Supporting Information List:

Supporting Information S1: PSF of FSE

The PSF of the FSE with a linear (LN) profile order is derived from Eq. (4) and (11):

$$PSF_{FSE}^{LN}(z) = \int_{-0.5}^{0.5} MTF_{FSE}^{LN}(k) e^{i2\pi kz} dk$$

$$= \int_{-0.5}^{0.5} M_{prep} \cdot E_2^{(k+0.5)N} e^{i2\pi kz} dk$$

$$= M_{prep} \cdot E_2^{0.5N} \int_{-0.5}^{0.5} e^{\ln E_2 kN} e^{i2\pi kz} dk$$

$$= M_{prep} \cdot E_2^{0.5N} \int_{-0.5}^{0.5} e^{(i2\pi z + N \ln E_2)k} dk$$

$$= M_{prep} \cdot E_2^{0.5N} \left. \frac{e^{(i2\pi z + N \ln E_2)k}}{i2\pi z + N \ln E_2} \right|_{-0.5}^{0.5}$$

$$= M_{prep} \cdot E_2^{0.5N} \frac{e^{0.5(i2\pi z + N \ln E_2)} - e^{-0.5(i2\pi z + \ln E_2)}}{i2\pi z + N \ln E_2}$$

$$= M_{prep} \cdot E_2^{0.5N} \frac{{E_2}^{0.5N} e^{i\pi z} - {E_2}^{-0.5N} e^{-i\pi z}}{i2\pi z + \ln E_2}$$

$$= M_{prep} \cdot \frac{E_2^N e^{i\pi z} - e^{-i\pi z}}{N \ln E_2 + i2\pi z} \tag{S1.1}$$

The PSF of the FSE with a low-high (LH) profile order is derived from Eq. (4) and (12):

$$PSF_{FSE}^{LH}(z) = \int_{-0.5}^{0.5} MTF_{FSE}^{LH}(k) e^{i2\pi kz} dk$$

$$= \int_{-0.5}^{0.5} M_{prep} \cdot E_2^{2|k|N} e^{i2\pi kz} dk$$

$$= M_{prep} \cdot \left(\int_{-0.5}^{0} E_2^{-2kN} e^{i2\pi kz} dk + \int_{0}^{0.5} E_2^{2kN} e^{i2\pi kz} dk \right)$$

$$= M_{prep} \cdot \left(\int_{-0.5}^{0} e^{2(i\pi z - N \ln E_2)k} dk + \int_{0}^{0.5} e^{2(i\pi z + N \ln E_2)k} dk \right)$$

$$= M_{prep} \cdot \left(\frac{e^{2(i\pi z - N \ln E_2)k}}{2(i\pi z - N \ln E_2)} \Bigg|_{-0.5}^{0} + \frac{e^{2(i\pi z + N \ln E_2)k}}{2(i\pi z + N \ln E_2)} \Bigg|_{0}^{0.5} \right)$$

$$= M_{prep} \cdot \left(\frac{1 - e^{N \ln E_2 - i\pi z}}{2(i\pi z - N \ln E_2)} + \frac{e^{i\pi z + N \ln E_2} - 1}{2(i\pi z + N \ln E_2)} \right)$$

$$= M_{prep} \cdot \left(\frac{e^{N \ln E_2 - i\pi z} - 1}{2(N \ln E_2 - i\pi z)} + \frac{e^{i\pi z + N \ln E_2} - 1}{2(N \ln E_2 + i\pi z)} \right)$$

$$= M_{prep} \cdot \frac{\left(E_2^N e^{-i\pi z} - 1\right)(N \ln E_2 + i\pi z) + \left(E_2^N e^{i\pi z} - 1\right)(N \ln E_2 - i\pi z)}{2(N \ln E_2 - i\pi z)(N \ln E_2 + i\pi z)}$$

$$= M_{prep} \cdot \frac{N \ln E_2 \left(E_2^N \left(e^{i\pi z} + e^{-i\pi z}\right) - 2\right) - i\pi x \varepsilon^N \left(e^{i\pi z} - e^{-i\pi z}\right)}{2N^2 \ln^2 E_2 + 2\pi^2 z^2}$$

$$= M_{prep} \cdot \frac{2E_2^N N \ln \varepsilon \cos(\pi z) + 2E_2^N \pi z \sin(\pi z) - 2N \ln \varepsilon}{2N^2 \ln^2 E_2 + 2\pi^2 z^2}$$

$$= M_{prep} \cdot \frac{E_2^N N \ln E_2 \cos(\pi z) + {E_2}^N \pi x \sin(\pi z) - N \ln E_2}{N^2 \ln^2 E_2 + \pi^2 z^2} \qquad \text{(S1.2)}$$

Supporting Information S2: ruCNR Optimization of FSE

To simplify Eq. (17), let $r = -N \ln E_2 = N \cdot TE/T_2$. We have:

$$ruCNR_{FSE} = \frac{1 - e^{-r}}{\sqrt{r}} \sqrt{T_2} \tag{S2.1}$$

The derivative of $ruCNR_{FSE}$ w.r.t. r is:

$$\begin{aligned}\frac{\partial ruCNR_{FSE}}{\partial r} &= \frac{\partial}{\partial r} \frac{1 - e^{-r}}{\sqrt{r}} \sqrt{T_2} = \sqrt{T_2} \frac{\partial}{\partial r} \frac{1 - e^{-r}}{\sqrt{r}} = \sqrt{T_2} \frac{e^{-r}\sqrt{r} - (1 - e^{-r}) \frac{1}{2\sqrt{r}}}{r} \\ &= \sqrt{T_2} \frac{2re^{-r} + e^{-r} - 1}{2r\sqrt{r}}\end{aligned} \tag{S2.2}$$

$ruCNR_{FSE}$ has an optimal solution when the numerator of the derivative is 0:

$2re^{-r} + e^{-r} - 1 = 0$

$(r + 0.5)e^{-r} = 0.5$

$(r + 0.5)e^{-r-0.5} = 0.5e^{-0.5}$

$(-r - 0.5)e^{-r-0.5} = -0.5/\sqrt{e}$

$-r - 0.5 = W_{-1}(-0.5/\sqrt{e})$

$r = -0.5 - W_{-1}(-0.5/\sqrt{e})$

$r \approx 1.26$

$N \cdot TE/T_2 = 1.26$

$$N \cdot TE = 1.26T_2 \tag{S2.3}$$

Here, $W_{-1}(x)$ is the negative branch of the Lambert W function. The properties of $W_{-1}(x)$ includes:

$$W_{-1}(x)e^{W_{-1}(x)} = x. \tag{S2.4}$$

$$W_{-1}(xe^{x}) = x. \tag{S2.5}$$

$$W_{-1}(x) \leq -1 \tag{S2.6}$$

$W_{-1}(x) = -1.$ only when $x = -1/e$ (S2.7)

As a result, the optimal $ruCNR_{FSE}$ is :

$$ruCNR_{FSE}^{*} = \frac{1 - e^{-1.26}}{\sqrt{1.26}}\sqrt{T_2} \approx 0.64\sqrt{T_2}. \tag{S2.8}$$

Supporting Information S3: MTF of FLASH

The solution of $M_z(n)$ can be derived from Eq. (20):

$$M_z(n+1) = M_z(n)\varepsilon + M^0(1-E_1)$$

$$M_z(n+1) = M_z(n)\varepsilon + M^0(1-\varepsilon)\frac{1-E_1}{1-\varepsilon}$$

$$M_z(n+1) - M^0\frac{1-E_1}{1-\varepsilon} = \left(M_z(n) - M^0\frac{1-E_1}{1-\varepsilon}\right)\varepsilon$$

$$M_z(n) - M^0\frac{1-E_1}{1-\varepsilon} = \left(M_{prep} - M^0\frac{1-E_1}{1-\varepsilon}\right)\varepsilon^{n-1}$$

$$M_z(n) = \left(M_{prep} - M^0\frac{1-E_1}{1-\varepsilon}\right)\varepsilon^{n-1} + M^0\frac{1-E_1}{1-\varepsilon} \tag{S3.1}$$

$$= M_{prep}\cdot\varepsilon^{n-1} + M^0\frac{1-E_1}{1-\varepsilon}\cdot(1-\varepsilon^{n-1}).$$

We can derive an effective decay time (T_1*) from Eq: (24):

$$M_{xy}(n) = M_{ss} + \left(M_{prep}E_2^*\sin\alpha - M_{ss}\right)\cdot\varepsilon^{n-1}$$

Let: $A = M_{ss}$, $B = \left(M_{prep}E_2^*\sin\alpha - M_{ss}\right)/\varepsilon$

$$M_{xy}(n) = A + B\cdot\varepsilon^n \tag{S3.2}$$

There is a relation between n and the decay time t: $t = n\cdot TR$, or $n = t/TR$. Therefore:

$$\varepsilon^n = e^{n\ln\varepsilon} = e^{\frac{t}{TR}\ln\varepsilon} = e^{-t\left(\frac{-\ln\varepsilon}{TR}\right)} = e^{-t\left(\frac{-\ln\left(e^{-TR/T_1}\cos\alpha\right)}{TR}\right)} = e^{-t\left(\frac{-\left(\ln e^{-TR/T_1}+\ln\cos\alpha\right)}{TR}\right)} =$$

$$= e^{-t\left(\frac{\frac{TR}{T_1}-\ln\cos\alpha}{TR}\right)} = e^{-t\left(\frac{1}{T_1}-\frac{\ln\cos\alpha}{TR}\right)} \tag{S3.3}$$

