Efficacy of "stay-at-home" policy and transmission of COVID-19 in Toronto, Canada: a mathematical modeling study

Pei Yuan[1,2*], Juan Li[1,3*], Elena Aruffo[1,2*], Qi Li[1,4] ,Tingting Zheng[1,5], Nick Ogden[6], Beate Sander[7,8], Jane Heffernan[1,2], Evgenia Gatov[9], Effie Gournis[8,9], Sarah Collier[9], Yi Tan[1,2], Jun Li[1,10] Julien Arino1[1,11] Jacques Bélair[1,12] James Watmough[1,13], Jude Dzevela Kong[1,2], Iain Moyles[1,2], Huaiping Zhu[1,2 †]

Affiliations:

[1] Centre for Diseases Modeling (CDM), York University, Toronto, Canada
[2] Department of Mathematics and Statistics, York University, Toronto, Canada
[3] Complex Systems Research Center, Shanxi University, Taiyuan, Shanxi, China
[4] Department of Mathematics, Shanghai Normal University, Shanghai, China
[5] College of Mathematics and System Science, Xinjiang University, Urumqi, Xinjiang, China
[6] Public Health Risk Sciences Division, National Microbiology Laboratory, Public Health Agency of Canada, Canada
[7] Toronto Health Economics and Technology Assessment (THETA) Collaborative, University Health Network, Toronto, Canada
[8] Dalla Lana School of Public Health, University of Toronto, Toronto, Canada
[9] Toronto Public Health, City of Toronto, Toronto, ON, Canada.
[10] School of Mathematics and Statistics, Xidian University, Xi'an, Shaanxi, China
[11] Department of Mathematics, University of Manitoba, Winnipeg, Manitoba, Canada
[12] Département de Mathématiques et de Statistique, Université de Montréal, Montréal, Québec, Canada
[13] Department of Mathematics and Statistics, University of New Brunswick, Fredericton, New Brunswick, Canada

* Co-first authors made the same contributions.

† Corresponding Author: huaiping@yorku.ca,
4700 Keele Street, Toronto, Ontario, Canada, M3J1P3

Summary

Background

In many parts of the world, restrictive non-pharmaceutical interventions (NPI) to reduce contact rates, including stay-at-home orders, limitations on gatherings, and closure of public places, are being lifted. Here we aim to capture the combination of use of NPI's and reopening measures to prevent an infection rebound.

Methods

We employ an SEAIR model with a household structure to capture the stay-at-home policy (SAHP). Considering the SAHP compliance rate and using confirmed case data for the City of Toronto, we evaluate basic and instantaneous reproduction numbers and simulate how the average household size, the stay-at-home rate, the efficiency and duration of SAHP implementation affect the outbreak trajectory.

Results

The estimated basic reproduction number was 2.36 (95% CI: 2.28, 2.45) in Toronto. With the SAHP, the contact rate outside the household fell by 39%. When people properly respect the SAHP, the outbreak can be quickly controlled, but extending its duration beyond two months had little effect. To avoid a large rebound of the epidemic, the average number of contacts per person per day should be kept below nine. This study suggests that fully reopening schools and workplaces are possible if the other NPIs can be strictly adhered to.

Interpretation

Our model confirmed that the SAHP implemented in Toronto had a great impact on controlling the spread of COVID-19. Given the lifting of restrictive NPIs, we estimated the thresholds values of the maximum number of contacts, probability of transmission and testing needed to ensure that the reopening will be safe. Our results also predicts the current situation of cases' increment possibly related to a high number of contacts.

Keywords: *COVID-19; transmission model; household structure; stay-at-home policy; non-pharmaceutical interventions; reopen*

1. Introduction

Although COVID-19 cases have been increasing daily[1], it was effectively mitigated via non-pharmaceutical interventions[2] (NPIs), i.e., social distancing (including stay-at-home policy, SAHP), isolation of cases, contact tracing, quarantine, hand washing, and use of protective equipment (PPE). For effective control, the Canadian Government has strongly encouraged residents to take any possible precautions to protect themselves[6], while Provinces and Territories have implemented restrictive closures of businesses, schools, work and public spaces to reduce the number of contacts. Ontario declared a state of emergency on March 17, the City of Toronto has also issued directions on a series of NPIs[7].

The SAHP reduced contacts outside the household, but possibly increased contacts among family members, leading to higher risk within a household[8]. The secondary infection rate in households can be as high as 30%[9]. However, the SAHP may be beneficial for control in the community[10]. Different studies investigated the transmission within households[9,11,12], which revealed the importance of within and between household transmission on the epidemic. Although keeping everyone at home will reduce the transmission, however, this may not be practical due to the essential operations of society. Moreover, an extended SAHP implementation might harm the physical and mental health of people and the economy[13,14]. Since individuals respond differently to the use of NPI's, the rate at which people "stay-at-home" is a function of changes in policies and behaviors. Also, the stay-at-home rates for symptomatic cases, or for traced contacts, are different from that of uninfected or asymptomatic individuals, since they must follow home isolation/quarantine after diagnosis or contacts' tracing[15]. Rates of diagnosis and isolation of cases, tracing, and quarantine of contacts, as well as public compliance to SAHP, are essential factors to determine the transmission and likelihood of epidemic resurgence after the lifting of restrictive closures[31].
To allow such a level of complexity, we developed a household-based transmission model to capture the differences in policy uptake behaviors. We aimed to evaluate the effect of SAHP on the transmission of COVID-19, accounting for average household size, the rates with which people respond and comply with the policy, and the length of

the policy implementation. Additionally, based on the average family size and epidemic data, we computed the reproduction numbers R_0 and R_t. We also investigated the threshold conditions on the number of contacts, testing, and use of NPIs to mitigate the epidemic and simulate the dynamic behavior under different reopening scenarios of relaxing SAHP. Our simulations propose reopening strategies for public health.

2. Method

2.1 Data and materials

We obtained daily new confirmed cases data, by episode date and reporting date in Toronto from Feb 24, 2020, to Jun 27, 2020 (see Figure 1A)[15,17]. Due to the lack of hospital resources, testing reagents, and the waiting time for testing, there is a time lag between the episode date and the reporting date (Figure 1A). We fit our model by the least-square method to estimate the parameters by using the cumulative confirmed case data by episode date and the cumulative number of deaths data in Toronto from Feb 24 to Jun 13 (period due to the incubation time plus the reporting delay).

In Toronto, testing has mainly been provided to individuals showing symptoms[15]. We define the following indicator

$$d_c(t) = \frac{Cumulative\ number\ of\ confirmed\ cases\ by\ report\ date\ on\ day\ t}{Cumulative\ number\ of\ confirmed\ cases\ by\ episode\ date\ on\ day\ t}.$$

as the ratio of symptomatic diagnosis' completion, with $d_c(t) \in [0,1]$. This quantity is used to inform the stay-at-home rate of detected infectious people (i.e., the rate at which they follow isolation recommendations). A delay in case diagnosis will result in a delay in implementing control measures, increasing the risk of transmission.

2.2 Compartmental model: description and assumptions

We develop a household-based transmission model following a Susceptible- Exposed-Asymptomatic (subclinical) - Infectious (prodromal phase) - Infectious (with symptoms) framework including two compartments, depending on the severity of the infection: hospitalization (H) and fully isolated (W). Given the importance of asymptomatic and pre-symptomatic infection in COVID-19[18], both stages are included.

To capture differences in social policy uptake, the population is divided into two subgroups: individuals following SAHP (be that associated with recommendations for all citizens to stay at home, or associated with orders to isolate at home for mild cases, and for at-home quarantine of contacts), and those not following it. Based on SAHP implementation, we assumed that the time needed to complete it follows a Gamma distribution. The movement between the SAHP compliant and non-compliant groups is modeled as policy and time vary, described by a stay-at-home rate $q(t)$ and a going out rate $g(t)$. We include a low stay-at-home rate before Toronto declared a state of emergency on March 12[7]. After that, some people chose to stay at home based on their behaviors and knowledge of the epidemic. We assume that τ is a random variable which describes how long it will take the five groups S_g, E_g, A_g, I_{g1}, I_{g2} to complete the stay-at-home process when conducting SAHP. Although I_{g2} is the symptomatic compartment, until confirmed, we assume that its stay-at-home rate is the same as the other g groups. After confirmed by testing, the "quarantine" rate of I_{g2} is defined as $q_{g2}(t) = q(t) + \varepsilon d_c(t)$ or, if the testing process is not included, as $q_{g2}(t) = q(t)$.

The flow diagram (Figure 2) describes the dynamics of our model. We include in Tables 1-3 about our model assumptions, variables, and parameters, respectively. Details on the model structure and its equations are provided in Appendix A.

2.3 Reproduction numbers

Model-free estimation of the reproduction number The R_0 is numerically estimated using an exponential growth method[19,20]based on the Toronto case data by episode date[15,17]. The instantaneous reproduction number R_t is also estimated by[21,22]

$$R_t = \frac{I_t}{\sum_{j=1}^{t} p_j I_{t-j}}.$$

where I_t is the new cases on day t and p_j is the discretized distribution of serial interval, assuming a Gamma distributed serial interval of 7.5 days (standard deviation 3.4 days[23]).

