frontiers

Supplementary Material

Robust weighted averaging accounts for recruitment into collective motion in human crowds

Trenton D. Wirth[1*] & William H. Warren[1]

[1]Department of Cognitive, Linguistic, and Psychological Sciences, Brown University, Providence, RI, USA

*** Correspondence:**
Corresponding Author
trenton_wirth@alumni.brown.edu

1 Supplementary Tables

Table SM1

Fixed Effects Structures of the Linear Mixed Effects (LME) analyses in Experiment 1: Participant final heading (A), the variable error of final heading (B) and heading over time (C). Each model contains a random effect for participant, and the intercepts represent the grand mean (see Formula). The included fixed effects for each model were chosen by removing insignificant terms using a chi-squared likelihood ratio test in a step-down procedure.

Experiment 1

A) LME Regression: Final Heading

Formula: Final Heading ~ 1 + Turn Angle + (1 + Turn Angle*Noise| Participant)

Fixed Effects	Estimate	SE	t-statistic	p-value	95% CI Lower	95% CI Upper
Intercept	14.804	0.452	32.740	$p < 0.001$	13.916	15.691
Turn Angle	1.108	0.076	14.598	$p < 0.001$	0.959	1.257

B) LME Regression: Variable Error

Formula: Variable Error ~ 1 + Noise + (1 + Turn Angle*Noise | Participant)

Fixed Effects	Estimate	SE	t-statistic	p-value	95% CI Lower	95% CI Upper
Intercept	7.905	0.466	16.974	$p < 0.001$	6.978	8.832
Noise	0.105	0.007	15.152	$p < 0.001$	0.092	0.119

C) LME Regression: Heading over Time

Formula: Heading Over Time ~ 1 + Turn Angle*Time + (1 + Noise*Time*Turn Angle| Participant)

Fixed Effects	Estimate	SE	t-statistic	p-value	95% CI Lower	95% CI Upper
Intercept	1.736	0.159	10.919	$p < 0.001$	1.424	2.048
Turn Angle	0.140	0.022	6.460	$p < 0.001$	0.097	0.182
Time	1.867	0.039	47.799	$p < 0.001$	1.791	1.944
Turn Angle:Time	0.142	0.007	21.366	$p < 0.001$	0.129	0.155

Table SM2

Fixed Effects Structures of the Linear Mixed Effects (LME) analyses in Experiment 2: Participant final heading (A), the variable error of final heading (B), and the participant heading over time (C). Each model contains a fully specified random effects structure (see Formula), and the model intercepts represent the grand mean.

Experiment 2

A) LME Regression: Final Heading

Formula: Final Heading ~ 1 + AngularDifference*%Majority + (1+ AngularDifference*%Majority| Participant)

Fixed Effects	Estimate	SE	t-statistic	p-value	95% CI Lower	95% CI Upper
Intercept	3.875	0.474	8.172	*p* < 0.001	2.945	4.805
Angular Difference	0.144	0.026	5.583	*p* < 0.001	0.093	0.195
%Majority	0.176	0.020	8.613	*p* < 0.001	0.136	0.217
Angular Difference* %Majority	0.005	0.002	2.746	0.006	0.002	0.009

B) LME Regression: Variable Error

Formula: Variable Error~AngularDifference*%Majority + (1 + AngularDifference*%Majority | Participant)

Fixed Effects	Estimate	SE	t-statistic	p-value	95% CI Lower	95% CI Upper
Intercept	5.860	0.614	9.537	*p* < 0.001	4.645	7.074
Angular Difference	0.171	0.029	5.991	*p* < 0.001	0.115	0.228
%Majority	-0.004	0.018	-0.234	0.815	-0.040	0.031
Angular Difference* %Majority	0.001	0.001	0.373	0.710	-0.002	0.003

C) LME Regression: Heading over Time

Formula: Heading over Time~AngularDifference:Time* Time*%Majority:Time* AngularDifference:%Majority:Time + (1 + AngularDifference*%Majority*Time | Participant)

Fixed Effects	Estimate	SE	t-statistic	p-value	95% CI Lower	95% CI Upper
Intercept	0.240	0.088	2.717	0.007	0.067	0.414
Time	0.559	0.070	7.999	p < 0.001	0.422	0.696
Angular Difference:Time	0.020	0.003	5.876	*p* < 0.001	0.013	0.027

%Majority:Time	0.030	0.003	9.580	p < 0.001	0.023	0.036
Angular Difference:%Majority:Time	0.001	0.0002	4.257	p < 0.001	0.001	0.002

Table SM3

Fixed Effects Structures of the Linear Mixed Effects (LME) analyses in Experiment 3: Participant final heading (A), the variable error of final heading (B), and participant heading over time. Each model contains a random effect for subject, and the intercepts represent the grand mean (see Formula).