From Eq. (S3.3) the effective decay time is:

$$T_1{}^* = \frac{1}{\frac{1}{T_1} - \frac{\ln\cos\alpha}{TR}} \tag{S3.4}$$

When Enrst angle is used, we have $\cos\alpha = e^{-TR/T_1}$. In this situration, the effective decay time is:

$$T^*_{1,Enrst} = \frac{1}{\frac{1}{T_1} - \frac{-TR/T_1}{TR}} = \frac{1}{\frac{1}{T_1} + \frac{1}{T_1}} = \frac{T_1}{2} \tag{S3.4}$$

Supporting Information S4: PSF of FLASH

The PSF of the FLASH with a linear (LN) profile order is derived from Eq. (4) and (25):

$$PSF_{FLASH}^{LN}(z) = \int_{-0.5}^{0.5} MTF_{FLASH}^{LN}(k) e^{i2\pi kz} dk$$

$$= \int_{-0.5}^{0.5} \left[\left(M_{prep} \cdot E_2^* \sin\alpha - M_{ss} \right) \cdot \varepsilon^{(k+0.5)N} + M_{ss} \right] e^{i2\pi kz} dk$$

$$= \left(M_{prep} \cdot E_2^* \sin\alpha - M_{ss} \right) \cdot \int_{-0.5}^{0.5} \varepsilon^{(k+0.5)N} e^{i2\pi kz} dk + M_{ss} \cdot \int_{-0.5}^{0.5} e^{i2\pi kz} dk$$

$$= \left(M_{prep} \cdot E_2^* \sin\alpha - M_{ss} \right) \cdot \varepsilon^{0.5N} \int_{-0.5}^{0.5} e^{(i2\pi z + N \ln \varepsilon)k} dk + M_{ss} \cdot \int_{-0.5}^{0.5} e^{i2\pi zk} dk$$

$$= \left(M_{prep} \cdot E_2^* \sin\alpha - M_{ss} \right) \cdot \varepsilon^{0.5N} \left. \frac{e^{(i2\pi z + N \ln \varepsilon)k}}{i2\pi z + N \ln \varepsilon} \right|_{-0.5}^{0.5} + M_{ss} \cdot \left. \frac{e^{i2\pi zk}}{i2\pi z} \right|_{-0.5}^{0.5}$$

$$= \left(M_{prep} \cdot E_2^* \sin\alpha - M_{ss} \right) \cdot \varepsilon^{0.5N} \frac{e^{0.5(i2\pi z + N \ln \varepsilon)} - e^{-0.5(i2\pi z + N \ln \varepsilon)}}{i2\pi z + N \ln \varepsilon} + M_{ss} \cdot \frac{e^{i\pi z} - e^{-i\pi z}}{i2\pi z}$$

$$= \left(M_{prep} \cdot E_2^* \sin\alpha - M_{ss} \right) \cdot \varepsilon^{0.5N} \frac{\varepsilon^{0.5N} e^{i\pi z} - \varepsilon^{-0.5N} e^{-i\pi z}}{i2\pi z + N \ln \varepsilon} + M_{ss} \cdot \frac{i2 \sin(\pi z)}{i2\pi z}$$

$$= \left(M_{prep} \cdot E_2^* \sin\alpha - M_{ss} \right) \cdot \frac{\varepsilon^{N} e^{i\pi z} - e^{-i\pi z}}{N \ln \varepsilon + i2\pi z} + M_{ss} \cdot \mathrm{sinc}(\pi z) \tag{S4.1}$$

The PSF of the FLASH with a low-high (LH) profile order is derived from Eq. (4) and (26):

$$PSF_{FLASH}^{LH}(z) = \int_{-0.5}^{0.5} MTF_{FLASH}^{LH}(k) e^{i2\pi kz} dk$$

$$= \int_{-0.5}^{0.5} \left[\left(M_{prep} \cdot E_2^* \sin\alpha - M_{ss} \right) \cdot \varepsilon^{2|k|N} + M_{ss} \right] e^{i2\pi kz} dk$$

$$= \left(M_{prep} \cdot E_2^* \sin\alpha - M_{ss} \right) \cdot \int_{-0.5}^{0.5} \varepsilon^{2|k|N} e^{i2\pi kz} dk + M_{ss} \cdot \int_{-0.5}^{0.5} e^{i2\pi kz} dk$$

$$= \left(M_{prep} \cdot E_2^* \sin\alpha - M_{ss} \right) \cdot \int_{-0.5}^{0.5} \varepsilon^{2|k|N} e^{i2\pi kz} dk + M_{ss} \cdot \mathrm{sinc}(\pi z)$$

$$= \left(M_{prep} \cdot E_2^* \sin\alpha - M_{ss}\right)\left(\int_{-0.5}^{0} \varepsilon^{-2kN} e^{i2\pi kz} dk + \int_{0}^{0.5} \varepsilon^{2kN} e^{i2\pi kz} dk\right) + M_{ss}\,\mathrm{sinc}(\pi z)$$

$$= \left(M_{prep} \cdot E_2^* \sin\alpha - M_{ss}\right)\left(\int_{-0.5}^{0} e^{2(i\pi z - N\ln\varepsilon)k} dk + \int_{0}^{0.5} e^{2(i\pi z + N\ln\varepsilon)k} dk\right) + M_{ss}\,\mathrm{sinc}(\pi z)$$

$$= \left(M_{prep} \cdot E_2^* \sin\alpha - M_{ss}\right)\left(\left.\frac{e^{2(i\pi z - N\ln\varepsilon)k}}{2(i\pi z - N\ln\varepsilon)}\right|_{-0.5}^{0} + \left.\frac{e^{2(i\pi z + N\ln\varepsilon)k}}{2(i\pi z + N\ln\varepsilon)}\right|_{0}^{0.5}\right) + M_{ss}\,\mathrm{sinc}(\pi z)$$

$$= \left(M_{prep} \cdot E_2^* \sin\alpha - M_{ss}\right)\left(\frac{1 - e^{N\ln\varepsilon - i\pi z}}{2(i\pi z - N\ln\varepsilon)} + \frac{e^{i\pi z + N\ln\varepsilon} - 1}{2(i\pi z + N\ln\varepsilon)}\right) + M_{ss}\,\mathrm{sinc}(\pi z)$$

$$= \left(M_{prep} \cdot E_2^* \sin\alpha - M_{ss}\right)\left(\frac{e^{N\ln\varepsilon - i\pi z} - 1}{2(N\ln\varepsilon - i\pi z)} + \frac{e^{i\pi z + N\ln\varepsilon} - 1}{2(N\ln\varepsilon + i\pi z)}\right) + M_{ss}\,\mathrm{sinc}(\pi z)$$

$$= \left(M_{prep} \cdot E_2^* \sin\alpha - M_{ss}\right)\frac{\left(\varepsilon^N e^{-i\pi z} - 1\right)(N\ln\varepsilon + i\pi z) + \left(\varepsilon^N e^{i\pi z} - 1\right)(N\ln\varepsilon - i\pi z)}{2(N\ln\varepsilon - i\pi z)(N\ln\varepsilon + i\pi z)}$$

$$+ M_{ss}\,\mathrm{sinc}(\pi z)$$

$$= \left(M_{prep} \cdot E_2^* \sin\alpha - M_{ss}\right)\frac{N\ln\varepsilon\left(\varepsilon^N\left(e^{i\pi z} + e^{-i\pi z}\right) - 2\right) - i\pi x\varepsilon^N\left(e^{i\pi z} - e^{-i\pi z}\right)}{2\pi^2 z^2 + 2N^2\ln^2\varepsilon}$$

$$+ M_{ss}\,\mathrm{sinc}(\pi z)$$

$$= \left(M_{prep} \cdot E_2^* \sin\alpha - M_{ss}\right)\frac{2\varepsilon^N N\ln\varepsilon\cos(\pi z) + 2\varepsilon^N \pi x\sin(\pi z) - 2N\ln\varepsilon}{2\pi^2 z^2 + 2N^2\ln^2\varepsilon} + M_{ss}\,\mathrm{sinc}(\pi z)$$

$$= \left(M_{prep} \cdot E_2^* \sin\alpha - M_{ss}\right)\frac{\varepsilon^N N\ln\varepsilon\cos(\pi z) + \varepsilon^N \pi x\sin(\pi z) - N\ln\varepsilon}{N^2\ln^2\varepsilon + \pi^2 z^2} + M_{ss}\,\mathrm{sinc}(\pi z)\,. \tag{S4.2}$$

Supporting Information S5: ruCNR Optimization of FLASH

The derivative of $ruCNR_{FLASH}$ w.r.t. the factor N according to Eq. (31):

$$\frac{\partial ruCNR_{FLASH}}{\partial N} = \frac{\partial}{\partial N}\frac{\varepsilon^N - 1}{\ln\varepsilon}\frac{\sqrt{\mathrm{TR}}}{\sqrt{N}}E_2^*\sin\alpha$$

$$= \frac{\sqrt{\mathrm{TR}}E_2^*\sin\alpha}{\ln\varepsilon}\frac{\partial}{\partial N}\frac{\varepsilon^N - 1}{\sqrt{N}}$$

$$= \frac{\sqrt{\mathrm{TR}}E_2^*\sin\alpha}{\ln\varepsilon}\frac{\partial}{\partial N}\frac{\varepsilon^N\ln\varepsilon\sqrt{N} - (\varepsilon^N - 1)\frac{1}{2\sqrt{N}}}{N}$$

$$= \frac{\sqrt{\mathrm{TR}}E_2^*\sin\alpha}{\ln\varepsilon}\frac{2N\varepsilon^N\ln\varepsilon - \varepsilon^N + 1}{2N\sqrt{N}} \quad \text{(S5.1)}$$

According to Eq. (S5.1), an optimal factor (N^*) maximizes $ruCNR_{FLASH}$ when the numerator of the derivative is 0, therefore, the part

$$2N^*\varepsilon^{N^*}\ln\varepsilon - \varepsilon^{N^*} + 1 = 0$$

$$2N^*e^{N^*\ln\varepsilon}\ln\varepsilon - e^{N^*\ln\varepsilon} = -1$$

$$(N^*\ln\varepsilon - 0.5)e^{N^*\ln\varepsilon} = -0.5$$

$$(N^*\ln\varepsilon - 0.5)e^{N^*\ln\varepsilon - 0.5} = -0.5e^{-0.5}$$

$$N^*\ln\varepsilon - 0.5 = W_{-1}(-0.5e^{-0.5})$$

$$N^* = \frac{W_{-1}(-0.5e^{-0.5}) + 0.5}{\ln\varepsilon} \approx \frac{-1.26}{\ln\varepsilon} \quad \text{(S5.2)}$$

Again, $W_{-1}(x)$ is the negative branch of the Lambert W function. The properties were described in Eq. (S2.4 - S2.7).

