Appendix A: Lack of identification of net age overstatement parameters

Using the same notation as in the text we have

$$\Pi^T = (1/\phi^{no})[\hat{\Theta}^S]^{-1}\Pi^o$$

and

$$\Delta^T = (1/\lambda^{no})[\hat{\Theta}^S]^{-1}\Delta^o$$

In a closed population the relation between the vectors for populations in two successive censuses and the vector of intercensal deaths is:

$$\Pi^T_{t+k} = \Pi^T_t - \Delta^T_{[t,t+k]} \tag{A.1}$$

Using the first two expressions in (A.1) yields:

$$(1/\phi^{no})[\hat{\Theta}^S]^{-1}\Pi^o_{t+k} = (1/\phi^{no})[\hat{\Theta}^S]^{-1}\Pi^o_t - (1/\lambda^{no})[\hat{\Theta}^S]^{-1}\Delta^o_{[t,t+k]} \tag{A.2}$$

From (A.2) we see that only (ϕ^{no}/λ^{no}) is identifiable with the available information.

Appendix B: Behavior of the age misreporting index $cmR^o_{x,[t_1,t_2]}$

The expression of the age misreporting index is

$$cmR^o_{x,[t_1,t_2]} = \frac{cmP^o_{x+k,t_2}/cmP^o_{x,t_1}}{1-(cmD^o_{x,[t_1,t_2]}/cmP^o_{x,t_1})}$$

a ratio of two different estimators of the same quantity, namely the cumulative probability of survival of the population aged x and over at time t_1 to age $(x+k)$ and over at time t_2. Use of cumulative quantities in the index is an important prerequisite since it minimizes the impact of age misreporting within the bounds of the cumulative quantities. Thus, erroneous transfers over age x do not affect population counts at ages x and over. These quantities are influenced only by transfers from ages younger than x into ages x and above or by transfers from ages x and above to ages younger than x. Because use of cumulative quantities complicates the algebra, we will redefine the expression *for single years of age* to obtain:

$$R^o_{x,[t_1,t_2]} = \frac{P^o_{x+k,t_2}/P^o_{x,t_1}}{1-(D_{x,[t_1,t_2]}/P^o_{x,t_1})}$$

or the ratio of a conventional survival ratio computed from two successive population counts to the survival ratio computed from the complement of a measure of the conditional probability of dying between the two censuses. If the population is stationary, the numerator is simply the ratio L_{x+k}/L_x in a life table and the denominator is the complement of the probability of dying in the intercensal period, namely, $1-(1-L_{x+k}/L_x)$. Taking logs on both sides we get,

$$\ln\left(R^o_{x,[t_1,t_2]}\right) \sim -I^N_{x,x+k} - \ln\left(1-\left[1-\exp\left(-I^D_{x,x+k}\right)\right]\right) \tag{B.1}$$

where $I^D_{x,x+k}$ and $I^N_{x,x+k}$ are estimators of the integrated hazards between x and $x+k$ consistent with the survival ratios in the denominator and numerator respectively. When the population is closed to migration, coverage is perfect and there is no net age overstatement, expression (B.1) equals 0 since $I^D_{x,x+k} = I^N_{x,x+k} = I^T_{x,x+k}$, that is, both estimators yield the correct value of integrated hazards. However, when there is (net) age overstatement expression (B.1) becomes

$$\ln\left(R^o_{x,[t_1,t_2]}\right) \sim \ln\left(\frac{h(x+k)}{h(x)}\right) - I^T_{x,x+k} - \ln\left(1-\frac{g(x)}{h(x)}\left[1-\exp(-I^T_{x,x+k})\right]\right) \tag{B.2}$$

where $h(x)$ is an increasing function of age that depends on age overstatement of populations and $g(x)$ is an increasing function of age that depends only on overstatement of ages at death. Thus, the behavior of the sequence of values $R_{x,[t_1,t_2]}$ depends on functions $g(x)$, $h(x)$ and $I^T_{x,x+k}$, where $I^T_{x,x+k}$ refers to the true value of $I_{x,x+k}$. In turn, these functions depend on the true population age distribution, age-specific mortality rates, and the age-specific propensities to misreport ages embedded in the standard pattern of age misreporting. To assess the effects of age misreporting on the index, we now examine the behavior of the functions $h(x)$ and $g(x)$.

Both $h(x)$ and $g(x)$ are functions of the propensity to overstate ages and of the (true)underlying population and deaths age distribution. Assume that the propensity to overstate ages (of populations or deaths) is age invariant (or increases with age) and that the following three conditions hold: (a) the (true) age distribution slopes sharply downward, (b) the age distribution of deaths increases with age, and (c) the rate of decrease of population with age is smaller than the rate of increase of deaths with age. Under these three conditions, almost universally verified in growing human populations, the ratio $h(x+k)/h(x)$ will always be larger than 1 and will increase with age, $g(x)$ will always be larger than 1 and increase with age, and the rate of increase in $g(x)$ will exceed the rate of increase in $h(x)$ so that $g(x) > h(x)$ almost everywhere in the age span.

When $g(x)$ and $h(x)$ are equal to 1, there is neither population nor death age overstatement or, if there is, their effects cancel each other out. Expression (B.2) can be simplified if we expand the log expression on the right as a Taylor series around a value of $g(x)/h(x) = 1$:

$$\begin{aligned} \ln\left(R^o_{x,[t_1,t_2]}\right) \sim \ln\left(\frac{h(x+k)}{h(x)}\right) - I^T_{x,x+k} + \left(\frac{g(x)}{h(x)} - 1\right)(1 + I^T_{x,x+k}) + I^T_{x,x+k} = \\ \ln\left(\frac{h(x+k)}{h(x)}\right) + \left(\frac{g(x)}{h(x)} - 1\right)(1 + I^T_{x,x+k}) \end{aligned} \tag{B.3}$$

When $I^T_{x,x+t}$ is small and $h(x+k)/h(x) = g(x)/h(x) = 1$ the above expression is close to 0.