Experiment 3

A) LME Regression: Final Heading

Formula: Final Heading ~ 1 + Subgroup % + (1 + Subgroup SD*Subgroup % | Participant)

Fixed Effects	Estimate	SE	t-statistic	p-value	95% CI Lower	95% CI Upper
Intercept	11.444	0.642	17.816	*p* < 0.001	10.184	12.704
Subgroup %	0.185	0.021	9.015	*p* < 0.001	0.145	0.225

B) LME Regression: Variable Error

Formula: Variable Error ~ 1 + Subgroup SD + Subgroup% + (1 + Subgroup SD*Aligned% | Participant)

Fixed Effects	Estimate	SE	t-statistic	p-value	95% CI Lower	95% CI Upper
Intercept	17.316	2.299	7.532	*p* < 0.001	12.78	21.85
Subgroup SD	0.356	0.132	2.695	0.019	0.10	0.62
Subgroup %	-0.205	0.027	-7.680	*p* < 0.001	-0.26	-0.15

C) LME Regression: Heading Over Time

Formula: Heading Over Time ~ 1 + Subgroup % + (1 + Subgroup SD*Subgroup % | Participant)

Fixed Effects	Estimate	SE	t-statistic	p-value	95% CI Lower	95% CI Upper
Intercept	2.456	0.232	10.579	*p* < 0.001	2.001	2.911
Time	1.863	0.088	21.214	*p* < 0.001	1.691	2.036
Subgroup%:Time	0.035	0.003	13.831	*p* < 0.001	0.030	0.039

Table SM4

Fixed Effects Structures of the Linear Mixed Effects (LME) analyses in Experiment SM1: Participant final heading (A) and the variable error of final heading (B). Each model contains a fully specified random effects structure, and the intercepts represent the grand mean (see Formula).

Experiment SM1

A) LME Regression: Final Heading

Formula: Final Heading ~ 1 + AngularDifference*%Majority + (1+AngularDifference*%Majority | Participant)

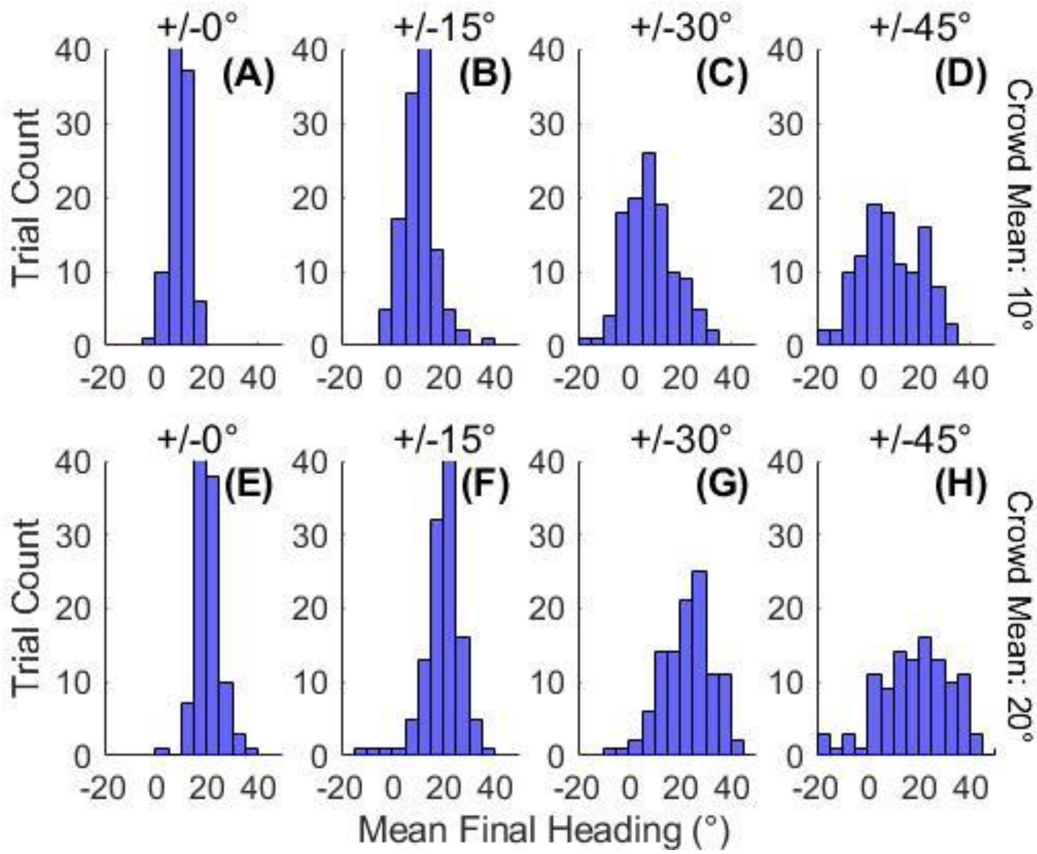
Fixed Effects	Estimate	SE	t-statistic	p-value	95% CI Lower	95% CI Upper
Intercept	3.772	0.659	5.726	$p < 0.001$	2.480	5.064
Angular Difference	0.216	0.047	4.597	$p < 0.001$	0.124	0.309
%Majority	0.155	0.020	7.574	$p < 0.001$	0.115	0.195
Angular Difference *%Majority	0.006	0.002	3.167	$p < 0.001$	0.002	0.010