Model-based estimation of the reproduction number Total infection data (including symptomatic and asymptomatic infections) generated by the model were used to estimate the instantaneous reproduction number in Toronto.

Risk index after reopening We define a risk index R_{reopen} to evaluate the risk of reopening by calculating the reproduction number without SAHP:

$$R_{reopen} = c_0\beta_g\left(\frac{1-a}{\gamma_a} + a\left(\tau_2 + \frac{1}{\gamma_m + \theta_h + \theta_i + q_{g2}}\right)\right) .$$

3. Results

3.1 Parameter estimation and data fitting

Our model fits very well with the Normalized Mean Square Error (NMSE) = 0.998(Figure 3A). The results of parameter estimation indicate that at most 65.1% of people stay at home due to SAHP, after which the contact rate dropped from an initial 11.58 to 7.1, with a reduction of 39%. After May 6, it increased to 8.65, and after the stage 1 reopening of the city on May 19[7], it gradually increased to 9.4, corresponding to an 18% and 24% increase compared to May 6, respectively (Figure 3B).

3.2 Estimation of reproduction numbers in Toronto

The estimation result of the model-free R_c is 1.45 (95% CI 1.43-1.48) (goodness of fit $R^2 = 0.905$), while the model-based R_c is 2.36 (2.28-2.45) ($R^2 = 0.971$). According to the episode data, R_t varied before and after the implementation of SAHP, which gradually decreased from 3.56 (95% CI 3.02-4.14) on March 12 to less than 1 on April 22 and to 0.84 (0.79-0.89) on May 6, corresponding to a 76% (71-81%) reduction in transmissibility (Figure 4A). After May 6, launching ActiveTO plan[15], R_t gradually surpassed 1, rising to 1.13 (1.07-1.20) on May 19[7](Figure 4B). After entering the first phase of the city restart on May 19[7], R_t showed a clear downward trend, and gradually decreased to 0.67 (0.61-0.73) on June 13, although the contact rate was expected to be higher.

3.3 Effect of stay-at-home policy

Overall, SAHP reduces the average contact rate outside the household, which affects the development of the epidemic. Indeed, the cumulative number of infections dropped significantly compared to without SAHP (Figure 5A, B). The aggregate number of infected persons without SAHP was 12.5 times larger than the ones conducting SAHP with a mean family size (n) of 3. When n is smaller, the effect of SAHP on the control of the epidemic is better. The cumulative number of infected people on May 6 with $n = 2$ is less than half of its value when $n = 3$. However, in early phases of implementation of SAHP, due to the higher risk of transmission within the family, the number of infections was higher than when there was no SAHP (Figure 5A). This phenomenon is more pronounced when n is large (Figure 5E). With $n = 2$, R_t decreases as the maximum compliance rate (Q) increases for SAHP of both one- and ten-days duration (Figure 5C,D). In contrast, with $n = 3$, R_t is seen to decrease with Q for a 10-day SAHP but increase with Q after a SAHP of one day (Figure 5E,F).

The higher the value of Q, the sooner people comply with SAHP, which led to a lower total number of infections and deaths by May 6 (Figure 6A, B). If Q increases from 55% to 75%, the cumulative number of infections by May 6 will decrease by 63.2% (from 14032 to 5167), and the cumulative number of deaths will decline by 57.4% (from 504 to 215) when fixed $\Delta T_Q = 9$ (Figure 6A). Furthermore, the maximum reduction in contact rate, 5.6, for $Q = 0.95$ (Figure 6C). If ΔT_Q is shortened to 3 days and Q=0.65, the cumulative number of infections and deaths by May 6 are reduced by 50.5% and 45.6%, respectively (Figure 6B). However, whether the epidemic continues to be controlled, or resurges, depends on the sustained compliance rate of SAHP.

The effect of an extended SAHP is not visible. When the duration of SAHP is increased from 65 days to 95 days, the cumulative number of infections and cumulative deaths by July 2 only decreased by 9.6% and 3.6%, respectively (Figure 6F).

3.4 Threshold of contact rate and safe reopening

After the city's reopening, when the symptomatic diagnosis' completion ratio $d_c(t)$ is 97% (40%) (Figure 7B and 7A), if the contact rate is maintained at 11.58 persons per day, probability of transmission per contact outside household (β_g) needs to be reduced by 5% (26%) to avoid epidemic resurgence; and if β_g is maintained at 1.9% (the current state); the contact rate needs to be reduced to 11 (9) (Figure 7A, B). When $d_c(t)$ is high and β_g =1.9% (2.2%), $R_{reopen} = 1.04$ (1.2), hence the city still face the risk of epidemic resurgence as the city reopens completely (Figure 7D, E). While if β_g declines to 1.6%, $R_{reopen} = 0.87$, then reopening is safe, which is also shown in Figure 7E.

Based on the current epidemic situation, combined with Toronto's restart plan[7], we projected the future trend of the epidemic and estimated risk presented by schools and workplaces reopening on September 1. After fully reopening, Toronto will face the resurgence when keeping the current transmission rate (Figure 7D). But we show a safe reopening of public places when reducing the contact rate to nine and maintaining current strict social distancing (Figure 7D).

4. Interpretation

Using our novel model with household structure, we analyze the effect of the SAHP on the transmission of the COVID-19 using Toronto as a case study. SAHP has helped to control the epidemic and prevent the collapse of the healthcare system. However, in cities, such as Wuhan (China), SAHP was not effective at the early stage of the lockdown. This phenomenon can be related to the average household size of 3.5 in Wuhan[24], larger than the size 2.4 of Toronto[16]. Hence, the implementation of SAHP needs to be adapted to local conditions. For areas with large average family size, additional measures, such as establishing temporary shelter hospitals, may be needed to reduce transmission[3]. Indeed, the smaller the average family size, the more obvious the mitigation effect. Moreover, a lower probability of transmission, provided by keeping mandatory, or highly recommended use of NPI's[25], particularly in indoor public places[26], which will contribute significantly to the epidemic control.

Our model-based R_0 estimation captures the asymptomatic transmission and is higher than the estimate derived by case data, which underestimate R_0. After Toronto reopened into the first phase on May 19[7], R_t gradually declined, possibly due to the strengthening of government regulations on personal protections' use[7]. Although the contact rate may increase after reopening, the enhancement of personal protection is expected to reduce the probability of infection (β_g), thereby reducing the risk of the epidemic rebounding.

We constructed a new indicator, symptomatic diagnosis' completion ratio (d_c) and its trend (Figure 1B) how public health's response became more efficient as the pandemic grew. We also observe that d_c affects the achievement of $R_{reopen} < 1$. Indeed, with a smaller d_c, public health will need to strengthen NPIs and decrease the number of contacts to avoid the resurgence of the epidemic.

The fully reopening without other strengthened NPIs will face resurgence. When the average number of contacts exceeds the threshold after public areas reopening, the number of cases will rapidly increase. This explains the current increasing trend in Ontario.

Limitations

Our scenario analyses were under the assumption that the average family size in Toronto is 3. However, the household composition is varied in different areas and regions. The mitigating effect of SAHP should be reexamined when applying to other regions or countries, but this is beyond the scope of this paper. Second, even though we capture some non-homogeneous mixing by introducing the household structure, the age structure is not incorporated. The transmission difference among adults and children is observed, and children are less likely to acquire and transmit the infection. Further study including the age structure to capture the heterogeneity in contact pattern by age group will be explored.

Conclusions

In conclusion, we explored the impact of SAHP by incorporating household structure and NPIs on the COVID-19 epidemic, using Toronto as an example. The effect of SAHP has been almost wholly manifested after two months from its implementation. If the period of SAHP is extended, the impact on mitigating becomes not evident. Hence, this policy may be relaxed when the epidemic is effectively alleviated, then combined with social distancing, wearing PPEs, increasing the detection and isolation rate of symptomatic infections (with associated contact tracing and quarantine), to maintain control of the epidemic and reduce the burden on the healthcare system. Given the current increment of cases, we encourage public health to consider a new assessment on the maximum number of students in a class, gathering participants, workers in closed spaces, etc. and use of NPI's, needed to keep the spread under control. Indeed, the epidemic can be controlled if all the measures are strengthened simultaneously. All these results confirm that factors such as the testing process, contacts and transmission play a crucial role in reducing the spread. Since relaxing one of them affects the others, it is important to take all into consideration when planning a partial or full reopening.

Data sharing statement:

The data use for this study are published by the City of Toronto and Berry, I. and publicly available at the following links: https://www.toronto.ca/home/covid-19/ and https://github.com/ishaberry/Covid19Canada.

Author Contributions:

Research design: H.Z., P.Y., JuanL, N.O., J.B., J.H.; Literature search: Q.L., E.A., P.Y., JuanL, T.Z., Y.T.; Data collection: T.Z., Q.L., Y.T., P.Y., JuanL.; Modeling: H.Z. and all; Model analysis: JunL., P.Y., JuanL., E.A., H.Z.; Simulations: P.Y., JuanL., E.A., T.Z., Y.T.,JunL; Draft preparation: P.Y., JuanL, E.A., Q.L., T.Z., Y.T., H.Z.; Writing-reviewing-editing: H.Z.,N.O, B.S., J.H., EvgeniaG., EffieG., S.C., J.A., J.B., J.W., J.D. K., I.M., Supervision: H.Z.