Expression (B.3) is the analytic support for inferences regarding the effects of age misreporting on the index $cmR_{x,[t_1,t_2]}$. Two caveats are important. First, although in our derivation we assumed stationarity, departures from it may complicate the algebra but leave implications of expression (B.3) intact. Second, to make the algebra tractable the derivations above do not refer to the cumulative functions, as required by the original index. The issue is whether the same inferences can carry over from the discrete to the cumulative functions. We now show informally that this is the case. We can think of the cumulative ratios as functions not of the exact integrated hazards, as in expressions (B.1)-(B.3) but rather as expressions of mean values of corresponding integrated hazards. Thus, in a stationary population, the survival ratio of the cumulative populations at ages x and $x+k$ is the ratio $T(x+k)/T(x)$ which can be written as $\int_{x+k}^{\infty}[\exp(-\int_0^y \mu(s)ds)]dx / \int_x^{\infty}[\exp(-\int_0^y \mu(s)ds)]dx$. Using the mean value theorem in numerator and denominator leads to the approximation $\exp(-\int_{x+i}^{x+k+i'} \mu(s)ds)$ or, more generally, $\exp(-\int_{x^*}^{x^{**}} \mu(s)ds)$ where $x^* > x$ and $x^{**} > x+k$. Upon taking logs in this expression we retrieve an integrated hazard over two ages that are not fixed ex ante (such as x and $x+k$) but, rather, between limits (ages) that are functions of the underlying force of mortality. Thus, the expressions above that use the symbols $I^N_{x,x+k}$, $I^D_{x,x+k}$ and $I^T_{x,x+k}$ for integrated hazards between two exact ages, can also accurately represent relations when the integrated hazards refer to those associated with cumulative quantities. In summary, all derivations based on discrete functions carry over to cumulative quantities. In particular, the following scenarios can be defined for the discrete and, by extension, the cumulative functions.

1. When there is systematic age overstatement of population counts ONLY, $h(x) > 1$ and

$g(x) = 1$, then expression (B.3) reduces to

$$\ln(R^o_{x,[t_1,t_2]}) \sim \ln\left(\frac{h(x+k)}{h(x)}\right) + (h^{-1}(x) - 1)(1 + I^T_{x,x+k}) < 0$$

This inequality holds because the positive term in the expression, e.g the factor that distorts the survival ratio based on population counts, is smaller than the negative factor that distorts the second estimator of the survival ratio based on intercensal death rates.

2. When there is systematic age overstatement of death counts ONLY, $h(x) = 1$ and $g(x) > 1$, the expression becomes

$$\ln(R^o_{x,[t_1,t_2]}) \sim \ln(g(x) - 1)(1 + I^T_{x,x+k}) > 0$$

and all terms in the expression are positive.

3. When there is systematic overstatement of BOTH population and death counts, $g(x) > h(x) > 1$, then

$$\ln(R^o_{x,[t_1,t_2]}) \sim \ln\left(\frac{h(x+k)}{h(x)}\right) + \left(\frac{g(x)}{h(x)} - 1\right)(1 + I^T_{x,x+k}) > 0$$

because, by assumption, all terms are positive.

These predicted impacts of age misreporting are consistent both in previous simulation studies (Condran et al., 1991; Palloni and Pinto, 2004; Grushka, 1996) and in our simulation.

Two important remarks are needed. First, empirical patterns of age overstatement of deaths and populations could offset each other and produce observed survival ratios close to 1 even though the underlying data are incorrect. That is, scenario (3) above is such that the log of the observed survival ratios $cmR_{x,[t_1,t_2]}$ could be 0 at all ages even when there is net age overstatement. Because of this, a diagnostic of conditions based on the observed value of $cmR_{x,[t_1,t_2]}$ (or its log) can *only* detect consistency (including error consistency) of age declaration in population and death counts, rather than suggest accuracy (Dechter and Preston, 1991).

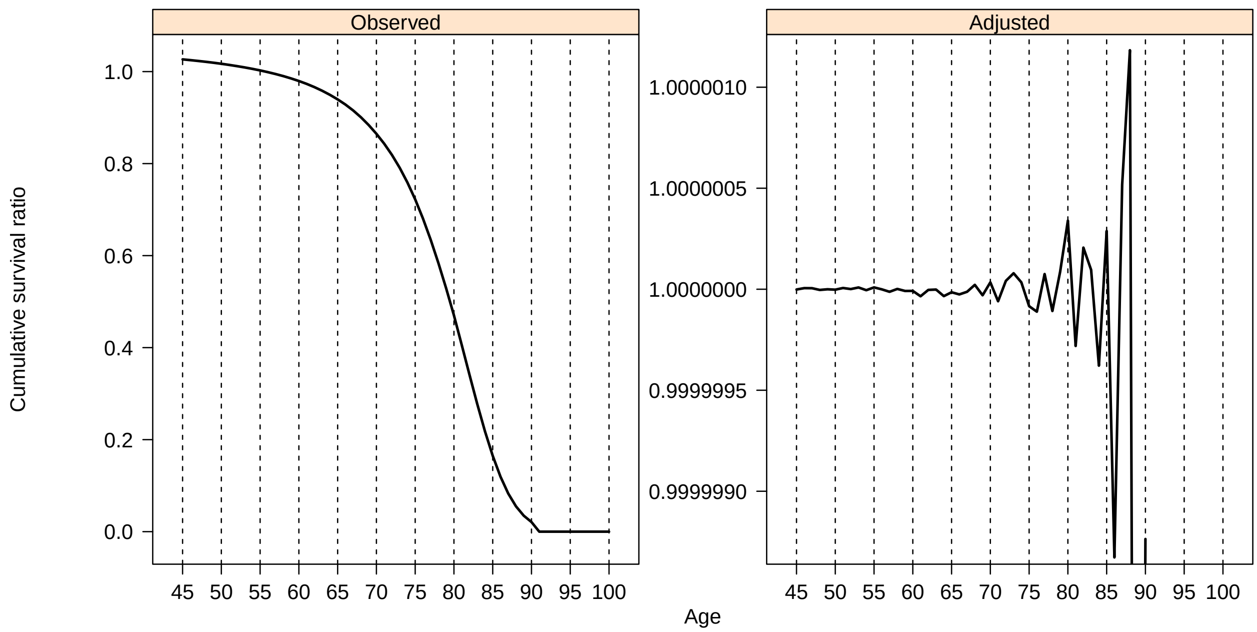
Second, and most important, throughout we assumed that both census and death counts had perfect coverage and that the sequence $cmR_{x,[t_1,t_2]}$ is only influenced by age misreporting. This is an unrealistic assumption at least for LAC countries. When there is defective census or death registration coverage the log of the discrete ratios of survival can be expressed as

