

# Mathematical modelling of SARS-CoV-2 variant outbreaks reveals their probability of extinction

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## Supplementary Information

In this appendix a simplified model is presented and analyzed to demonstrate the counter-intuitive non-monotonous relationship between extinction probability and reproduction number. Consider a discrete time random process  $x : N \rightarrow R_+ \cup 0$  with initial distribution on  $\{0, 1\}$ . The process constitutes a simple model of the number of infected, and its initial value  $x(0) \in \{0, 1\}$  constitutes a simple model of the infection being extinct or not. The evolution of  $x$  is defined by  $x(n) = \exp(a)x(n-1)$ , where  $a$  can be viewed as a growth rate and  $\exp(a)$  is equivalent to the reproduction number. If  $x(0) = 0$ , then  $x(n) = 0$  for all  $n \geq 0$ , whereas if  $x(0) = 1$ , then  $x(n) = \exp(a)^n = \exp(an)$ . At time  $N > 0$  a testing procedure is modelled with the assumption, that each individual of (an effectively infinite population) is tested with probability  $p$ , which means that the probability of finding 0 infected is therefore  $(1-p)^{x(N)}$ . To model the higher (a-priori) extinction probability of lower growth rates we set the probability of extinction ( $P(x(0) = 0)$ ) as

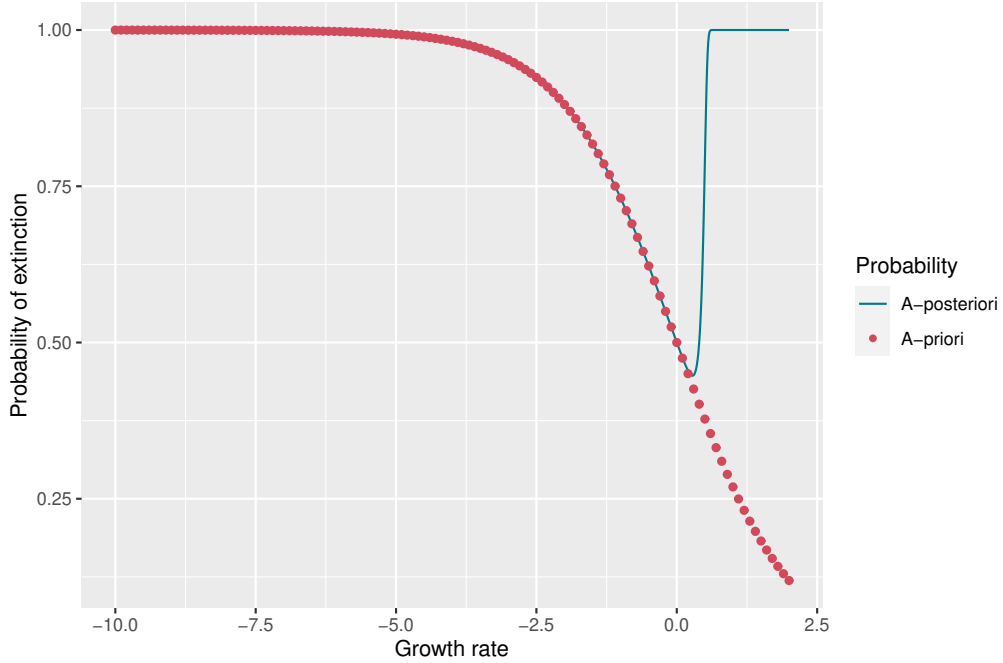
$$P(x(0) = 0) = \frac{1}{1 + \exp(a/\alpha)},$$

where  $\alpha > 0$  is a shaping parameter. Thus

$$\begin{aligned} \lim_{a \rightarrow -\infty} P(x(0) = 0) &= 1, \\ \lim_{a \rightarrow \infty} P(x(0) = 0) &= 0. \end{aligned}$$

Finally, we can compute the conditional probability  $P_{e|0}$  of extinction given 0 infected were found at the test at time  $N$  by

$$\begin{aligned} P_{e|0} &= \frac{P(x(0) = 0)}{P(x(0) = 0) + P(x(0) = 1)(1-p)^{x(N)}} \\ &= \frac{\frac{1}{1 + \exp(a/\alpha)}}{\frac{1}{1 + \exp(a/\alpha)} + \frac{\exp(a/\alpha)}{1 + \exp(a/\alpha)}(1-p)^{\exp(aN)}} \\ &= \frac{1}{1 + \exp(a/\alpha)(1-p)^{\exp(aN)}} \end{aligned}$$



Supplementary Figure 1: The a-priori and a-posteriori extinction probabilities,  $P(x(0) = 0)$  and  $P_{e|0}$ , respectively, for varying growth rates  $a$ .

The conditional (a-posterior) probability  $P_{e|0}$  is shown in Figure 1 for  $\alpha = 1, N = 15, p = 10^{-3}$  and varying growth rates. Figure 1 clearly indicates the (counter-intuitive) non-monotonous relationship between growth rate and the a-posterior probability of extinction  $P_{e|0}$ .  $P_{e|0}$  is also minimal for a growth rate slightly higher than 0 as is the case for the elaborate model.