Supplementary Material for "Two-phase analysis and study design for survival models with error-prone exposures"

Kyunghee Han[1], Thomas Lumley[2], Bryan E. Shepherd[3] and Pamela A. Shaw[1]

[1]University of Pennsylvania, USA

[2]University of Auckland, New Zealand

[3]Vanderbilt University, USA

A Technical details

A.1 Regularity conditions for Theorem 1

We assume there exists $J \geq 1$ such that $P(Y \leq t_J)$ with probability one. Let $\boldsymbol{\theta}_0 \in \Theta_0$ be the true parameter for some open $\Theta_0 \subset \mathbf{R}^{J+d}$ and further assume the following conditions:

A1. $\mathrm{E}|L_1(\boldsymbol{\theta}; Y, \Delta, \mathbf{X})| < \infty$ at $\boldsymbol{\theta}_0$.

A2. $L_1(\boldsymbol{\theta}; Y, \Delta, \mathbf{X})$ is three-times continuously differentiable and bounded away from 0 near $\boldsymbol{\theta}_0$ (a.s.).

A3. $I_V = -\mathrm{E}\big[\frac{\partial^2}{\partial\boldsymbol{\theta}\partial\boldsymbol{\theta}^\top} \log L_1(\boldsymbol{\theta}; Y, \Delta, \mathbf{X})\big]$ is positive definite at $\boldsymbol{\theta}_0$.

A4. $\pi(y, \delta, \mathbf{z}) > 0$ for all $(y, \delta, \mathbf{z})$.

Since we have formulated discrete-time survival models where the censoring time C is bounded away from infinity, the right-censored survival time Y has an upper-bound t_J almost surely so that finite numbers of baseline hazards suffice to specify likelihood for the discrete-time survival model (1). Without loss of generality, we write $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_J)^\top$ by the transformation of baseline hazards. Then, the above assumptions A1–A4 are the regularity conditions for the maximum likelihood estimation of finite-dimensional parameters $\boldsymbol{\theta} = (\boldsymbol{\alpha}, \boldsymbol{\beta})$, and therefore, the proof follows Theorem 1 in Reilly and Pepe (1995) which we refer the reader to for details.

A.2 Some derivations for numerical implementation

For the implementation of the optimal design in our numerical simulations, we empirically estimated certain parameters by generating a large independent sample of N_0 individuals. Let $\mathcal{X}^\star_{N_0} = \{(Y_i^\star, \Delta_i^\star, \mathbf{X}_i^\star, \mathbf{Z}_i^\star) : 1 \leq i \leq N_0\}$ be an external random sample of $(Y, \Delta, \mathbf{X}, \mathbf{Z})$ independent on $\mathcal{X}_N$ or $\mathcal{X}_{\mathrm{I},N}$. We estimate unknown quantities in the optimal sampling design (7) by

$$
\begin{aligned}
\widehat{I}_V^{\star} &= -\frac{1}{N_0}\sum_{i=1}^{N_0}\frac{\partial^2}{\partial\boldsymbol{\theta}\partial\boldsymbol{\theta}^\top}\log L_1(\boldsymbol{\theta}; Y_i^\star, \Delta_i^\star, \mathbf{X}_i^\star), \\
\widehat{\mathrm{Var}}^\star\big(U_1(\boldsymbol{\theta})|y,\delta,z\big) &= \frac{N^\star(y,\delta,\mathbf{z})}{N^\star(y,\delta,\mathbf{z})-1}\big\{\hat{\mu}_2^\star(\boldsymbol{\theta}; y,\delta,\mathbf{z}) - \hat{\mu}_1^\star(\boldsymbol{\theta}; y,\delta,\mathbf{z})^2\big\},
\end{aligned}
\tag{A.1}
$$

where $N^\star(y,\delta,\mathbf{z}) = \sum_{i=1}^{N_0}\mathbb{I}(Y_i^\star = y, \Delta_i^\star = \delta, \mathbf{Z}_i^\star = \mathbf{z})$ and

$$
\hat{\mu}_\ell^\star(\boldsymbol{\theta}; y,\delta,\mathbf{z}) = \frac{1}{N^\star(y,\delta,\mathbf{z})}\sum_{i=1}^{N_0} U_1(\boldsymbol{\theta}; Y_i^\star, \Delta_i^\star, \mathbf{X}_i^\star)^\ell \cdot \mathbb{I}(Y_i^\star = y, \Delta_i^\star = \delta, \mathbf{Z}_i^\star = \mathbf{z}) \tag{A.2}
$$

for $\ell = 1, 2$.

We now introduce some derivations for Newton-Raphson algorithm. Recall that the discrete-time survival model (1) under the odds transformation $g_1(u) = \frac{u}{1-u}$ is given by $\mathrm{logit}(\lambda_j(\mathbf{x})) = \alpha_j + \boldsymbol{\beta}^\top\mathbf{x}$, where $\alpha_j = \mathrm{logit}(\lambda_{0j})$ is the logit transformation of the baseline hazard. The maximum likelihood estimates of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are obtained by solving score equations of (4) with

$$
\begin{aligned}
\frac{\partial}{\partial\alpha_j}Q_N(\boldsymbol{\alpha},\boldsymbol{\beta}) &= \sum_{i\in\mathcal{I}}\hat{\pi}(Y_i,\Delta_i,\mathbf{Z}_i)^{-1}\mathbb{I}(j\le J(i))\left(D_{ij} - \frac{e^{\alpha_j+\boldsymbol{\beta}^\top\mathbf{X}_i}}{1+e^{\alpha_j+\boldsymbol{\beta}^\top\mathbf{X}_i}}\right) = 0, \\
\frac{\partial}{\partial\boldsymbol{\beta}}Q_N(\boldsymbol{\alpha},\boldsymbol{\beta}) &= \sum_{i\in\mathcal{I}}\hat{\pi}(Y_i,\Delta_i,\mathbf{Z}_i)^{-1}\sum_{j=1}^{J(i)}\left(D_{ij} - \frac{e^{\alpha_j+\boldsymbol{\beta}^\top\mathbf{X}_i}}{1+e^{\alpha_j+\boldsymbol{\beta}^\top\mathbf{X}_i}}\right)\mathbf{X}_i = \mathbf{0}.
\end{aligned}
\tag{A.3}
$$