$$\ln(R_{x,[t_1,t_2]}) \sim \ln\left(\frac{C_{t_2}}{C_{t_1}}\right) + \ln\left(\frac{h(x+k)}{h(x)}\right) - \left(\frac{CD_{[t_1,t_2]} \cdot g(x)}{C_{t_1} \cdot h(x)} - 1\right)(1 + I^T_{x,x+k}) \tag{B.4}$$

where C_{t_1}, C_{t_2} and $CD_{[t_1,t_2]}$ are the completeness of the first and second census and the average completeness of intercensal death registration, respectively. These quantities are computed as the ratio of the true to the observed counts.As conventionally done in the literature, we assume that completeness of population and death registration are age invariant. The evaluation study described below leads to a choice of procedures that are robust to violations of the age invariance assumption. This enables us to follow standard practice and restrict C_i to be age invariant.

Thus, unless all $C_{t_1} = C_{t_2}$ and $CD_{[t_1,t_2]} = C_{t_1}$, it is impossible to separate the influences of age overstatement and of defective completeness from the observed sequence of values $cmR_{x,[t_1,t_2]}$ alone. Even if there is no age misreporting at all, expression (B.4) can yield non-zero values and mimic age patterns that result naturally from age overstatement alone. In particular, when death registration is perfect, e.g. $CD_{[t_1,t_2]} = 1$, but $C_{t_1} \neq C_{t_2}$, the value of the index will drift away from 0 in an age dependent fashion. This is shown in Figure B.1 that displays the cumulative survival ratios for a case where $CD_{[t_1,t_2]} = 1$ but $C_{t_1}/C_{t_2} < 1$ (left panel) and then when the census figures are properly adjusted for completeness of enumeration so that $C_{t_1}/C_{t_2} = 1$ (right panel).

Figure B.1: Behavior of index of age misstatement with differential censuses completeness.



Appendix C: Simulated populations

C.1 Objectives

The creation of a large set of simulated populations has two objectives. The first is to explore and estimate relations between observed quantities, e.g. the sequence $cmR_{x,[t_1,t_2]}$ and unknown parameters that the investigator desires to estimate, e.g. parameters controlling the levels of age misreporting of population and deaths. The second aim is to assess the performance of techniques to (i) adjust for relative completeness of death registration and (ii) evaluate the integrated strategy to estimate life tables in populations with defective coverage and systematic age misreporting.

The set of simulated population is designed to include as many populations as could be constructed with a large combination of several parameters. The intent is to have a sufficiently large set so that the an investigator can safely invoke the assumption that the defective age distributions and counts of deaths and population that she is studying and seeks to adjust *belongs to the simulated sets*, e.g. that there is at least one simulated population characterized but deaths and population age distributions that are arbitrarily close to the observed one.

The evaluation study is an extension of work described in Palloni and Pinto (2004). It is different from another evaluation study by Hill and colleagues (Hill et al., 2009) in that the simulated populations include consideration of systematic age misreporting that follow the standard pattern of age misreporting.

C.2 Simulated populations

The simulated populations depend on three sets of functions. The first are demographic parameters that uniquely identify age distribution of deaths and populations, *conditional of age patterns of mortality and fertility.* The second identify the age patterns of mortality and fertility. Finally, the third set of functions define the distortions of counts and age distributions of populations and deaths. We discuss these in turn.

C.2.1 Demographic parameters, initial populations and population trajectories

Five master (female) populations were created, one stable and four non-stable populations, that represent trajectories followed during a 100 year period, from 1900 to 2000. The stable population has a $GRR = 3.03$ and $E(0) = 45$ with a natural rate of increase $r = 0.025$. This model stable population roughly corresponds to the average of LAC populations in the period 1940-60, e.g. not yet heavily perturbed by large scale net migration, as is the case in Argentina, Brazil, Cuba, and Uruguay, or fertility changes, as in the cases of Argentina and Uruguay. We also include four non-stable populations profiles that follow (approximately) the mortality and fertility schedules for Costa Rica, Mexico, Guatemala Argentina, and Uruguay during the period 1900-2000. The initial stable distribution for the first three non stable populations are set to be equal to the stable populations with parameters r and $E(0)$ equal to those estimated around 1900 in the corresponding countries (Costa Rica, Mexico and Guatemala). In contrast, the initial age distributions corresponding to the fourth non stable profile (Argentina and Uruguay) are set equal to the observed average age distribution in population censuses within the period 1850-1900. Thus, the initial populations (and deaths) distributions roughly correspond to the actual initial starting populations experienced in the LAC region. The decade-specific demographic parameters in the four non stable populations for the period of interest (1900-2000) are displayed in Table C.1

