Supplementary Table

Data Collection Points Summarized by Study and Treatment Types

Clinical Trial 1: NCT01049516 Prolonged Exposure for Posttraumatic Stress Disorder (PTSD) Among Operation Iraqi Freedom/Operation Enduring Freedom (OIF/OEF) Personnel				**Clinical Trial 2: NCT01286415** Group Cognitive Processing Therapy for Combat-related Posttraumatic Stress Disorder (PTSD)		**Clinical Trial 3: NCT02173561** Group vs. Individual Cognitive Processin Therapy for Combat-related PTSD	
MPE	MCC	SPE	PCT	Group CPT	Group PCT	Individual CPT	Group CPT
Baseline	Baseline	Baseline	Baseline	Baseline	Baseline	Baseline	Baseline
Session 3	Post	Session 2	Session 2	Session 1	Session 1	Session 1	Session 1
Session 5	Week 4 Assessment	Session 3	Session 3	Session 3	Session 3	Session 3	Session 3
Session 7	NP1	F/U 1	F/U 1	Session 5	Session 5	Session 5	Session 5
Session 9	NP2	Session 4	Session 4	Session 7	Session 7	Session 7	Session 7
Post	NP3	Session 5	Session 5	Session 9	Session 9	Session 9	Session 9
F/U 2		F/U 2	F/U 2	Session 11	Session 11	Session 11	Session 11
F/U 3		Session 6	Session 6	Post	Post	Post	Post
F/U 4		Session 7	Session 7	6-Month F/U	6-Month F/U	6-Month F/U	6-Month F/U
F/U 5		Session 8	Session 8	12-Month F/U	12 Month- F/U	12-Month F/U	12-Month F/U
6-Month F/U		Session 9	Session 9			NP1	NP1
12-Month F/U		Post	Post			NP2	NP2
		F/U 4	F/U 4				
		F/U 5	F/U 5				
		6-Month F/U	6-Month F/U				
		12-Month F/U	12-Month F/U				

Note: MPE = Massed Prolonged Exposure; MCC = Minimal Contact Control; SPE = Spaced Prolonged Exposure; PCT = Present-Centered Therapy; CPT = Cognitive Processing Therapy; Post = posttreatment; F/U = follow up; NP = nonprotocol visits (due to crisis or emergency, the patient and therapist agreed to spend the session on the crisis/emergency event as opposed to completing what was planned for that scheduled session).

Supplementary Methods

The approach utilized involves 10 different multilevel models. We break them into two sets. The first set is where second derivatives/accelerations for each item on the Scale for Suicidal Ideation (SSI) item are treated as the dependent variables:

$$BS\ddot{S}I1_{it} = \gamma_{10} + \gamma_{11}BS\dot{S}I1_{ij} + \gamma_{12}BS\dot{S}I2_{ij} + \gamma_{13}BS\dot{S}I3_{ij} + \gamma_{14}BS\dot{S}I4_{ij} + \gamma_{15}BS\dot{S}I5_{ij} + \gamma_{16}BSSI1_{ij} + \gamma_{17}BSSI2_{ij} + \gamma_{18}BSSI3_{ij} + \gamma_{19}BSSI4_{ij} + \gamma_{110}BSSI5_{ij} + \omega_{1j} + e_{1ij}$$

$$BS\ddot{S}I2_{it} = \gamma_{20} + \gamma_{21}BS\dot{S}I1_{ij} + \gamma_{22}BS\dot{S}I2_{ij} + \gamma_{23}BS\dot{S}I3_{ij} + \gamma_{24}BS\dot{S}I4_{ij} + \gamma_{25}BS\dot{S}I5_{ij} + \gamma_{26}BSSI1_{ij} + \gamma_{27}BSSI2_{ij} + \gamma_{28}BSSI3_{ij} + \gamma_{29}BSSI4_{ij} + \gamma_{210}BSSI5_{ij} + \omega_{2j} + e_{2ij}$$

$$BS\ddot{S}I3_{it} = \gamma_{30} + \gamma_{31}BS\dot{S}I1_{ij} + \gamma_{32}BS\dot{S}I2_{ij} + \gamma_{33}BS\dot{S}I3_{ij} + \gamma_{34}BS\dot{S}I4_{ij} + \gamma_{35}BS\dot{S}I5_{ij} + \gamma_{36}BSSI1_{ij} + \gamma_{37}BSSI2_{ij} + \gamma_{38}BSSI3_{ij} + \gamma_{39}BSSI4_{ij} + \gamma_{310}BSSI5_{ij} + \omega_{3j} + e_{3ij}$$

