for the parameters in (2) are $(\hat{\beta}_{10}, \hat{\beta}_{11}, \hat{\beta}_{20}, \hat{\beta}_{21})$.

4.4 Variance-Covariance Estimation

The variance-covariance matrix of $(\hat{\beta}_{10}, \hat{\beta}_{11}, \hat{\beta}_{20}, \hat{\beta}_{21})$ can be estimated by the lower-right 4×4 submatrix of $n^{-1}V_n^{-1}\Sigma_n(V_n^{-1})^T$. Σ_n and V_n are defined as follows.

For ease of notation in this section, let $B_t = A_t - \hat{p}(X_t)$. Define

$$
\begin{aligned}
U_N(\rho) &= \begin{bmatrix} \sum_{t=1}^T I_t 1_{X_t=0}\{p_t(H_t)-\rho_0\} \\ \sum_{t=1}^T I_t 1_{X_t=1}\{p_t(H_t)-\rho_1\} \end{bmatrix}, \\
U_M(\eta,\xi,\rho) &= \sum_{t=1}^T \sum_{m=1}^{120} I_t W_t(\rho)\left\{e^{-A_t Z_t^T \eta} M_{t+m} - e^{Z_t{}^T \xi}\right\} \begin{bmatrix} Z_t \\ B_t Z_t \end{bmatrix}, \\
U_1(\alpha,\beta,\xi,\eta,\rho) &= \sum_{t=1}^T \sum_{m=1}^{120} I_t W_t(\rho) e^{-Z_t{}^T\xi - A_t Z_t^T \eta} M_{t+m} D_t(\rho) \begin{bmatrix} R_{1tm}(\alpha,\beta) \\ R_{2tm}(\alpha,\beta) \end{bmatrix}.
\end{aligned}
$$

We consider the following dimensions: $\dim(\rho) = 2$, $\dim(\eta) = \dim(\xi) = q$, $\dim(\alpha) = 2q$, $\dim(\beta) = 4$. Let

$$
U(\rho,\eta,\xi,\alpha,\beta) = \begin{bmatrix} U_N(\rho) \\ U_M(\eta,\xi,\rho) \\ U_1(\alpha,\beta,\xi,\eta,\rho) \end{bmatrix}_{(4q+6)\times 1}.
$$

Define Σ_n by

$$
\Sigma_n = \mathbb{P}\left\{ U(\hat{\alpha}, \hat{\beta}, \hat{\xi}, \hat{\eta}, \hat{\rho}) U(\hat{\alpha}, \hat{\beta}, \hat{\xi}, \hat{\eta}, \hat{\rho})^T \right\}.
$$

To define V_n, we introduce the following additional notation. Let q be the dimension of Z_t. Let $\vec{X}_t = (X_t, 1-X_t)^T$. Let $\beta_1 = (\beta_{10}, \beta_{11})^T$ and $\beta_2 = (\beta_{20}, \beta_{21})^T$. Recall that D_t is

$$
D_t = \begin{bmatrix} L_t & 0_{q\times 1} \\ 0_{q\times 1} & L_t \\ B_t \vec{X}_t & 0_{2\times 1} \\ 0_{2\times 1} & B_t \vec{X}_t \end{bmatrix}_{(2q+4)\times 2}.
$$

Define V_n by $V_n = \mathbb{P}(V)$, with

$$
V = \begin{bmatrix} V_{11} & 0 & 0 & 0 & 0 \\ V_{21} & V_{22} & V_{23} & 0 & 0 \\ V_{31} & V_{32} & V_{33} & V_{34} & V_{35} \end{bmatrix}_{(4q+6)\times(4q+6)},
$$

where

$$V_{11} = \begin{bmatrix} -\sum_{t=1}^{T} I_t 1_{X_t=0} & 0 \\ 0 & -\sum_{t=1}^{T} I_t 1_{X_t=1} \end{bmatrix}$$