C.2.2 Age patterns of fertility and mortality

The calculation of yearly and age specific counts of populations and deaths during the 1900-2000 periods follows standard population projection techniques and demands specification of patterns of fertility and mortality. For mortality we chose the West and South models in the Coale-Demeny family of life tables. For fertility, we adopt a unique age pattern of fertility identical to the one used in the computations of the Coale-Demeny stable population models (Coale et al., 1983). We assume throughout that each type of demographic transition in the non stable populations preserves the age patterns of mortality and fertility. Since all calculations are in single years age group both the Coale-Demeny life tables and fertility patterns were transformed into single years schedules of mortality and fertility, respectively. The transformation of the life tables functions into single years functions was carried out by strictly adhering to separator factors adopted by Coale and Demeny. The single-year fertility functions was derived using splines.

In summary, we create 10 stable and non-stable populations (five masters for the West and five masters for the South mortality models) that span a 100 year period from 1900 to 2000 and represent a very broad set of experiences, from those preserving population stability throughout, to those that remain stable up until 1950 or thereabouts, to those shifting to quasi-stability from 1930 up to 1980 and, finally, to those with little or no stability at all from the outset.

C.2.3 Distortions

We introduce three different types of distortions. The first is related to defective completeness of death and population counts. We let C_{t_1} and C_{t_2} to take on values between .8 and 1 in intervals of .5 whereas $CD_{[t_1,t_2]}$ takes values between .7 and 1 in intervals of .5. This yields a total of 175 different combination of defective completeness. When combined with 10 master populations, we generate a total of 1,750 populations. The unknown parameters controlling the levels of net age overstatement of population and death counts were assigned values ranging between 0 and 3 in intervals of .5 for a total of 36 possible patterns of age misreporting. When combined with the previous 1,750 populations they generate 63,000 populations. Finally, to represent age varying completeness of population and death registration we define two patterns, one with higher understatement at ages 45-54 and 70+ (concave upward) and another with higher understatement at ages over 70 (J-shaped). When combined with 10 master populations and 36 patterns of age misreporting we obtain 720 additional populations. Altogether there are a total of 63,720 simulated populations.

C.3 Evaluation of techniques to adjust for relative completeness of death registration

An important advantage of the set of simulated populations is that it enables us to study the impact of distortions of different nature on population indicators. It is through examination of relations observed in the simulated population that we noted that the two unknown parameters controlling levels of age net overstatement could, under some assumptions, be recovered from the sequence of values $cmR_{x,[t_1,t_2]}$. Most importantly, however, the simulated populations can be used to assess the robustness of alternative techniques to estimate relative completeness of death registration to violations of assumptions. This is an important step as the procedures to estimate parameters of severity of age misreporting do not perform well on data distorted by defective relative completeness.

C.3.1 Techniques to adjust for relative completeness of death registration

The set of techniques to detect and adjust for faulty completeness evaluated in this study are summarized in Table C.2.We considered a longer list of techniques but, with two exceptions, chose to test only those that *did not* rely on the assumption of stability or quasi-stability. The table identifies techniques using the names of researcher(s) who proposed them (or modified an original version), highlights key assumptions on which they rely and the information required to implement each of them. These techniques share important commonalities and all but two (Brass No 1 and Preston-Hill No 1) do not invoke the assumption of stability. Yet they differ in at least one feature that, under suitable empirical conditions, potentially grants them a competitive advantage over other methods.

The combination of highly heterogeneous demographic conditions, diversity in the weaknesses of national vital statistics and population censuses counts, and variability of adjustment techniques, each relying on specialized assumptions, makes the choice of adjustments for any particular case a non-trivial matter. Ideally, one would like to be able to choose a very small set of techniques that, under given empirical conditions, produce optimal estimates. To support this endeavor our evaluation study assesses the performance of candidate techniques by applying them to the 63,720 thousands simulated. We then compute multiple error measures under the simulated set of conditions that violate (or not) assumptions on which the techniques rest. Although others could have been chosen, the results of the evaluation we describe here are based on one error measure, namely, the mean absolute value of the proportionate error, MAPE. For any given technique we observe a distribution of MAPE that corresponds to well defined conditions (e.g. violation (or not) of assumptions). For example, suppose we use a technique T in all populations that do not violate any of the assumptions on which the technique relies. We would not expect the numerical value of the parameter estimated by T to always be identical to the population parameter as computations rely on a number of approximations whose impact may vary depending on the nature of the population being examined (stable versus non stable, under model West or under model South, etc.). Thus MAPE are truly random and should have a mean equal to 0. Assume now that the technique is applied in populations that violate a subset of its assumptions. In this case, MAPE will have both a random and a systematic component and will be distributed with a non-zero mean identical to the expected value of the bias associated with the technique given conditions that violate assumptions. More generally, we can compute not just the expected value (and bias) but also the entire distribution of MAPE for each technique and for each set of conditions that violate assumptions we care to specify. In particular, we can calculate medians, quartiles and the probability that MAPE is less than .05.

C.3.2 Results of the evaluation study

Table C.3 displays a number of statistics for the quantity MAPE associated with each techniques we chose to evaluate under three different scenarios.To simplify the table we display statistics for only a subset of all the techniques we studied. Each of this is identified by an acronym defined in Appendix D. Panel A is for scenarios that include population either stable or non-stable and where the completeness of the two successive census may be different but there is no age misreporting. Panels B and C are for scenarios where the populations may experience higher age misreporting in death counts than in population counts (age misreporting 1 and Panel 2) or higher age misreporting in population counts than in death counts (age misreporting 2).This is one of many tables we were able to assemble that targeted different subsets of assumptions and display errors associated with violations in those assumptions. Other tables that display errors when different classes of assumptions are violated could be built.