B) LME Regression: Variable Error

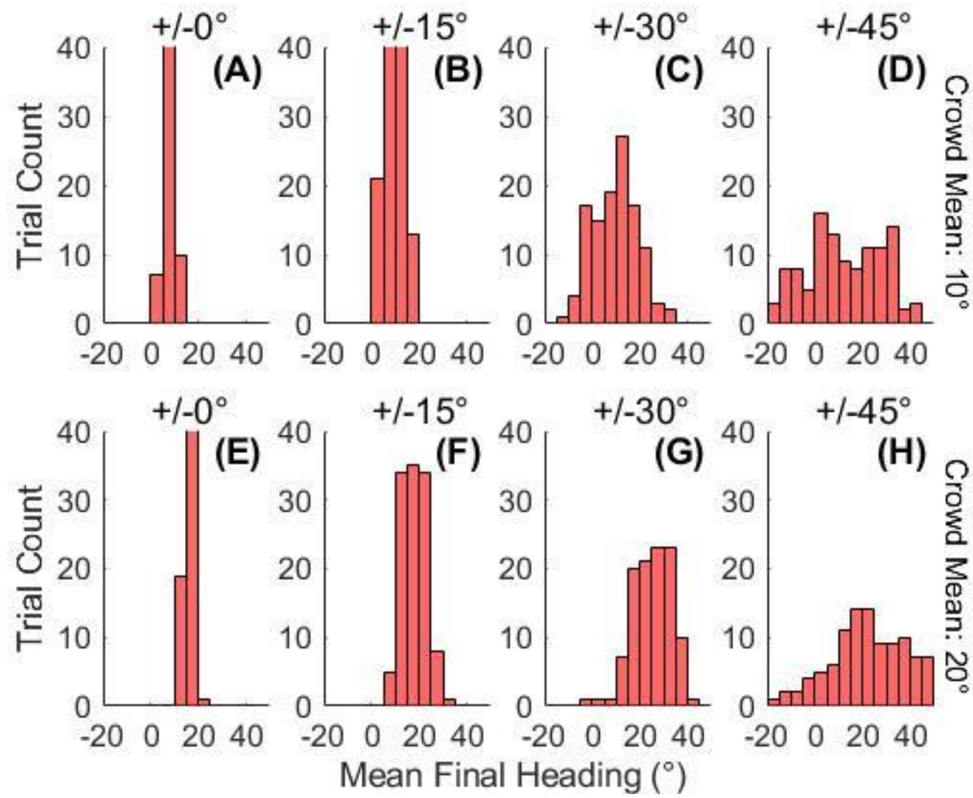
Formula: Variable Error~AngularDifference*%Majority + (1+AngularDifference*%Majority | Participant)

Fixed Effects	Estimate	SE	t-statistic	p-value	95% CI Lower	95% CI Upper
Intercept	5.025	0.383	13.107	$p < 0.001$	4.269	5.781
Angular Difference	0.161	0.023	6.904	$p < 0.001$	0.115	0.207
%Majority	-0.038	0.020	-1.954	0.052	-0.077	0.000
Angular Difference *%Majority	-0.002	0.002	-0.860	0.391	-0.006	0.002