$$V_{21} = \sum_{t=1}^{T}\sum_{m=1}^{120} I_t \hat{W}_t \left\{ e^{-A_t Z_t^T \hat{\eta}} M_{t+m} - e^{{Z_t}^T \hat{\xi}} \right\} \begin{bmatrix} \frac{2A_t-1}{\hat{p}(X_t)^{A_t}\{1-\hat{p}(X_t)\}^{1-A_t}} Z_t \\ \left\{ \frac{2A_t-1}{\hat{p}(X_t)^{A_t}\{1-\hat{p}(X_t)\}^{1-A_t}} B_t - 1 \right\} Z_t \end{bmatrix} \vec{X}_t^T$$

$$V_{22} = \sum_{t=1}^{T}\sum_{m=1}^{120} I_t \hat{W}_t \left\{ -e^{{Z_t}^T \hat{\xi}} \right\} \begin{bmatrix} Z_t \\ B_t \, Z_t \end{bmatrix} {Z_t}^T$$

$$V_{23} = \sum_{t=1}^{T}\sum_{m=1}^{120} I_t \hat{W}_t \left\{ e^{-A_t Z_t^T \hat{\eta}} M_{t+m} \right\} \begin{bmatrix} Z_t \\ B_t \, Z_t \end{bmatrix} (-A_t Z_t^T)$$

$$V_{31} = \sum_{t=1}^{T}\sum_{m=1}^{120} I_t \hat{W}_t e^{-{Z_t}^T \hat{\xi} - A_t Z_t^T \hat{\eta}} M_{t+m} \frac{2A_t - 1}{\hat{p}(X_t)^{A_t} \{1-\hat{p}(X_t)\}^{1-A_t}} D_t \begin{bmatrix} R_{1tm}(\hat{\alpha}, \hat{\beta}) \\ R_{2tm}(\hat{\alpha}, \hat{\beta}) \end{bmatrix} \vec{X}_t^T$$

$$+ \sum_{t=1}^{T}\sum_{m=1}^{120} I_t \hat{W}_t e^{-{Z_t}^T \hat{\xi} - A_t Z_t^T \hat{\eta}} M_{t+m} \begin{bmatrix} 0_{q\times 2} \\ 0_{q\times 2} \\ -R_{1tm}(\hat{\alpha}, \hat{\beta}) \vec{X}_t \vec{X}_t^T \\ -R_{2tm}(\hat{\alpha}, \hat{\beta}) \vec{X}_t \vec{X}_t^T \end{bmatrix}$$

$$V_{32} = \sum_{t=1}^{T}\sum_{m=1}^{120} I_t \hat{W}_t e^{-{Z_t}^T \hat{\xi} - A_t Z_t^T \hat{\eta}} M_{t+m} \begin{bmatrix} R_{1tm}(\hat{\alpha}, \hat{\beta}) L_t \\ R_{2tm}(\hat{\alpha}, \hat{\beta}) L_t \\ R_{1tm}(\hat{\alpha}, \hat{\beta}) B_t \, \vec{X}_t \\ R_{2tm}(\hat{\alpha}, \hat{\beta}) B_t \, \vec{X}_t \end{bmatrix} (-{Z_t}^T)$$

$$V_{33} = \sum_{t=1}^{T}\sum_{m=1}^{120} I_t \hat{W}_t e^{-{Z_t}^T \hat{\xi} - A_t Z_t^T \hat{\eta}} M_{t+m} \begin{bmatrix} R_{1tm}(\hat{\alpha}, \hat{\beta}) L_t \\ R_{2tm}(\hat{\alpha}, \hat{\beta}) L_t \\ R_{1tm}(\hat{\alpha}, \hat{\beta}) B_t \, \vec{X}_t \\ R_{2tm}(\hat{\alpha}, \hat{\beta}) B_t \, \vec{X}_t \end{bmatrix} (-A_t Z_t^T)$$