Correspondingly the maximized $ruCNR_{FLASH}$ is:

$$ruCNR_{FLASH}(N^*) = \frac{\varepsilon^{N^*} - 1}{\ln \varepsilon} \frac{\sqrt{\mathrm{TR}}}{\sqrt{N^*}} E_2^* \sin \alpha$$

$$= \frac{e^{N^* \ln \varepsilon} - 1}{\ln \varepsilon} \frac{\sqrt{-\mathrm{TR} \ln \varepsilon}}{\sqrt{-W_{-1}(-0.5e^{-0.5}) - 0.5}} E_2^* \sin \alpha$$

$$= \frac{e^{W_{-1}(-0.5e^{-0.5})+0.5} - 1}{-\sqrt{-W_{-1}(-0.5e^{-0.5}) - 0.5}} \frac{\sqrt{\mathrm{TR}}}{\sqrt{-\ln \varepsilon}} E_2^* \sin \alpha$$

$$= \frac{e^{0.5} \frac{-0.5e^{-0.5}}{W_{-1}(-0.5e^{-0.5})} - 1}{-\sqrt{-W_{-1}(-0.5e^{-0.5}) - 0.5}} \frac{\sqrt{\mathrm{TR}}}{\sqrt{-\ln \varepsilon}} E_2^* \sin \alpha$$

$$= \frac{\frac{-0.5 - W_{-1}(-0.5e^{-0.5})}{W_{-1}(-0.5e^{-0.5})}}{-\sqrt{-W_{-1}(-0.5e^{-0.5}) - 0.5}} \frac{\sqrt{\mathrm{TR}}}{\sqrt{-\ln \varepsilon}} E_2^* \sin \alpha$$

$$= \frac{\sqrt{-W_{-1}(-0.5e^{-0.5}) - 0.5}}{-W_{-1}(-0.5e^{-0.5})} \frac{\sqrt{\mathrm{TR}}}{\sqrt{-\ln \varepsilon}} E_2^* \sin \alpha$$

$$\approx 0.64 \frac{\sqrt{\mathrm{TR}}}{\sqrt{-\ln \varepsilon}} E_2^* \sin \alpha \qquad \text{(S5.3)}$$

The derivative of $ruCNR_{FLASH}(N^*)$ w.r.t. flip angle α is:

$$\frac{\partial ruCNR_{FLASH}(N^*)}{\partial \alpha} = \frac{\partial}{\partial \alpha} 0.64 \frac{\sqrt{\mathrm{TR}}}{\sqrt{-\ln \varepsilon}} E_2^* \sin \alpha$$

$$= 0.64\sqrt{\mathrm{TR}} E_2^* \frac{\partial}{\partial \alpha} \frac{\sin \alpha}{\sqrt{-\ln \varepsilon}}$$

$$= \frac{0.64\sqrt{\mathrm{TR}} E_2^*}{-\ln \varepsilon} \left(\cos \alpha \sqrt{-\ln \varepsilon} - \sin \alpha \frac{\partial \varepsilon}{\partial \alpha} \frac{\partial}{\partial \varepsilon} \sqrt{-\ln \varepsilon} \right)$$

$$= \frac{0.64\sqrt{\mathrm{TR}} E_2^*}{-\ln \varepsilon} \left(\cos \alpha \sqrt{-\ln \varepsilon} - \sin \alpha \, (-E_1 \sin \alpha) \frac{-1/\varepsilon}{2\sqrt{-\ln \varepsilon}} \right)$$

$$= \frac{0.64\sqrt{\mathrm{TR}} E_2^*}{-\ln \varepsilon} \left(\cos \alpha \sqrt{-\ln \varepsilon} - E_1 \sin^2 \alpha \frac{1}{2\varepsilon\sqrt{-\ln \varepsilon}} \right)$$

$$= \frac{0.64\sqrt{\mathrm{TR}}E_2^*}{2(-\ln\varepsilon)^{1.5}\cos\alpha}(-2\cos^2\alpha\ln\varepsilon - \sin^2\alpha) \tag{S5.4}$$

Therefore, the flip angle that maximize ruCNR satisfies:

$$2\cos^2\alpha^*\ln\varepsilon^* + \sin^2\alpha^* = 0$$

$$\cos^2\alpha^*(2\ln E_1 + \ln\cos^2\alpha^*) + (1-\cos^2\alpha^*) = 0$$

$$\cos^2\alpha^*(\ln\cos^2\alpha^* + 2\ln E_1 - 1) = -1 \tag{S5.5}$$

Let $x = \ln\cos^2\alpha^*$, then:

$$e^x(x + 2\ln E_1 - 1) = -1$$

$$(x + 2\ln E_1 - 1)e^{x+2\ln E_1 - 1} = e^{2\ln E_1 - 1} = -E_1^2/e$$

$$x + 2\ln E_1 - 1 = W_{-1}(-E_1^2/e)$$

$$x = W_{-1}(-E_1^2/e) - 2\ln E_1 + 1$$

$$\cos^2\alpha^* = e^x = e^{W_{-1}(-E_1^2/e)-2\ln E_1+1} = \frac{e}{E_1^2}\frac{-E_1^2/e}{W_{-1}(-E_1^2/e)} = \frac{1}{-W_{-1}(E_1^2e)}$$

$$\alpha^* = \arccos\frac{1}{\sqrt{-W_{-1}(-E_1^2/e)}}$$

$$\ln\varepsilon^* = \ln E_1 + \ln\cos\alpha^* = \ln E_1 + 0.5\,(W_{-1}(-E_1^2/e) - 2\ln E_1 + 1)$$

$$\ln\varepsilon^* = 0.5(W_{-1}(-E_1^2/e) + 1)$$

$$N^* \approx \frac{-1.26}{\ln\varepsilon^*} = \frac{-1.26}{0.5(W_{-1}(-E_1^2/e)+1)} \approx \frac{-2.51}{W_{-1}(-E_1^2/e)+1} \tag{S5.6}$$

The corresponding optimal $ruCNR_{FLASH}^*$:

$$ruCNR_{FLASH}^* = 0.64\frac{\sqrt{\mathrm{TR}}}{\sqrt{-\ln\varepsilon^*}}E_2^*\sin\alpha$$

$$= \frac{0.64E_2^*\sqrt{\text{TR}}}{\sqrt{-0.5(W_{-1}(-E_1^2/e)+1)}}\sqrt{1-\cos^2\alpha^*}$$

$$= \frac{0.64E_2^*\sqrt{\text{TR}}}{\sqrt{-0.5(W_{-1}(-E_1^2/e)+1)}}\sqrt{1-\frac{1}{-W_{-1}(E_1^2 e)}}$$

$$= \frac{0.64E_2^*\sqrt{\text{TR}}}{\sqrt{-0.5(W_{-1}(-E_1^2/e)+1)}}\sqrt{\frac{W_{-1}(-E_1^2/e)+1}{W_{-1}(E_1^2 e)}}$$

$$= \frac{0.64E_2^*\sqrt{\text{TR}}}{\sqrt{0.5}}\frac{1}{\sqrt{-W_{-1}(-E_1^2/e)}}$$

$$\approx \frac{0.90E_2^*\sqrt{\text{TR}}}{\sqrt{-W_{-1}(-E_1^2/e)}} \tag{S5.7}$$

The derivative of $ruCNR^*_{FLASH}$ w.r.t. TR is.