The main messages from the table are the following. First, even under the worst conditions (Panels B and C) Brass-Hill method (br2Ce) delivers an optimal performance for estimation of relative completeness of two censuses. Note that with probability 1 it will produce an estimate that is within 5 percent of the true value of the parameter.

Second, estimates from the multiple variants of Bennett-Horiuchi technique (bh1Co5-bh2Co75) perform quite badly, even in the absence of age misreporting (Panel A) and so do all the other methods *except variants of Bennett-Horiuchi with adjusted rates of growth* (bh1Co5-mix-bh2Co75-mix). This is a consequence of the fact that changing completeness of the censuses bounding an intercensal interval, biases the age specific rates of growth. This is a problem to which the adjusted Bennett-Horiuchi technique is much less sensitive to. The third finding is that, under the most general and worst conditions (Panels B), even the optimal method (adjusted Bennett-Horiuchi technique) does not have an impeccable record, as one would not expect its estimates to be with 5 percent of the true value in less than 30 percent of the cases. A similar result obtains in Panel C. In both cases though the performance of the method is satisfactory as the median error is less than 7 percent.

C.3.3 Robustness to error of older adult age misreporting

The key result of the evaluation study is this: if one excludes populations with defective census completeness, the *optimal* choice of techniques to adjust for relative completeness of death registration is always one of the variants of Bennett-Horiuchi method. Importantly, however, the Bennett-Horiuchi technique does not perform well unless a correction is introduced to adjust for different completeness of population counts in the first and second census. This could, of course, be a serious limitation were it not for a second result of our evaluation study. The finding is that we confirm a result first reported by Ken Hill (Hill et al., 2009) namely, that the modified Brass technique (Brass-Hill) to estimate relative completeness of death registration also produces a robust estimate of relative censuses completeness, namely, of the ratio C_{t_1}/C_{t_2}. In their original study Hill and colleagues included a limited set of distortions due to age misreporting. However, the same finding is replicated in our study based on simulations of a much larger array of distortions due to age misreporting. It follows that estimates from Brass-Hill and Bennett-Horiuchi are sufficient to correct the observed values of the ratios cmR_x.Both the estimate from Brass-Hill and Bennett-Horiuchi techniques are *mean optimal*, in the following sense: the average error of the estimates they produce are lower than those of other techniques under all conditions spawned by the simulated populations. It does not mean that, once these techniques are applied to observed data, the adjusted mortality rates (and derived functions of the life table) will also be best estimates. This is because the sensitivity to violations of assumptions of techniques we included in the evaluation study varies depending on the *particular subset of assumptions that are violated*. The Brass-Hill and Bennett-Horiuchi estimates are, on average best, but they may not be the best choice were we to restrict examination to populations where a few assumptions are not met (age misreporting) but others are (censuses differential completeness). Another way of saying this is that the strategy we propose based on the evaluation study can only aspire to identify a mean global optimal, rather than a mean local optimal, candidate technique among alternative possible ones. This is discussed in more detail in Appendix C and is highlighted in the discussion section.

Overall, these are remarkably fortunate results for they suggest that, after all, it is possible to adjust the sequence cmR_x for defective completeness of population and death registration *even if the observed data are contaminated by age misreporting.*"Fortunate results" may be an overstatement. Insensitivity of some techniques that adjust for defective death and population to errors of age misreporting is more or less expected due to the utilization of cumulative rather than age-specific counts of population and deaths. If this were not the case, a quest to correct the data for systematic age misreporting would be futile unless coverage of census and death counts are perfect (or equally bad).

Table C.1: Parameters of the non-stable populations.[1]

	I			II			III			IV		
Year	$E(0)$	GRR	r	$E(0)$	GRR	r	$E(0)$	GRR	r	$E(0)$	GRR	r
1900	34.70	3.60	0.05	26.30	6.20	0.04	22.10	5.80	0.03	45.40	1.80	0.02
1910	35.10	3.40	0.05	29.60	5.70	0.04	25.40	5.70	0.03	48.90	1.70	0.02
1920	35.10	3.20	0.05	32.90	5.20	0.04	28.70	5.20	0.03	51.30	1.60	0.02
1930	42.20	2.60	0.05	36.20	4.70	0.04	32.00	4.70	0.03	54.40	1.50	0.02
1940	46.90	2.50	0.05	41.80	4.20	0.04	37.40	3.80	0.03	59.60	1.40	0.02
1950	55.60	2.40	0.05	50.70	3.40	0.04	40.20	3.50	0.03	66.30	1.30	0.02
1960	62.60	2.30	0.05	58.50	3.30	0.04	47.00	3.30	0.03	68.40	1.40	0.02
1970	65.40	2.10	0.05	62.60	3.20	0.04	53.90	3.10	0.03	68.80	1.50	0.02
1980	72.60	1.70	0.05	67.70	2.10	0.04	58.20	3.00	0.03	71.00	1.30	0.02
1990	75.70	1.50	0.05	71.50	1.50	0.04	62.60	2.60	0.03	72.80	1.20	0.02
2000	77.30	1.30	0.05	73.40	1.20	0.04	65.90	2.20	0.03	75.20	1.10	0.02

[1] Non-stable I-III mimic population trajectories in Costa Rica, Mexico and Guatemala; Non-stable IV mimics Argentina and Uruguay's trajectories

Table C.2: Methods to adjust for completeness of death registration: assumptions and required data.[1]

