Cost-effective surveillance of invasive species using info-gap theory

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Supplementary Information

Section S1: Derivation of the robustness function

The total number of Surveillance System Components (SSCs) to be used should be the sum of SSCs at different locations L depending on the relative area A_L^j / A_t^j (A_L^j is the area of zone j at location L, A_t^j is the total area of zone j). For example, if the gecko scat analysis used in Zone 1 is calculated to be one unit, when this specific SSC is allocated to six different locations in Zone 1, each of the locations should be allocated one unit and thus there will be six units in total for all locations. The number of SSCs applied at six different locations will be rounded up to the next highest value taking the relative area into consideration. Function 'ceil' is used in this

research to round up to the next highest value. For example, for the values 4.2 and 4.8, the final value would be five. Define the quantity of surveillance type i used in zone j is N_i^j , so the quantity of surveillance type *i* used at location *L* is:

$$
n_i^L = \sum_j \text{ceil}(N_i^j \times \frac{A_L^j}{A_i^j}) \ (n_i^L \ge 0, N_i^j \ge 0, A_L^j \ge 0, A_i^j \ge 0) \quad (S1)
$$

where,

$$
N_i^j = \frac{\log(\beta^{\frac{1}{(1-\sigma_i^j F_i^j)C_i(1/R^j)}})}{K \log(1-\sigma_i^j F_i^j)} (K \ge 0, \sigma_i^j \le 1, F_i^j \le 1, R^j \le 1, C_i > 0)
$$
 (S2)

Therefore, the total surveillance cost on the whole island is:

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\n
$$
r = \sum_{i} (\sum_{L} n_{i}^{L} \times C_{i}) = \sum_{i} \{ (\sum_{L} (\sum_{j} \text{ceil}(N_{i}^{j} \times \frac{A_{L}^{j}}{A_{i}^{j}})) \times C_{i} \}
$$
\n
$$
= \sum_{i} \{ (\sum_{L} (\sum_{j} \text{ceil}(\frac{\log(\beta^{(\overline{1-\sigma_{i}^{j}F_{i}^{j}})C_{i}(1/R^{j})}{K \log(1-\sigma_{i}^{j}F_{i}^{j})})} \times \frac{A_{L}^{j}}{A_{i}^{j}})) \times C_{i} \}
$$

(S3)

As referred in the body of the paper, we will refer generically to the 40 uncertain parameters (K, σ_i^j, F_i^j) and R^j for $i = 1, ..., 6$ and $j = 0, 1, 2$) with the vector $x = (x_1, ..., x_{40})$ and sometimes refer explicitly to K, σ_i^j, F_i^j and R^j for clarity or simplicity. We also have initial estimates of K, σ_i^j, F_i^j and R^j , denoted by the 40-vector \tilde{x} .

The fractional-error info-gap model of uncertainty is defined as:

$$
U(h) = \{x : |\frac{x_n - \tilde{x}_n}{\tilde{x}_n} | \le h, x_n \ge 0, n = 1 : 40; \sigma_i^j \le 1, F_i^j \le 1, R^j \le 1, i = 1 : 6, j = 0 : 2\}, h \ge 0
$$
\n
$$
(S4)
$$

Then the info-gap's robustness function is defined as:

$$
\hat{h}(r_c) = \max\{h : (\max_{K,\sigma,F,R \in U(h)} r) \le r_c\} \quad (S5)
$$

where \hat{h} is the greatest horizon of uncertainty up to which the system model obeys the performance requirement.

Let $m(h)$ denote the inner maximum in the definition of the robustness function. The sets of the info-gap model, $U(h)$ become more inclusive as h increases. This implies that $m(h)$ increases as *h* increases. From the definition of the robustness we see that $\hat{h}(r_c)$ is the greatest value of *h* at which $m(h) \le r_c$. This means that a plot of h vs $m(h)$ is equivalent to a plot of $\hat{h}(r_c)$ vs r_c . In other words, $m(h)$ is the inverse function of $\hat{h}(r_c)$. Once we derive an algebraic expression for $m(h)$ we can plot the robustness curve; we do not need to invert $m(h)$, which can be problematic. We now proceed to derive $m(h)$. Plots of robustness curves in the body of the manuscript are based on the expressions for $m(h)$.

Let $m(h)$ denote the inner maximum,

h) denote the inner maximum,
\n
$$
m(h) = \max r = \max \sum_{i} \{ (\sum_{L} (\sum_{j} \text{ceil}(\frac{\log(\beta^{\frac{1}{(1-\sigma_{i}^{j}F_{i}^{j})C_{i}(1/R^{j})}}{\text{Klog}(1-\sigma_{i}^{j}F_{i}^{j})})} \times \frac{A_{L}^{j}}{A_{t}^{j}})) \times C_{i} \} \le r_{c} \quad (S6)
$$

Since both σ_i^j and F_i^j are less than one, the value of $\log(1 - \sigma_i^j F_i^j)$ must be negative. And K is positive, thus the denominator must be negative. $(1 - \sigma_i^j F_i^j) \times C_i \times (1/R^j)$ is positive and

 $\beta = 0.2$, 1 $\beta^{\overline{(1-\sigma_i^j F_i^j)C_i(1/R^j)}}$ < 1, thus the value of numerator is negative. In order to get maximum of *r* , the negative denominator should be as large as possible and negative numerator as small as possible. Therefore, K, σ, F should be minimum and R be maximum.

