

## Supplementary Materials for

### **Strong interlayer interactions in bilayer and trilayer moiré superlattices**

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Supplementary Text  
Figs. S1 and S2

## Supplementary Text

### Moiré superlattice under uniaxial heterostrain

The layered materials in moiré superlattice samples are often under strain, which can be introduced during the sample preparation process. Such strain affects the reciprocal wavevector of the corresponding material, and therefore the moiré wavevectors. In the case where both layers are strained equally, the change in moiré wavevector is minimal (given the magnitude of strain is typically less than 1%). In contrast, when the two layers are strained differently (i.e., heterostrain), the moiré wavevector can have significant changes. In the following discussion, we focus on heterostrained WS<sub>2</sub>/WSe<sub>2</sub> superlattices, where WS<sub>2</sub> is under a uniaxial strain while WSe<sub>2</sub> remains unstrained.

We assign  $\mathbf{k}_1$ ,  $\mathbf{k}_2$ , and  $\mathbf{k}_3$  to be the reciprocal wavevectors of WS<sub>2</sub>, and  $\mathbf{k}'_1$ ,  $\mathbf{k}'_2$ , and  $\mathbf{k}'_3$  to be the reciprocal wavevectors of WSe<sub>2</sub>.

$$\mathbf{k}_1 = \begin{pmatrix} k_{\text{WS}_2} \\ 0 \end{pmatrix}, \mathbf{k}_2 = \begin{pmatrix} k_{\text{WS}_2} \times \cos(60) \\ k_{\text{WS}_2} \times \sin(60) \end{pmatrix}, \mathbf{k}_3 = \begin{pmatrix} k_{\text{WS}_2} \times \cos(120) \\ k_{\text{WS}_2} \times \sin(120) \end{pmatrix} \text{ and } k_{\text{WS}_2} = \frac{4\pi}{\sqrt{3} * a_{\text{WS}_2}}$$

$$\mathbf{k}'_1 = \begin{pmatrix} k_{\text{WSe}_2} \\ 0 \end{pmatrix}, \mathbf{k}'_2 = \begin{pmatrix} k_{\text{WSe}_2} \times \cos(60) \\ k_{\text{WSe}_2} \times \sin(60) \end{pmatrix}, \mathbf{k}'_3 = \begin{pmatrix} k_{\text{WSe}_2} \times \cos(120) \\ k_{\text{WSe}_2} \times \sin(120) \end{pmatrix} \text{ and } k_{\text{WSe}_2} = \frac{4\pi}{\sqrt{3} * a_{\text{WSe}_2}}$$

The moiré wavevectors (shown in Fig. S1A) are then given by

$$\mathbf{K}_1 = \mathbf{k}_1 - \mathbf{k}'_1, \mathbf{K}_2 = \mathbf{k}_2 - \mathbf{k}'_2, \mathbf{K}_3 = \mathbf{k}_3 - \mathbf{k}'_3$$

A uniaxial strain with the magnitude of  $\varepsilon$ , along the direction of  $\theta_s$  respective to the  $x$  axis, which can be described by the following matrix:

$$\mathbf{S}(\varepsilon, \theta_s) = \begin{pmatrix} \cos(\theta_s) & \sin(\theta_s) \\ -\sin(\theta_s) & \cos(\theta_s) \end{pmatrix} \begin{pmatrix} \frac{1}{1+\varepsilon} & 0 \\ 0 & \frac{1}{1-\delta\varepsilon} \end{pmatrix} \begin{pmatrix} \cos(\theta_s) & -\sin(\theta_s) \\ \sin(\theta_s) & \cos(\theta_s) \end{pmatrix}$$

Where  $\delta$  is the Poisson ratio of the material being strained ( $\delta = 0.22$  for WS<sub>2</sub>).

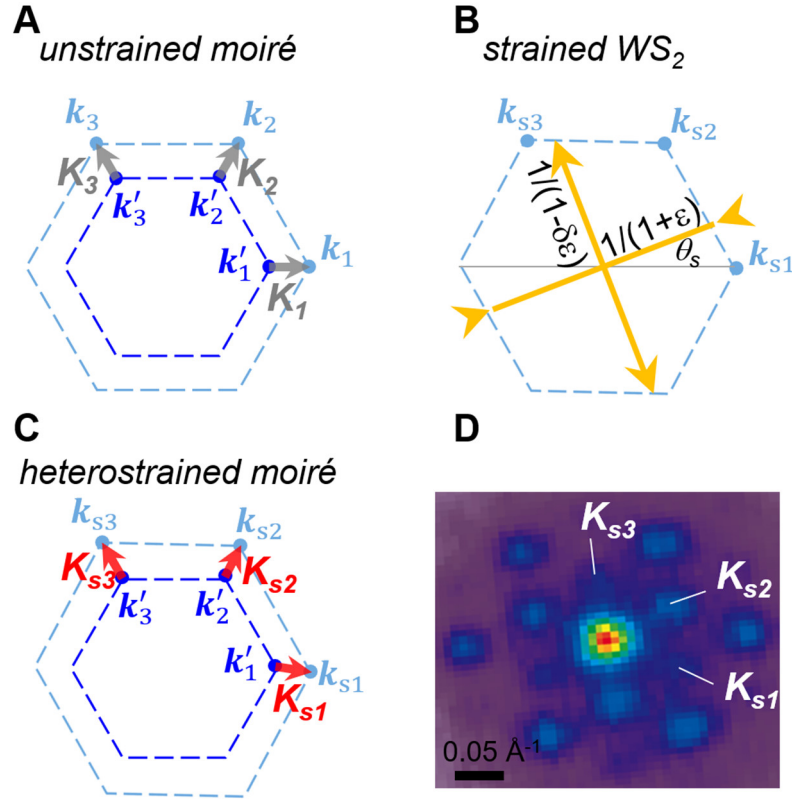
The reciprocal wavevectors of strained WS<sub>2</sub> (Fig. S1B) become