Conflicts of Interest: The authors declare no conflict of interest.

Funding: This research was supported by Canadian Institutes of Health Research (CIHR), Canadian COVID-19 Math Modelling Task Force (NO, BS, JH, JA, JB, JW, JD, HZ), the Natural Sciences and Engineering Research Council of Canada (JH, JA, JB, JW, JD, IM, HZ) and York University Research Chair program (HZ) and the Chinese Scholarship Council (PY, QL, TZ, YT).

References

1. WHO. Coronavirus disease 2019 (COVID-19) situation report. July 8, 2020. https://www.who.int/emergencies/diseases/novel-coronavirus-2019/situation-reports/ (accessed July 15, 2020).
2. Ferguson NM, Laydon D, Gilani GN, et al. Report 9: Impact of non-pharmaceutical interventions (NPIs) to reduce COVID-19 mortality and healthcare demand. Imperial College COVID-19 Response Team. March 16, 2020. https://www.imperial.ac.uk/media/imperial-college/medicine/sph/ide/gida-fellowships/Imperial-College-COVID19-NPI-modelling-16-03-2020.pdf (accessed May 26, 2020).
3. Li J, Yuan P, Heffernan J, et al. Observation wards and control of the transmission of COVID-19 in Wuhan. *Bull World Health Organ* 2020; published online Apr 9. DOI: 10.2471/BLT.20.258152 (preprint).
4. Tang B, Xia F, Tang SY, et al. The effectiveness of quarantine and isolation determine the trend of the COVID-19 epidemics in the final phase of the current outbreak in China. *Int J Infect Dis* 2020; **95**: 288-293.
5. Traini MC, Caponi C, Ferrari R, et al. A study of SARS-CoV-2 epidemiology in Italy: from early days to secondary effects after social distancing. *J Infect Dis* 2020; published online Jul 30. DOI: 10.1080/23744235.2020.1797157.
6. Government of Canada. Coronavirus disease (COVID-19). 2020. https://www.canada.ca/en/public-health/services/diseases/coronavirus-disease-covid-19.html (accessed May 31, 2020).
7. Toronto. News Releases & Media Advisories. 2020. https://www.toronto.ca/news/ (accessed July 30).
8. Prem K, Liu Y, Russell TW, et al. The effect of control strategies to reduce social mixing on outcomes of the COVID-19 epidemic in Wuhan, China: a modelling study. *Lancet Public Health*, 2020; **5**: e261–e270.
9. Wang Z, Ma W, Zheng X, Wu G, Zhang R. Household transmission of SARS-CoV-2. *J Infect* 2020; **81**: 179-182.
10. Sjödin H, Wilder-Smith A, Osman S, et al. Only strict quarantine measures can

curb the coronavirus disease (COVID-19) outbreak in Italy, 2020. *Euro Surveill* 2020, **25**: 2000280.

11. Pellis L, Ferguson NM, Fraser C. Epidemic growth rate and household reproduction number in communities of households, schools and workplaces. *J Math Biol* 2011; **63**: 691–734.

12. House T, Keeling MJ. Deterministic epidemic models with explicit household structure. Mathematical Biosciences. *Math Biosci* 2008; **213**: 29-39.

13. Brooks SK, Webster RK, Smith LE, et al. The psychological impact of quarantine and how to reduce it: rapid review of the evidence. *Lancet* 2020; **395**: 912-920.

14. Mandel A, Veetil V. The economic cost of COVID lockdowns: an out-of-equilibrium analysis. *Econ Dis Cli Cha* 2020; published online Jun 19. DOI: 10.1007/s41885-020-00066-z.

15. City of Toronto. COVID-19. 2020. https://www.toronto.ca/home/covid-19/ (accessed July 20, 2020).

16. Statistics Canada. Census Profile, 2016 Census. 2016. https://www12.statcan.gc.ca/census-recensement/2016/dp-pd/prof/details/Page.cfm?Lang=E&Geo1=CSD&Code1=3520005&Geo2=PR&Data=Count&B1=All (accessed March 20, 2020).

17. Berry I. ishaberry/Covid19Canada. 2020. https://github.com/ishaberry/Covid19Canada (accessed May 20, 2020).

18. Zou, L, Ruan F, Huang M, et al. SARS-CoV-2 Viral Load in Upper Respiratory Specimens of Infected Patients. *N Engl J Med* 2020; **382**: 1177–1179.

19. Wallinga J, Lipsitch M. How generation intervals shape the relationship between growth rates and reproductive numbers. *Proc Royal So B* 2007; **274**: 599-604.

20. Obadia T, Haneef R, Boëlle PY. The R0 package: a toolbox to estimate reproduction numbers for epidemic outbreaks. *BMC Medical Inform Decis Mak* 2012; **12**: 147.

21. Wallinga J, Teunis P. Different epidemic curves for severe acute respiratory syndrome reveal similar impacts of control measures. *Am J Epidemiol* 2004; **160**: 509.

22. Cori A, Ferguson N M, Fraser C, et al. A new framework and software to estimate time-varying reproduction numbers during epidemics. *Am J Epidemiol* 2013; **178**: 1505-1512.

23. Li Q, Guan X, Wu P, et al. Early transmission dynamics in Wuhan,China, of novel coronavirus-infected pneumonia. *N Engl J Med* 2020; **382**: 1199–207.

24. Hubei Provincial Bureau of Statistics. 2018 Wuhan Statistic Yearbook. June 6, 2019. http://tjj.hubei.gov.cn/tjsj/sjkscx/tjnj/gsztj/whs/ (accessed July 20, 2020).

25. Leung NHL, Chu DKW, Shiu EYC. et al. Respiratory virus shedding in exhaled breath and efficacy of face masks. *Nat Med* 2020; **26**: 676–680.

26. Centers for Disease Control and Prevention. Recommendations for Cloth Face Covers. 2020. https://www.cdc.gov/coronavirus/2019-ncov/prevent-getting-sick/cloth-face-cover.html (accessed April 8, 2020).

27. City of Toronto. Toronto at a Glance. 2018. https://www.toronto.ca/city-government/data-research-maps/toronto-at-a-glance/ (accessed April 22, 2020).

28. Li R, Pei S, Chen B, et al. Substantial undocumented infection facilitates the rapid dissemination of novel coronavirus (SARS-CoV2). *Science* 2020; **368**: 489-493.

29. Tang B, Scarabel F, Bragazzi N L, et al. De-Escalation by Reversing the Escalation with a Stronger Synergistic Package of Contact Tracing, Quarantine, Isolation and Personal Protection: Feasibility of Preventing a COVID-19 Rebound in Ontario, Canada, as a Case Study. *Biology (Basel)* 2020; **9**:100.

30. Government of Ontario. Reopening Ontario in stages. 2020. https://www.ontario.ca/page/reopening-ontario-whats-each-stage (accessed June 28, 2020).

31. Ng V, Fazil A, Waddell LA, Bancej C, Turgeon P, Otten A, Atchessi N, Ogden N. Projected effects of nonpharmaceutical public health interventions to prevent resurgence of SARS-CoV-2 transmission in Canada. CMAJ. 2020 Aug 9:cmaj.200990.

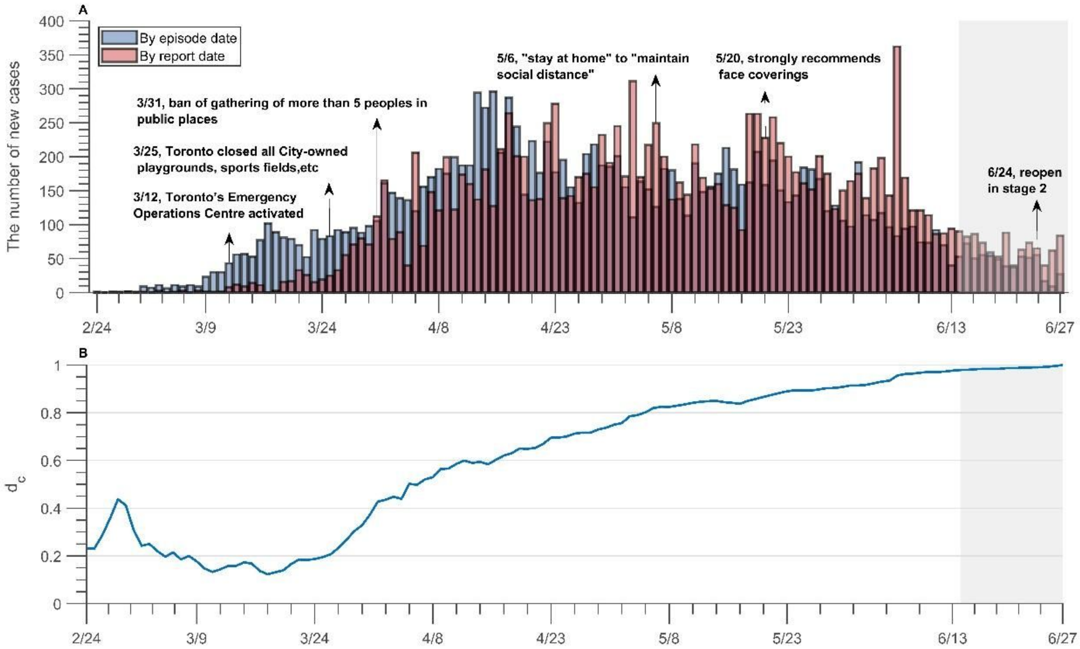
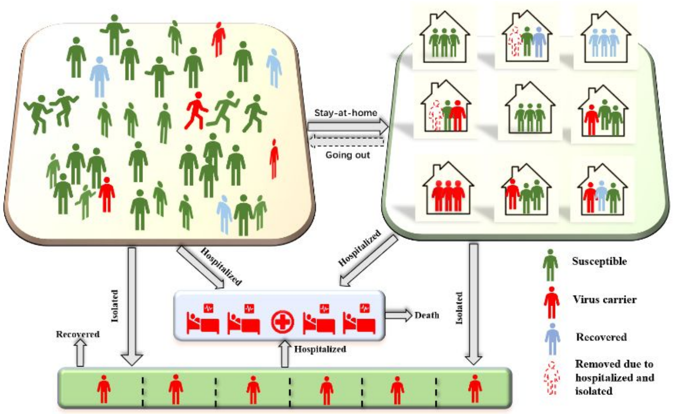
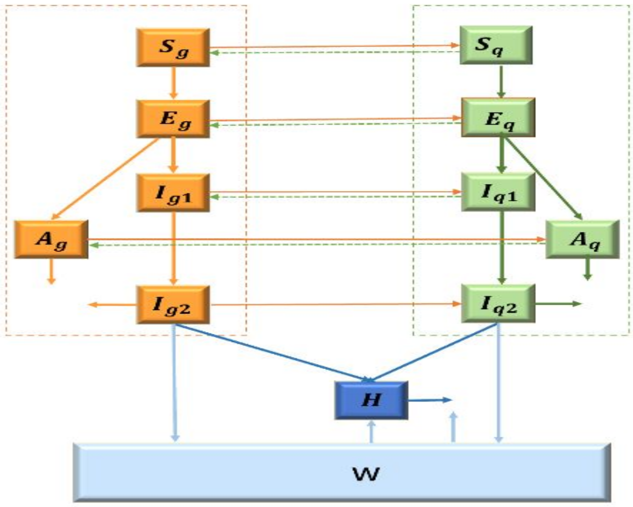