$$\frac{\partial ruCNR^*_{FLASH}}{\partial \text{TR}} = \frac{\partial}{\partial \text{TR}}\frac{0.90E_2^*\sqrt{\text{TR}}}{\sqrt{-W_{-1}(-E_1^2/e)}} = 0.90E_2^*\frac{\partial}{\partial TR}\frac{\sqrt{\text{TR}}}{\sqrt{-W_{-1}(-E_1^2/e)}}$$

$$= 0.90E_2^*\frac{\frac{1}{2\sqrt{TR}}\sqrt{-W_{-1}(-E_1^2/e)}-\sqrt{\text{TR}}\frac{1}{2\sqrt{-W_{-1}(-E_1^2/e)}}\frac{\partial}{\partial \text{TR}}\left(-W_{-1}(-E_1^2/e)\right)}{-W_{-1}(-E_1^2/e)}$$

$$= 0.90E_2^*\frac{-W_{-1}(-E_1^2/e)+TR\frac{W_{-1}(-E_1^2/e)}{-E_1^2/e\left(1+W_{-1}(-E_1^2/e)\right)}\frac{\partial(-E_1^2/e)}{\partial \text{TR}}}{-2W_{-1}(-E_1^2/e)\sqrt{-T_R W_{-1}(-E_1^2/e)}}$$

$$= 0.90E_2^*\frac{-W_{-1}(-E_1^2/e)+TR\frac{W_{-1}(-E_1^2/e)}{-E_1^2/e\left(1+W_{-1}(-E_1^2/e)\right)}\frac{-2E_1}{e}\frac{\partial E_1}{\partial \text{TR}}}{-2W_{-1}(-E_1^2/e)\sqrt{-T_R W_{-1}(-E_1^2/e)}}$$

$$= 0.90E_2^*\frac{-W_{-1}(-E_1^2/e)+\text{TR}\frac{2W_{-1}(-E_1^2/e)}{E_1\left(1+W_{-1}(-E_1^2/e)\right)}\left(-\frac{1}{T_1}\right)e^{-\frac{\text{TR}}{T_1}}}{-2W_{-1}(-E_1^2/e)\sqrt{-\text{TR}W_{-1}(-E_1^2/e)}}$$

$$= 0.90E_2^*\frac{-1-\frac{TR}{T_1}\frac{2}{1+W_{-1}(-E_1^2/e)}}{-2\sqrt{-\text{TR}W_{-1}(-E_1^2/e)}}$$

$$= 0.90E_2^* \frac{-\left(1 + W_{-1}(-E_1^2/e)\right) + 2\ln(E_1)}{-2\left(1 + W_{-1}(-E_1^2/e)\right)\sqrt{-\mathrm{TR} \cdot W_{-1}(-E_1^2/e)}} \tag{S5.8}$$

Due to the property of the negative branch of Lambert W function (Eq. (S2.6-S2.7)), i.e., $W_{-1}(-E_1^2/e) < -1$, we have $-\left(1 + W_{-1}(-E_1^2/e)\right) > 0$, and hence, the denominator is positive. Now we are going to prove that the numerator is also positive. According to Eq. (S2.4):

$$W_{-1}(-E_1^2/e) = \frac{-E_1^2/e}{e^{W_{-1}(-E_1^2/e)}} = -e^{-\left(1+W_{-1}(-E_1^2/e)\right)}E_1^2 < -1$$

$$e^{-\left(1+W_{-1}(-E_1^2/e)\right)}E_1^2 > 1$$

$$-\left(1 + W_{-1}(-E_1^2/e)\right) + 2\ln(E_1) > 0 \tag{S5.9}$$

As a result, the optimal $ruCNR_{FLASH}$ is monotonically increasing with TR.

Supporting Information S6: Sacrificed ruCNR of FLASH

We assume the The sacrificed $ruCNR$ is η times the optimal $ruCNR$.

$$ruCNR_{FLASH}(N^*) = \eta \cdot ruCNR^*_{FLASH}(TR)$$

By using Eq. S5.3 and Eq. S5.7 we have:

$$0.64\frac{\sqrt{\mathrm{TR}}}{\sqrt{-\ln \varepsilon}}E_2^* \sin \alpha = \eta \frac{0.90E_2^*\sqrt{\mathrm{TR}}}{\sqrt{-W_{-1}(-E_1^2/e)}}$$

$$\frac{\sin \alpha}{\sqrt{-\ln \varepsilon}} = \eta \frac{1.41}{\sqrt{-W_{-1}(-E_1^2/e)}}$$

$$W_{-1}(-E_1^2/e)\sin^2 \alpha = 2\eta^2 \ln \varepsilon$$

$$W_{-1}(-E_1^2/e)(1 - \cos^2 \alpha) = 2\eta^2 \ln \varepsilon$$

$$W_{-1}(-E_1^2/e)\left(1 - \frac{\varepsilon^2}{E_1^2}\right) = 2\eta^2 \ln \varepsilon$$

$$\frac{W_{-1}(-E_1^2/e)}{\eta^2} - \frac{W_{-1}(-E_1^2/e)}{\eta^2 E_1^2}\varepsilon^2 = 2 \ln \varepsilon \qquad \text{(S6.1)}$$

Define a known parameter:

$$x = \frac{W_{-1}(-E_1^2/e)}{\eta^2 E_1^2} \qquad \text{(S6.2)}$$

We can rewrite and simplify Eq. S6.1

$$E_1^2 - x\varepsilon^2 = 2 \ln \varepsilon$$

$$-2 \ln \varepsilon + xE_1^2 = xe^{2 \ln \varepsilon}$$

$$(-2 \ln \varepsilon + xE_1^2)e^{-2 \ln \varepsilon} = x$$

$$(-2 \ln \varepsilon + xE_1^2)e^{-2 \ln \varepsilon + xE_1^2} = xe^{xE_1^2}$$

$$-2\ln\varepsilon + xE_1^2 = W_{-1}\left(xe^{xE_1^2}\right)$$

$$-2\ln\varepsilon = W_{-1}\left(xe^{xE_1^2}\right) - xE_1^2$$

$$\ln\varepsilon = \frac{1}{2}\left(xE_1^2 - W_{-1}\left(xe^{xE_1^2}\right)\right) \qquad \text{(S6.3)}$$

We are now able to derive the optimal flip angle with Eq. S6.3 using the property of the Lambert function Eq. (S2.4):

$$\varepsilon = \sqrt{e^{xE_1^2 - W_{-1}\left(xe^{xE_1^2}\right)}} = \sqrt{e^{xE_1^2}e^{-W_{-1}\left(xe^{xE_1^2}\right)}} = \sqrt{e^{xE_1^2}\frac{W_{-1}\left(-xe^{xE_1^2}\right)}{xe^{xE_1^2}}} = \sqrt{\frac{W_{-1}\left(xe^{xE_1^2}\right)}{x}}$$

$$\cos\alpha_{FLASH}^{\eta} = \frac{\varepsilon}{E_1} = \sqrt{\frac{W_{-1}\left(xe^{xE_1^2}\right)}{xE_1^2}}$$

$$\alpha_{FLASH}^{\eta} = \arccos\left(\sqrt{\frac{W_{-1}\left(xe^{xE_1^2}\right)}{xE_1^2}}\right) \qquad \text{(S6.4)}$$

The optimal N can is calculated from Eq. 5.2 and Eq. S6.3:

$$N_{FLASH}^{\eta} = \frac{-1.26}{\ln\varepsilon} = \frac{2.51}{W_{-1}\left(xe^{xE_1^2}\right) - xE_1^2} \qquad \text{(S6.5)}$$

Supporting Information S7: PSF of bSSFP

The PSF of the bSSFP with a linear (LN) profile order is derived from Eq. (4) and (39).

$$PSF_{bSSFP}^{LN}(z) = \int_{-0.5}^{0.5} MTF_{bSSFP}^{LN}(k) e^{i2\pi kz} dk$$

$$= \int_{-0.5}^{0.5} \left[\left(M_{prep} \cdot \sin(\alpha/2) - M_{ss} \right) \cdot \lambda^{(k+0.5)N} + M_{ss} \right] e^{i2\pi kz} dk$$

$$= \left(M_{prep} \cdot \sin(\alpha/2) - M_{ss} \right) \cdot \lambda^{0.5N} \int_{-0.5}^{0.5} e^{(i2\pi z+\lambda)k} dk + M_{ss} \cdot \int_{-0.5}^{0.5} e^{i2\pi zk} dk$$

$$= \left(M_{prep} \cdot \sin(\alpha/2) - M_{ss} \right) \cdot \lambda^{0.5N} \frac{e^{(i2\pi z+\ln\lambda)k}}{i2\pi z + \ln\lambda} \Bigg|_{-0.5}^{0.5} + M_{ss} \frac{e^{i2\pi zk}}{i2\pi z} \Bigg|_{-0.5}^{0.5}$$

$$= \left(M_{prep} \cdot \sin(\alpha/2) - M_{ss} \right) \cdot \lambda^{0.5N} \frac{e^{0.5(i2\pi z+\ln\lambda)} - e^{-0.5(i2\pi z+\ln\lambda)}}{i2\pi z + \ln\lambda} + M_{ss} \cdot \frac{e^{i\pi z} - e^{-i\pi z}}{i2\pi z}$$

$$= \left(M_{prep} \cdot \sin(\alpha/2) - M_{ss} \right) \cdot \lambda^{0.5N} \frac{\lambda^{0.5N} e^{i\pi z} - \lambda^{-0.5N} e^{-i\pi z}}{i2\pi z + \ln\lambda} + M_{ss} \cdot \frac{i2\sin(\pi z)}{i2\pi z}$$

$$= \left(M_{prep} \cdot \sin(\alpha/2) - M_{ss} \right) \cdot \frac{\lambda^{N} e^{i\pi z} - e^{-i\pi z}}{N\ln\lambda + i2\pi z} + M_{ss} \cdot \mathrm{sinc}(\pi z) \tag{S7.1}$$

