Supplementary material for Health Savings Accounts: Consumer Contribution Strategies & Policy Implications

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This supplement provides a proof for the dynamic policy contribution (4) stated in the main body of the paper.

The proposed contribution is adapted from the solution to the following problem: Consider the infinite horizon version of problem (3), i.e. $T = \infty$. A closed form solution exists under the following idealized conditions:

A1 Funds can be withdrawn from the HSA for non-qualified expenses, penalty-free.

A2 There is a one-to-one correspondence between cost percentiles X_t and OOP expenses V_t.

A3 The distribution of household costs is stationary in time (no annual inflation).

Proposition 1. *Suppose $T = \infty$ and that A1-A3 hold. Then the solution to (3) is*

$$C_t = v_t - (1+w)W_{t-1},$$

where v_t is the unique cost threshold satisfying

$$\mathbb{P}(V_t \leq v_t | X_{t-1}) = Q = \frac{h-1}{h-\delta(1+w)}.$$

To adapt the solution above to the more realistic setting where A1-A3 do not hold and $T < \infty$, we make the following changes to arrive at (4). First, because there is significant penalty for withdrawing funds from the HSA for non-qualified expenses, we prohibit C_t from becoming negative by setting it to $\max\{0, v_t - (1+w)W_{t-1}\}$ in (4). Second, because the OOP expenses V_t is capped by the OOP maximum M_t, the distribution of V_t may have an atomic mass at M_t that invalidates A2. Hence if $\mathbb{P}(V_t < M_t | X_{t-1}) < Q$, the modified threshold v_t defined in (5) becomes M_t. Third, to capture cost inflation, our cost evolution model explicitly accounts for it. Finally, the performance of the policy is evaluated over a finite contribution period using a simulation.

Proof of Proposition 1. Consider a household in cost percentile X_{t-1} in the previous year and with HSA balance W. It transitions to cost percentile X_t in the following year and incurs $V(X_t)$ in OOP expenses. For notational brevity we use X, Z to denote X_{t-1}, X_t, and $p(Z|X)$ is the probability of transition from X to Z. Then Bellman's optimality equation is

$$\begin{aligned}
B(W,X) &= \delta \min_c \left\{ c + \int_{V(Z)<(1+w)W+c} p(Z|X) B((1+w)W + c - V(Z), Z) dZ \right. \\
&\quad \left. + \int_{V(Z)\geq(1+w)W+c} p(Z|X)[B(0,Z) + h(V(Z) - (1+w)W - c)] dZ \right\} \\
&= -\delta(1+w)W + \delta \min_v \left\{ v + \int_0^{z(v)} p(Z|X) B(v - V(Z), Z) dZ \right. \\
&\quad \left. + \int_{z(v)}^1 p(Z|X)[B(0,Z) + h(V(Z) - v)] dZ \right\}
\end{aligned}$$

where $v = (1+w)W + c$, and $z(v)$ is the percentile for the OOP expense v: $V(z(v)) = v$. This shows that the solution is of the form $B(W,X) = \bar{B}(X) - \delta(1+w)W$. Substituting back into the above yields a Bellman equation for $\bar{B}(X)$:

$$\begin{aligned}
\bar{B}(X) &= \delta \min_v \left\{ v + \int_0^{z(v)} p(Z|X) \left(\bar{B}(Z) - \delta(1+w)(v - V(Z)) \right) dZ + \int_{z(v)}^1 p(Z|X)[\bar{B}(Z) + h(V(Z) - v)] dZ \right\} \\
&= \delta E[\bar{B}(Z)|X] + \delta \min_v \left\{ v - \delta(1+w) \int_0^{z(v)} p(Z|X)(v - V(Z)) dZ + h \int_{z(v)}^1 p(Z|X)(V(Z) - v) dZ \right\}.
\end{aligned}$$

Since $E[\bar{B}(Z)|X]$ is a constant, the optimal value for v satisfies the first order condition

$$p(V(Z) \leq v | X) = p(Z \leq z(v) | X) = \frac{h-1}{h - \delta(1+w)},$$

and hence

$$c = v - (1+w)W.$$

In addition, from the second order condition we obtain the necessary condition $h > \delta(1+w)$. $\square$