We numerically solve the system of equations (A.3) with the Newton-Raphson algorithm using the associated Hessian matrix whose components consist of

$$
\begin{aligned}
\frac{\partial^2}{\partial\alpha_j\partial\alpha_{j'}}Q_N(\boldsymbol{\alpha},\boldsymbol{\beta}) &= -\sum_{i\in\mathcal{I}}\hat{\pi}(Y_i,\Delta_i,\mathbf{Z}_i)^{-1}\mathbb{I}(j\le J(i))\frac{e^{\alpha_j+\boldsymbol{\beta}^\top\mathbf{X}_i}}{\big(1+e^{\alpha_j+\boldsymbol{\beta}^\top\mathbf{X}_i}\big)^2}\mathbb{I}(j=j'), \\
\frac{\partial^2}{\partial\alpha_j\partial\boldsymbol{\beta}}Q_N(\boldsymbol{\alpha},\boldsymbol{\beta}) &= -\sum_{i\in\mathcal{I}}\hat{\pi}(Y_i,\Delta_i,\mathbf{Z}_i)^{-1}\mathbb{I}(j\le J(i))\frac{e^{\alpha_j+\boldsymbol{\beta}^\top\mathbf{X}_i}}{\big(1+e^{\alpha_j+\boldsymbol{\beta}^\top\mathbf{X}_i}\big)^2}\mathbf{X}_i, \\
\frac{\partial^2}{\partial\boldsymbol{\beta}\partial\boldsymbol{\beta}^\top}Q_N(\boldsymbol{\alpha},\boldsymbol{\beta}) &= -\sum_{i\in\mathcal{I}}\hat{\pi}(Y_i,\Delta_i,\mathbf{Z}_i)^{-1}\sum_{j=1}^{J(i)}\frac{e^{\alpha_j+\boldsymbol{\beta}^\top\mathbf{X}_i}}{\big(1+e^{\alpha_j+\boldsymbol{\beta}^\top\mathbf{X}_i}\big)^2}\mathbf{X}_i\mathbf{X}_i^\top.
\end{aligned}
\tag{A.4}
$$

Similarly, suppose the complementary log transformation $g_2(u) = -\log(1-u)$ defines the true survival model (1). Then it can be easily seen that $\mathrm{logit}(\lambda_j(\mathbf{x})) = \exp\big(e^{\alpha_j+\boldsymbol{\beta}^\top\mathbf{x}}\big) - 1$, where $\alpha_j = \log(-\log(1-\lambda_{0j}))$ is the complementary log-log transformation of the baseline hazard. The maximum likelihood estimates of $\boldsymbol{\theta} = (\boldsymbol{\alpha}, \boldsymbol{\beta})$ are obtained by solving score equations of (4) with

$$\begin{aligned}
\frac{\partial}{\partial \alpha_j} Q_N(\boldsymbol{\alpha}, \boldsymbol{\beta}) &= \sum_{i \in \mathcal{I}} \hat{\pi}(Y_i, \Delta_i, \mathbf{Z}_i)^{-1} \mathbb{I}(j \leq J(i)) \left(D_{ij} \frac{e^{\alpha_j + \boldsymbol{\beta}^\top \mathbf{X}_i}}{1 - e^{\exp(\alpha_j + \boldsymbol{\beta}^\top \mathbf{X}_i)}} - e^{\alpha_j + \boldsymbol{\beta}^\top \mathbf{X}_i} \right) = 0, \\
\frac{\partial}{\partial \boldsymbol{\beta}} Q_N(\boldsymbol{\alpha}, \boldsymbol{\beta}) &= \sum_{i \in \mathcal{I}} \hat{\pi}(Y_i, \Delta_i, \mathbf{Z}_i)^{-1} \sum_{j=1}^{J(i)} \left(D_{ij} \frac{e^{\alpha_j + \boldsymbol{\beta}^\top \mathbf{X}_i}}{1 - e^{\exp(\alpha_j + \boldsymbol{\beta}^\top \mathbf{X}_i)}} - e^{\alpha_j + \boldsymbol{\beta}^\top \mathbf{X}_i} \right) \mathbf{X}_i = \mathbf{0}.
\end{aligned} \tag{A.5}$$

We numerically solve the system of equations (A.5) with the Newton-Raphson algorithm using the associated Hessian matrix whose components consist of

$$\begin{aligned}
\frac{\partial^2}{\partial \alpha_j \partial \alpha_{j'}} Q_N(\boldsymbol{\alpha}, \boldsymbol{\beta}) &= -\sum_{i \in \mathcal{I}} \hat{\pi}(Y_i, \Delta_i, \mathbf{Z}_i)^{-1} \mathbb{I}(j \leq J(i)) \Bigg\{ D_{ij} \frac{e^{\alpha_j + \boldsymbol{\beta}^\top \mathbf{X}_i}}{1 - e^{\exp(\alpha_j + \boldsymbol{\beta}^\top \mathbf{X}_i)}} \times \\
&\qquad \left(1 - \frac{e^{\alpha_j + \boldsymbol{\beta}^\top \mathbf{X}_i} e^{-\exp(\alpha_j + \boldsymbol{\beta}^\top \mathbf{X}_i)}}{1 - e^{\exp(\alpha_j + \boldsymbol{\beta}^\top \mathbf{X}_i)}} \right) - e^{\alpha_j + \boldsymbol{\beta}^\top \mathbf{X}_i} \Bigg\} \mathbb{I}(j = j'), \\
\frac{\partial^2}{\partial \alpha_j \partial \boldsymbol{\beta}} Q_N(\boldsymbol{\alpha}, \boldsymbol{\beta}) &= -\sum_{i \in \mathcal{I}} \hat{\pi}(Y_i, \Delta_i, \mathbf{Z}_i)^{-1} \mathbb{I}(j \leq J(i)) \Bigg\{ D_{ij} \frac{e^{\alpha_j + \boldsymbol{\beta}^\top \mathbf{X}_i}}{1 - e^{\exp(\alpha_j + \boldsymbol{\beta}^\top \mathbf{X}_i)}} \times \\
&\qquad \left(1 - \frac{e^{\alpha_j + \boldsymbol{\beta}^\top \mathbf{X}_i} e^{-\exp(\alpha_j + \boldsymbol{\beta}^\top \mathbf{X}_i)}}{1 - e^{\exp(\alpha_j + \boldsymbol{\beta}^\top \mathbf{X}_i)}} \right) - e^{\alpha_j + \boldsymbol{\beta}^\top \mathbf{X}_i} \Bigg\} \mathbf{X}_i, \\
\frac{\partial^2}{\partial \boldsymbol{\beta} \partial \boldsymbol{\beta}^\top} Q_N(\boldsymbol{\alpha}, \boldsymbol{\beta}) &= -\sum_{i \in \mathcal{I}} \hat{\pi}(Y_i, \Delta_i, \mathbf{Z}_i)^{-1} \sum_{j=1}^{J(i)} \Bigg\{ D_{ij} \frac{e^{\alpha_j + \boldsymbol{\beta}^\top \mathbf{X}_i}}{1 - e^{\exp(\alpha_j + \boldsymbol{\beta}^\top \mathbf{X}_i)}} \times \\
&\qquad \left(1 - \frac{e^{\alpha_j + \boldsymbol{\beta}^\top \mathbf{X}_i} e^{-\exp(\alpha_j + \boldsymbol{\beta}^\top \mathbf{X}_i)}}{1 - e^{\exp(\alpha_j + \boldsymbol{\beta}^\top \mathbf{X}_i)}} \right) - e^{\alpha_j + \boldsymbol{\beta}^\top \mathbf{X}_i} \Bigg\} \mathbf{X}_i \mathbf{X}_i^\top.
\end{aligned} \tag{A.6}$$