Method	Assumptions	Required Data
Brass (Brass and CELADE, 1975)(B)	1-2-3-4-5	B
Brass (1979b,a)Brass-Hill (BHill[2])	2-3-4	A
Martin (1980)(Martin[3])	1-2-3-4-6	B
Bennett and Horiuchi (1981) No 1 (BH_1)	1-2-3-4	A
Bennet-Horiuchi No 2 (BH_2)	1-2-3-4	A
Bennet-Horicuhi No 3 (BH_3)	1-2-3-4	A
Bennet-Horiuchi No 4 (BH_4)	1-2-3-4	A
Bennet-Horiuchi No 5 (2SBH_4)	1-2-3-4	A
Preston and Hill (1980) No 1 (PH_1)	1-2-3-4-5	A
Preston-Hill No 2 (PH_2)	1-2-3-4	A
Preston and Bennett (1983) (PB)	1-2-3-4	A
Preston and Lahiri (1991) No 1 (PL_1)	1-2-3-4	A
Preston-Lahiri No 2 (PL_2)	1-2-3-4	A

[1]See appendix D for definitions of the four variants of Bennett-Horiuchi method and the two variants of Preston-Lahiri method.

[2]BHill is method used to retrieve estimates of the ratio of completeness of the first relative to the second census.

[3]Martin is a variant of Brass classic method that relaxes the assumption of stability and assumes instead past mortality decline.

KEYS FOR ASSUMPTIONS

1. Identical completeness of census counts in both census
2. Closed to migration
3. No age misreporting
4. Invariant completeness by age
5. Stability
6. Quasi stability

KEYS FOR REQUIRED DATA

A. Two censuses and intercensal deaths

B. One census and one to three years of deaths by age

Table C.3: Evaluation of performance of techniques to adjust for relative completeness of censuses and death enumeration.[1]

Method	Panel A: Stable/non-stable-time varying completeness				Panel B: Stable/non-stable-time varying completeness Age misreporting (1)				Panel C: Stable/non-stable-time varying completeness Age misreporting (2)			
	Median	**IR**	**psCV**	$P(e \leq .05)$	**Median**	**IR**	**psCV**	$P(e \leq .05)$	**Median**	**IR**	**psCV**	$P(e \leq .05)$
br2Ce	0.00090	0.00111	1.22865	1	0.00471	0.00268	0.57023	1	0.00313	0.00197	0.62894	1
ph5Ce	0.00058	0.00082	1.41164	1	0.00698	0.00169	0.24210	1	0.00066	0.00079	1.19297	1
ph10Ce	0.00074	0.00068	0.91928	1	0.00751	0.00184	0.24544	1	0.00066	0.00097	1.46997	1
ph15Ce	0.00110	0.00083	0.75621	1	0.00802	0.00193	0.24106	1	0.00136	0.00103	0.75301	1
bh1Co5	0.29675	0.25504	0.85943	0	0.28939	0.26050	0.90019	0	0.29929	0.30292	1.01213	0
bh1Co75	0.29798	0.25597	0.85901	0	0.29036	0.26676	0.91872	0	0.30097	0.26768	0.88938	0
bh2Co5	0.29711	0.25514	0.85874	0	0.28918	0.25797	0.89209	0	0.29969	0.29992	1.00076	0
bh2Co75	0.29798	0.25597	0.85901	0	0.29036	0.26676	0.91872	0	0.30097	0.26768	0.88938	0
bh1Co5_mix	0.05409	0.06675	1.23406	0.95	0.05729	0.07375	1.28721	0.27	0.05550	0.06880	1.23974	0.5
bh1Co75_mix	0.05523	0.06775	1.22679	0.95	0.05729	0.07553	1.31845	0.27	0.05382	0.06782	1.26013	0.43
bh2Co5_mix	0.05428	0.06691	1.23282	0.95	0.05744	0.07418	1.29146	0.27	0.05561	0.06904	1.24147	0.5
bh2Co75_mix	0.05512	0.06828	1.23884	0.95	0.05747	0.07549	1.31346	0.26	0.05385	0.06808	1.26435	0.43
br1Co	0.02735	0.06853	2.50537	0.65	0.06797	0.08792	1.29356	0.34	0.08056	0.11322	1.40545	0.4
br2Co	0.00549	0.00560	1.01895	1	0.04759	0.02030	0.42655	0.57	0.04974	0.02592	0.52109	0.5
pbCo	0.68908	0.86442	1.25445	0	0.78071	0.81245	1.04065	0.005	0.78071	0.75151	0.96259	0
plCo5	0.48008	0.90139	1.87757	0.0014	0.48294	0.91790	1.90068	0.0007	0.48536	0.95293	1.96337	0.0007
ph5Co	0.05374	0.04835	0.89970	0.44	0.06458	0.08109	1.25567	0.46	0.05334	0.04700	0.88100	0.45
ph10Co	0.05315	0.04662	0.87716	0.45	0.06515	0.08431	1.29406	0.46	0.05369	0.04706	0.87666	0.45
ph15Co	0.05252	0.04556	0.86758	0.47	0.06559	0.08679	1.32327	0.46	0.05331	0.04874	0.91427	0.46

[1] See Appendix D for definitions of each method.
List of acronyms:
IR=Interquartile range;
psCV=Interquartile range/Median;
$P(e \leq .05)$=Probability of error less than .05

Appendix D: Techniques to adjust for relative completeness of population censuses and death registration

Although we assessed performance of a larger number of techniques, Table C.2 reports results for those that are best known. In some cases a technique can be applied using different data configurations (different age groups, different age groupings etc.). We consider these variants separately rather than using one heading for all. The following are the acronyms used to identify them as well as the original sources