$$\mathbf{k}_{s1} = \mathbf{S}(\varepsilon, \theta_s) \mathbf{k}_1, \mathbf{k}_{s2} = \mathbf{S}(\varepsilon, \theta_s) \mathbf{k}_2, \mathbf{k}_{s3} = \mathbf{S}(\varepsilon, \theta_s) \mathbf{k}_3$$

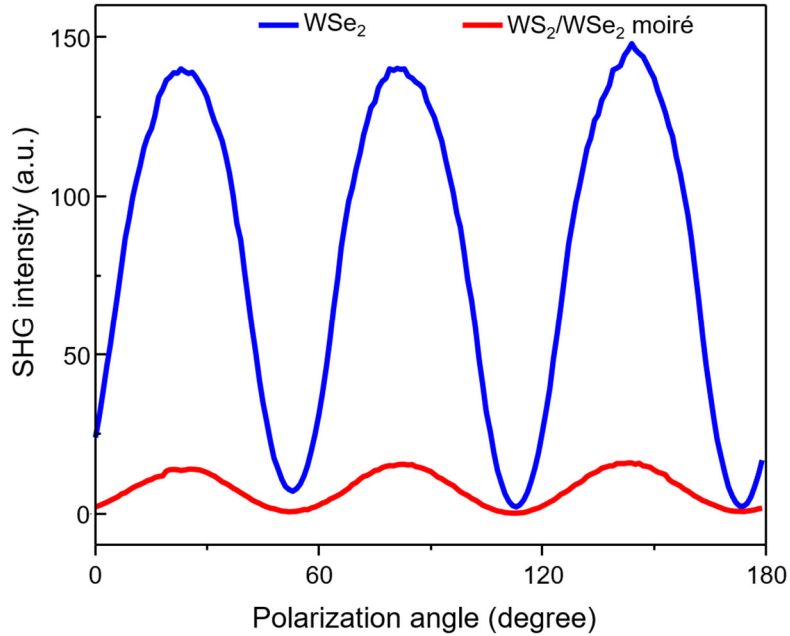
Thus the moiré wavevectors in the heterostrained WS<sub>2</sub>/WSe<sub>2</sub> superlattices (Fig. S1C) become

$$\mathbf{K}_{s1} = \mathbf{k}_{s1} - \mathbf{k}'_1, \mathbf{K}_{s2} = \mathbf{k}_{s2} - \mathbf{k}'_2, \mathbf{K}_{s3} = \mathbf{k}_{s3} - \mathbf{k}'_3$$

We numerically fit the magnitude of  $\mathbf{K}_{s1}$ ,  $\mathbf{K}_{s2}$ , and  $\mathbf{K}_{s3}$  to find  $\varepsilon$  and  $\theta_s$ . For the sample shown in Fig. 3B (also shown in Fig. S1D), the numeric fit based on the magnitude of  $\mathbf{K}_{s1}$ ,  $\mathbf{K}_{s2}$ , and  $\mathbf{K}_{s3}$  gives  $\varepsilon = 0.7\%$  and  $\theta_s = 106.4^\circ$ .



**Fig. S1. Moiré wavevectors under uniaxial heterostrain.** (A) Unstrained reciprocal wavevectors of  $\text{WS}_2$  (light blue) and  $\text{WSe}_2$  (blue), respectively, and the corresponding moiré wavevectors (gray). (B) Reciprocal wavevectors of  $\text{WS}_2$  under a uniaxial strain of magnitude  $\varepsilon$ , along the direction  $\theta_s$  with respect to  $x$  axis. (C) Moiré wavevectors in heterostrained  $\text{WS}_2/\text{WSe}_2$  moiré superlattice, and the corresponding moiré wavevectors (red). (D) Moiré wavevectors identified in Fig. 3B for numeric fitting find  $\varepsilon$  and  $\theta_s$ .



**Figure S2. Polarization-resolved SHG intensity.** Second-harmonic intensity as a function of the polarization angle of the excitation light taken at WSe<sub>2</sub> area (blue) and WS<sub>2</sub>/WSe<sub>2</sub> area (red), respectively, on the sample shown in Figure 1C. The lower intensity measured from the WS<sub>2</sub>/WSe<sub>2</sub> area indicates that the second-harmonic radiation from monolayer WSe<sub>2</sub> and WS<sub>2</sub> is added destructively, suggesting a 60° twist angle between WS<sub>2</sub> and WSe<sub>2</sub>.