B Additional numerical results

B.1 Supplemental tables for the simulation study

Table B.1: Relative performance for the estimation of β_1 is compared for (i) the complete case analysis with simple random sampling (CC-SRS), (ii) the mean score method with simple random sampling (MS-SRS), (iii) a design-based estimation with balanced sample, equivalent to the mean score (MS-BAL), (iv & v) the mean score estimation with adaptive sampling (MS-A) and the optimal sampling design (MS-O), for varying sample sizes. Results for the full cohort estimator based on complete data are provided as a benchmark. Mean squared error (MSE) and its bias-variance decomposition are estimated from 1000 Monte Carlo replications, where the censoring rate was 30%. The adaptive and optimal sampling designs were for efficient estimation of X_1 with $\beta_1 = \log(1.5) \approx 0.405$. In all scenarios, we took equal proportions for the pilot and adaptive samples.

Sampling	Estimation	Criterion	Estimation performance by sample sizes					
			$N = 4000$			$n = 400$		
			$n = 200$	$n = 400$	$n = 800$	$N = 2000$	$N = 4000$	$N = 8000$
Full cohort	CC	$\sqrt{\text{MSE}}$	0.079	0.079	0.079	0.112	0.079	0.056
		Bias	0.004	0.004	0.004	0.000	0.004	0.001
		$\sqrt{\text{Var}}$	0.079	0.079	0.079	0.112	0.079	0.056
SRS	CC	$\sqrt{\text{MSE}}$	0.408	0.284	0.198	0.281	0.284	0.282
		Bias	0.013	0.012	0.011	0.013	0.012	0.015
		$\sqrt{\text{Var}}$	0.408	0.284	0.197	0.281	0.284	0.282
	MS	$\sqrt{\text{MSE}}$	0.268	0.177	0.132	0.191	0.177	0.170
		Bias	0.067	0.016	0.012	0.014	0.016	0.017
		$\sqrt{\text{Var}}$	0.259	0.176	0.131	0.190	0.176	0.169
Balanced	MS	$\sqrt{\text{MSE}}$	0.302	0.216	0.153	0.219	0.216	0.202
		Bias	0.023	0.017	0.015	0.010	0.017	0.018
		$\sqrt{\text{Var}}$	0.301	0.215	0.152	0.219	0.215	0.201
Adaptive	MS	$\sqrt{\text{MSE}}$	0.279	0.168	0.114	0.181	0.168	0.159
		Bias	0.038	0.024	0.011	0.012	0.024	0.017
		$\sqrt{\text{Var}}$	0.276	0.166	0.113	0.181	0.166	0.158
Oracle	MS	$\sqrt{\text{MSE}}$	0.213	0.155	0.112	0.166	0.155	0.137
		Bias	0.025	0.001	0.001	0.005	0.001	0.002
		$\sqrt{\text{Var}}$	0.212	0.155	0.112	0.166	0.155	0.137

Table B.2: Relative performance for the estimation of β_1 is compared for (i) the complete case analysis with simple random sampling (CC-SRS), (ii) the mean score method with simple random sampling (MS-SRS), (iii) a design-based estimation with balanced sample, equivalent to the mean score (MS-BAL), (iv & v) the mean score estimation with adaptive sampling (MS-A) and the optimal sampling design (MS-O), for varying sample sizes. Results for the full cohort estimator based on complete data are provided as a benchmark. Mean squared error (MSE) and its bias-variance decomposition are estimated from 1000 Monte Carlo replications, where the censoring rate was 70%. The adaptive and optimal sampling designs were for efficient estimation of X_1 with $\beta_1 = \log(1.5) \approx 0.405$. In all scenarios, we took equal proportions for the pilot and adaptive samples.

Sampling	Estimation	Criterion	Estimation performance by sample sizes					
			$N = 4000$			$n = 400$		
			$n = 200$	$n = 400$	$n = 800$	$N = 2000$	$N = 4000$	$N = 8000$
Full cohort	CC	$\sqrt{\text{MSE}}$	0.119	0.119	0.119	0.175	0.119	0.086
		Bias	0.000	0.000	0.000	0.004	0.000	0.003
		$\sqrt{\text{Var}}$	0.119	0.119	0.119	0.175	0.119	0.086
SRS	CC	$\sqrt{\text{MSE}}$	0.630	0.420	0.299	0.442	0.420	0.419
		Bias	0.025	0.016	0.002	-0.005	0.016	0.018
		$\sqrt{\text{Var}}$	0.630	0.420	0.299	0.442	0.420	0.418
	MS	$\sqrt{\text{MSE}}$	0.477	0.304	0.208	0.328	0.304	0.287
		Bias	0.177	0.078	0.018	0.065	0.078	0.081
		$\sqrt{\text{Var}}$	0.443	0.294	0.207	0.322	0.294	0.273
Balanced	MS	$\sqrt{\text{MSE}}$	0.535	0.330	0.225	0.347	0.330	0.327
		Bias	0.092	0.026	0.005	0.033	0.026	0.029
		$\sqrt{\text{Var}}$	0.527	0.329	0.224	0.346	0.329	0.326
Adaptive	MS	$\sqrt{\text{MSE}}$	0.411	0.250	0.178	0.259	0.250	0.225
		Bias	0.018	0.007	0.001	0.003	0.007	0.003
		$\sqrt{\text{Var}}$	0.410	0.250	0.178	0.259	0.250	0.225
Oracle	MS	$\sqrt{\text{MSE}}$	0.315	0.215	0.174	0.250	0.215	0.224
		Bias	0.023	0.010	0.000	0.017	0.010	0.008
		$\sqrt{\text{Var}}$	0.314	0.214	0.174	0.249	0.214	0.224

Table B.3: Relative performance for the estimation of regression parameters is compared for (i) the complete case analysis with simple random sampling (CC-SRS), (ii) the mean score method with simple random sampling (MS-SRS), (iii) a design-based estimation with balanced sample, equivalent to the mean score (MS-BAL), (iv & v) the mean score estimation with adaptive sampling (MS-A) and the optimal sampling design (MS-O), where $n = 200$ subsample was drawn from the full cohort size of $N = 4000$. Results for the full cohort estimator based on complete data are provided as a benchmark. Mean squared error (MSE) and its bias-variance decomposition are estimated from 1000 Monte Carlo replications, where the censoring rate was 50%. The adaptive and optimal sampling designs were for efficient estimation of X_1 with $\beta_1 = \log(1.5) \approx 0.405$.