Figure 1 COVID-19 cases in Toronto by report date and episode date. (A)The daily new case of infection by episode date and first report date. (B) The change of d_c over time for the city of Toronto from Feb. 24 to June 27, 2020. d_c = symptomatic diagnosis' completion ratio.



(a)



(b)

Figure 2 Modeling with household structure. (a) shows the activity and response of different groups. (b) Schematic diagram of the dynamics of COVID-19 in Toronto. Solid lines indicate movement between classes. Dashed lines represent the virus transmission routes.

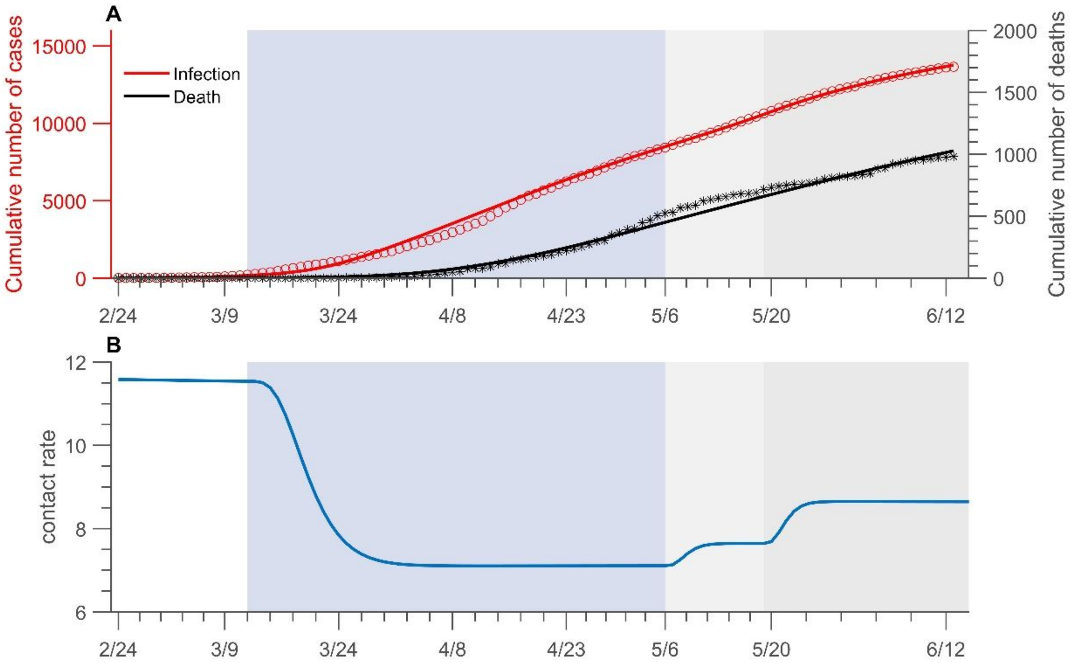


Figure 3: Cumulative COVID-19 incidence and deaths in Toronto (A) and the change of contact rate over time. Data fitting of COVID-19 infection in Toronto from Feb 24 to Jun 13, 2020. The red circles (infection) and black stars (death, right panel) represent real data. The solid curves are from model simulations. Shaded bars show the dates that SAHP implemented, (light blue), preopening (light grey), reopening stage 1 (medium grey). All dates are in 2020.

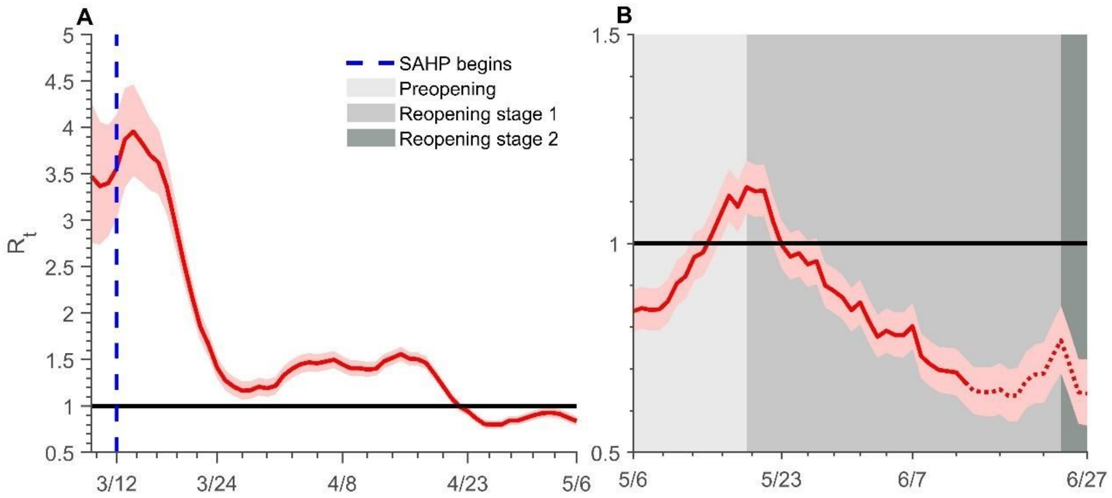


Figure 4: Transmissibility of COVID-19 in Toronto. Estimates of daily R_t of COVID-19 over time (A) from Mar 8 to May 6 and (B) from May 6 to Jun 27, with 95% CIs represented by the pink shaded area. The dates after Jun 13 are indicated by a red dotted line. The dark solid line indicates the critical threshold of $R_t = 1$. The blue dashed line is the time that the SAHP activated. Shaded bars show the dates of preopening (light grey), reopening stage 1 (medium grey) and stage 2 (dark grey). All dates are in 2020. R_t= instantaneous reproduction number.