The PSF of the bSSFP with a low-high profile order is derived from Eq. (4) and (40):

$$PSF_{bSSFP}^{LH}(z) = \int_{-0.5}^{0.5} MTF_{bSSFP}^{LN}(k) e^{i2\pi kz} dk$$

$$= \int_{-0.5}^{0.5} \left[\left(M_{prep} \cdot \sin(\alpha/2) - M_{ss} \right) \cdot \lambda^{2|k|N} + M_{ss} \right] e^{i2\pi kz} dk$$

$$= \left(M_{prep} \cdot \sin(\alpha/2) - M_{ss} \right) \cdot \int_{-0.5}^{0.5} \lambda^{2|k|N} e^{i2\pi kz} dk + M_{ss} \cdot \int_{-0.5}^{0.5} e^{i2\pi kz} dk$$

$$= \left(M_{prep} \cdot \sin(\alpha/2) - M_{ss} \right) \left(\int_{-0.5}^{0} \lambda^{-2kN} e^{i2\pi kz} dk + \int_{0}^{0.5} \lambda^{2kN} e^{i2\pi kz} dk \right) + M_{ss} \cdot \mathrm{sinc}(\pi z)$$

$$= \left(M_{prep} \cdot \sin(\alpha/2) - M_{ss} \right) \left(\int_{-0.5}^{0} e^{2(i\pi z - N\ln\lambda)k} dk + \int_{0}^{0.5} e^{2(i\pi z + N\ln\lambda)k} dk \right) + M_{ss} \cdot \mathrm{sinc}(\pi z)$$

$$= \left(M_{prep} \cdot \sin(\alpha/2) - M_{ss}\right) \cdot \left(\frac{e^{2(i\pi z - N \ln \lambda)k}}{2(i\pi z - N \ln \lambda)} \Bigg|_{-0.5}^{0} + \frac{e^{2(i\pi z + N \ln \lambda)k}}{2(i\pi z + N \ln \lambda)} \Bigg|_{0}^{0.5} \right) + M_{ss} \cdot \mathrm{sinc}(\pi z)$$

$$= \left(M_{prep} \cdot \sin(\alpha/2) - M_{ss}\right) \cdot \left(\frac{1 - e^{N \ln \lambda - i\pi z}}{2(i\pi z - N \ln \lambda)} + \frac{e^{i\pi z + N \ln \lambda} - 1}{2(i\pi z + N \ln \lambda)} \right) + M_{ss} \cdot \mathrm{sinc}(\pi z)$$

$$= \left(M_{prep} \cdot \sin(\alpha/2) - M_{ss}\right) \cdot \left(\frac{e^{N \ln \lambda - i\pi z} - 1}{2(N \ln \lambda - i\pi z)} + \frac{e^{i\pi z + N \ln \lambda} - 1}{2(N \ln \lambda + i\pi z)} \right) + M_{ss} \cdot \mathrm{sinc}(\pi z)$$

$$= \left(M_{prep} \cdot \sin(\alpha/2) - M_{ss}\right) \cdot \frac{\left(\lambda^N e^{-i\pi z} - 1\right)(N \ln \lambda + i\pi z) + \left(\lambda^N e^{i\pi z} - 1\right)(N \ln \lambda - i\pi z)}{2(N \ln \lambda - i\pi z)(N \ln \lambda + i\pi z)}$$

$$+ M_{ss} \cdot \mathrm{sinc}(\pi z)$$

$$= \left(M_{prep} \cdot \sin(\alpha/2) - M_{ss}\right) \cdot \frac{N \ln \lambda \left(\varepsilon^N \left(e^{i\pi z} + e^{-i\pi z}\right) - 2\right) - i\pi x \lambda^N \left(e^{i\pi z} - e^{-i\pi z}\right)}{2\pi^2 z^2 + 2N^2 \ln^2 \lambda}$$

$$+ M_{ss} \cdot \mathrm{sinc}(\pi z)$$

$$= \left(M_{prep} \cdot \sin(\alpha/2) - M_{ss}\right) \cdot \frac{2\lambda^N N \ln \lambda \cos(\pi z) + 2\lambda^N \pi x \sin(\pi z) - 2N \ln \lambda}{2\pi^2 z^2 + 2N^2 \ln^2 \lambda} + M_{ss} \cdot \mathrm{sinc}(\pi z)$$

$$= \left(M_{prep} \cdot \sin(\alpha/2) - M_{ss}\right) \cdot \frac{\lambda^N N \ln \lambda \cos(\pi z) + \lambda^N \pi z \sin(\pi z) - N \ln \lambda}{N^2 \ln^2 \lambda + \pi^2 x z^2} + M_{ss} \cdot \mathrm{sinc}(\pi z) \qquad \text{(S7.2)}$$

Supporting Information S8: ruCNR Optimization of bSSFP

The derivative of $ruCNR_{bSSFP}$ w.r.t. the factor N according to Eq. (45):

$$\frac{\partial ruCNR_{bSSFP}}{\partial N} = \frac{\partial}{\partial N}\frac{\lambda^N - 1}{\ln\lambda}\frac{\sqrt{\mathrm{TR}}}{\sqrt{N}}\sin(\alpha/2)$$

$$= \frac{\sqrt{\mathrm{TR}}\sin(\alpha/2)}{\ln\lambda}\frac{\partial}{\partial N}\frac{\lambda^N - 1}{\sqrt{N}}$$

$$= \frac{\sqrt{\mathrm{TR}}\sin(\alpha/2)}{\ln\lambda}\frac{\partial}{\partial N}\frac{\lambda^N \ln\lambda\sqrt{N} - (\lambda^N - 1)\frac{1}{2\sqrt{N}}}{N}$$

$$= \frac{\sqrt{\mathrm{TR}}\sin(\alpha/2)}{\ln\lambda}\frac{2N\lambda^N \ln\lambda - \lambda^N + 1}{2N\sqrt{N}} \quad \text{(S8.1)}$$

An optimal bSSFP factor (N^*) maximizes $ruCNR_{bSSFP}$ when the numerator of the derivative is 0:

$$2N^*\lambda^{N^*}\ln\lambda - \lambda^{N^*} + 1 = 0$$

$$2N^*e^{N^*\ln\lambda}\ln\lambda - e^{N^*\ln\lambda} + 1 = 0$$

$$(2N^*\ln\lambda - 1)e^{N^*\ln\lambda} = -1$$

$$(N^*\ln\lambda - 0.5)e^{N^*\ln\lambda} = -0.5$$

$$(N^*\ln\lambda - 0.55)e^{N^*\ln\lambda - 0.5} = -0.5e^{-0.5}$$

$$N^*\ln\lambda - 0.5 = W_{-1}(-0.5e^{-0.5})$$

$$N^* = \frac{W_{-1}(-0.5e^{-0.5}) + 0.5}{\ln\lambda} \approx \frac{-1.26}{\ln\lambda} \quad \text{(S8.2)}$$

Correspondingly the maximized $ruCNR_{bSSFP}$ is:

$$CNR_{bSSFP}(N^*) = \frac{\lambda^{N^*} - 1}{\ln\lambda}\frac{\sqrt{\mathrm{TR}}}{\sqrt{N^*}}\sin(\alpha/2)$$

$$= \frac{e^{N^* \ln\lambda} - 1}{\ln\lambda} \frac{\sqrt{-\mathrm{TR}\ln\lambda}}{\sqrt{-W_{-1}(-0.5e^{-0.5}) - 0.5}} \sin(\alpha/2)$$

$$= \frac{e^{W_{-1}(-0.5e^{-0.5})+0.5} - 1}{-\sqrt{-W_{-1}(-0.5e^{-0.5}) - 0.5}} \frac{\sqrt{\mathrm{TR}}}{\sqrt{-\ln\lambda}} \sin(\alpha/2)$$

$$= \frac{e^{0.5} \frac{-0.5e^{-0.5}}{W_{-1}(-0.5e^{-0.5})} - 1}{-\sqrt{-W_{-1}(-0.5e^{-0.5}) - 0.5}} \frac{\sqrt{\mathrm{TR}}}{\sqrt{-\ln\lambda}} \sin(\alpha/2)$$

$$= \frac{\frac{-0.5 - W_{-1}(-0.5e^{-0.5})}{W_{-1}(-0.5e^{-0.5})}}{-\sqrt{-W_{-1}(-0.5e^{-0.5}) - 0.5}} \frac{\sqrt{\mathrm{TR}}}{\sqrt{-\ln\lambda}} \sin(\alpha/2)$$

$$= \frac{\sqrt{-W_{-1}(-0.5e^{-0.5}) - 0.5}}{-W_{-1}(-0.5e^{-0.5})} \frac{\sqrt{\mathrm{TR}}}{\sqrt{-\ln\lambda}} \sin(\alpha/2)$$

$$\approx 0.64 \frac{\sqrt{\mathrm{TR}}}{\sqrt{-\ln\lambda}} \sin(\alpha/2) \qquad \text{(S8.3)}$$