Sample size	Sampling	Estimation	Criterion	Estimation performance by regression coefficient									
				α_1	α_2	α_3	α_4	α_5	α_6	β_1	β_2	β_3	β_4
$n = 200$, $N = 4000$	SRS	CC	$\sqrt{\text{MSE}}$	0.547	0.520	0.447	0.429	0.409	0.396	0.470	0.555	0.214	0.203
			Bias	-0.086	-0.076	-0.064	-0.034	-0.033	-0.008	0.017	-0.017	0.013	-0.009
			$\sqrt{\text{Var}}$	0.540	0.514	0.443	0.428	0.408	0.396	0.470	0.555	0.214	0.202
		MS	$\sqrt{\text{MSE}}$	0.479	0.381	0.330	0.316	0.306	0.301	0.332	0.602	0.236	0.219
			Bias	-0.267	-0.166	-0.112	-0.086	-0.074	-0.056	0.053	-0.021	0.020	-0.017
			$\sqrt{\text{Var}}$	0.398	0.343	0.310	0.304	0.297	0.295	0.330	0.601	0.235	0.218
	Balanced	MS	$\sqrt{\text{MSE}}$	0.405	0.401	0.399	0.398	0.398	0.405	0.400	0.859	0.317	0.299
			Bias	-0.023	-0.011	-0.009	-0.002	0.009	0.034	0.042	-0.034	0.025	-0.017
			$\sqrt{\text{Var}}$	0.404	0.400	0.399	0.398	0.398	0.403	0.398	0.858	0.317	0.298
	Adaptive	MS	$\sqrt{\text{MSE}}$	0.332	0.324	0.323	0.323	0.325	0.326	0.374	0.720	0.255	0.251
			Bias	-0.018	-0.007	-0.006	-0.001	0.006	0.024	0.049	-0.065	0.025	-0.028
			$\sqrt{\text{Var}}$	0.331	0.324	0.323	0.323	0.325	0.325	0.371	0.717	0.254	0.249
	Oracle	MS	$\sqrt{\text{MSE}}$	0.258	0.253	0.255	0.252	0.251	0.254	0.253	0.519	0.198	0.189
			Bias	-0.013	0.003	0.002	0.005	0.007	0.018	0.009	-0.027	0.001	-0.017
			$\sqrt{\text{Var}}$	0.257	0.253	0.255	0.252	0.251	0.253	0.252	0.518	0.198	0.188
Full cohort analysis			$\sqrt{\text{MSE}}$	0.107	0.096	0.091	0.085	0.080	0.079	0.094	0.116	0.044	0.042
			Bias	-0.010	0.000	-0.003	-0.002	-0.005	-0.002	0.003	-0.001	-0.000	0.000
			$\sqrt{\text{Var}}$	0.107	0.096	0.091	0.085	0.080	0.079	0.094	0.116	0.044	0.042

$(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6) = (-3.410, -3.027, -2.641, -2.249, -1.849, -1.435)$;
$(\beta_1, \beta_2, \beta_3, \beta_4) = (\log(1.5), \log(0.7), \log(1.3), -\log(1.3)) \approx (0.405, -0.357, 0.262, -0.262)$.

Table B.4: Relative performance for the estimation of regression parameters is compared for (i) the complete case analysis with simple random sampling (CC-SRS), (ii) the mean score method with simple random sampling (MS-SRS), (iii) a design-based estimation with balanced sample, equivalent to the mean score (MS-BAL), (iv & v) the mean score estimation with adaptive sampling (MS-A) and the optimal sampling design (MS-O), where $n = 400$ subsample was drawn from the full cohort size of $N = 4000$. Results for the full cohort estimator based on complete data are provided as a benchmark. Mean squared error (MSE) and its bias-variance decomposition are estimated from 1000 Monte Carlo replications, where the censoring rate was 50%. The adaptive and optimal sampling designs were for efficient estimation of X_1 with $\beta_1 = \log(1.5) \approx 0.405$.

Sample size	Sampling	Estimation	Criterion	Estimation performance by regression coefficient									
				α_1	α_2	α_3	α_4	α_5	α_6	β_1	β_2	β_3	β_4
$n = 400$, $N = 4000$	SRS	CC	$\sqrt{\text{MSE}}$	0.377	0.347	0.319	0.295	0.288	0.280	0.330	0.404	0.146	0.143
			Bias	-0.050	-0.045	-0.037	-0.014	-0.025	-0.008	0.014	0.000	0.006	-0.004
			$\sqrt{\text{Var}}$	0.374	0.344	0.316	0.295	0.287	0.280	0.329	0.404	0.146	0.143
		MS	$\sqrt{\text{MSE}}$	0.253	0.226	0.216	0.209	0.204	0.203	0.220	0.423	0.154	0.147
			Bias	-0.081	-0.046	-0.039	-0.032	-0.030	-0.022	0.035	0.005	0.007	-0.005
			$\sqrt{\text{Var}}$	0.240	0.222	0.213	0.207	0.202	0.202	0.217	0.423	0.154	0.147
	Balanced	MS	$\sqrt{\text{MSE}}$	0.284	0.279	0.276	0.276	0.277	0.277	0.278	0.582	0.200	0.204
			Bias	-0.014	-0.003	-0.004	-0.000	0.003	0.016	0.024	-0.007	0.006	-0.015
			$\sqrt{\text{Var}}$	0.284	0.279	0.276	0.276	0.277	0.277	0.277	0.582	0.200	0.204
	Adaptive	MS	$\sqrt{\text{MSE}}$	0.210	0.200	0.201	0.201	0.198	0.201	0.197	0.426	0.147	0.147
			Bias	-0.013	-0.003	-0.005	-0.003	-0.002	0.005	0.007	-0.014	0.006	-0.009
			$\sqrt{\text{Var}}$	0.210	0.200	0.200	0.201	0.198	0.200	0.197	0.426	0.147	0.147
	Oracle	MS	$\sqrt{\text{MSE}}$	0.189	0.184	0.182	0.177	0.178	0.178	0.182	0.354	0.134	0.132
			Bias	-0.003	0.007	0.005	0.006	0.005	0.011	0.003	-0.014	-0.005	-0.008
			$\sqrt{\text{Var}}$	0.189	0.184	0.182	0.177	0.178	0.177	0.182	0.354	0.133	0.132
Full cohort analysis			$\sqrt{\text{MSE}}$	0.107	0.096	0.091	0.085	0.080	0.079	0.094	0.116	0.044	0.042
			Bias	-0.010	0.000	-0.003	-0.002	-0.005	-0.002	0.003	-0.001	-0.000	0.000
			$\sqrt{\text{Var}}$	0.107	0.096	0.091	0.085	0.080	0.079	0.094	0.116	0.044	0.042

$(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6) = (-3.410, -3.027, -2.641, -2.249, -1.849, -1.435)$;
$(\beta_1, \beta_2, \beta_3, \beta_4) = (\log(1.5), \log(0.7), \log(1.3), -\log(1.3)) \approx (0.405, -0.357, 0.262, -0.262)$.