1. br2Ce = Brass-Hill 2 census completeness
2. ph5Ce = Preston-Hill census completeness (start at age 5) (Preston and Hill, 1980)
3. ph10Ce = ibid (start at age 10)
4. ph15Ce = ibid (start at age 15)
5. martin = Martin death completeness (Martin, 1980)
6. bh1Co5 = Bennett-Horiuchi I death completeness (forward accumulation starting at age 5) (Bennett and Horiuchi, 1981)
7. bh1Co75 = Bennett-Horiuchi I death completeness (backward accumulation starting at age 75)
8. bh2Co5 = Bennett-Horiuchi II death completeness (forward accumulation starting at age 5)
9. bh2Co75 = Bennett-Horiuchi II death completeness (backward accumulation starting at age 75)
10. bh1Co5_mix = adjusted Bennett-Horiuchi I death completeness (forward accumulation starting at age 5)
11. bh1Co75_mix = adjusted Bennett-Horiuchi I death completeness (backward accumulation starting at age 75)
12. bh2Co5_mix = mix adjusted Bennett-Horiuchi II death completeness (forward accumulation starting at age 5)
13. bh2Co75_mix = adjusted Bennett-Horiuchi II death completeness (backward accumulation starting at age 5)
14. br1Co = Brass 1 death completeness (Brass and CELADE, 1975)
15. br2Co = Brass 2 death completeness

16. pbCo = Preston-Bennett death completeness (Preston and Bennett, 1983)

17. plCo5 = Preston-Lahiri death completeness(start age 5) (Preston and Lahiri, 1991)

18. phCo5 = Preston-Hill death completeness (start age 5) (Preston and Hill, 1980)

19. ph10Co = Preston-Hill death completeness (start age 10)

20. ph15Co = Preston-Hill death completeness (start age 15)

Appendix E: Sensitivity analysis

E.1 Objectives

In this appendix we assess sensitivity of adjusted estimates of population and death counts to violation of the to key assumptions, namely, one involving the pattern of age misreporting and the identify assumption.

We use a subset of simulated populations consisting of an initially stable population under Model South distorted by 175 patterns of defective completeness. We then apply a new standard of age misreporting and introduce, as we did in the original simulation, 36 combinations of values of levels of age misreporting, 6 for death and 6 for population counts. This leads to a total of 6,300 simulated populations characterized by patterns of age misreporting different from the Costa Rican one. We then apply the proposed adjustment procedure (which requires to invoke the assumption of a Costa Rican patterns of age misreporting) and retrieve an adjusted sequence of values $cmR_{x,[t_1,t_2]}$, estimates of unknown parameters for levels of age misreporting, and adjusted life tables. We then compare selected statistics of the adjusted life tables with the true life table that generated the data. The differences between the two are a measure of the errors associated with misidentification of the age pattern of age misreporting.

E.2 New pattern of age misreporting

Without additional constraints, the number of potential candidates to become alternative standard for net age overstatement is infinite. To narrow down the set of plausible candidates we modify separately the probabilities of net overstatement and the conditional probabilities of overstating by n years.

First, we choose a standard for the probabilities of net overstatement that satisfies two conditions:

1. Condition 1: it has approximately the same probabilities of net overreporting at ages 45 and and 100 as the Costa Rican standard. This condition constrains the level parameters to be within the same range or parameter space as those compatible with the Costa Rican standard, e.g. $(0-3)$.

2. Condition 2: the new standard probabilities increase much more rapidly with age than in the Costa Rican standard. This will reflect situations were the standard pattern producing the data imply much worse age misreporting than is embedded in the Costa Rican standard.

The function that defines the probabilities of net overstatement is $P(x) = .18 * (1 - S(x)) + .15$ where $S(x)$ is a Gompertz survival function with level parameter $\alpha = .030$ and slope parameter $\beta = .80$. It attains a value equal to 1 at age 45 and a median value at age 58. Other transformations of the function $S(x)$ are of course possible. The function we use here distorts in significant ways the shape of the Costa Rican standard (from linear to logistic). It also maximizes differences in probabilities between ages 45 and 90 while simultaneously allowing room for level parameters to increase (decreases) these probabilities to the same maximum and minimum levels allowed by the Costa Rican standard.

Second, the conditional probabilities of misreporting age by n years follows a nearly symmetrical binomial distribution with binomial probability $p = .50$. This is in stark contrast with the (approximately) negative binomial distribution embedded in the Costa Rican standard.

Figure E.1 displays the unconditional and conditional probabilities embedded in the alternative standard

E.2.1 Effects of using an incorrect standard of age misreporting

Results of the sensitivity exercise are in Figure E.2. The figure displays the cumulative distribution of relative errors in estimates of life expectancy at age 45 (top panel) and 60 (bottom panel). These figures reveal two properties of the resulting estimates. First, the bulk of errors (over 95 percent) are positive, namely, the estimated values of life expectancy are higher than the true ones. This is consistent with the fact that the standard that generated the simulated populations has significantly higher probabilities of net overstatement than the Costa Rican standard used to retrieve estimates of parameters. Thus, the outcome of using a standard probabilities of overstatement than rise much slower with age than the one that generates the data will be to under adjust mortality rates and overestimate life expectancy at adult ages. Second, the distribution of errors for life expectancy at age 45 is quite benign as they are less than 5 percent in about 80 percent of cases. In contrast, the errors are more serious for life expectancy at age 60 as only in 35 percent of cases are the errors below 5 percent.

Two final caveats. First, although the alternative pattern of age misreporting used in this sensitivity exercise departs significantly from the Costa Rican standard, it is still based on the assumption of net overstatement. But this may not be a universal feature of age misreporting. In their work on age misreporting, Preston and colleagues find that net understatement is not uncommon among US African Americans and has been found elsewhere (Preston et al., 1999)(Preston personal communication). Even though net understatement, like net overstatement, *must* lead to underestimates of old age mortality, its presence in observed data invalidates the use of a pattern of age misreporting based on net overstatement.