The derivative of $ruCNR_{bSSFP}(N^*)$ w.r.t. flip angle α is:

$$\frac{\partial ruCNR_{bSSFP}(N^*)}{\partial\alpha} = \frac{\partial}{\partial\alpha} 0.64 \frac{\sqrt{\mathrm{TR}}}{\sqrt{-\ln\lambda}} \sin(\alpha/2)$$

$$= 0.64\sqrt{\mathrm{TR}} \frac{\partial}{\partial\alpha} \frac{\sin(\alpha/2)}{\sqrt{-\ln\lambda}}$$

$$= \frac{0.64\sqrt{\mathrm{TR}}}{-\ln\lambda} \left(\frac{1}{2}\cos(\alpha/2)\sqrt{-\ln\lambda} - \sin(\alpha/2) \frac{\partial\lambda}{\partial\alpha} \frac{\partial}{\partial\lambda} \sqrt{-\ln\lambda}\right)$$

$$= \frac{0.64\sqrt{\mathrm{TR}}}{-\ln\lambda} \left(\frac{1}{2}\cos(\alpha/2)\sqrt{-\ln\lambda} - \sin(\alpha/2) \frac{-1/\lambda}{2\sqrt{-\ln\lambda}} \frac{\partial}{\partial\alpha} [E_2 \sin^2(\alpha/2) + E_1 \cos^2(\alpha/2)]\right)$$

$$= \frac{0.64\sqrt{\mathrm{TR}}}{2\ln\lambda\sqrt{-\ln\lambda}} \left(\lambda\ln\lambda\cos(\alpha/2) - \sin(\alpha/2) \frac{\partial}{\partial\alpha} \left[E_2 \frac{1-\cos\alpha}{2} + E_1 \frac{1+\cos\alpha}{2}\right]\right)$$

$$= \frac{0.64\sqrt{\mathrm{TR}}}{2\ln\lambda\sqrt{-\ln\lambda}} \left(\lambda\ln\lambda\cos(\alpha/2) - \frac{1}{2}\sin(\alpha/2) [E_2 \sin\alpha - E_1 \sin\alpha]\right)$$

$$= \frac{0.64\sqrt{\mathrm{TR}}}{2\ln\lambda\sqrt{-\ln\lambda}} \left(\lambda\ln\lambda\cos(\alpha/2) - \frac{E_2 - E_1}{2}\sin(\alpha/2) (2\sin(\alpha/2)\cos(\alpha/2))\right)$$

$$= \frac{0.64\sqrt{\text{TR}}}{2\ln\lambda\sqrt{-\ln\lambda}}\cos(\alpha/2)\left(\lambda\ln\lambda - (E_2 - E_1)\sin^2(\alpha/2)\right)$$

$$= \frac{0.64\sqrt{\text{TR}}}{2\ln\lambda\sqrt{-\ln\lambda}}\cos(\alpha/2)\left(\lambda\ln\lambda - (E_2\sin^2(\alpha/2) + E_1\cos^2(\alpha/2)) + E_1\right)$$

$$= \frac{0.64\sqrt{\text{TR}}}{2\ln\lambda\sqrt{-\ln\lambda}}\cos(\alpha/2)\left(\lambda\ln\lambda - \lambda + E_1\right) \tag{S8.4}$$

It is obvious that $\frac{0.64\sqrt{\text{TR}}}{2\ln\lambda\sqrt{-\ln\lambda}}\cos(\alpha/2) < 0$ when $\alpha < 180°$, so we need to consider $\lambda\ln\lambda - \lambda + E_1$ only. When T_1, T_2, and TR are fixed, the parameters E_1 and E_2 are also fixed. In this case, according to Eq. (38), λ is an affine combination of E_1 and E_2, for $\sin^2(\alpha/2) + \cos^2(\alpha/2) = 1$. Therefore,

$$\lambda = E_2\sin^2(\alpha/2) + E_1\cos^2(\alpha/2) \in [E_2, E_1] < 1 \tag{S8.5}$$

Now we prove that $\lambda\ln\lambda - \lambda + E_1$ is monotonically decreasing with λ, i.e., its derivative is negative:

$$\frac{\partial(\lambda\ln\lambda - \lambda + E_1)}{\partial\lambda} = \ln\lambda + \lambda\frac{1}{\lambda} - 1 = \ln\lambda < 0 \tag{S8.6}$$

Therefore it reaches minimum and maximum value when λ is E_1, and E_2, respectively:

$$\lambda\ln\lambda - \lambda + E_1 \in [E_1\ln E_1, E_2\ln E_2 - E_2 + E_1] \tag{S8.7}$$

Because $E_1\ln E_1 \le 0$, if $E_2\ln E_2 - E_2 + E_1 < 0$, $nCNR_{bSSFP}(N^*)$ is monotonically increasing with α (the derivative of $ruCNR_{bSSFP}(N^*)$ w.r.t. α is positive and decreases to 0 at $\alpha = 180°$). In this case the optimal $ruCNR_{bSSFP}$ is achieved when $\lambda = E_2$ and:

$$\alpha^*(TR) = 180°$$

$$N^*_{bSSFP}(\mathrm{TR}) = \frac{-1.26}{\ln\lambda} = \frac{-1.26}{\ln E_2} = 1.26\frac{T_2}{\mathrm{TR}} \tag{S8.8}$$

$$nCNR^*_{bSSFP}(\mathrm{TR}) = 0.64\frac{\sqrt{\mathrm{TR}}}{\sqrt{-\ln\lambda}}\sin(\alpha^*/2) = 0.64\frac{\sqrt{\mathrm{TR}}}{\sqrt{-\ln E_2}}\sin(\alpha^*/2) = 0.64\sqrt{T_2}$$

If $E_2 \ln E_2 - E_2 + E_1 > 0$, there is an optimal flip angle $\alpha^* < 180°$ when the derivative of $ruCNR_{bSSFP}(N^*)$ w.r.t. α is 0, i.e., $\lambda\ln\lambda - \lambda + E_1 = 0$. To simplify the calculation, we define $x = \ln\lambda$:

$$e^x x - e^x + E_1 = 0$$

$$(x-1)e^x = -E_1$$

$$(x-1)e^{x-1} = -E_1/e$$

$$x - 1 = W_{-1}(-E_1/e)$$

$$x = W_{-1}(-E_1/e) + 1$$

$$\lambda = e^x = e^{W_{-1}(-E_1/e)+1} = e^{W_{-1}(-E_1/e)}e = \frac{-E_1/e}{W_{-1}(-E_1/e)}e = \frac{-E_1}{W_{-1}(-E_1/e)}$$

$$E_2\sin^2\frac{\alpha^*}{2} + E_1\cos^2\frac{\alpha^*}{2} = \frac{-E_1}{W_{-1}(-E_1/e)}$$

$$E_2\frac{1-\cos\alpha^*}{2} + E_1\frac{1+\alpha^*}{2} = \frac{-E_1}{W_{-1}(-E_1/e)}$$

$$(E_1 - E_2)\cos\alpha^* + E_1 + E_2 = \frac{-2E_1}{W_{-1}(-E_1/e)}$$

$$\cos\alpha^* = \frac{2E_1 + (E_1+E_2)W_{-1}(-E_1/e)}{(E_2-E_1)W_{-1}(-E_1/e)}$$

$$\alpha^*_{bSSFP}(\mathrm{TR}) = \arccos\left(\frac{2E_1 + (E_1+E_2)W_{-1}(-E_1/e)}{(E_2-E_1)W_{-1}(-E_1/e)}\right) \tag{S8.9}$$

$$N^*_{bSSFP}(\mathrm{TR}) = \frac{-1.26}{\ln\lambda} = \frac{-1.26}{x} = \frac{-1.26}{W_{-1}(-E_1/e)+1}$$

According to Eq. (S8.7), the condition to accept Eq. (S8.9) rather than (S8.8) is:

$$E_2 \ln E_2 - E_2 + E_1 > 0$$

$$\left(-\frac{TR}{T_2}\right) e^{-TR/T_2} - e^{-TR/T_2} + e^{-\frac{TR}{T_1}} > 0$$

$$e^{-\frac{TR}{T_1}} > (TR/T_2 + 1) e^{-\frac{TR}{T_2}}$$

$$e^{\left(\frac{1}{T_2}-\frac{1}{T_1}\right)TR} > \frac{TR}{T_2} + 1$$

$$(TR + T_2) e^{\left(\frac{1}{T_1}-\frac{1}{T_2}\right)TR} < T_2$$

$$\left(\frac{1}{T_1} - \frac{1}{T_2}\right)(TR + T_2) e^{\left(\frac{1}{T_1}-\frac{1}{T_2}\right)TR} > \frac{T_2 - T_1}{T_1}$$

$$\left(\frac{1}{T_1} - \frac{1}{T_2}\right)(TR + T_2) e^{\left(\frac{1}{T_1}-\frac{1}{T_2}\right)(TR+T_2)} > \frac{T_2 - T_1}{T_1} e^{\frac{T_2-T_1}{T_1}}$$

$$\left(\frac{1}{T_1} - \frac{1}{T_2}\right)(TR + T_2) < W_{-1}\left(\frac{T_2 - T_1}{T_1} e^{\frac{T_2-T_1}{T_1}}\right)$$

$$TR > \frac{T_1 T_2}{T_2 - T_1} W_{-1}\left(\frac{T_2 - T_1}{T_1} e^{\frac{T_2-T_1}{T_1}}\right) - T_2 \tag{S8.10}$$

Here we use a property that $W_{-1}(\cdot)$ is monotonically decreasing. We define the threshold value:

$$\overline{TR} = \frac{T_1 T_2}{T_2 - T_1} W_{-1}\left(\frac{T_2 - T_1}{T_1} e^{\frac{T_2-T_2}{T_1}}\right) - T_2. \tag{S8.11}$$