Table B.5: Relative performance for the estimation of regression parameters is compared for (i) the complete case analysis with simple random sampling (CC-SRS), (ii) the mean score method with simple random sampling (MS-SRS), (iii) a design-based estimation with balanced sample, equivalent to the mean score (MS-BAL), (iv & v) the mean score estimation with adaptive sampling (MS-A) and the optimal sampling design (MS-O), where $n = 800$ subsample was drawn from the full cohort size of $N = 4000$. Results for the full cohort estimator based on complete data are provided as a benchmark. Mean squared error (MSE) and its bias-variance decomposition are estimated from 1000 Monte Carlo replications, where the censoring rate was 50%. The adaptive and optimal sampling designs were for efficient estimation of X_1 with $\beta_1 = \log(1.5) \approx 0.405$.

Sample size	Sampling	Estimation	Criterion	Estimation performance by regression coefficient									
				α_1	α_2	α_3	α_4	α_5	α_6	β_1	β_2	β_3	β_4
$n = 800$ $N = 4000$	SRS	CC	$\sqrt{\text{MSE}}$	0.262	0.239	0.220	0.206	0.199	0.191	0.228	0.286	0.101	0.101
			Bias	-0.030	-0.018	-0.016	-0.007	-0.016	-0.005	0.005	0.002	0.005	-0.004
			$\sqrt{\text{Var}}$	0.260	0.238	0.220	0.206	0.198	0.191	0.228	0.286	0.101	0.101
		MS	$\sqrt{\text{MSE}}$	0.169	0.156	0.151	0.149	0.146	0.144	0.155	0.292	0.104	0.102
			Bias	-0.022	-0.008	-0.009	-0.008	-0.009	-0.004	0.004	0.001	0.007	-0.004
			$\sqrt{\text{Var}}$	0.167	0.156	0.151	0.148	0.145	0.144	0.155	0.292	0.103	0.102
	Balanced	MS	$\sqrt{\text{MSE}}$	0.202	0.196	0.193	0.191	0.189	0.190	0.194	0.396	0.145	0.141
			Bias	-0.017	-0.006	-0.009	-0.007	-0.007	0.000	0.010	0.008	-0.002	0.002
			$\sqrt{\text{Var}}$	0.201	0.195	0.193	0.190	0.189	0.190	0.194	0.396	0.145	0.141
	Adaptive	MS	$\sqrt{\text{MSE}}$	0.154	0.149	0.145	0.142	0.139	0.140	0.147	0.280	0.099	0.102
			Bias	-0.010	-0.001	-0.003	-0.003	-0.003	0.001	0.006	-0.010	0.005	-0.003
			$\sqrt{\text{Var}}$	0.154	0.149	0.145	0.142	0.139	0.140	0.147	0.280	0.099	0.102
	Oracle	MS	$\sqrt{\text{MSE}}$	0.147	0.139	0.137	0.135	0.131	0.131	0.133	0.261	0.096	0.095
			Bias	-0.004	0.006	0.003	0.004	0.003	0.007	0.005	-0.018	0.002	-0.005
			$\sqrt{\text{Var}}$	0.147	0.139	0.137	0.135	0.131	0.131	0.133	0.260	0.096	0.095
Full cohort analysis			$\sqrt{\text{MSE}}$	0.107	0.096	0.091	0.085	0.080	0.079	0.094	0.116	0.044	0.042
			Bias	-0.010	0.000	-0.003	-0.002	-0.005	-0.002	0.003	-0.001	-0.000	0.000
			$\sqrt{\text{Var}}$	0.107	0.096	0.091	0.085	0.080	0.079	0.094	0.116	0.044	0.042

$(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6) = (-3.410, -3.027, -2.641, -2.249, -1.849, -1.435)$;
$(\beta_1, \beta_2, \beta_3, \beta_4) = (\log(1.5), \log(0.7), \log(1.3), -\log(1.3)) \approx (0.405, -0.357, 0.262, -0.262)$.

Table B.6: Relative performance for the estimation of regression parameters is compared for (i) the complete case analysis with simple random sampling (CC-SRS), (ii) the mean score method with simple random sampling (MS-SRS), (iii) a design-based estimation with balanced sample, equivalent to the mean score (MS-BAL), (iv & v) the mean score estimation with adaptive sampling (MS-A) and the optimal sampling design (MS-O), where $n = 400$ subsample was drawn from the full cohort size of $N = 2000$. Results for the full cohort estimator based on complete data are provided as a benchmark. Mean squared error (MSE) and its bias-variance decomposition are estimated from 1000 Monte Carlo replications, where the censoring rate was 50%. The adaptive and optimal sampling designs were for efficient estimation of X_1 with $\beta_1 = \log(1.5) \approx 0.405$.

