

## 1 SUPPLEMENTARY MATERIAL

### 1.1 MUSHA Full Kinematic Model

The full kinematic model of MUSHA hand is equal to:

$$\begin{aligned}
 x_{r,T} &= \cos(q_W)(a_{2,T} \cos(q_{2,T}) + a_{3,T} \cos(q_{3,T})) \\
 y_{r,T} &= \sin(q_W)(a_{2,T} \cos(q_{2,T}) + a_{3,T} \cos(q_{3,T})) \\
 z_{r,T} &= a_{2,T} \sin(q_{2,T}) + a_{3,T} \sin(q_{3,T}) \\
 a_{2,T} &= \begin{cases} \left(\frac{6L_{2,T}}{5q_{2,T}}\right) \sin\left(\frac{5q_{2,T}}{6}\right) \\ L_{2,T} \text{ if } q_{2,T} = 0 \end{cases} \\
 a_{3,T} &= \begin{cases} \left(\frac{6L_{3,T}}{5q_{3,T}}\right) \sin\left(\frac{5q_{3,T}}{6}\right) \\ L_{3,T} \text{ if } q_{3,T} = 0 \end{cases} \\
 x_{r,I} &= \cos\left(q_W + \frac{2\pi}{3}\right)(a_{PIP} \cos(q_{PIP}) + a_{3,I} \cos(q_{3,I})) \\
 y_{r,I} &= \sin\left(q_W + \frac{2\pi}{3}\right)(a_{PIP} \cos(q_{PIP}) + a_{3,I} \cos(q_{3,I})) \\
 z_{r,I} &= a_{PIP} \sin(q_{PIP}) + a_{3,I} \sin(q_{3,I}) \\
 a_{PIP} &= \begin{cases} \left(\frac{6L_{PIP}}{5q_{PIP}}\right) \sin\left(\frac{5q_{PIP}}{6}\right) \\ L_{PIP} \text{ if } q_{PIP} = 0 \end{cases} \\
 a_{3,I} &= \begin{cases} \left(\frac{6L_{3,I}}{5q_{3,I}}\right) \sin\left(\frac{5q_{3,I}}{6}\right) \\ L_{3,I} \text{ if } q_{3,I} = 0 \end{cases} \\
 x_{r,M} &= \cos\left(q_W + \frac{4\pi}{3}\right)(a_{PIP} \cos(q_{PIP}) + a_{3,M} \cos(q_{3,M})) \\
 y_{r,M} &= \sin\left(q_W + \frac{4\pi}{3}\right)(a_{PIP} \cos(q_{PIP}) + a_{3,M} \cos(q_{3,M})) \\
 z_{r,M} &= a_{PIP} \sin(q_{PIP}) + a_{3,M} \sin(q_{3,M}) \\
 a_{3,M} &= \begin{cases} \left(\frac{6L_{3,M}}{5q_{3,M}}\right) \sin\left(\frac{5q_{3,M}}{6}\right) \\ L_{3,M} \text{ if } q_{3,M} = 0 \end{cases}
 \end{aligned} \tag{1}$$

## 1.2 MUSHA Differential Model

The full differential kinematic model of MUSHA hand is equal to  $\dot{p}_r = J_r \dot{q}_r$  with:

$$\begin{aligned}
J_{r11} &= -s_W \frac{6}{5} \left( \frac{1}{q_{2,T}} s_{\frac{5}{6}2,T} c_{2,T} + \frac{1}{q_{3,T}} s_{\frac{5}{6}3,T} c_{3,T} \right) \\
J_{r12} &= c_W \frac{6}{5} \left( \frac{1}{q_{2,T}} (c_{2,T} \frac{5}{6} c_{\frac{5}{6}2,T} - s_{2,T} s_{\frac{5}{6}2,T}) - \frac{1}{q_{2,T}^2} c_{2,T} s_{\frac{5}{6}2,T} \right) \\
J_{r13} &= c_W \frac{6}{5} \left( \frac{1}{q_{3,T}} (c_{3,T} \frac{5}{6} c_{\frac{5}{6}3,T} - s_{3,T} s_{\frac{5}{6}3,T}) - \frac{1}{q_{3,T}^2} c_{3,T} s_{\frac{5}{6}3,T} \right) \\
J_{r21} &= c_W \frac{6}{5} \left( \frac{1}{q_{2,T}} s_{\frac{5}{6}2,T} c_{2,T} + \frac{1}{q_{3,T}} s_{\frac{5}{6}3,T} c_{3,T} \right) \\
J_{r22} &= s_W \frac{6}{5} \left( \frac{1}{q_{2,T}} (s_{2,T} s_{\frac{5}{6}2,T} - c_{2,T} \frac{5}{6} c_{\frac{5}{6}2,T}) + \frac{1}{q_{2,T}^2} c_{2,T} s_{\frac{5}{6}2,T} \right) \\
J_{r23} &= s_W \frac{6}{5} \left( \frac{1}{q_{3,T}} (s_{3,T} s_{\frac{5}{6}3,T} - c_{3,T} \frac{5}{6} c_{\frac{5}{6}3,T}) + \frac{1}{q_{3,T}^2} c_{3,T} s_{\frac{5}{6}3,T} \right) \\
J_{r32} &= c_{2,T} \frac{5}{6q_{2,T}} s_{\frac{5}{6}2,T} + s_{2,T} \left( \frac{1}{q_{2,T}} c_{\frac{5}{6}2,T} - \frac{6}{5q_{2,T}^2} s_{\frac{5}{6}2,T} \right) \\
J_{r33} &= c_{3,T} \frac{5}{6q_{3,T}} s_{\frac{5}{6}3,T} + s_{3,T} \left( \frac{1}{q_{3,T}} c_{\frac{5}{6}3,T} - \frac{6}{5q_{3,T}^2} s_{\frac{5}{6}3,T} \right) \\
J_{r41} &= -s_W + \frac{2\pi}{3} \frac{6}{5} \left( \frac{1}{q_{PIP}} s_{\frac{5}{6}2,I} c_{PIP} + \frac{1}{q_{3,I}} s_{\frac{5}{6}3,I} c_{3,I} \right) \\
J_{r44} &= c_W + \frac{2\pi}{3} \frac{6}{5} \left( \frac{1}{q_{PIP}} (c_{PIP} \frac{5}{6} c_{\frac{5}{6}PIP} - s_{PIP} s_{\frac{5}{6}PIP}) \right) - c_W + \frac{2\pi}{3} \frac{6}{5} \left( \frac{1}{q_{PIP}^2} c_{PIP} s_{\frac{5}{6}PIP} \right) \\
J_{r45} &= c_W + \frac{2\pi}{3} \frac{6}{5} \left( \frac{1}{q_{3,I}} (c_{3,I} \frac{5}{6} c_{\frac{5}{6}3,I} - s_{3,I} s_{\frac{5}{6}3,I}) - \frac{1}{q_{3,I}^2} c_{3,I} s_{\frac{5}{6}3,I} \right) \\
J_{r51} &= c_W + \frac{2\pi}{3} \frac{6}{5} \left( \frac{1}{q_{PIP}} s_{\frac{5}{6}p} c_{PIP} + \frac{1}{q_{3,I}} s_{\frac{5}{6}3,I} c_{3,I} \right) \\