$$BS\ddot{S}I4_{it} = \gamma_{40} + \gamma_{41}BS\dot{S}I1_{ij} + \gamma_{42}BS\dot{S}I2_{ij} + \gamma_{43}BS\dot{S}I3_{ij} + \gamma_{44}BS\dot{S}I4_{ij} + \gamma_{45}BS\dot{S}I5_{ij} + \gamma_{46}BSSI1_{ij} + \gamma_{47}BSSI2_{ij} + \gamma_{48}BSSI3_{ij} + \gamma_{49}BSSI4_{ij} + \gamma_{410}BSSI5_{ij} + \omega_{4j} + e_{4ij}$$

$$BS\ddot{S}I5_{it} = \gamma_{50} + \gamma_{51}BS\dot{S}I1_{ij} + \gamma_{52}BS\dot{S}I2_{ij} + \gamma_{53}BS\dot{S}I3_{ij} + \gamma_{54}BS\dot{S}I4_{ij} + \gamma_{55}BS\dot{S}I5_{ij} + \gamma_{56}BSSI1_{ij} + \gamma_{57}BSSI2_{ij} + \gamma_{58}BSSI3_{ij} + \gamma_{59}BSSI4_{ij} + \gamma_{510}BSSI5_{ij} + \omega_{5j} + e_{5ij}$$

Equations 1-5

Where *i* is a given instance for a given individual, *j*, we have added the first subscript to indicate the equation number. The gammas are the fixed effects with the omegas representing random intercepts and *e* as a different error term for each equation. We use the single dot over a variable to indicate a first derivative and a double dot to indicate the second derivative.

For the second set of equations, the first derivatives/velocities are the dependent variables.

$$\dot{BSSI1}_{\iota t} = \gamma_{60} + \gamma_{61}BSSI1_{\iota j} + \gamma_{62}BSSI2_{\iota j} + \gamma_{63}BSSI3_{\iota j} + \gamma_{64}BSSI4_{\iota j} + \gamma_{65}BSSI5_{\iota j} + \omega_{6j} + e_{6\iota j}$$
$$\dot{BSSI2}_{\iota t} = \gamma_{70} + \gamma_{71}BSSI1_{\iota j} + \gamma_{72}BSSI2_{\iota j} + \gamma_{73}BSSI3_{\iota j} + \gamma_{74}BSSI4_{\iota j} + \gamma_{75}BSSI5_{\iota j} + \omega_{7j} + e_{7\iota j}$$
$$\dot{BSSI3}_{\iota t} = \gamma_{80} + \gamma_{81}BSSI1_{\iota j} + \gamma_{82}BSSI2_{\iota j} + \gamma_{83}BSSI3_{\iota j} + \gamma_{84}BSSI4_{\iota j} + \gamma_{85}BSSI5_{\iota j} + \omega_{8j} + e_{8\iota j}$$
$$\dot{BSSI4}_{\iota t} = \gamma_{90} + \gamma_{91}BSSI1_{\iota j} + \gamma_{92}BSSI2_{\iota j} + \gamma_{93}BSSI3_{\iota j} + \gamma_{94}BSSI4_{\iota j} + \gamma_{95}BSSI5_{\iota j} + \omega_{9j} + e_{9\iota j}$$
$$\dot{BSSI5}_{\iota t} = \gamma_{100} + \gamma_{101}BSSI1_{\iota j} + \gamma_{102}BSSI2_{\iota j} + \gamma_{103}BSSI3_{\iota j} + \gamma_{104}BSSI4_{\iota j} + \gamma_{105}BSSI5_{\iota j} + \omega_{10j} + e_{10\iota j}$$

Equations 6-10

Note that only the zeroth derivatives are treated as simultaneous predictors. Due to the sheer volume of effects being estimated, models were run separately rather than simultaneously to ease estimation. This makes an assumption that any dependency amongst dependent variables (with the exclusion of the person dependency captured through the multilevel model) is accounted for by the predictors.

The fixed coefficients with the exclusion of the intercepts are used to construct a Jacobian matrix, a matrix of partial derivatives. It is well documented that the eigenvalues (sometimes called characteristic roots) of the Jacobian matrix identify the overall dynamic of a system (Abraham & Shaw, 2005; Butner, Deits-Lebehn, et al., 2017; Butner, Wiltshire, et al., 2017). However, the expansion of the Jacobian matrix for modeling second order equations (second derivatives as the criteria) are less well documented.