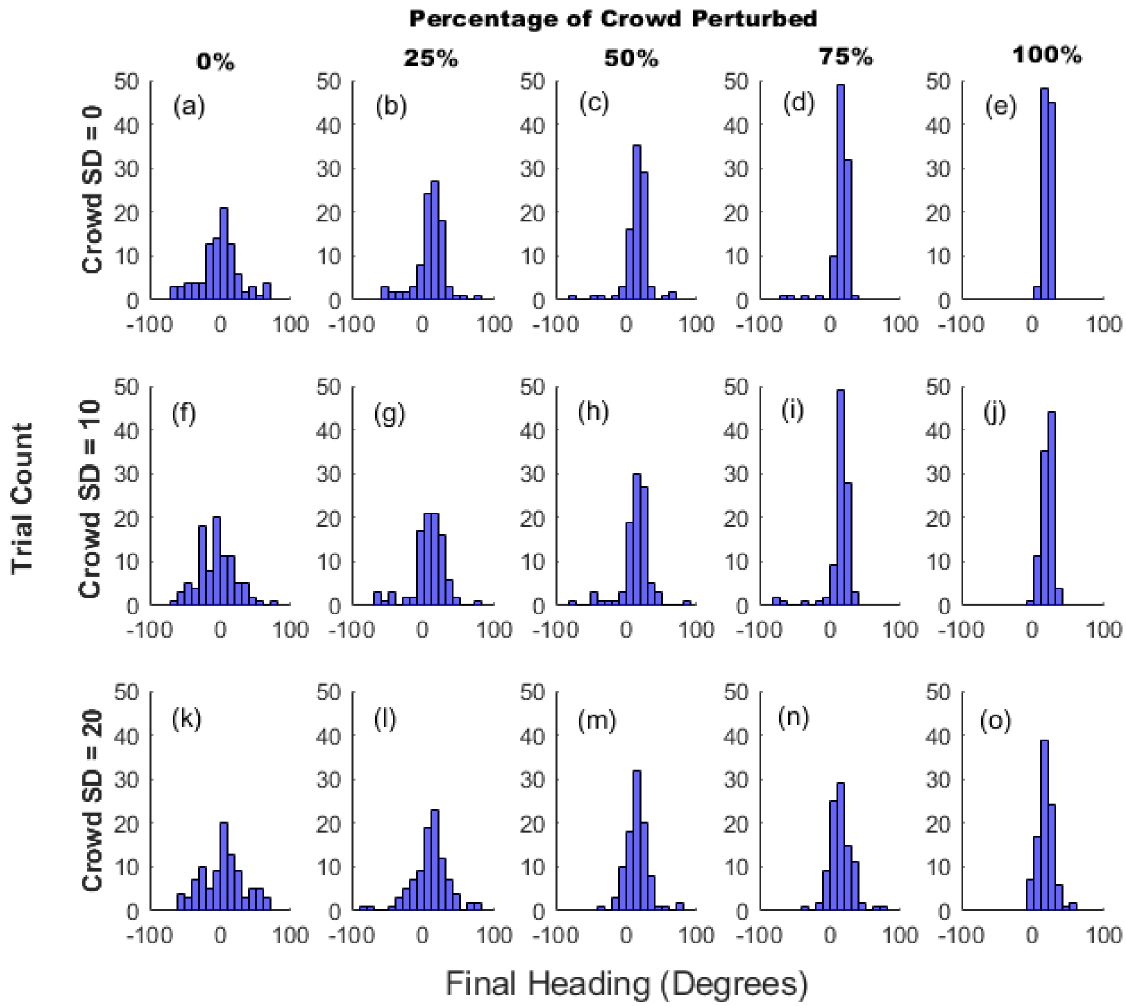
2 Supplementary Figures



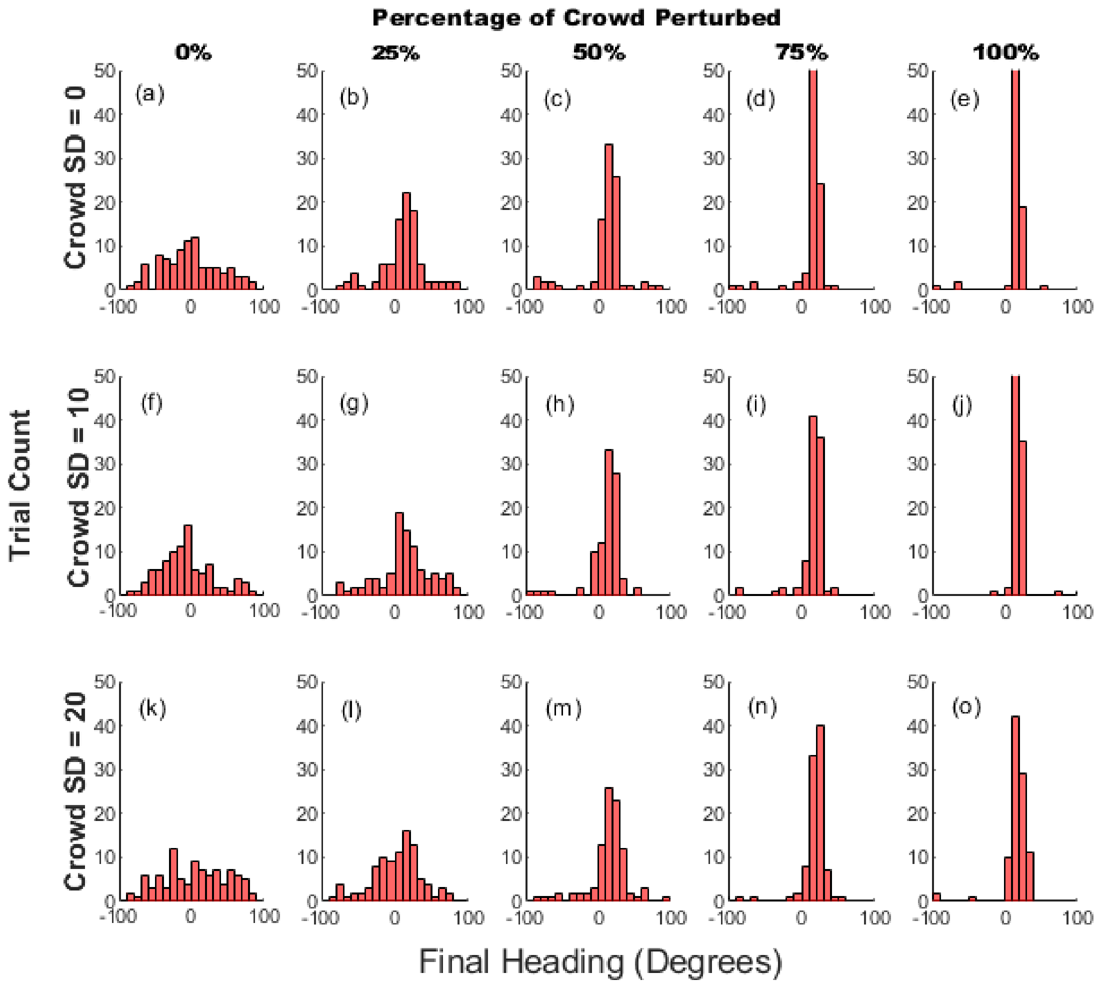
SM Figure 1. Final heading histograms from Experiment 1, with the representative panels corresponding to their column and row labels. The y-axis of each panel corresponds to the number of trials in each column-bin. The top row (A-D) represents the data for a crowd mean heading of 10°, and the bottom row (E-H) represents the data for a crowd mean heading of 20°.