$$V_{34} = \sum_{t=1}^{T}\sum_{m=1}^{120} I_t \hat{W}_t e^{-Z_t{}^T\hat{\xi} - A_t Z_t^T \hat{\eta}} M_{t+m} \begin{bmatrix} -e^{L_t{}^T\hat{\alpha}_1} L_t L_t{}^T & 0_{q\times q} \\ 0_{q\times q} & -e^{L_t{}^T\hat{\alpha}_2} L_t L_t{}^T \\ -e^{L_t{}^T\hat{\alpha}_1} B_t \vec{X}_t L_t{}^T & 0_{q\times q} \\ 0_{q\times q} & -e^{L_t{}^T\hat{\alpha}_2} B_t \vec{X}_t L_t{}^T \end{bmatrix}$$

$$\begin{aligned} V_{35} &= \sum_{t=1}^{T}\sum_{m=1}^{120} I_t \hat{W}_t e^{-Z_t{}^T\hat{\xi} - A_t Z_t^T \hat{\eta}} M_{t+m} \\ &\times \begin{bmatrix} -e^{-A_t\vec{X}_t^T\hat{\beta}_1} 1(Y_{t+m}=1) A_t L_t \vec{X}_t^T & 0_{q\times 2} \\ 0_{q\times 2} & -e^{-A_t\vec{X}_t^T\hat{\beta}_2} 1(Y_{t+m}=2) A_t L_t \vec{X}_t^T \\ -e^{-A_t\vec{X}_t^T\hat{\beta}_1} 1(Y_{t+m}=1) A_t B_t \vec{X}_t \vec{X}_t^T & 0_{q\times 2} \\ 0_{q\times 2} & -e^{-A_t\vec{X}_t^T\hat{\beta}_2} 1(Y_{t+m}=2) A_t B_t \vec{X}_t \vec{X}_t^T \end{bmatrix} \end{aligned}$$

5 Weighted-Average-Ratio Interpretation of the Estimators

Our estimators $\hat{\beta}_{kx}$ are consistent for the true β_{kx} in (2). When (2) is an incorrect model (e.g., when the relative risk varies depending on t or m), the "true" β_{kx} are not well defined. In such cases, our estimator can be interpreted as a projection onto model (2).

In particular, estimators $\hat{\beta}_{k0}, \hat{\beta}_{k1}$ are consistent estimators of the following quantities:

$$\beta'_{k0} = \log \frac{\sum_t^T \sum_{m=1}^{120} E\left\{1(Y_{t+m}(\bar{A}_{t-1},1,\bar{0}_{m-1})=k) \mid I_t=1, X_t=0\right\} P(I_t=1,X_t=0)}{\sum_t^T \sum_{m=1}^{120} E\left\{1(Y_{t+m}(\bar{A}_{t-1},0,\bar{0}_{m-1})=k) \mid I_t=1, X_t=0\right\} P(I_t=1,X_t=0)}, \quad (8)$$

$$\beta'_{k1} = \log \frac{\sum_t^T \sum_{m=1}^{120} E\left\{1(Y_{t+m}(\bar{A}_{t-1},1,\bar{0}_{m-1})=k) \mid I_t=1, X_t=1\right\} P(I_t=1,X_t=1)}{\sum_t^T \sum_{m=1}^{120} E\left\{1(Y_{t+m}(\bar{A}_{t-1},0,\bar{0}_{m-1})=k) \mid I_t=1, X_t=1\right\} P(I_t=1,X_t=1)}. \quad (9)$$

Each can be interpreted as a ratio of two weighted averages. For example, consider $k=1$ (the outcome being classified as probably stressed), and

$$\beta'_{10} = \log \frac{\sum_t^T E\left\{\sum_{m=1}^{120} 1(Y_{t+m}(\bar{A}_{t-1},1,\bar{0}_{m-1})=1) \mid I_t=1, X_t=0\right\} P(I_t=1,X_t=0)}{\sum_t^T E\left\{\sum_{m=1}^{120} 1(Y_{t+m}(\bar{A}_{t-1},0,\bar{0}_{m-1})=1) \mid I_t=1, X_t=0\right\} P(I_t=1,X_t=0)}.$$