Jacobian Matrix for Second Order Equations

The construction of the Jacobian matrix consists of treating the rows as the dependent variables and the columns as the independent variables, where the order of variables is always the same and the main diagonal consists of how the zeroth derivatives predict its own first derivatives. This matrix is then treated as the input matrix for an eigenvalue/eigenvector (spectral) decomposition, a matrix algebra calculation. However, when we add in equations that have second derivatives as the outcomes, the form has to change slightly.

To illustrate the Jacobian matrix for a second order equation, we will consider a much simpler case where we merely model the first and second derivative of y as a function of itself. This would involve two equations (EQ), in this case treated as regressions (again for simplicity):

$$\ddot{y}_i = b_{10} + b_{11}\dot{y}_i + b_{12}y_i + e_{1i} \qquad \text{EQ 11}$$

$$\dot{y}_i = b_{20} + b_{21}y_i + e_{2i} \qquad \text{EQ 12}$$

As before, *i* is for instance. Thus, these are a pair of time series equations. The framework for the Jacobian matrix is made up of dependent variables as rows and predictors as columns. In this case, we can think of the Jacobian matrix as having the form of

Criterion	Predictors	
	Velocity	Value
Acceleration	b_{11}	b_{12}
Velocity	1	b_{21}

Notice that we treat velocity as a perfect predictor of itself in that for every one change in velocity, the velocity (same variable) changes by exactly one unit. The order of placement for predictors in the Jacobian matrix is important. First order derivatives as predictors of the second order derivative must be placed on the main diagonal, as they are a form of attraction/repulsion. When modeling a second derivative, the velocity term is often called damping, which indicates the rate of decay in amplitude as a function of exponential time (Butner et al., 2005). These values must be on the main diagonal, as when the eigenvalue is taken we are maximizing the overall system attraction/repulsion where both have the same meaning.

The value/position term can be thought of as a frequency term when the equation is constructed through forces from physics (Butner et al., 2005). However, an alternative description is angular attraction, or attraction that is around rather than towards or away. In other words, the coefficient of a value predicting its own second derivative can be thought of as how a variable couples with itself to form an indication of system momentum. We therefore express the term on the off diagonal in line with the placement of coupling relationships.

These two equations generate two eigenvalues that can be only about the cyclicity, only about attraction/repulsion, and combinations of the two. The order of the eigenvalues will be expressed in terms of dominance, which effects are largest. Further, the generation of imaginary eigenvalues will represent the cyclic nature, while the real portions will illustrate attraction/repulsion.

Three circumstances can occur as a function of cycles and damping/attraction when both are negative (angular and point attraction, respectively). An underdamped system is where cycles are able to complete before all of the amplitude has died away. That is, the angular attraction is more powerful than the point attraction. A critically damped system is where the system most rapidly approaches the steady state/point of attraction. An overdamped system is where the frequency becomes imaginary, showing no signs of cycling at all. Instead, it displays an exponential decay towards the steady state/point of attraction. The point attraction overpowers the angular attraction. Using only EQ 11, these three circumstances are determined by the comparison of b_{11} to b_{12}:

Underdamped:	$b_{11}^2 < 4b_{12}$
Critically damped:	$b_{11}^2 = 4b_{12}$
Over damped:	$b_{11}^2 > 4b_{12}$

When modeling both EQ 11 and 12, these values then convert to the real and imaginary eigenvalues, respectively. To exemplify this, Figures S1, S2, and S3 show a bluescale tile map of the imaginary portion of the first eigenvalue, the real portion of the first eigenvalue, and the real portion of the second eigenvalue, respectively. In all cases, b_{12} (the frequency/angular attraction term) was fixed to -.8 while b_{11} and b_{21} (the steady state/point attraction terms) were varied between -4 and 4 (creating a total of 10,000 combinations). The diagonal band from the lower left to the upper right represent circumstances where the system is underdamped, showing an imaginary eigenvalue close to .8 (note that the formula for frequency and damping are not entirely independent, represented by the fuzzy edges of the diagonal). The area beyond is where the two steady state/point attraction terms are stronger, consistent with overdamping and generating an imaginary eigenvalue of zero.

Figures S2 and S3 show similar patterns, now for the real components. Within the underdamped range (from lower left to upper right), the first real eigenvalue appropriately shows the damping/point attraction in addition to the imaginary component. Once, in the overdamped range, however, we instead see a combined attraction with the diagonal line (from upper left to lower right), merely indicating which of the two steady state/point of attraction terms are dominant. S3 is merely the inverse of S2, showing the same pattern within the underdamped range, but the other real eigenvalue in the critical and overdamped range.