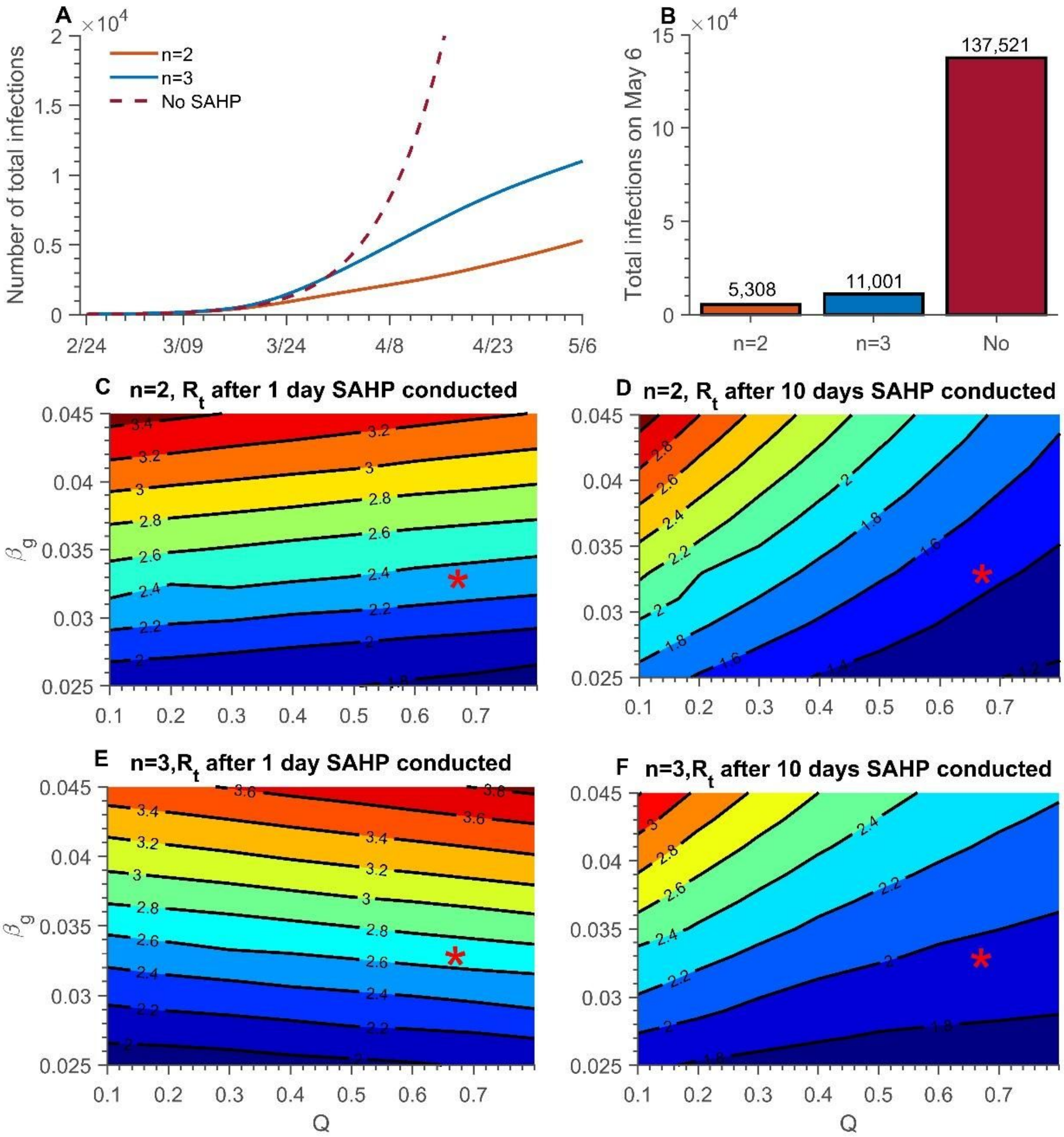


Figure 5: Effect of SAHP and different average household size. (A) The cumulative infection over time from Feb 24 to May 6 and (B) cumulative infection on May 6, without SAHP (dark red) and $n = 2$ (orange) and $n = 3$ (blue). Contour plot of R_t with different β_g and Q (C) one day after SAHP conducted, $n = 2$; (D) ten days after SAHP conducted, $n = 2$; (E) one day after SAHP conducted, $n = 3$; (F) after ten days after SAHP conducted, $n = 3$. $n =$ average household size. $\beta_g =$ probability of transmission per contact outside household. $Q =$ maximum compliance rate.

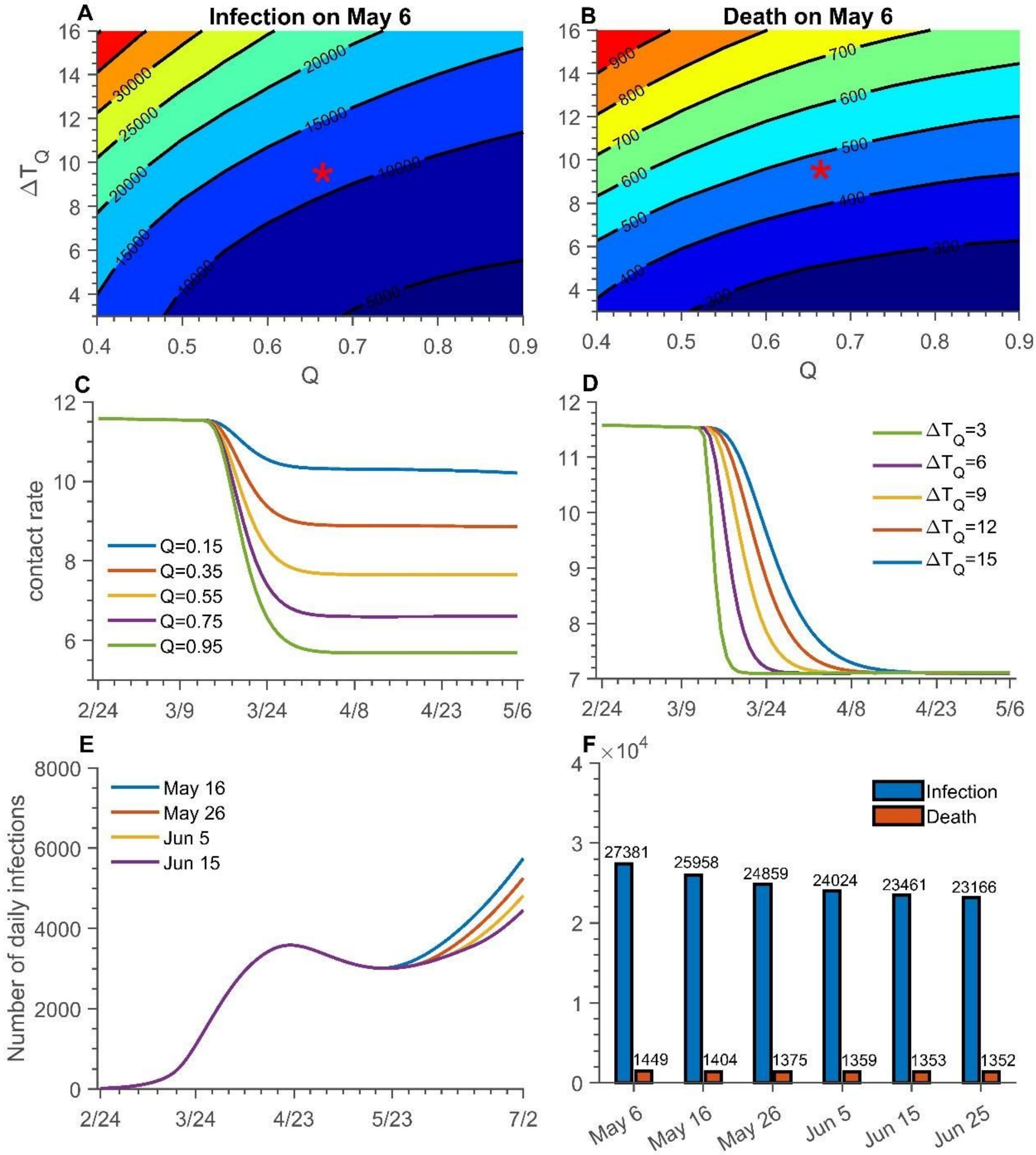


Figure 6: Effect of SAHP with maximum compliance rate, average completing time and length of SAHP. Contour plot of cumulative infection (A) and death (B) on May 6 with different ΔT_Q and Q. Red star represents the parameter values estimated from data and are used in simulations C, D, E, F. The contact rate change over time (C) under different Q with $\Delta T_Q = 9$ and (D) under different $\Delta T_Q$with $Q = 0.65$. (E) The number of daily infection and (F) cumulative number of infections (blue bar) and deaths (orange bar) on Jul 2 with different length of SAHP, 55days (May 6), 65 days (May 16, blue), 75 days (May 26, orange), 85 days (Jun 5, yellow), 95 days (Jun 15, purple), 105 days (Jun 25). Q= maximum compliance rate. $\Delta T_Q =$ average completion time.