Note that $\sum_{m=1}^{120} 1(Y_{t+m}(\bar{A}_{t-1},1,\bar{0})=1)$ is the number of minutes classified as probably stressed after receiving an intervention at time t. So

$$E\left\{\sum_{m=1}^{120} 1(Y_{t+m}(\bar{A}_{t-1},1,\bar{0})=1) \mid I_t=1, X_t=0\right\}$$

is the conditional mean of the number of minutes classified as *probably stressed* after receiving an intervention at time t among individuals who are available and classified as *probably not stressed* at time t. Therefore, $e^{\beta'_{10}}$ is the ratio of two weighted averages: The numerator is the weighted average, across time, of the conditional mean of the number of minutes classified as probably stressed after receiving an intervention among individuals who are currently available and classified as probably not stressed; the denominator is the weighted average, across time, of the conditional mean of the number of minutes classified as probably stressed after *not* receiving an intervention among individuals who are currently available and classified as probably not stressed. The weight, $P(I_t = 1, X_t = 0)$, at each decision point is the probability of being available and classified as probably not stressed.

6 Choosing Z_t and L_t

6.1 Choosing Z_t

In order to justify the MAR assumption for the missing data in our primary outcome, we will investigate which covariates prior to an available decision point (i.e., at t when $I_t = 1$) are predictive of missing minutes within the primary outcome. Note that missing minutes within the proximal outcome are either due to bad quality stress episode data or completely missing episode data.

We will consider the following for candidate covariates that may predict missing episodes:

- x_1: ID (Integer with values in $\{1, \ldots, 48\}$)
- x_2: Day in MRT (Integer with values in $\{1, \ldots, 10\}$)
- x_3: Indicator that user is within a probably stressed episode at time of available decision point (Binary with values in $\{1, 0\}$)
- x_4: Length (in minutes) of the episode from start to peak. Note that the episode includes the available decision point which is one minute following the peak.
- x_5: Indicator that the previous episode is missing (Binary with values in $\{1, 0\}$). *Previous* refers to the episode prior to the episode that contains the available decision point.
- x_6: Indicator that the previous episode is classified as probably stressed (Binary with values in $\{1, 0\}$).
- x_7: Indicator that the previous episode is classified as probably not stressed (Binary with values in $\{1, 0\}$).
- x_8: Length (in minutes) of the previous episode from its start to end.
- x_9: Previous day's proportion of minutes within 12 hour day that the user is physical active.
- x_{10}: Previous day's proportion of minutes within 12 hour day with bad quality REP Data.
- x_{11}: Previous day's proportion of minutes within 12 hour day with bad quality ECG Data.

- x_{12}: Number of interventions sent on the previous day
- x_{13}: BMI on day 1 of study (continuous variable from 18 to 46)
- x_{14}: Gender (0 = female, 1 = male)
- x_{15}: Age (integer from 20 to 63)
- x_{16}: Age started smoking (Integer from 20 to 63)
- x_{17}: Total Fagerstrom score (Integer from 0 to 9)
- x_{18}: Weekday (1 = weekday, 0 = weekend)
- x_{19}: Hour of day (integer with values in $\{1, \ldots, 23\}$)
- x_{20}: Indicator for morning (1 = morning, 0 = other)
- x_{21}: Indicator for afternoon (1 = afternoon, 0 = other)
- x_{22}: Indicator for night (1 = night, 0 = other)

We will use a logistic regression model for the binary outcome: missing or not missing episode, and we will train and test the predictive performance of this model with cross validation (i.e., we train on one portion of data and test on another that the model has not yet seen and we do this multiple times). For predictive performance, we will use the weighted F1 score[5]. We will consider the 5 most influential[6] covariates from the best performing model (i.e., the model with the highest weighted F1-score ≥ 0.6. If we achieve such a model, these covariates will be added to Z_t for the primary analysis.