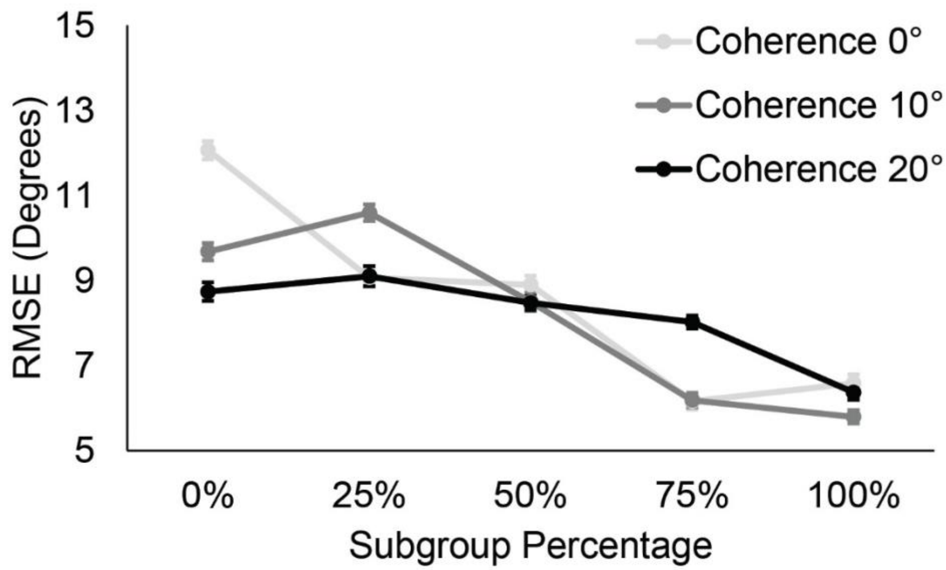
SM Figure 2. Model simulated final heading histograms for Experiment 1. The y-axis of each panel corresponds to the number of trials in each column-bin. The top row (A-D) represents the simulated data for a crowd mean heading of 10°, and the bottom row (E-H) represents the simulated data for a crowd mean heading of 20°.