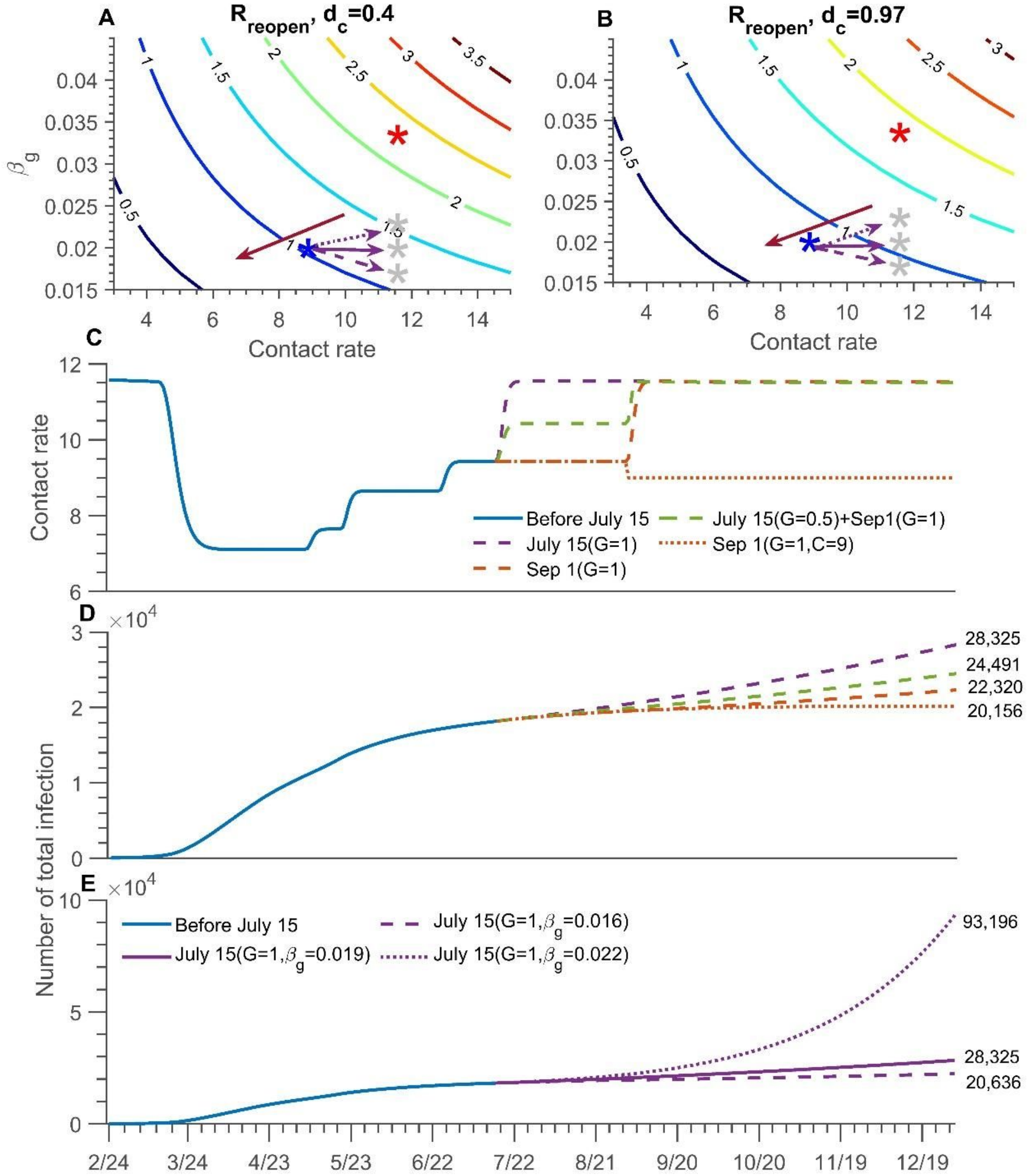


Figure 7: The risk of reopening, and different reopening scenarios. Contour plot of R_{out} with different β_g and C (A) d_c= 0.4; (B) d_c= 0.97. The red star is the initial status of β_g and C, and the blue is the current state. The grey star is the possible state after completely reopen in simulations E. The red arrow shows the low-risk direction with the safe reopening. (C) The change of contact rate and (D) the cumulative infection over time with different ways to reopen, fully reopening on July 15 (purple dash line), partially reopening on July 15 and then fully reopening on September 1 (green dash line), fully reopening on September 1 (orange dash line), fully reopening on September 1 and maintain contact rate is 9 (orange dot line). (E) The number of cumulative infections over time with $\beta_g = 0.016$ (dash line), 0.019 (solid line, current state), 0.022 (dot line) when fully reopening on July 15. $R_{reopen} =$ the transmission risk after fully reopening. d_c= completion ratio of symptomatic diagnosis. $\beta_g =$ probability of transmission per contact outside the household. $C =$ contact rate. $G =$ going out rate.

Table 1 Model assumptions

General setting	a	No birth, death or immigration.
	b	We divide the population into two groups: one consisting of individuals who follows SAHP (marked by subscript q) and another consisting of individuals who do not opt for this intervention (marked by subscript g). Due to influences of self-protection consciousness and severity of the epidemic, people are assumed to move from one group to another with stay-at-home rate (denoted by $q(t)$) or going out rate (denoted by $g(t)$).
	c	Each subpopulation is further the divided into Susceptible ($S_i(t)$), Exposed ($E_i(t)$), Asymptomatic (subclinical) infection ($A_i(t)$), Infectious pre-symptomatic (will eventually show symptoms) ($I_{i1}(t)$) and Infectious symptomatic ($I_{i2}(t)$).
	d	Both $A_i(t)$ and $I_{i1}(t)$ are infectious virus carriers. Individuals in $A_i(t)$ will never show symptoms, while individuals in $I_{i1}(t)$ develop into symptomatic classes ($I_{i2}(t)$) after a specified period of time.
	e	Mild symptomatic infections ($I_{i2}(t)$), may choose to either isolate themselves at home (or other places). If the quarantine is respected well enough, these infections will be fully isolated and, consequently, will not contribute to the spread of the virus. Otherwise, they are still a source of infection until recovery.
	f	Two further compartments for severe infections: the fully isolated ($W(t)$), and the hospitalized (H(t)) who are all severely affected. Neither of these compartments contribute to infection transmission.
Household structure setting	g	All households contain n ($n = 3$) individuals and family members are homogeneously mixing i.e, contacting each other randomly.
	h	The infection rate of the asymptomatic and symptomatic infectious individuals to the susceptible is the same among the household.
	i	Two members in a family cannot be infected at the same time t.
	j	Every family except for those with symptomatic members has an equal opportunity to be released from quarantine after the SAHP is relaxed.
	k	Households with infected symptomatic individuals will continue to be quarantined after the SAHP is relaxed.
	l	For family members following SAHP, susceptible $S_q(t)$ will only be infected by infectious individuals in the home $A_q(t), I_{q1}(t)\ or\ I_{q2}(t)$.
	m	When no infections in a household, the family will be safe and will no longer be involved in the transmission of COVID-19.

Note: See appendix for model details and derivation process

Table 2 Identification of the variables and their initial values

Variables	Descriptions	Fixed initial Values	Sources
$I_{g2}(t)$	The number of SAHP non-compliant infected individuals with symptoms.	**10**	Data
$S_q(t)$	The number of SAHP compliant susceptible individuals.	(n-1)*3	Calculated
$E_q(t)$	The number of SAHP compliant exposed individuals	0	
$A_q(t)$	The number of SAHP compliant inapparent (subclinical) infected individuals	0	
$I_{q1}(t)$	The number of SAHP compliant infected individuals that do not symptoms that will become symptomatic	0	
$I_{q2}(t)$	The number of SAHP compliant infected with symptoms.	3	**17**
$H(t)$	The number of patients in hospitals	0	
$W(t)$	The number of isolation patients	0	
P	Total number of populations in Toronto	2956024	**27**
Variables	**Descriptions**	**Initial Values**	**Sources**
$S_g(t)$	The number of SAHP non-compliant susceptible individuals	2955988	Estimated
$E_g(t)$	The number of SAHP non-compliant exposed individuals	20	Estimated
$A_g(t)$	The number of SAHP non-compliant inapparent infected at day t. (that will never develop symptoms)	1	Estimated
$I_{g1}(t)$	The number of SAHP non-compliant infected without symptoms at day t. (that will become symptomatic)	2	Estimated

Table 3 Parameter estimation for COVID-2019 in Toronto

Parameters	Descriptions	Fixed Values	Sources
τ_1	Average time spent in the exposed classes, E_g,E_q,days	4	23,28
τ_2	Average time period spent in I_{g1}, I_{g1},days	3	28
a	Proportion of infected with apparent infection	0.953	3
γ_a	Recovery rate of inapparent infected	0.07	3
γ_m	Recovery rate of patients with mild symptoms	1/14	1
γ	Recovery rate of patients in hospitals	0.0357	1(1/42-1/21)
c_0	Contact rate before SAHP implemented,1/day	11.58	29
T_1	Time when the SAHP is implemented	March 12	7
T_2	Time when SAHP is relaxed	May 6	7
T_3	Time when the reopening of stage 1 begins	May 19	30
T_4	Time of reopening of stage 2 begins	June 24	7
n	Average number of household population	2-3	16
$q(t)$	Stay-at-home rate of S_g, E_g, A_g and I_{g1}	-	
$d_c(t)$	Completion rate of diagnosis of all symptomatic infections	-	15
$g(t)$	Proportion of households of going out after relaxing the SAHP	-	
$q_{g2}(t)$	Quarantined rate of I_{g2}	-	
$Q(t)$	The proportion of population in stay-at-home state to the total population at time t	-	
Parameters	**Descriptions**	**Estimated Values**	**Sources**
β_g	Probability of transmission per contact outside household	3.2984e-02	Estimated
μ	Exponential decreasing rate of contact rate due to SAHP	7.5000e-01	Estimated
β_q	Infection rate of SAHP compliant susceptible by the infectious one without symptoms	1.5030e-02	Estimated
q	Stay-at-home rate of S_g, E_g, A_g and I_{g1} before SAHP implemented	3.0001e-04	Estimated
ε	Adjust parameter	7.0000e-01	Estimated
g	Going out rate of S_g, E_g, A_g and I_{g1} during the period of SAHP implemented	1.0000e-04	Estimated
θ_h	Proportion of hospitalization of I_{g2}	0.0152	Estimated
θ_i	Isolation rate of confirmed cases	3.9978e-02	Estimated
d	Disease-induced death rate in hospitals	3.4000e-02	Estimated
Q	Maximum compliance rate induced by SAHP	6.5058e-01	Estimated
ΔT_Q	Average completing time for all those who conducted the SAHP	9	Estimated
G_0	Maximum going out rate in the period of May 6 to May 19	1.5000e-01	Estimated
G_1	Maximum going out rate in the period of May 20 to Jun 24	3.0000e-01	Estimated
G_2	Maximum going out rate in the period of reopen stage 2 starting Jun 24	3.0000e-01	Assumed
ΔT_G	Average time required for all stay-at-home people to go out after reopening	3	Assumed

Appendix A

Modeling

1. Description

This study considers the entire population of Toronto with the "stay at home" policy (SAHP) that was enacted on March 12th and gradually relaxed after May 6[7], as well as the document "A Framework for Reopening our Province" Ontario released on April 27[30]. The province will gradually reopen all workplaces and public spaces. Stage 1, which began on May 19, allowed the opening of select workplaces and some small gatherings. On Jun 24, the city of Toronto enters Stage 2 of reopening, opening more workplaces and outdoor spaces, allowing gatherings of up to 10 people[7]. We divide the population into two groups: one consisting of individuals who follow SAHP (marked by subscript q) and another consisting of individuals who do not opt for this intervention (marked by subscript g). Due to influences of self-protection consciousness and severity of the epidemic, people are assumed to move from one group to another with stay-at-home rate (denoted by $q(t)$) or going-out rate (denoted by $g(t)$). We note that we omit demographic components, such as immigration, birth, and natural death.