Note that the derivation in Eq. (S8.10) is reversible, i.e., the two inequalities $E_2 \ln E_2 - E_2 + E_1 > 0$ and $\mathrm{TR} > \overline{\mathrm{TR}}$ are equivalent. Therefore, $E_2 \ln E_2 - E_2 + E_1 \leq 0$ is also equivalent to $\mathrm{TR} \leq \overline{\mathrm{TR}}$. We can combine the two conditions with a max function where the left (right) side is larger if $\mathrm{TR} \leq \overline{\mathrm{TR}}$ ($\mathrm{TR} > \overline{\mathrm{TR}}$):

$$N^*_{bSSFP}(\mathrm{TR}) = 1.26\max\left\{\frac{T_2}{TR}, \frac{-1}{1 + W_{-1}(-E_1/e)}\right\},$$

$$\alpha^*_{bSSFP}(\mathrm{TR}) = \arccos\left(\max\left\{-1, \frac{2E_1 + (E_1 + E_2)W_{-1}(-E_1/e)}{(E_2 - E_1)W_{-1}(-E_1/e)}\right\}\right), \tag{S8.12}$$

The corresponding maximum $ruCNR_{bSSFP}$ is:

$$nCNR^*_{bSSFP}(\mathrm{TR}) = 0.64\frac{\sqrt{\mathrm{TR}}}{\sqrt{-\ln\lambda}}\sin\left(\frac{\alpha^*}{2}\right)$$

$$= 0.64\min\left\{\sqrt{\mathrm{T}_2}, \frac{\sqrt{\mathrm{TR}}}{\sqrt{-W_{-1}\left(-\frac{E_1}{e}\right) - 1}}\right\}\frac{\sqrt{1 - \cos\alpha^*}}{\sqrt{2}}$$

$$= \frac{0.64}{\sqrt{2}}\min\left\{\sqrt{\mathrm{T}_2}, \frac{\sqrt{\mathrm{TR}}}{\sqrt{-W_{-1}\left(-\frac{E_1}{e}\right) - 1}}\right\}\sqrt{1 - \max\left\{-1, \frac{2E_1 + (E_1 + E_2)W_{-1}\left(-\frac{E_1}{e}\right)}{(E_2 - E_1)W_{-1}\left(-\frac{E_1}{e}\right)}\right\}}$$

$$= \frac{0.64}{\sqrt{2}}\min\left\{\sqrt{\mathrm{T}_2}, \frac{\sqrt{\mathrm{TR}}}{\sqrt{-W_{-1}\left(-\frac{E_1}{e}\right) - 1}}\right\}\max\left\{\sqrt{2}, \sqrt{1 - \frac{2E_1 + (E_1 + E_2)W_{-1}\left(-\frac{E_1}{e}\right)}{(E_2 - E_1)W_{-1}\left(-\frac{E_1}{e}\right)}}\right\}$$

$$= \frac{0.64}{\sqrt{2}}\min\left\{\sqrt{\mathrm{T}_2}, \frac{\sqrt{\mathrm{TR}}}{\sqrt{-W_{-1}\left(-\frac{E_1}{e}\right) - 1}}\right\}\max\left\{\sqrt{2}, \sqrt{\frac{-2E_1W_{-1}\left(-\frac{E_1}{e}\right) - 2E_1}{(E_2 - E_1)W_{-1}\left(-\frac{E_1}{e}\right)}}\right\}$$

$$= 0.64\min\left\{\sqrt{\mathrm{T}_2}, \frac{\sqrt{\mathrm{TR}}}{\sqrt{-W_{-1}\left(-\frac{E_1}{e}\right) - 1}}\right\}\max\left\{1, \sqrt{\frac{E_1}{E_2 - E_1}\frac{-W_{-1}\left(-\frac{E_1}{e}\right) - 1}{W_{-1}\left(-\frac{E_1}{e}\right)}}\right\}$$

$$= \begin{cases} 0.64\sqrt{T_2} & \text{if } TR \le \overline{TR} \\ 0.64\dfrac{\sqrt{\mathrm{TR}\cdot E_1}}{\sqrt{(E_2 - E_1)W_{-1}(-E_1/e)}} & \text{if } TR > \overline{TR} \end{cases} \tag{S8.13}$$

Supporting Information S9: Sacrificed ruCNR of bSSFP

According to Eq. (49) the sacrificed $ruCNR$ is η times the optimal $ruCNR$. There are multiple pairs of N and α solutions for each η.

$$ruCNR_{bSSFP}(N^*) = \eta \cdot ruCNR_{bSSFP}^*(\text{TR})$$

When $\text{TR} < \overline{\text{TR}}$, the left side of the max and min opterators in N_{bSSFP}^*, α_{bSSFP}^*, and $ruCNR_{bSSFP}^*$ are used. The one that has the minimum α satisfies Eq. (S8.2) and (S8.3):

$$0.64\frac{\sqrt{\text{TR}}}{\sqrt{-\ln\lambda}}\sin(\alpha/2) = 0.64\eta\sqrt{T_2}$$

$$\frac{\sqrt{\text{TR}}}{\sqrt{-\ln\lambda}}\sin(\alpha/2) = \eta\sqrt{T_2}$$

$$-\frac{\text{TR}}{\ln\lambda}\sin^2(\alpha/2) = \eta^2 T_2$$

$$-\frac{\text{TR}}{\eta^2 T_2}\sin^2(\alpha/2) = \ln\lambda$$

$$\frac{\ln E_2}{\eta^2}\sin^2(\alpha/2) = \ln\lambda$$

$$e^{\frac{\ln E_2}{\eta^2}\sin^2(\alpha/2)} = \lambda = E_2\sin^2(\alpha/2) + E_1\cos^2(\alpha/2) = (E_2 - E_1)\sin^2(\alpha/2) + E_1 \tag{S9.1}$$

To simplify the calculation, let $x = \sin^2(\alpha/2)$, we can rewrite Eq. (S9.1) and solve for x:

$$e^{\frac{\ln E_2}{\eta^2}x} = (E_2 - E_1)x + E_1$$

$$[(E_2 - E_1)x + E_1]e^{-\frac{\ln E_2}{\eta^2}x} = 1$$

$$\left(x + \frac{E_1}{E_2 - E_1}\right)e^{-\frac{\ln E_2}{\eta^2}x} = \frac{1}{E_2 - E_1}$$

$$-\frac{\ln E_2}{\eta^2}\left(x-\frac{E_1}{E_1-E_2}\right)e^{-\frac{\ln E_2}{\eta^2}x}=\frac{\ln E_2}{\eta^2(E_1-E_2)}$$

$$-\frac{\ln E_2}{\eta^2}\left(x-\frac{E_1}{E_1-E_2}\right)e^{-\frac{\ln E_2}{\eta^2}\left(x-\frac{E_1}{E_1-E_2}\right)}=\frac{\ln E_2}{\eta^2(E_1-E_2)}e^{\frac{E_1\ln E_2}{\eta^2(E_1-E_2)}}$$

$$-\frac{\ln E_2}{\eta^2}\left(x-\frac{E_1}{E_1-E_2}\right)=W_{-1}\left(\frac{\ln E_2}{\eta^2(E_1-E_2)}e^{\frac{E_1\ln E_2}{\eta^2(E_1-E_2)}}\right)$$

$$x=-\frac{\eta^2}{\ln E_2}W_{-1}\left(\frac{\ln E_2}{\eta^2(E_1-E_2)}e^{\frac{E_1\ln E_2}{\eta^2(E_1-E_2)}}\right)+\frac{E_1}{E_1-E_2} \tag{S9.2}$$

The optimal α (α^{η}_{bSSFP}) and optimal N^{η}_{bSSFP} can then be solved. Here we use Eq. (S9.2) for the optimal N:

$$\cos\alpha=1-2\sin^2(\alpha/2)=1+\frac{2\eta^2}{\ln E_2}W_{-1}\left(\frac{\ln E_2}{\eta^2(E_1-E_2)}e^{\frac{E_1\ln E_2}{\eta^2(E_1-E_2)}}\right)-\frac{2E_1}{E_1-E_2}$$

$$=\frac{2\eta^2}{\ln E_2}W_{-1}\left(\frac{\ln E_2}{\eta^2(E_1-E_2)}e^{\frac{E_1\ln E_2}{\eta^2(E_1-E_2)}}\right)-\frac{E_1+E_2}{E_1-E_2}$$

$$\alpha^{\eta}_{bSSFP}(TR)=\arccos\left(\frac{2\eta^2}{\ln E_2}W_{-1}\left(\frac{\ln E_2}{\eta^2(E_1-E_2)}e^{\frac{E_1\ln E_2}{\eta^2(E_1-E_2)}}\right)-\frac{E_1+E_2}{E_1-E_2}\right) \tag{S9.3}$$

$$\lambda=E_2\sin^2\left(\frac{\alpha}{2}\right)+E_1\cos^2\left(\frac{\alpha}{2}\right)=E_2\frac{1-\cos\alpha}{2}+E_1\frac{1+\cos\alpha}{2}$$

$$=\frac{1}{2}(E_1-E_2)\cos\alpha+\frac{1}{2}(E_1+E_2)$$

$$=\frac{1}{2}(E_1-E_2)\frac{2\eta^2}{\ln E_2}W_{-1}\left(\frac{\ln E_2}{\eta^2(E_1-E_2)}e^{\frac{E_1\ln E_2}{\eta^2(E_1-E_2)}}\right)-\frac{1}{2}(E_1+E_2)+\frac{1}{2}(E_1+E_2)$$

$$=\frac{2\eta^2(E_1-E_2)}{\ln E_2}W_{-1}\left(\frac{\ln E_2}{\eta^2(E_1-E_2)}e^{\frac{E_1\ln E_2}{\eta^2(E_1-E_2)}}\right)$$

$$=\frac{2\eta^2(E_1-E_2)}{\ln E_2}\frac{\frac{\ln E_2}{\eta^2(E_1-E_2)}e^{\frac{E_1\ln E_2}{\eta^2(E_1-E_2)}}}{\exp\left[W_{-1}\left(\frac{\ln E_2}{\eta^2(E_1-E_2)}e^{\frac{E_1\ln E_2}{\eta^2(E_1-E_2)}}\right)\right]}=e^{\frac{E_1\ln E_2}{\eta^2(E_1-E_2)}-W_{-1}\left(\frac{\ln E_2}{\eta^2(E_1-E_2)}e^{\frac{E_1\ln E_2}{\eta^2(E_1-E_2)}}\right)}$$

$$\ln\lambda = \frac{E_1 \ln E_2}{\eta^2(E_1 - E_2)} - W_{-1}\left(\frac{\ln E_2}{\eta^2(E_1 - E_2)} e^{\frac{E_1 \ln E_2}{\eta^2(E_1 - E_2)}}\right)$$

$$N_{bSSFP}^{\eta}(\mathrm{TR}) = -\frac{1.26}{\ln\lambda} = \frac{1.26}{W_{-1}\left(\frac{\ln E_2}{\eta^2(E_1 - E_2)} e^{\frac{E_1 \ln E_2}{\eta^2(E_1 - E_2)}}\right) - \frac{E_1 \ln E_2}{\eta^2(E_1 - E_2)}} \tag{S9.4}$$