6.2 Choosing L_t

We will investigate which covariates, if introduced into the analysis, will likely reduce variability in the estimation of β. Natural control covariates under consideration are pre-decision point measures of the outcome. For example,

- The minutes in the prior 120 minutes from the available decision point that are within a probably stressed episode; and
- The minutes in the prior 120 minutes from the available decision point that are physically active.

In addition, we will consider the following identifying covariates:

- ID (Integer with values in 1, . . . , 48)
- Day in MRT (Integer with values in 1, . . . , 10)

[5]The weighted F1 score is defined as the weighted average of $\{2 \times (\text{precision} \times \text{recall})\}/(\text{precision} + \text{recall})$ for both of the classes in the binary outcome. Precision is the fraction of relevant instances among the retrieved instances, while recall is the fraction of the total amount of relevant instances that were actually retrieved.

[6]We will use the magnitude and sign of the coefficients to detect the influence of the covariates given that the data was standardised prior to the model fit.

- Indicator that user is within a probably stressed episode at time of available decision point (Binary with values in $\{1, 0\}$)
- Minute identifier for the corresponding minute following an available decision point (Integer with values in $\{1, \ldots, 120\}$)
- Weekday (1 = weekday, 0 = weekend)
- Hour of day (integer with values in $\{1, \ldots, 23\}$)
- Indicator for Morning (1 = morning, 0 = other)
- Indicator for Afternoon (1 = afternoon, 0 = other)
- Indicator for Night (1 = night, 0 = other)

All numerical covariates will be converted to their standard scores. Each row within this data set corresponds to a minute's outcome within the 120 minutes following an available decision point (i.e., there are up to 120 minutes, or 120 rows in this data set, corresponding to an available decision point).

To learn which of these variables explain the minute level outcomes we will use a multi-class logistic regression model for the multi-class outcome: *physically active minute*, *probably stressed minute* and *probably not stressed minute*.

We will train and test the predictive performance of a multi-class logistic regression model with cross validation (i.e., we train on one portion of data and test on another that the model has not yet seen and we do this multiple times). For predictive performance, we will use the weighted F1 score[7]. We will consider the 5 most influential[8] covariates from the best performing model (i.e., the model with the highest weighted F1-score ≥ 0.6. If we achieve such a model, these covariates will be added to L_t for the primary analysis.

References

[1] Audrey Boruvka et al. "Assessing time-varying causal effect moderation in mobile health". In: *Journal of the American Statistical Association* 113.523 (2018), pp. 1112–1121.

[2] Walter Dempsey et al. "The stratified micro-randomized trial design: sample size considerations for testing nested causal effects of time-varying treatments". In: *Annals of Applied Statistics* 14.2 (2020), pp. 661–684.

[7] The weighted F1 score is defined as the weighted average of $\{2 \times (\text{precision} \times \text{recall})\}/(\text{precision} + \text{recall})$ for the three classes in the multi-class outcome. Precision is the fraction of relevant instances among the retrieved instances, while recall is the fraction of the total amount of relevant instances that were actually retrieved.

[8] An influential covariate in this analysis can be thought of as a usable covariate for distinguishing between two classs "one-vs-rest".

[3] Peng Liao et al. "Just-in-time but not too much: Determining treatment timing in mobile health". In: *Proceedings of the ACM on interactive, mobile, wearable and ubiquitous technologies* 2.4 (2018), pp. 1–21.

[4] Tianchen Qian et al. "Estimating Time-Varying Causal Excursion Effect in Mobile Health with Binary Outcomes". In: *Biometrika* (2020).

[5] Stephen L Rathbun et al. "Survival analysis with time varying covariates measured at random times by design". In: *Journal of the Royal Statistical Society: Series C (Applied Statistics)* 62.3 (2013), pp. 419–434.

[6] James M Robins. "Causal inference from complex longitudinal data". In: *Latent variable modeling and applications to causality*. Springer, 1997, pp. 69–117.

[7] Hillol Sarker et al. "Finding significant stress episodes in a discontinuous time series of rapidly varying mobile sensor data". In: *Proceedings of the 2016 CHI conference on human factors in computing systems*. 2016, pp. 4489–4501.