A detailed description of dynamical transmission of COVID-19 is described in the flowchart (Fig. 3). Let $N_i(t)$ $(i = g, q)$ be the total number of individuals in each sub-group, g, q, at time t. Each subpopulation is further the divided into Susceptible ($S_i(t)$), Exposed ($E_i(t)$), Asymptomatic (subclinical) infection ($A_i(t)$), Infectious pre-symptomatic (will eventually show symptoms) ($I_{i1}(t)$) and Infectious symptomatic ($I_{i2}(t)$). Both $A_i(t)$ and $I_{i1}(t)$ are considered to be infectious virus carriers. We assume that individuals in $A_i(t)$ will never show symptoms, while individuals in $I_{i1}(t)$ develop into symptomatic classes ($I_{i2}(t)$) after a specified period of time. Mild symptomatic infections in classes ($I_{i2}(t)$), may choose to either isolate themselves at home (or other places). If the quarantine is respected well enough, these infections will be fully isolated and, consequently, will not contribute to the spread of the virus. Otherwise, they are still a source of infection until recovery.

As the disease progresses, some mild infections may become severe and require hospitalization. We include two further compartments: the fully isolated ($W(t)$), and the hospitalized (H(t)) who are all severely affected. It is assumed that neither of these compartments contribute to infection transmission. Through a numerical analysis of H(t) and W(t) relevant parameters, we will present a pre-estimation of the ratio of mild to severe infections during the epidemic. We will also explore the influences of some measures (such as hospital capacity, testing and isolation) on the development of the disease.

Based on the classical SEIR framework, a household-based transmission model will be proposed to describe the impact of SAHP on the development of the epidemic. Considering that an infected person quarantined at home is interacting only with family members, the number of contacts is limited, so we will use the standard incidence rate in modelling.

Although home transmission is relatively strong, it only involves limited family members. To reflect this, and capture disease transmission within families, we separate people who follow the SAHP into households.

For family members following SAHP, susceptible individuals ($S_q(t)$) will only be infected by infectious individuals in the home $A_q(t),\ I_{q1}(t)\ or\ I_{q2}(t)$. When no cases are reported in a household, the family will be safe and will no longer be involved in the transmission of COVID-19. Additionally, infections who are completely isolated will not be involved in transmission.

2. Rates definition

Next, we will present the dynamical models for SAHP non-compliant, SAHP compliant and isolation population, respectively. First, we will describe the key rates on which the model is based.

- **Stay-at-home rate**

Before the government implemented SAHP on March 12[7], due to the impact of self-prevention awareness and the severity of the epidemic, a small number of people would consciously stay at home, so we assume that the stay-at-home rate is a very small constant, which we express as

$$q(t) = q,\ \ t \leq T_1,$$

where T_1 is the time when the SAHP is implemented, and q is the average daily stay-at-home rate before the policy is put into action.

After the SAHP was implemented, some people chose to stay at home based on their own behaviors and their knowledge of the epidemic. We denote the maximum compliance rate (Q_1) as the maximum proportion of the number of people in the group that will carry out SAHP, which is used to reflect the degree of the behavioral tendency of the population to change their original daily lifestyle and accept the SAHP under the requirements of prevention and control policies after the outbreak. The implementation of SAHP will directly affect the stay-at-home rate

$$q(t) = q(Q_1, t),\quad t > T_1.$$

Then we have

$$q(t) = \{q,\quad t \leq T_1, q_q(t),\ \ t > T_1.$$

We assume that τ is a random variable which describes how long it will take the five groups S_g, E_g, A_g, I_{g1}, I_{g2} to complete the stay-at-home process when conducting SAHP. Although I_{g2} is the symptomatic compartment, it should be the same as the other four categories before tested and confirmed. Hence, τ follows a Gamma distribution

$$\tau \sim Gamma(k, \theta)$$

with $$f(\tau) = \{\frac{1}{\Gamma(k)\theta^k}\tau^{k-1}\, exp\, exp\left(-\frac{\tau}{\theta}\right),\ \ \tau \geq 0, 0,\qquad \tau < 0,$$

where $k = 5, \Gamma(k) = \int_0^{+\infty} \tau^{k-1}e^{\tau}d\tau, k > 0.$

The expectation of $\tau = E(\tau) = k * \theta = \Delta T_Q$ (ΔT_Q is the average completing time for all those who conducted the SAHP), and $f(\tau)$ is the probability that those in the five groups will accomplish stay-at-home process in τ days.

The total population that may conduct SAHP of Toronto at T_1 is $P_1 = S_g(T_1) +$

$A_g(T_1) + E_g(T_1) + I_{g1}(T_1) + I_{g2}(T_1)$. The number of people who accomplished stay-at-home process on $T_1 + \tau$ days was $\Delta P_1(T_1 + \tau) = Q_1 * P_1 * f(\tau) = (Q_1 * f(\tau)) * P_1$. Let $Q(T_1 + \tau)$ be the daily stay-at-home rate on day $T_1 + \tau$, then $Q_1(T_1 + \tau) = Q_1 * f(\tau)$. And it satisfies $\int_0^\infty Q_1(T_1 + \tau)d\tau = Q_1$.

We also assume that each group has the same daily stay-at-home ratio, $q_q(T_1 + \tau)$, which is the daily stay-at-home rate of the people who began to stay at home on day $T_1 + \tau$. Then the number of people newly stay-at-home on that day is

$$q_q(T_1 + \tau)\left(S_g(T_1 + \tau) + A_g(T_1 + \tau) + E_g(T_1 + \tau) + I_{g1}(T_1 + \tau) + I_{g2}(T_1 + \tau)\right).$$

The newly stay-at-home number on day $T_1 + \tau$ is equal to the number of people conducting SAHP on that day, i.e.,

$$q_q(T_1 + \tau)\left(S_g(T_1 + \tau) + A_g(T_1 + \tau) + E_g(T_1 + \tau) + I_{g1}(T_1 + \tau) + +I_{g2}(T_1 + \tau)\right) = \Delta P(T_1 + \tau), \quad \tau > 0.$$

Hence, we have

$$q_q(T_1 + \tau) = \frac{Q_1 * P_1 * f(\tau)}{S_g(T_1+\tau)+A_g(T_1+\tau)+E_g(T_1+\tau)+I_{g1}(T_1+\tau)+I_{g2}(T_1+\tau)}, \quad \tau > 0.$$

Let $t = T_1 + \tau$, then

$$q_q(t) = \frac{Q_1 * P_1 * f(t - T_1)}{S_g(t)+A_g(t)+E_g(t)+I_{g1}(t)+I_{g2}(t)}, \quad t > T_1.$$

According to the relative policies of Toronto, people who are detected to be COVID-19 positive need to stay at home and self-isolate for 14 days[15].Combined with the flowchart shown in Fig.3, there are three different ways to allocate infectious patients: to be hospitalized, to isolate at home, or to isolate in a place other than home. Due to the strengthening effect of testing, the stay-at-home rate of the infected cases with symptoms is much higher than others. Here, we modify the quarantined rate of I_{g2} (separately rewritten as $q_{g2}(t)$) to be $q_{g2}(t) = q(t) + \varepsilon d_c(t)$, where $d_c(t)$ is the completion rate of diagnosis of all symptomatic infections. $d_c(t)$ obtained from the onset data and the reported data shown in Section 2, and ε is an adjustment parameter to describe the impact of testing on the quarantine rate of I_{g2}. Here, it is assumed that $q_{g2}(t) = q(t)$ if there is no testing.