To expand these notions to more than one simultaneous set of second order equations, all that remains is the values to be inserted into the Jacobian matrix for when one velocity predicts another velocity. Consistent with the assumption that any dependency amongst the derivatives is captured as a function of the set of predictors, these values are fixed to zero. The eigenvalue procedure is then applied to the entire matrix.

Supplemental References

Abraham, R. H., & Shaw, C. D. (2005). *Dynamics: The geometry of behavior* (4th ed.). Aerial Press.

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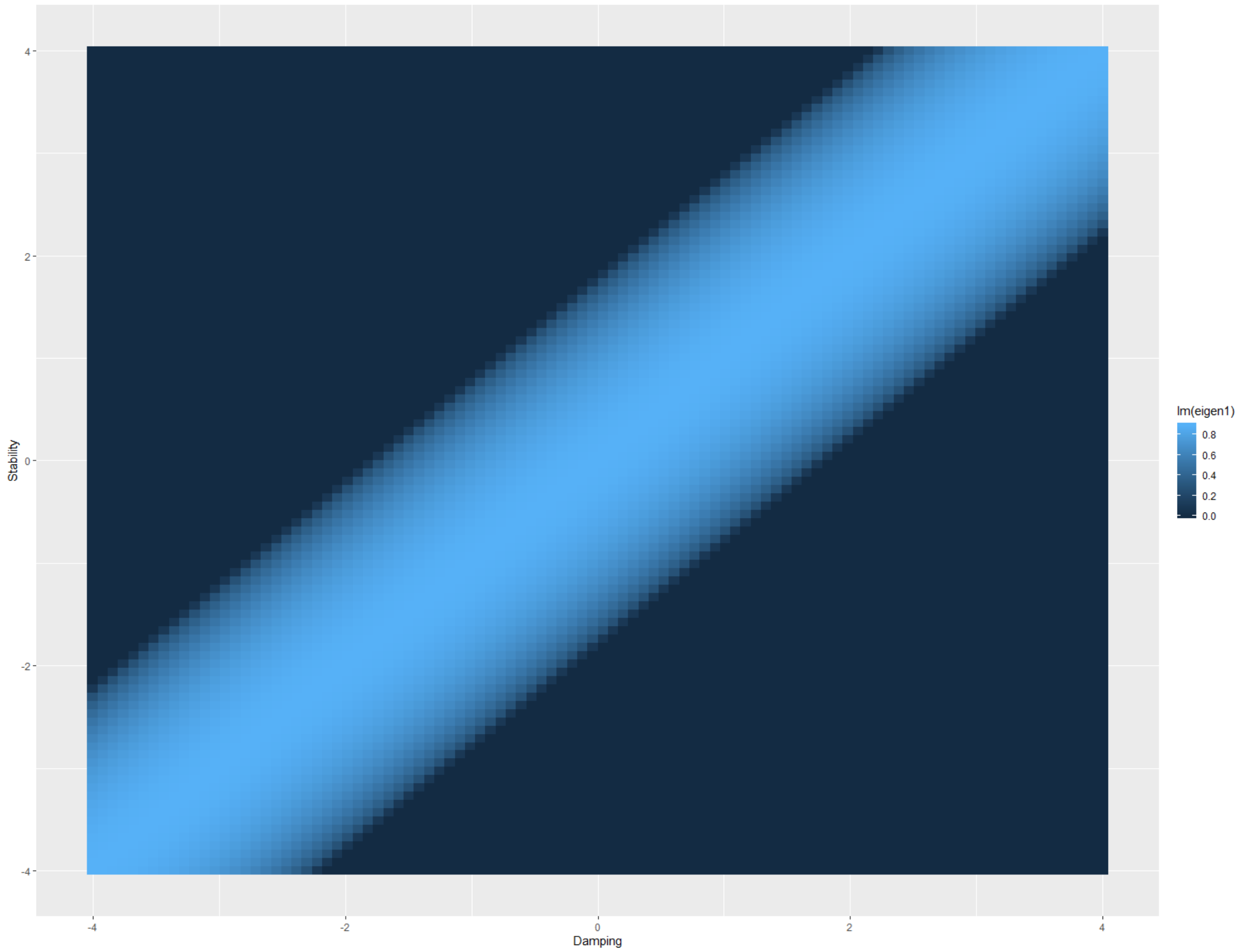
Butner, J. E., Deits-Lebehn, C., Crenshaw, A. O., Wiltshire, T. J., Perry, N. S., Kent de Grey, R. G., Hogan, J. N., Smith, T. W., Baucom, K. J. W., & Baucom, B. R. W. (2017). A multivariate dynamic systems model for psychotherapy with more than one client. *Journal of Counseling Psychology, 64*(6), 616–625. https://doi.org/10.1037/cou0000238