It is defined here:

$$
\xi^+ = \begin{cases} \xi, & 0 \le \xi \le 1 \\ & 0, \xi < 0 \\ & 1, \xi > 1 \end{cases}
$$
 (S7)

Then

$$
\sigma_i^j = (1 - h)^+ \tilde{\sigma}_i^j \quad (S8)
$$

$$
F_i^j = (1 - h)^+ \tilde{F}_i^j \quad (S9)
$$

$$
R^j = ((1 + h)\tilde{R}^j)^+ \quad (S10)
$$

$$
K = (1 - h)^+ \tilde{K} \quad (S11)
$$

Therefore,

$$
m(h) = \max r = \max \sum_{i} \{ (\sum_{L} (\sum_{j} \text{ceil}(N_{i}^{j} \times \frac{A_{L}^{j}}{A_{i}}))) \times C_{i} \}
$$

= $\sum_{i} \{ (\sum_{L} (\sum_{j} \text{ceil}(\frac{\log(\beta^{\overline{(1-(1-h)^{+} \tilde{\sigma}_{i}^{j} (1-h)^{+} \tilde{F}_{i}^{j} C_{i} [1/(1+h)\tilde{R}^{j})^{+}})}{(1-h)^{+} \tilde{K} \log[1-(1-h)^{+} \tilde{\sigma}_{i}^{j} (1-h)^{+} \tilde{F}_{i}^{j}]} \times \frac{A_{L}^{j}}{A_{i}^{j}})) \times C_{i} \} \le r_{c}$

(S12)

Section S2: Derivation of the opportuneness function

The opportuneness function in this model is defined as:

$$
\hat{\beta}(r_w) = \min\{h : (\min_{K,\sigma,F,R \in U(h)} r) \le r_w\} \quad (S13)
$$

Let $w(h)$ denote the inner minimum,

h) denote the inner minimum,
\n
$$
w(h) = \min r = \min \sum_{i} \{ (\sum_{L} (\sum_{j} \text{ceil} (\frac{\log(\beta^{(1-\sigma_i^j F_i^j)C_i(I/R^j)})}{K \log(1-\sigma_i^j F_i^j)}) \times \frac{A_L^j}{A_L^j})) \times C_i \} \le r_w \quad (S14)
$$

Opposite to this robustness function referred to above in Supplementary Section S1, K, σ, F should be maximum and R be minimum, shown as follows:

$$
\sigma_i^j = ((1+h)\tilde{\sigma}_i^j)^+ \quad \text{(S15)}
$$
\n
$$
F_i^j = ((1+h)\tilde{F}_i^j)^+ \quad \text{(S16)}
$$
\n
$$
R^j = (1-h)^+ \tilde{R}^j \quad \text{(S17)}
$$
\n
$$
K = (1+h)\tilde{K} \quad \text{(S18)}
$$

Therefore,

$$
\text{refore,}
$$
\n
$$
w(h) = \min r = \sum_{i} \{ (\sum_{L} (\sum_{j} \text{ceil}(\frac{\log(\beta^{\overline{(1-(1+h)\tilde{\sigma}_{i}^{j})^{+}((1+h)\tilde{F}_{i}^{j})^{+}}\Gamma_{i}[\Gamma/(1-h)^{+}\tilde{R}^{j})]}{(1+h)\tilde{K}[\log(1-((1+h)\tilde{\sigma}_{i}^{j})^{+}((1+h)\tilde{F}_{i}^{j})^{+}]}\times \frac{A_{L}^{j}}{A_{L}^{j}})) \} \times C_{i} \} \le r_{w}
$$

As can be seen in the opportuneness function, with the increase of *h* , the range of uncertainty increases, and the min *r* decreases. Thus $w(h)$ decreases as *h* increases. $\hat{\beta}(r_w)$ is the least value of h at which $w(h) \le r_w$. A plot of h vs $w(h) \le r_w$ is identical to that of $\hat{\beta}(r_w)$ vs r_w . Thus $w(h)$ is the inverse function of $\hat{\beta}(r_{w})$. Plots of opportuneness curves in the body of the paper are based on the expressions for $w(h)$.

Table S1. Relative importance weights (RIWs) for entry points for an Asian House Gecko incursion on Barrow Island. Data was based on expert elicitation that was coordinated by Chevron Australia

Table S2. Relative importance weights (RIWs) for establishment of the Asian House Gecko on $\rm Barrow$ Island $^{\rm l}$

Table S3. Surveillance area of each location on Barrow Island. Data was provided by Chevron Australia.

^a The surface area of Z1 does not refer to the entire Zone 1 area on the quarantine invasion risk map (Fig. 1), but only the specified habitat where AHG would be detected (i.e. building area).

Table S4. Definition of Surveillance System Components (SSCs). Data was provided by Chevron Australia.

^aThe maximum number of passive workers is 1,000. This number could vary depending on the activities being undertaken on the island.

Table S5. σ , footprint and unit cost of various Surveillance System Components (SSCs) at different locations and zones. Data was based on expert elicitation that was coordinated by Chevron Australia.

* Z0 only occurs at the MOF. ^a Sigma is the detection probability of Surveillance System

Components given invasive species present in the footprint. ^b Footprint is the area in which an AHG can be detected with a single unit of SSC. ^c Cost is per unit of Surveillance System Components.

References

- 1. Wintle, B. & Burgman, M. *Expert Elicitation for Barrow Island Surveillance System Revision, Project Report.* (2015).
- 2. Thomas, M. L. *et al.* Many eyes on the ground: citizen science is an effective early detection tool for biosecurity. *Biol. Invasions* **19**, 2751-2765 (2017).