Sample size	Sampling	Estimation	Criterion	Estimation performance by regression coefficient									
				α_1	α_2	α_3	α_4	α_5	α_6	β_1	β_2	β_3	β_4
$n = 400$, $N = 2000$	SRS	CC	$\sqrt{\text{MSE}}$	0.381	0.342	0.323	0.288	0.281	0.284	0.324	0.419	0.145	0.147
			Bias	-0.049	-0.044	-0.039	-0.025	-0.020	-0.019	-0.011	0.029	0.006	-0.005
			$\sqrt{\text{Var}}$	0.378	0.340	0.321	0.287	0.280	0.283	0.324	0.418	0.145	0.147
		MS	$\sqrt{\text{MSE}}$	0.267	0.245	0.232	0.224	0.217	0.223	0.237	0.434	0.150	0.156
			Bias	-0.072	-0.050	-0.044	-0.036	-0.028	-0.031	0.020	0.027	0.005	-0.004
			$\sqrt{\text{Var}}$	0.258	0.240	0.228	0.221	0.215	0.221	0.236	0.433	0.150	0.156
	Balanced	MS	$\sqrt{\text{MSE}}$	0.278	0.275	0.267	0.266	0.263	0.266	0.276	0.550	0.200	0.198
			Bias	-0.024	-0.018	-0.019	-0.014	-0.006	-0.003	0.026	-0.005	0.013	-0.005
			$\sqrt{\text{Var}}$	0.277	0.274	0.266	0.265	0.263	0.266	0.275	0.550	0.199	0.198
	Adaptive	MS	$\sqrt{\text{MSE}}$	0.233	0.220	0.211	0.210	0.207	0.210	0.214	0.421	0.152	0.148
			Bias	-0.019	-0.014	-0.016	-0.012	-0.005	-0.007	0.004	0.006	0.010	-0.016
			$\sqrt{\text{Var}}$	0.232	0.219	0.210	0.209	0.207	0.210	0.214	0.421	0.152	0.147
	Oracle	MS	$\sqrt{\text{MSE}}$	0.215	0.204	0.195	0.195	0.186	0.195	0.202	0.383	0.134	0.136
			Bias	-0.021	-0.012	-0.014	-0.010	-0.004	-0.008	-0.012	0.021	0.001	-0.004
			$\sqrt{\text{Var}}$	0.214	0.203	0.194	0.194	0.186	0.195	0.202	0.382	0.134	0.136
Full cohort analysis			$\sqrt{\text{MSE}}$	0.152	0.136	0.124	0.121	0.112	0.118	0.129	0.172	0.062	0.061
			Bias	-0.009	-0.005	-0.008	-0.006	-0.002	-0.008	-0.004	0.010	0.001	-0.001
			$\sqrt{\text{Var}}$	0.152	0.136	0.124	0.121	0.112	0.118	0.129	0.171	0.062	0.061

$(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6) = (-3.410, -3.027, -2.641, -2.249, -1.849, -1.435)$;
$(\beta_1, \beta_2, \beta_3, \beta_4) = (\log(1.5), \log(0.7), \log(1.3), -\log(1.3)) \approx (0.405, -0.357, 0.262, -0.262)$.

Table B.7: Relative performance for the estimation of regression parameters is compared for (i) the complete case analysis with simple random sampling (CC-SRS), (ii) the mean score method with simple random sampling (MS-SRS), (iii) a design-based estimation with balanced sample, equivalent to the mean score (MS-BAL), (iv & v) the mean score estimation with adaptive sampling (MS-A) and the optimal sampling design (MS-O), where $n = 400$ subsample was drawn from the full cohort size of $N = 8000$. Results for the full cohort estimator based on complete data are provided as a benchmark. Mean squared error (MSE) and its bias-variance decomposition are estimated from 1000 Monte Carlo replications, where the censoring rate was 50%. The adaptive and optimal sampling designs were for efficient estimation of X_1 with $\beta_1 = \log(1.5) \approx 0.405$.

Sample size	Sampling	Estimation	Criterion	Estimation performance by regression coefficient									
				α_1	α_2	α_3	α_4	α_5	α_6	β_1	β_2	β_3	β_4
$n = 400$, $N = 8000$	SRS	CC	$\sqrt{\text{MSE}}$	0.381	0.334	0.316	0.289	0.281	0.280	0.321	0.419	0.155	0.149
			Bias	-0.042	-0.037	-0.034	-0.020	-0.014	-0.004	-0.001	0.001	0.012	-0.007
			$\sqrt{\text{Var}}$	0.379	0.332	0.314	0.288	0.280	0.280	0.321	0.419	0.154	0.149
		MS	$\sqrt{\text{MSE}}$	0.236	0.211	0.208	0.201	0.199	0.200	0.200	0.435	0.162	0.157
			Bias	-0.071	-0.043	-0.034	-0.028	-0.022	-0.017	0.027	-0.000	0.015	-0.009
			$\sqrt{\text{Var}}$	0.225	0.206	0.205	0.199	0.198	0.200	0.198	0.435	0.161	0.157
	Balanced	MS	$\sqrt{\text{MSE}}$	0.272	0.271	0.271	0.268	0.269	0.270	0.265	0.572	0.211	0.207
			Bias	-0.012	-0.009	-0.007	-0.004	0.004	0.014	0.021	-0.002	0.007	-0.017
			$\sqrt{\text{Var}}$	0.272	0.271	0.271	0.268	0.269	0.270	0.264	0.572	0.211	0.207
	Adaptive	MS	$\sqrt{\text{MSE}}$	0.196	0.194	0.195	0.193	0.194	0.195	0.190	0.417	0.153	0.148
			Bias	-0.009	-0.008	-0.007	-0.005	-0.001	0.004	0.002	-0.014	0.008	0.001
			$\sqrt{\text{Var}}$	0.196	0.193	0.195	0.193	0.194	0.195	0.189	0.417	0.153	0.148
	Oracle	MS	$\sqrt{\text{MSE}}$	0.178	0.174	0.170	0.170	0.171	0.172	0.174	0.362	0.134	0.133
			Bias	-0.006	-0.004	-0.002	-0.002	0.002	0.005	-0.005	-0.010	-0.001	0.008
			$\sqrt{\text{Var}}$	0.178	0.174	0.170	0.170	0.171	0.172	0.174	0.362	0.134	0.133
Full cohort analysis			$\sqrt{\text{MSE}}$	0.076	0.071	0.063	0.060	0.057	0.055	0.067	0.085	0.030	0.031
			Bias	-0.002	-0.001	-0.001	-0.001	0.001	0.001	0.000	-0.003	0.001	0.000
			$\sqrt{\text{Var}}$	0.076	0.071	0.063	0.060	0.057	0.055	0.067	0.085	0.030	0.031

$(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6) = (-3.410, -3.027, -2.641, -2.249, -1.849, -1.435)$;
$(\beta_1, \beta_2, \beta_3, \beta_4) = (\log(1.5), \log(0.7), \log(1.3), -\log(1.3)) \approx (0.405, -0.357, 0.262, -0.262)$.

B.2 Two-phase analysis with the modified sampling design

Table B.8: Modification of the proposed mean score sampling design for the discrete-time survival analysis of the National Wilms Tumor Study to include all individuals regardless of the time to censoring ($N = 3915$). Similarly to Table 2, we compare five different methods (CC-SRS, MS-SRS, MS-BAL, MS-A, MS-O) as well as two modified designs by under-sampling the censored individuals not relapsing before the end of the follow-up period in the balanced and the pilot sample.[†,‡] The optimal sampling design was estimated using the full cohort data. The MS-A and MS-O designs are for efficient estimation of the interaction effect between unfavorable histology and disease stage. Results from the full cohort analysis with complete data are presented as a benchmark. Mean squared error and its bias-variance decomposition are estimated using 1000 phase two samples of $n = 400$ from the full cohort ($N = 3915$).