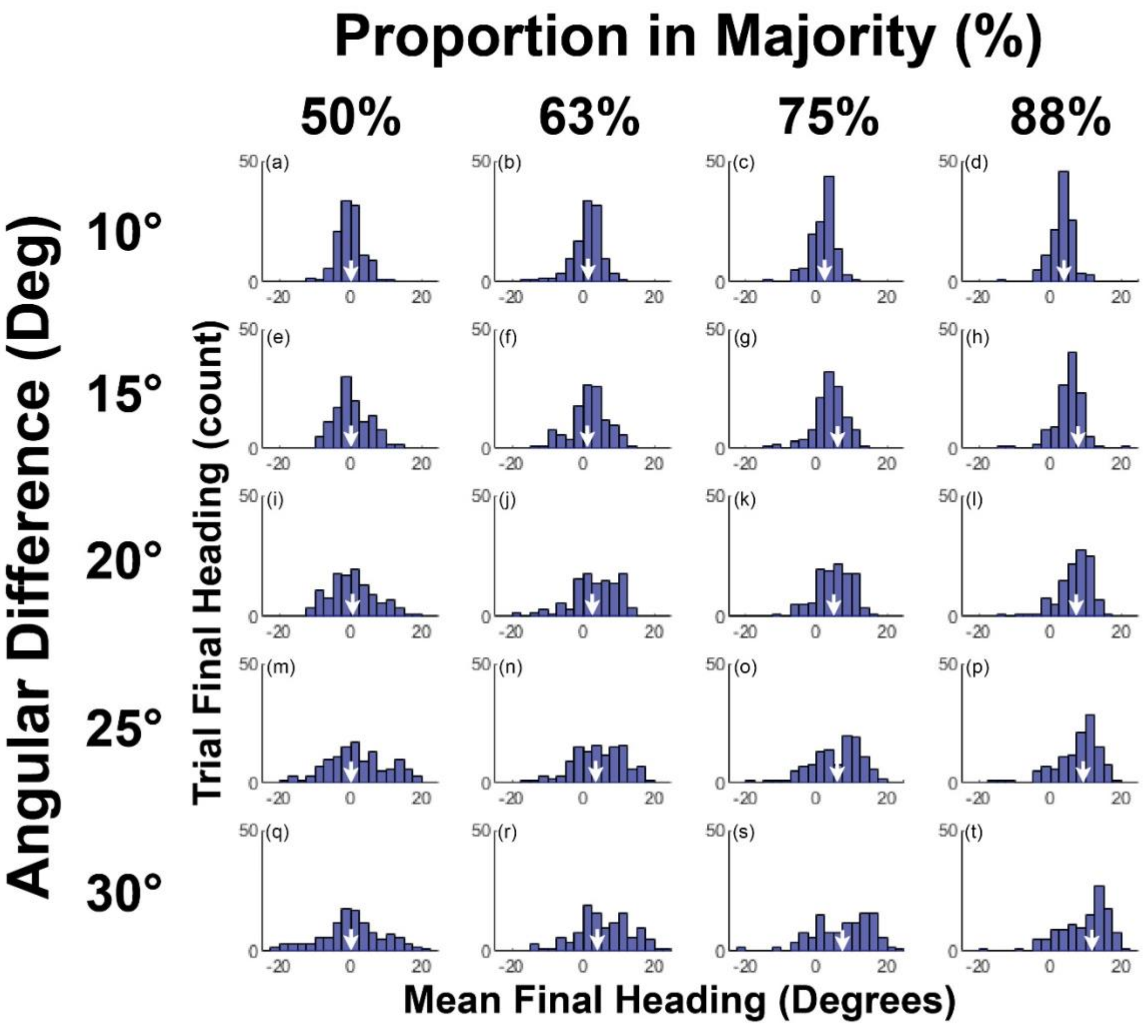
SM Figure 3. Final heading histograms from Experiment 3, with the representative panels corresponding to their column and row labels. The y-axis of each panel corresponds to the number of trials in each column-bin.



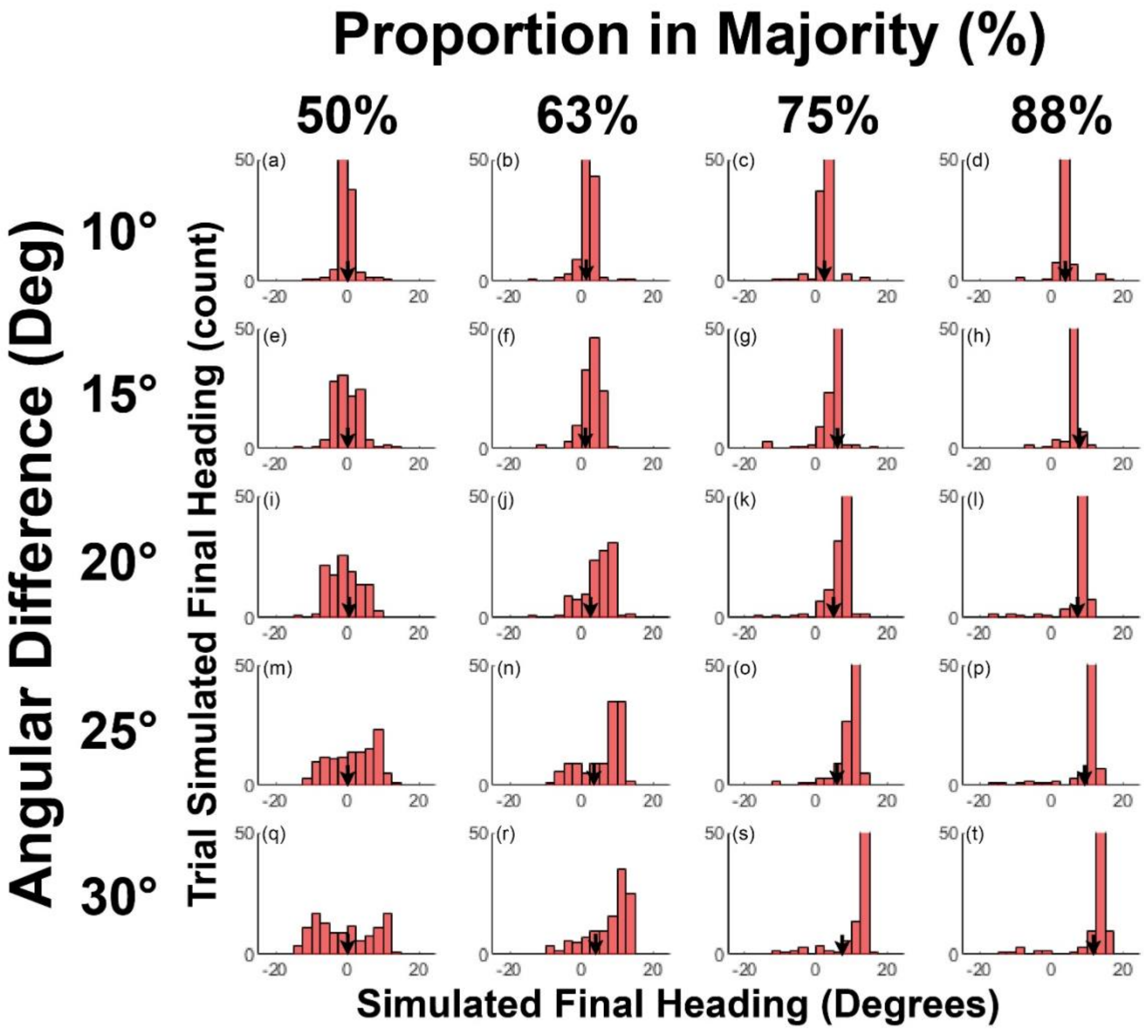
SM Figure 4. Model simulated final heading histograms for Experiment 3. The y-axis of each panel corresponds to the number of trials in each column-bin.