Butner, J. E., Wiltshire, T. J., & Munion, A. K. (2017). Modeling multi-agent self-organization through the lens of higher order attractor dynamics. *Frontiers in Psychology: Cognitive Science, 8,* 380-395. https://doi.org/10.3389/fpsyg.2017.00380

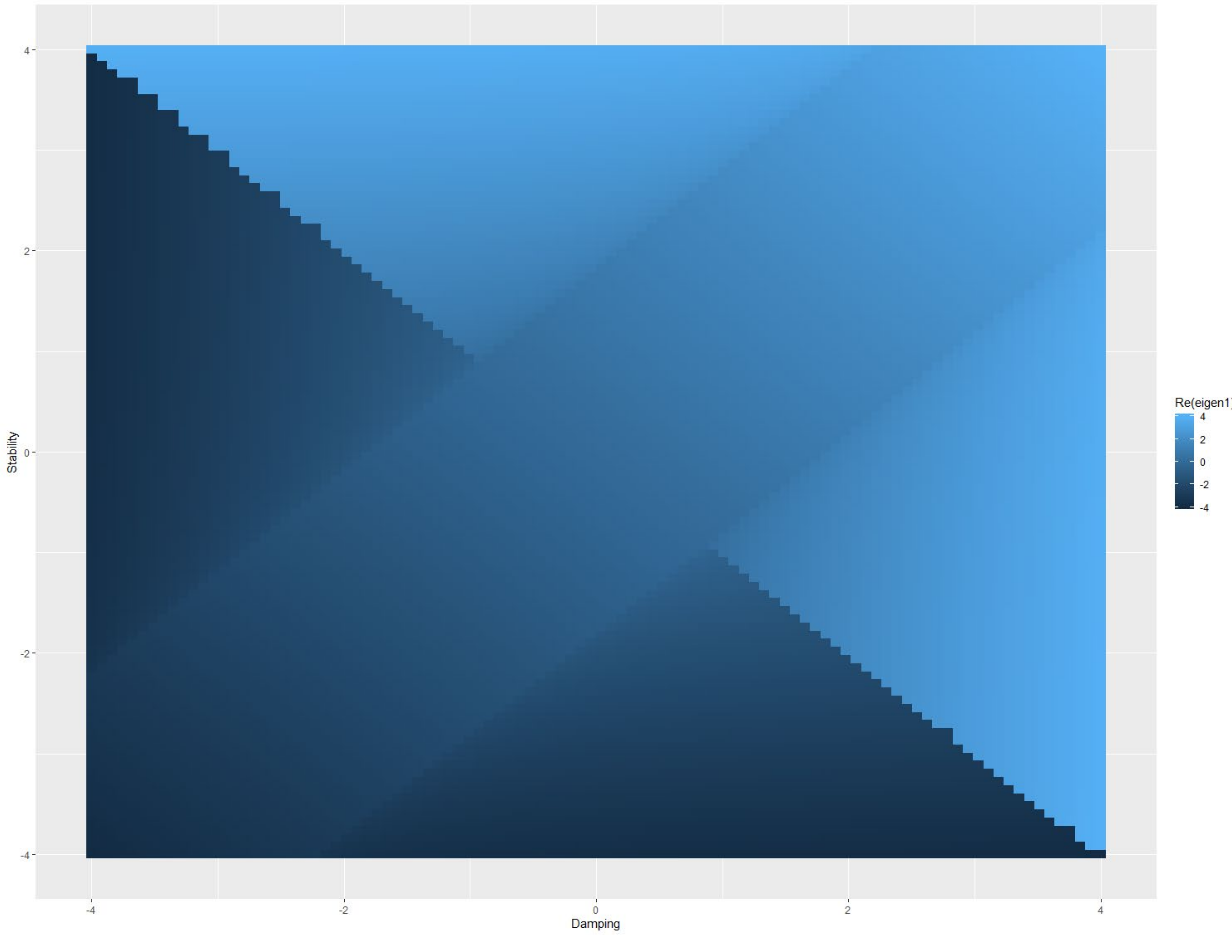
Supplementary Figure 1

Tile Map of the Imaginary Portion of the First Eigenvalue



Supplementary Figure 2

Tile Map of the Real Portion of the First Eigenvalue



Supplementary Figure 3

Tile Map of the Real Portion of the Second Eigenvalue