When $\mathrm{TR} > \overline{\mathrm{TR}}$, the right side of the max and min opterators in N_{bSSFP}^{*}, α_{bSSFP}^{*}, and $ruCNR_{bSSFP}^{*}$ are used. In this case, $\alpha_{bSSFP}^{*} < 180°$. A sacrificed $ruCNR$ is also applicable to reduce the flip angles. Similarly, we use Eq. (S8.2) and (S8.3) to focus on the optimal solution that has minimal α.

$$CNR_{bSSFP}(N^*) = \eta CNR_{bSSFP}^{*}(\mathrm{TR})$$

$$0.64\sqrt{\frac{\mathrm{TR}}{-\ln\lambda}}\sin(\alpha/2) = \eta 0.64\sqrt{\frac{\mathrm{TR}\cdot E_1}{(E_2 - E_1)W_{-1}(-E_1/e)}}$$

$$\sqrt{\frac{TR}{-\ln\lambda}}\sin(\alpha/2) = \eta\sqrt{\frac{\mathrm{TR}\cdot E_1}{(E_2 - E_1)W_{-1}(-E_1/e)}}$$

$$\frac{\mathrm{TR}}{-\ln\lambda}\sin^2(\alpha/2) = \eta^2\frac{\mathrm{TR}\cdot E_1}{(E_2 - E_1)W_{-1}(-E_1/e)}$$

$$(E_2 - E_1)\sin^2(\alpha/2)\,W_{-1}(-E_1/e) = -\eta^2 E_1 \ln\lambda$$

$$(E_2\sin^2(\alpha/2) + E_1\cos^2(\alpha/2) - E_1)W_{-1}(-E_1/e) = -\eta^2 E_1 \ln\lambda$$

$$(\lambda - E_1)W_{-1}(-E_1/e) = -\eta^2 E_1 \ln\lambda \tag{S9.5}$$

To simplify the calculation, let $x = \ln\lambda$, we can rewrite Eq. (S9.5) and solve for x:

$$(e^x - E_1)W_{-1}(-E_1/e) = -\eta^2 E_1 x$$

$$\eta^2 E_1 x - E_1 W_{-1}(-E_1/e) = -e^x W_{-1}(-E_1/e)$$

$$[\eta^2 E_1 x - E_1 W_{-1}(-E_1/e)]e^{-x} = -W_{-1}(-E_1/e)$$

$$[-\eta^2 E_1 x + E_1 W_{-1}(-E_1/e)]e^{-x} = W_{-1}(-E_1/e)$$

$$\left[-x+\frac{W_{-1}(-E_1/e)}{\eta^2}\right]e^{-x}=\frac{W_{-1}(-E_1/e)}{\eta^2 E_1}$$

$$\left[-x+\frac{W_{-1}(-E_1/e)}{\eta^2}\right]e^{-x+\frac{W_{-1}(-E_1/e)}{\eta^2}}=\frac{W_{-1}(-E_1/e)}{\eta^2 E_1}e^{\frac{W_{-1}(-E_1/e)}{\eta^2}}$$

$$-x+\frac{W_{-1}(-E_1/e)}{\eta^2}=W_{-1}\left(\frac{W_{-1}(-E_1/e)}{\eta^2 E_1}e^{\frac{W_{-1}(-E_1/e)}{\eta^2}}\right)$$

$$x=ln\lambda=\frac{W_{-1}(-E_1/e)}{\eta^2}-W_{-1}\left(\frac{W_{-1}(-E_1/e)}{\eta^2 E_1}e^{\frac{W_{-1}(-E_1/e)}{\eta^2}}\right) \quad \text{(S9.6)}$$

The optimal α (α^{η}_{bSSFP}) and optimal N^{η}_{bSSFP} can then be solved. Here we use Eq. (S8.2) for the optimal N:

$$N^{\eta}_{bSSFP}=\frac{1.26}{-\ln\lambda}=\frac{1.26}{-\frac{W_{-1}(-E_1/e)}{\eta^2}+W_{-1}\left(\frac{W_{-1}(-E_1/e)}{\eta^2 E_1}e^{\frac{W_{-1}(-E_1/e)}{\eta^2}}\right)}$$

$$=\frac{1.26}{W_{-1}\left(\frac{W_{-1}(-E_1/e)}{\eta^2 E_1}e^{\frac{W_{-1}(-E_1/e)}{\eta^2}}\right)-\frac{W_{-1}(-E_1/e)}{\eta^2}}$$

$$\lambda=e^{\frac{W_{-1}(-E_1/e)}{\eta^2}}e^{-W_{-1}\left(\frac{W_{-1}(-E_1/e)}{\eta^2 E_1}e^{\frac{W_{-1}(-E_1/e)}{\eta^2}}\right)}=e^{\frac{W_{-1}(-E_1/e)}{\eta^2}}\frac{W_{-1}\left(\frac{W_{-1}(-E_1/e)}{\eta^2 E_1}e^{\frac{W_{-1}(-E_1/e)}{\eta^2}}\right)}{\frac{W_{-1}(-E_1/e)}{\eta^2 E_1}e^{\frac{W_{-1}(-E_1/e)}{\eta^2}}}$$

$$=\frac{\eta^2 E_1 W_{-1}\left(\frac{W_{-1}(-E_1/e)}{\eta^2 E_1}e^{\frac{W_{-1}(-E_1/e)}{\eta^2}}\right)}{W_{-1}(-E_1/e)}$$

$$\lambda=E_2\sin^2\frac{\alpha}{2}+E_1\cos^2\frac{\alpha}{2}=E_2\frac{1-\cos\alpha}{2}+E_1\frac{1+\cos\alpha}{2}=-\frac{E_2-E_1}{2}\cos\alpha+\frac{E_1+E_2}{2}$$

$$\cos\alpha=\frac{E_1+E_2}{E_2-E_1}-\frac{2}{E_2-E_1}\lambda=\frac{(E_1+E_2)W_{-1}(-E_1/e)-2\eta^2 E_1 W_{-1}\left(\frac{W_{-1}(-E_1/e)}{\eta^2 E_1}e^{\frac{W_{-1}(-E_1/e)}{\eta^2}}\right)}{(E_2-E_1)W_{-1}(-E_1/e)}$$

$$\alpha_{bSSFP}^{\eta} = \arccos \frac{(E_1 + E_2)W_{-1}(-E_1/e) - 2\eta^2 E_1 W_{-1}\left(\frac{W_{-1}(-E_1/e)}{\eta^2 E_1} e^{\frac{W_{-1}(-E_1/e)}{\eta^2}}\right)}{(E_2 - E_1)W_{-1}(-E_1/e)}$$

$$= \arccos\left(\frac{2\eta^2 E_1}{(E_1 - E_2)W_{-1}(-E_1/e)} W_{-1}\left(\frac{W_{-1}(-E_1/e)}{\eta^2 E_1} e^{\frac{W_{-1}(-E_1/e)}{\eta^2}}\right) - \frac{E_1 + E_2}{E_1 - E_2}\right) \tag{S9.7}$$

The α can be reduced when both $\mathrm{TR} \le \overline{\mathrm{TR}}$ and $\mathrm{TR} > \overline{\mathrm{TR}}$. Now we define a parameter:

$$x = \begin{cases} \frac{\ln E_2}{\eta^2(E_1 - E_2)} & \text{if } \mathrm{TR} \le \overline{\mathrm{TR}} \\ \frac{W_{-1}(-E_1/e)}{\eta^2 E_1} & \text{if } \mathrm{TR} > \overline{\mathrm{TR}} \end{cases} \tag{S9.8}$$

With this parameter, Eq. (S8.3) and (S8.6) can be combined for $\mathrm{TR} \le \overline{\mathrm{TR}}$ and $\mathrm{TR} > \overline{\mathrm{TR}}$:

$$N_{bSSFP}^{\eta} = \frac{1.26}{W_{-1}(xe^{xE_1}) - xE_1}$$

$$\alpha_{bSSFP}^{\eta} = \arccos\left(\frac{2W_{-1}\left(xe^{xE_1}\right)/x - E_1 - E_2}{E_1 - E_2}\right). \tag{S9.9}$$