SM Figure 5. Subject Mean RMSE for Experiment 3 for each condition, where line darkness corresponds to the level of coherence (SD, degrees) and the subgroup percentage is represented in by each data point. Error bars represent Standard Error.



SM Figure 6. Final heading histograms for data in Experiment SM1. The columns correspond to the proportion in the majority, and the rows correspond to the angular separation of the crowd. The white arrow in each panel represents the unweighted crowd mean for that condition. The y-axis of each panel corresponds to the number of trials in each column-bin.



SM Figure 7. Model simulated final heading histograms for data in Experiment SM1. The columns correspond to the proportion in the majority, and the rows correspond to the angular separation of the crowd. The black arrow in each panel represents the unweighted crowd mean for that condition. The y-axis of each panel corresponds to the number of trials in each column-bin.

3 Supplementary Data (Experiment SM1)

When a crowd splits, do pedestrians continue to average the headings of all neighbors, or undergo a transition to follow one subgroup? In Experiment 2 we observed averaging when the two groups were continually crossing streams. In Experiment SM1 we repeated the experiment with a virtual crowd that split into two groups of only 4 or 8 neighbors, which spatially separated over time. In addition, we explored finer ranges of angular difference between the two groups (10-30° in increments of 5°) and proportion in the majority (50%, 63%, 75%, or 88%).

3.1 Results

The results demonstrate that participants robustly average the neighbors in both groups, even with the crowd separating at an angular difference of 30°. This result is apparent in the histograms of SM Figure 6: the final headings are clustered around the mean heading of the crowd (white arrows) in every condition. If participants were following the majority, the distributions would be shifted farther to the right, matching the turn angle in each row (for 63%, 75%, 88% majorities).

Similar to Experiment 2, the results show that participants averaged the headings of their neighbors in every condition. They thus followed the mean walking direction of the crowd, even with an angular difference of 30° between groups. However, an analysis of variability suggests that the heading response became less stable as the angular difference between groups increased.

Histograms of final heading on every trial are depicted by condition in SM Figure 6, collapsing across crowd size. Prior to statistical analysis, patterns in the data are already visually apparent. First, the heading distributions are unimodal and shift rightward as the majority increases (left to right panels) and as the angular difference increases (top to bottom panels). This pattern is consistent with an averaging rule. Second, the distributions appear to become narrower as the majority increases (left to right panels), but broader as the angular difference increases (top to bottom). This indicates that the attractiveness of mean heading increased with the proportion of the crowd in the majority, but decreased with the angular difference between the two groups. We analyze each in turn.

3.2 Final Heading

A mixed effects linear regression was used to analyze final heading on all trials (fitlme, Matlab R2019b). Final heading was regressed onto the fixed effects of angular difference (α), proportion in the majority, and their interaction term. In addition, a fully specified random intercept was included to account for between subject differences (Table SM4). The main effects and interaction term were tested by comparing models in a step-down procedure that removes the tested term from the full model, using likelihood ratio chi-squared tests. For the statistical models, both angular difference and proportion in the majority were treated as continuous, and each of the variables was centered on its mean value.

Prior to testing these effects, we tested whether crowd size contributed significantly to the variance in final heading. Visual inspection of the data in SM Figure 6 demonstrated that crowd size did not appear to do so. Statistical examination demonstrated that crowd size does not contribute to the variance of the data ($\chi2(4) = 4.65$, $p = 0.325$).