Sampling	Estimation	Criterion	Baseline hazard in complementary log-log scale						Regression coefficient				
			0.5yr	1yr	1.5yr	2yr	2.5yr	3yr	UH[1]	Stage[2]	Age[3]	dTmr[4]	U∗S[5]
Full cohort analysis		Ref.	-4.074	-3.916	-4.373	-5.037	-5.378	-5.737	1.087	0.287	0.064	0.031	0.632
SRS	CC	$\sqrt{\text{MSE}}$	0.469	0.460	0.479	0.706	1.537	2.601	0.640	0.340	0.050	0.034	0.763
		Bias	-0.074	-0.067	-0.074	-0.136	-0.283	-0.694	-0.025	-0.011	-0.001	0.001	0.070
		$\sqrt{\text{Var}}$	0.463	0.455	0.473	0.693	1.511	2.507	0.640	0.340	0.050	0.034	0.759
	MS	$\sqrt{\text{MSE}}$	0.433	0.428	0.429	0.580	1.464	2.587	0.635	0.355	0.051	0.037	0.815
		Bias	-0.028	-0.023	-0.035	-0.109	-0.330	-0.829	-0.102	-0.005	0.001	0.001	0.131
		$\sqrt{\text{Var}}$	0.432	0.428	0.428	0.570	1.426	2.450	0.627	0.355	0.051	0.037	0.804
Balanced	MS	$\sqrt{\text{MSE}}$	0.512	0.504	0.493	0.485	0.482	0.481	0.460	0.437	0.068	0.043	0.712
		Bias	-0.123	-0.105	-0.088	-0.077	-0.071	-0.067	0.048	0.046	0.018	0.004	-0.051
		$\sqrt{\text{Var}}$	0.497	0.492	0.485	0.479	0.477	0.476	0.457	0.435	0.066	0.042	0.710
Balanced (†modified)	MS	$\sqrt{\text{MSE}}$	0.436	0.428	0.419	0.412	0.410	0.409	0.403	0.371	0.061	0.036	0.614
		Bias	-0.090	-0.075	-0.062	-0.054	-0.049	-0.046	0.038	0.009	0.016	0.003	0.002
		$\sqrt{\text{Var}}$	0.426	0.421	0.415	0.409	0.407	0.406	0.401	0.371	0.059	0.036	0.614
Adaptive	MS	$\sqrt{\text{MSE}}$	0.350	0.343	0.336	0.331	0.329	0.327	0.326	0.313	0.045	0.028	0.519
		Bias	-0.071	-0.060	-0.052	-0.047	-0.044	-0.042	-0.010	0.002	0.009	0.003	0.056
		$\sqrt{\text{Var}}$	0.342	0.337	0.332	0.327	0.326	0.325	0.326	0.313	0.044	0.027	0.516
Adaptive (‡modified)	MS	$\sqrt{\text{MSE}}$	0.342	0.336	0.329	0.324	0.323	0.322	0.290	0.266	0.042	0.027	0.448
		Bias	-0.060	-0.052	-0.045	-0.040	-0.038	-0.036	-0.003	-0.014	0.009	0.002	0.057
		$\sqrt{\text{Var}}$	0.337	0.332	0.326	0.322	0.320	0.320	0.290	0.265	0.041	0.027	0.445
Oracle	MS	$\sqrt{\text{MSE}}$	0.269	0.267	0.264	0.261	0.261	0.260	0.234	0.232	0.035	0.022	0.379
		Bias	-0.007	-0.009	-0.008	-0.006	-0.006	-0.005	0.006	0.008	0.002	-0.000	0.030
		$\sqrt{\text{Var}}$	0.269	0.267	0.264	0.261	0.261	0.260	0.234	0.232	0.034	0.022	0.377

[1] Unfavorable histology versus favorable; [2] disease stage III/IV versus I/II;
[3] year at diagnosis; [4] tumor diameter (cm); [5] interaction effect between UH and Stage.

Table B.9: Modification of the proposed mean score sampling design for the two-phase continuous-time Cox model analysis of time to relapse in the National Wilms Tumor Study to include all individuals regardless of the time to censoring ($N = 3915$). Similarly to Table 4, we used inverse probability weights (IPW) for the two-phase analysis for four different sampling designs for the second phase; (i) simple random sampling (SRS), (ii) balanced sampling, (iii & iv) the proposed adaptive and oracle sampling designs as well as two modified designs by under-sampling the censored individuals not relapsing before the end of the follow-up period in the balanced and the pilot sample.[†,‡] Here, the adaptive and oracle designs were determined by the mean score method for the discrete-time survival analysis. The target parameter for the mean score design was the interaction between unfavorable histology and late stage disease. Mean squared error and its bias-variance decomposition are estimated from 1000 phase two subsamples of $n = 400$ from the full cohort. Reference parameters estimates are from the full cohort analysis using the continuous-time Cox model with complete data on all subjects.

Sampling	Criterion	Estimation performance by regressor				
		UH[1]	Stage[2]	Age[3]	dTmr[4]	U∗S[5]
Full cohort analysis	Ref.	1.050	0.297	0.065	0.021	0.620
SRS	$\sqrt{\text{MSE}}$	0.444	0.322	0.049	0.033	0.648
	Bias	-0.092	0.009	-0.000	0.000	0.115
	$\sqrt{\text{Var}}$	0.434	0.322	0.049	0.033	0.638
Balanced	$\sqrt{\text{MSE}}$	0.478	0.519	0.074	0.049	0.752
	Bias	0.061	0.032	0.016	0.006	-0.037
	$\sqrt{\text{Var}}$	0.474	0.518	0.072	0.048	0.751
Balanced (†modified)	$\sqrt{\text{MSE}}$	0.440	0.434	0.065	0.043	0.643
	Bias	0.039	0.012	0.011	0.008	0.025
	$\sqrt{\text{Var}}$	0.438	0.434	0.065	0.043	0.643
Adaptive	$\sqrt{\text{MSE}}$	0.338	0.373	0.052	0.034	0.541
	Bias	-0.001	0.005	0.005	0.002	0.035
	$\sqrt{\text{Var}}$	0.338	0.373	0.052	0.034	0.540
Adaptive (†modified)	$\sqrt{\text{MSE}}$	0.328	0.325	0.050	0.031	0.493
	Bias	-0.023	0.000	0.008	0.002	0.053
	$\sqrt{\text{Var}}$	0.327	0.325	0.049	0.031	0.491
Oracle	$\sqrt{\text{MSE}}$	0.255	0.250	0.041	0.025	0.396
	Bias	-0.001	-0.006	0.004	0.001	0.043
	$\sqrt{\text{Var}}$	0.255	0.250	0.041	0.025	0.393

[1] Unfavorable histology versus favorable; [2] disease stage III/IV versus I/II;
[3] year at diagnosis; [4] tumor diameter (cm); [5] interaction effect between UH and Stage.