J_{r54} &= s_W + \frac{2\pi}{3} \frac{6}{5} \left( \frac{1}{q_{PIP}} (s_{PIP} s_{\frac{5}{6}PIP} - c_{PIP} \frac{5}{6} c_{\frac{5}{6}PIP}) \right) + s_W + \frac{2\pi}{3} \frac{6}{5} \left( \frac{1}{q_{PIP}^2} c_{PIP} s_{\frac{5}{6}PIP} \right) \\
J_{r55} &= s_W + \frac{2\pi}{3} \frac{6}{5} \left( \frac{1}{q_{3,I}} (s_{3,I} s_{\frac{5}{6}3,I} - c_{3,I} \frac{5}{6} c_{\frac{5}{6}3,I}) + \frac{1}{q_{3,I}^2} c_{3,I} s_{\frac{5}{6}3,I} \right) \\
J_{r65} &= c_{3,I} \frac{5}{6q_{3,I}} s_{\frac{5}{6}3,I} + s_{3,I} \left( \frac{1}{q_{3,I}} c_{\frac{5}{6}3,I} - \frac{6}{5q_{3,I}^2} s_{\frac{5}{6}3,I} \right) \\
J_{r71} &= -s_W + \frac{4\pi}{3} \frac{6}{5} \left( \frac{1}{q_{2,M}} s_{\frac{5}{6}2,M} c_{2,M} + \frac{1}{q_{3,M}} s_{\frac{5}{6}3,M} c_{3,M} \right) \\
J_{r74} &= c_W + \frac{4\pi}{3} \frac{6}{5} \left( \frac{1}{q_{PIP}} (c_{PIP} \frac{5}{6} c_{\frac{5}{6}PIP} - s_{PIP} s_{\frac{5}{6}PIP}) \right) - c_W + \frac{4\pi}{3} \frac{6}{5} \left( \frac{1}{q_{PIP}^2} c_{PIP} s_{\frac{5}{6}PIP} \right) \\
J_{r76} &= c_W + \frac{4\pi}{3} \frac{6}{5} \frac{1}{q_{3,M}} (c_{3,M} \frac{5}{6} c_{\frac{5}{6}3,M} - s_{3,M} s_{\frac{5}{6}3,M}) - c_W + \frac{4\pi}{3} \frac{6}{5} \frac{1}{q_{3,M}^2} c_{3,M} s_{\frac{5}{6}3,M} \\
J_{r81} &= c_W + \frac{4\pi}{3} \frac{6}{5} \frac{1}{q_{PIP}} s_{\frac{5}{6}p} c_{PIP} + \frac{1}{q_{3,M}} s_{\frac{5}{6}3,M} c_{3,M} \\
J_{r84} &= s_W + \frac{4\pi}{3} \frac{6}{5} \frac{1}{q_{PIP}} (s_{PIP} s_{\frac{5}{6}PIP} - c_{PIP} \frac{5}{6} c_{\frac{5}{6}PIP}) + c_W + \frac{4\pi}{3} \frac{6}{5} \frac{1}{q_{PIP}^2} c_{PIP} s_{\frac{5}{6}PIP} \\
J_{r86} &= s_W + \frac{4\pi}{3} \frac{6}{5} \frac{1}{q_{3,M}} (s_{3,M} s_{\frac{5}{6}3,M} - c_{3,M} \frac{5}{6} c_{\frac{5}{6}3,M}) + s_W + \frac{4\pi}{3} \frac{6}{5} \frac{1}{q_{3,M}^2} c_{3,M} s_{\frac{5}{6}3,M} \\
J_{r94} &= J_{64} = c_{PIP} \frac{5}{6q_{PIP}} s_{\frac{5}{6}PIP} + s_{PIP} \left( \frac{1}{q_{PIP}} c_{\frac{5}{6}PIP} - \frac{6}{5q_{PIP}^2} s_{\frac{5}{6}2p} \right) \\
J_{r96} &= c_{3,M} \frac{5}{6q_{3,M}} s_{\frac{5}{6}3,M} + s_{3,M} \left( \frac{1}{q_{3,M}} c_{\frac{5}{6}3,M} - \frac{6}{5q_{3,M}^2} s_{\frac{5}{6}3,M} \right)
\end{aligned} \tag{2}$$

other terms of jacobian matrix are null.