Given that crowd size did not contribute to the variance in final heading, we tested the statistical model seen in Table SM4 A. Chi-squared likelihood ratio tests reveal a significant effect of angular difference ($\chi2(1) = 154.38$, $p < 0.001$), a significant effect of the percentage in the majority group ($\chi2(1) = 301.37$, $p < 0.001$), as well as a significant interaction between angular difference and percentage in the majority ($\chi2(1) = 23.083$, $p < 0.001$). This means that for each additional degree of angular difference the final heading increased 0.217° ± 0.017° (SE), and for each additional percentage point in the majority group the final heading increased by 0.155° ± 0.009° (SE). In addition, the interaction accounts for an increase in the final heading of 0.006° ± 0.0012° (SE) for every additional degree of angular difference and percentage point in the majority.

The estimates can be used to interpret the effects of each main effect and the interaction term across the entire experiment: Going from 50% to 88% of the crowd in the majority group accounts for an increase of 5.9° in final heading, going from 10° to 30° of angular difference accounts for an increase of 4.3° in final heading, and going from panel (a) to panel (t) (in SM Figure 6) accounts for an additional 4.6° increase in the final heading response. Overall, we find that the heading response shifts with an increase in both the angular difference and the percentage in the majority, as well as their interaction.

3.3 Variable Error

A similar mixed effects linear regression approach was used to analyze the within-subject standard deviation, which was regressed onto the fixed effects of angular difference, proportion in the majority, and their interaction term. Crowd size had no impact on the within-subject standard deviation ($\chi2(4) = 3.08$, $p = 0.545$). Chi-squared likelihood ratio tests demonstrate a significant effect of both angular difference ($\chi2(1) = 60.063$, $p < 0.001$) and percentage in the majority ($\chi2(1) = 15.122$, $p < 0.001$). The interaction between them was, however, not significant ($\chi2(1) = 1.710$, $p = 0.191$).

For every degree of increase in the angular difference, there was a 0.161° ± 0.019° (SE) increase in the within-subject standard deviation, amounting to a total increase of ~3.2° in the SD from 10° to 30°. This means that as the angular difference between sub-groups grew larger, final heading became less consistent, i.e., less stable. Were this trend to continue, it could reach a threshold in angular difference at which the mean crowd heading becomes unstable, and behavior transitions from averaging to following one group.

For every percentage point increase in the majority group, the within-subject standard deviation decreased by 0.038° ± 0.010° (SE), amounting to a reduction of ~1.4° in the SD from 50% to an 88% majority. This indicates that as the percentage in the majority increased, there was a slight increase in the attraction to the crowd mean. What might explain a more stable heading response with a larger majority? This effect might be due to reduced noise in the crowd itself, as observed in Experiment 1: as the majority grew from 50% to 88%, the SD of heading in the crowd decreased from 5.2˚ to 3.4˚ (with $\alpha = 10°$), and from 15.8˚ to 10.4˚ (with $\alpha = 30°$). Alternatively, it is possible that as the subgroups separate spatially during a trial, averaging a larger and a smaller group is more reliable than averaging similar groups. The latter hypothesis is tested by Experiment 2, where spatial separation is removed from the two groups.

3.4 Simulation

Simulations of heading were generated using Rio, Dachner & Warren's (2018) weighted averaging model (see main body for details). For each trial, the participant's initial position and heading defined the initial conditions, and the positions and velocities of the virtual crowd, together with the participant's recorded walking speed, were taken as input on each time step. The output was a time series of simulated heading for every trial in the experiment, which was analyzed similar to the human time series.

Histograms of final heading from the trial simulations of Experiment SM1 are plotted in SM Figure 7, which can be directly compared with the human histograms in SM Figure 6. It is immediately apparent that the model is less noisy than human participants, for these heading histograms are much narrower than the human histograms. This is to be expected, as the model does not account for the inherent noise of gait oscillations, neural noise, etc., but reflects only the noise in the stimulus due to the jittered positions and randomized perturbations of the virtual humans.

The mean RMSE in Experiment SM1 was 3.80°. To provide a benchmark for performance, we compare the model's performance to a null model that makes no response (i.e., heading remains 0°) and the participant mean time series. The mean RMSE of the null model is 4.19°.

To compare these two "model" fits, we calculated Bayes Factor comparing the mean absolute error (MAE) values generated between the participant mean time series and the model simulation to the MAE generated between the participant mean time series and the null prediction. The test provides substantial evidence for the alternative hypothesis ($BF_{12} = 3.10$), indicating that the averaging model fits the data better than the null model.

3.5 Discussion

Experiment SM1 provides further evidence and confirmation of the effect observed in Experiment 2: participants average over splitting crowds in a wide variety of scenarios. Experiment SM1 adds to the conversation by demonstrating that the while two groups may be spatially distinct, participants will tend to average over their directions in a way that is predicted by our weighted averaging model.