- **Going out rate**

Let T_2 $(T_2 > T_1)$ be the day on which the SAHP is announced to be relaxed. That is, some people would be encouraged to go outside home after that day. Similar to the formula design process of $q(t)$, we now determine $g(t)$, the proportion of households that are not stay-at-home versus all households, which is given by

$$g(t) = \begin{cases} 0, & t \le T_1, \\ g, & T_1 \le t \le T_2, \\ \dfrac{G * h(t - T_2) * (S_q(T_2) + A_q(T_2) + E_q(T_2) + I_{q1}(T_2))}{S_q(t) + A_q(t) + E_q(t) + I_{q1}(t)}, & t > T_2, \end{cases}$$

where g is a small positive constant, G is the maximum proportion of the population who will not continue to stay at home compared to the total size of the stay-at-home population at time T_2,

$$h(\tau) = \{\frac{1}{\Gamma(k)\theta^k}\tau^{k-1} \, exp\, exp\left(-\frac{\tau}{\theta}\right), \ \tau \ge 0, 0, \qquad \tau < 0,$$

$k = 4$ and $\tau = \Delta T_G$, where ΔT_G is the average completion time for all people who stay at home (except those with symptoms) to go outside.

3. Models

Stay-at-home and Isolation

According to the infection and development process of the disease in the human body, at time t, an individual in a household can belong to one of the following categories: $S_q(t)$, $E_q(t)$, $A_q(t)$, $I_{q1}(t)$, $I_{q2}(t)$, $H_q(t)$ or $W_q(t)$, or may be recovered, Corresponding to each disease class, we assign the number of individuals in each household to be i, j, k, l, m, x, y, z, respectively, and limit households to a size of n such that $n = i + j + k + l + m + x + y + z$. Therefore, each household at most consists of n different categories of individuals. Based on the classification and combination of individuals in households, all possible types of households in Toronto are C_{8+n-1}^{n}.

For each household type, the dynamics are determined by eight processes: within-household transmission; disease progression from Exposed to Asymptomatic infection or Infection without symptoms; disease progression from Infection without symptom to Infected with symptoms; recovery from Asymptomatic infection; recovery from Infected with symptoms; hospitalization of Infected with symptoms; isolation of Infected with symptoms; and newly entered stay-at-home. Then the variation of the number of households $P_{i,j,k,l,m,x,y,z}$ with respect to time t can be given by

$$\begin{aligned}\dot{P}_{i,j,k,l,m,x,y,z}(t) = \beta_q\big[&-i(k+l+m)P_{i,j,k,l,m,x,y,z}(t) \\ &+ (i+1)(k+l+m)P_{i+1,j-1,k,l,m,x,y,z}(t)\big] \quad + \frac{1}{\tau_1}[-jP_{i,j,k,l,m,x,y,z}(t) + (1 \\ &- a)(j+1)P_{i,j+1,k-1,l,m,x,y,z}(t) + a(j+1)P_{i,j+1,k,l-1,m,x,y,z}(t)] \\ &+ \frac{1}{\tau_2}[-lP_{i,j,k,l,m,x,y,z}(t) + (l+1)P_{i,j,k,l+1,m-1,x,y,z}(t)] \\ &+ \gamma_a\big[-kP_{i,j,k,l,m,x,y,z}(t) \\ &+ (k+1)P_{i,j,k+1,l,m,x,y,z-1}(t)\big] \quad + \gamma_m\big[-mP_{i,j,k,l,m,x,y,z}(t) \\ &+ (m+1)P_{i,j,k,l,m+1,x,y,z-1}(t)\big] \\ &+ \theta_h\big[-mP_{i,j,k,l,m,x,y,z}(t) + (m+1)P_{i,j,k,l,m+1,x-1,y,z}(t)\big] \\ &+ \theta_i\big[-mP_{i,j,k,l,m,x,y,z}(t) + (m+1)P_{i,j,k,l,m+1,x,y-1,z}(t)\big] \\ &+ \Delta P_{i,j,k,l,m,x,y,z}(t) \quad - g(t)P_{i,j,k,l,m,x,y,z}(t),\end{aligned}$$

where $P_{i,j,k,l,m,x,y,z}(t) \geq 0$ should be satisfied, $g(t)P_{i,j,k,l,m,x,y,z}(t)$ should be ignored for $m \neq 0$, and $\Delta P_{i,j,k,l,m,x,y,z}(t)$ is the number of new stay-at-home households with i susceptible, j exposed, k asymptomatic (subclinical) infection, l infectious without no symptoms, m infected with symptoms, x hospitalized, y isolated and z removed members,

$$\Delta P_{i,j,k,l,m,x,y,z}(t) = \left[\frac{1}{n}(q(t)\left(S_g(t) + E_g(t) + A_g(t) + I_{g1}(t)\right) + q_{g2}(t)I_{g2})\right]F,$$

where [.] is an integral function to return the value of a number rounded downwards to the nearest integer and F is the probability of each type of newly added quarantine household when $n = 2$,

$$F = \begin{cases} \dfrac{n_{ii}(n_{ii}-1)}{A_{n_q}^{n}}, ii = 1,2,3,4,5, & if\ one\ of\ i,j,k,l,m\ is\ not\ 0, \\ \dfrac{n_{jj}n_{ii}}{A_{n_q}^{n}}, ii, jj = 1,2,3,4,5, ii \neq jj, & if\ two\ of\ i,j,k,l,m\ is\ not\ 0, \\ 0, & others. \end{cases}$$

when $n = 3$,

$$F=\begin{cases}\dfrac{n_{ii}(n_{ii}-1)(n_{ii}-2)}{A_{n_q}^{n}}, ii=1,2,3,4,5, & if\ one\ of\ i,j,k,l,m\ is\ not\ 0,\\ \dfrac{C_n^2 n_{jj}n_{ii}(n_{ii}-1)}{A_{n_q}^{n}}, ii,jj=1,2,3,4,5, ii\neq jj, & if\ two\ of\ i,j,k,l,m\ is\ not\ 0,\\ \dfrac{A_n^3 n_{ii}n_{jj}n_{kk}}{A_{n_q}^{n}},\ \ ii,jj,kk=1,2,3,4,5, & if\ three\ of\ i,j,k,l,m\ is\ not\ 0,\\ 0, & others.\end{cases}$$

with $n_q=\sum_{ii=1}^{5} n_{ii},\ \ n_q\geq n, n_1=[q(t)S_g(t)], n_2=[q(t)E_g(t)], n_3=[q(t)A_g(t)],$

$$n_4=[q(t)I_{g1}(t)], n_5=[q_{g2}(t)I_{g2}].$$

With the above, we have the model describing the dynamics of the groups with stay-at-home and isolation as

$$\begin{cases}S_q^{'}=\sum\limits_{i,j,k,l,m,x,y,z} i\dot{P}_{i,j,k,l,m,x,y,z}(t),\\ E_q^{'}=\sum\limits_{i,j,k,l,m,x,y,z} j\dot{P}_{i,j,k,l,m,x,y,z}(t),\\ A_q^{'}=\sum\limits_{i,j,k,l,m,x,y,z} k\dot{P}_{i,j,k,l,m,x,y,z}(t),\\ I_{q1}^{'}=\sum\limits_{i,j,k,l,m,x,y,z} l\dot{P}_{i,j,k,l,m,x,y,z}(t),\\ I_{q2}^{'}=\sum\limits_{i,j,k,l,m,x,y,z} m\dot{P}_{i,j,k,l,m,x,y,z}(t),\\ H'=\theta_h(I_{g2}+I_{q2})+\sigma W-(\gamma+d)H,\\ W'=\theta_i(I_{g2}+I_{q2})-\gamma_m W-\sigma W,\end{cases}$$

where all parameters are positive, the interpretation of the variables and parameters are summarized in Table 2 and 3.

SAHP non-compliant population

$$\begin{cases}S_g^{'}=-\beta_g c_0 e^{-\mu(1-N_g/P)}(A_g+I_{g1}+I_{g2})\dfrac{S_g}{N_g}-q(t)S_g+g(t)\sum\limits_{i,j,k,l,m=0,x,y,z} iP_{i,j,k,l,m,x,y,z}(t),\\ E_g^{'}=\beta_g c_0 e^{-\mu(1-N_g/P)}(A_g+I_{g1}+I_{g2})\dfrac{S_g}{N_g}-\dfrac{1}{\tau_1}E_g-q(t)E_g+g(t)\sum\limits_{i,j,k,l,m=0,x,y,z} jP_{i,j,k,l,m,x,y,z}(t),\\ A_g^{'}=(1-a)\dfrac{1}{\tau_1}E_g-\gamma_a A_g-q(t)A_g+g(t)\sum\limits_{i,j,k,l,m=0,x,y,z} kP_{i,j,k,l,m,x,y,z}(t),\\ I_{g1}^{'}=a\dfrac{1}{\tau_1}E_g-\dfrac{1}{\tau_2}I_{g1}-q(t)I_{g1}+g(t)\sum\limits_{i,j,k,l,m=0,x,y,z} lP_{i,j,k,l,m,x,y,z}(t),\\ I_{g2}^{'}=\dfrac{1}{\tau_2}I_{g1}-q_{g2}(t)I_{g2}-\theta_h I_{g2}-\theta_i I_{g2}-\gamma_m I_{g2}.\end{cases}$$

where $N_g(t)=S_g(t)+A_g(t)+E_g(t)+I_{g1}(t)+I_{g2}(t)$, $P_{i,j,k,l,m,x,y,z}$ is the number of households with i susceptible, j exposed, k asymptomatic (subclinical) infection, l infectious without no symptoms, m infected with symptoms, x hospitalized, y isolated and z recovered members. The contact rate is $C=c_0e^{-\mu(1-N_g/P)}$. all parameters are positive, and the interpretation of other variables and parameters are given in Tables 2 and 3.