Table B.10: Modification of the proposed mean score sampling design for the two-phase continuous-time Cox model analysis with the exponential survival function. We used inverse probability weights (IPW) for the two-phase analysis for four different sampling designs for the second phase; (i) simple random sampling (SRS), (ii) balanced sampling, (iii & iv) the proposed adaptive and oracle sampling designs as well as two modified designs by under-sampling individuals censored before the maximum follow-up period t_6 in the balanced and the pilot sample.†,‡ Here, the adaptive and oracle designs were determined by the mean score method for the discrete-time survival analysis. The target parameter for the mean score design was $\beta_1 = \log(1.5) \approx 0.405$. Mean squared error and its bias-variance decomposition are estimated from 1000 Monte Carlo replications, where the censoring rate was 70%. The full cohort analysis provides a benchmark for performance, the continuous-time Cox model was fit using the complete data on all subjects.

Sampling	Criterion	Estimation performance by sample sizes					
		$N = 4000$			$n = 400$		
		$n = 200$	$n = 400$	$n = 800$	$N = 2000$	$N = 4000$	$N = 8000$
Full cohort	$\sqrt{\text{MSE}}$	0.174	0.174	0.174	0.252	0.174	0.125
	Bias	0.004	0.004	0.004	0.011	0.004	-0.001
	$\sqrt{\text{Var}}$	0.174	0.174	0.174	0.252	0.174	0.125
SRS	$\sqrt{\text{MSE}}$	0.512	0.395	0.286	0.424	0.395	0.393
	Bias	-0.027	-0.012	0.004	0.027	-0.012	0.010
	$\sqrt{\text{Var}}$	0.511	0.395	0.286	0.423	0.395	0.393
Balanced	$\sqrt{\text{MSE}}$	0.826	0.506	0.340	0.499	0.506	0.504
	Bias	0.160	0.084	0.032	0.069	0.084	0.069
	$\sqrt{\text{Var}}$	0.810	0.499	0.338	0.495	0.499	0.499
Balanced (†modified)	$\sqrt{\text{MSE}}$	0.637	0.429	0.274	0.403	0.429	0.408
	Bias	0.139	0.044	0.035	0.057	0.044	0.069
	$\sqrt{\text{Var}}$	0.621	0.427	0.271	0.399	0.427	0.402
Adaptive	$\sqrt{\text{MSE}}$	0.791	0.347	0.231	0.395	0.347	0.331
	Bias	0.118	0.015	0.026	0.046	0.015	0.015
	$\sqrt{\text{Var}}$	0.782	0.347	0.230	0.392	0.347	0.330
Adaptive (‡modified)	$\sqrt{\text{MSE}}$	0.599	0.297	0.223	0.337	0.297	0.287
	Bias	0.090	0.026	0.012	0.039	0.026	0.000
	$\sqrt{\text{Var}}$	0.592	0.296	0.223	0.334	0.296	0.287
Oracle	$\sqrt{\text{MSE}}$	0.384	0.272	0.218	0.320	0.272	0.266
	Bias	-0.003	0.004	0.005	0.015	0.004	-0.005
	$\sqrt{\text{Var}}$	0.384	0.272	0.218	0.319	0.272	0.266

$(\beta_1, \beta_2, \beta_3, \beta_4) = (\log(1.5), \log(0.7), \log(1.3), -\log(1.3))$.

Table B.11: The phase two sampling design for a single dataset analysis in the simulation study for Table B.10. We demonstrated the stratified sampling size for the proposed adaptive and oracle sampling designs as well as the modified design by under-sampling individuals censored before the maximum follow-up period t_6 in the pilot sample.†,‡ The full cohort panel shows the information of the phase one study.

Sampling	Stratification		$Y = t_1$	$Y = t_2$	$Y = t_3$	$Y = t_4$	$Y = t_5$	$Y = t_6$	Sum
Full cohort	$\Delta = 0$	$Z = 1$	31	19	15	13	14	291	383
		$Z = 2$	56	56	24	34	30	637	837
		$Z = 3$	71	60	39	45	37	742	994
		$Z = 4$	61	60	48	35	27	572	803
	Sum		219	195	126	127	108	2242	3017
	$\Delta = 1$	$Z = 1$	17	26	14	20	16	17	110
		$Z = 2$	49	47	32	30	37	20	215
		$Z = 3$	73	82	64	47	49	40	355
		$Z = 4$	79	59	52	35	45	33	303
	Sum		218	214	162	132	147	110	983
Adaptive	$\Delta = 0$	$Z = 1$	4	7	4	4	4	25	48
		$Z = 2$	4	4	4	4	4	36	56
		$Z = 3$	4	4	4	4	4	22	42
		$Z = 4$	4	4	4	4	6	45	67
	Sum		16	19	16	16	18	128	213
	$\Delta = 1$	$Z = 1$	6	4	5	4	4	4	27
		$Z = 2$	5	6	4	6	7	5	33
		$Z = 3$	27	27	6	10	4	4	78
		$Z = 4$	18	8	7	4	7	5	49
	Sum		56	45	22	24	22	18	187
Adaptive (†modified)	$\Delta = 0$	$Z = 1$	2	2	2	2	2	8	18
		$Z = 2$	2	2	2	2	2	60	70
		$Z = 3$	2	3	2	2	2	45	56
		$Z = 4$	2	2	2	2	4	44	56
	Sum		8	9	8	8	10	157	200
	$\Delta = 1$	$Z = 1$	6	6	5	6	6	5	34
		$Z = 2$	8	7	8	6	8	6	43
		$Z = 3$	16	24	17	9	9	6	81
		$Z = 4$	11	7	6	6	6	6	42
	Sum		41	44	36	27	29	23	200
Oracle	$\Delta = 0$	$Z = 1$	1	1	1	1	1	24	29
		$Z = 2$	1	2	1	2	3	63	72
		$Z = 3$	1	2	2	3	4	90	102
		$Z = 4$	1	3	3	3	3	67	80
	Sum		4	8	7	9	11	244	283
	$\Delta = 1$	$Z = 1$	3	3	1	2	2	2	13
		$Z = 2$	9	6	4	3	3	1	26
		$Z = 3$	12	13	8	5	5	3	46
		$Z = 4$	12	7	5	3	3	2	32
	Sum		36	29	18	13	13	8	117