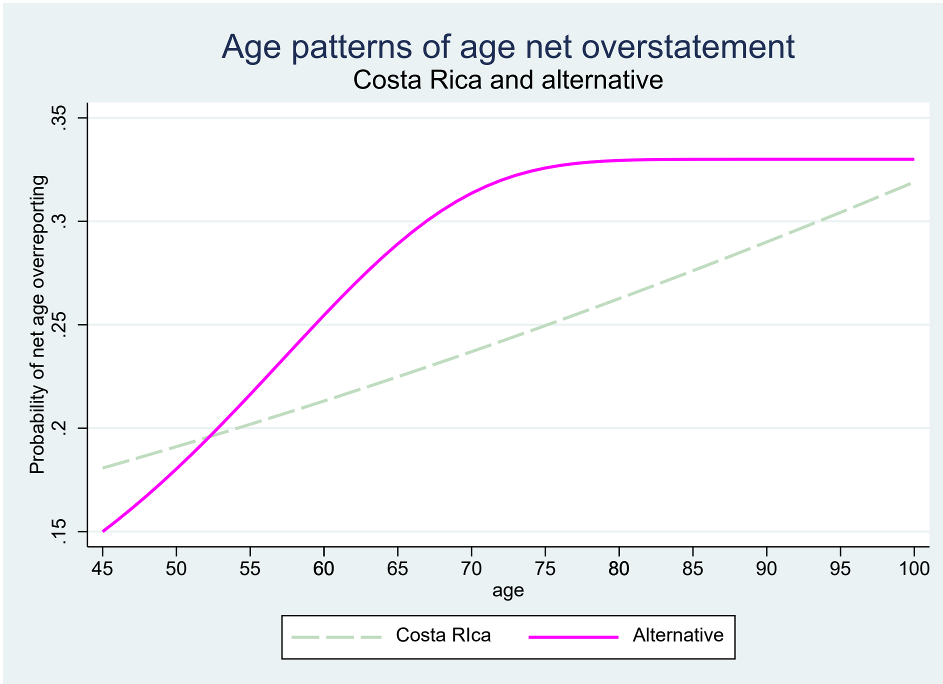
Second, it is also possible that in some populations overstating ages by more than 10 years can be a frequent occurrence, rather than a rare event. If so, the conditional distribution of n assumed throughout will depart in significant ways from the true distribution as this must have a much thicker right tail. In these cases the investigator should estimate separately the density of the random variable n and redefine the standard of age misreporting accordingly.

E.2.2 Failure of the "identity" assumption: deaths and population age misreporting follow different patterns

Because there are no known data on which to base the construction of a standard pattern of age misreporting of death counts, we assumed throughout that this was identical to the standard pattern of age misreporting of population counts. The only defense against potential problems caused by violation of the assumption is to examine the behavior of selected indicators. First, as is the case when there is misidentification of the standard pattern of population age misreporting (see above), the quantities in error will be estimates of the unknown level parameters λ^{no}, ϕ^{no}. If departures from the identity assumption are significant, estimates of the level parameters will be implausible, e.g. they will fall outside the range contained in the simulated population set and/or the fit of sequence $cmR_{x,[t_1,t_2]}$ to the data will be deficient (even if estimates are within the legitimate range).

Of course, if the investigator suspects or has ancillary evidence that misreporting of age in death counts is light, the parameter ϕ^{no} could be set to zero, only parameter must be estimated, and the identity assumption is unnecessary.

Figure E.1: Alternative standard of age misreporting: unconditional and conditional probabilities of age misreporting



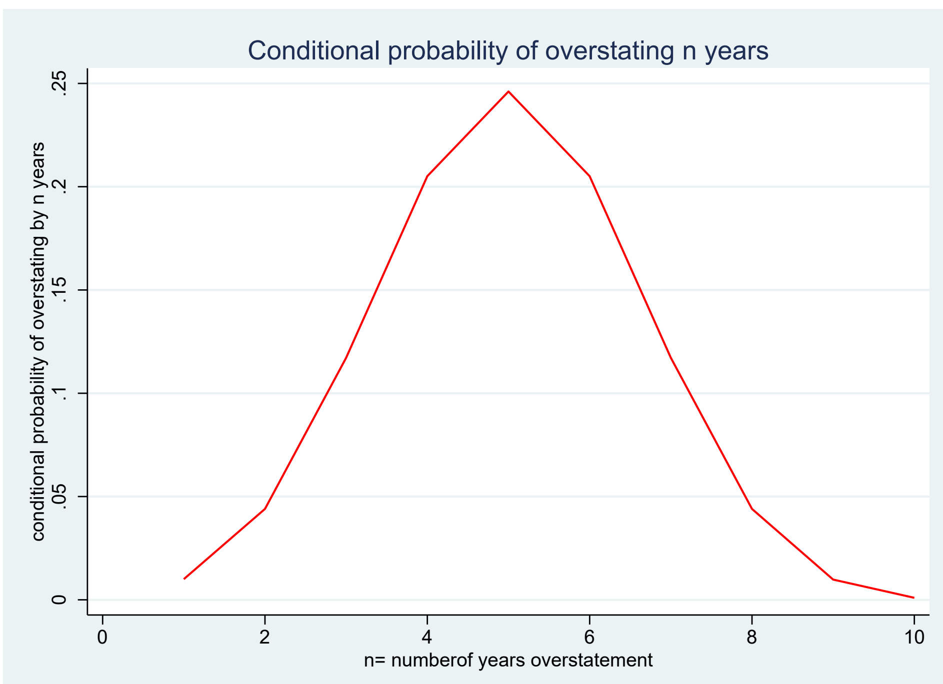
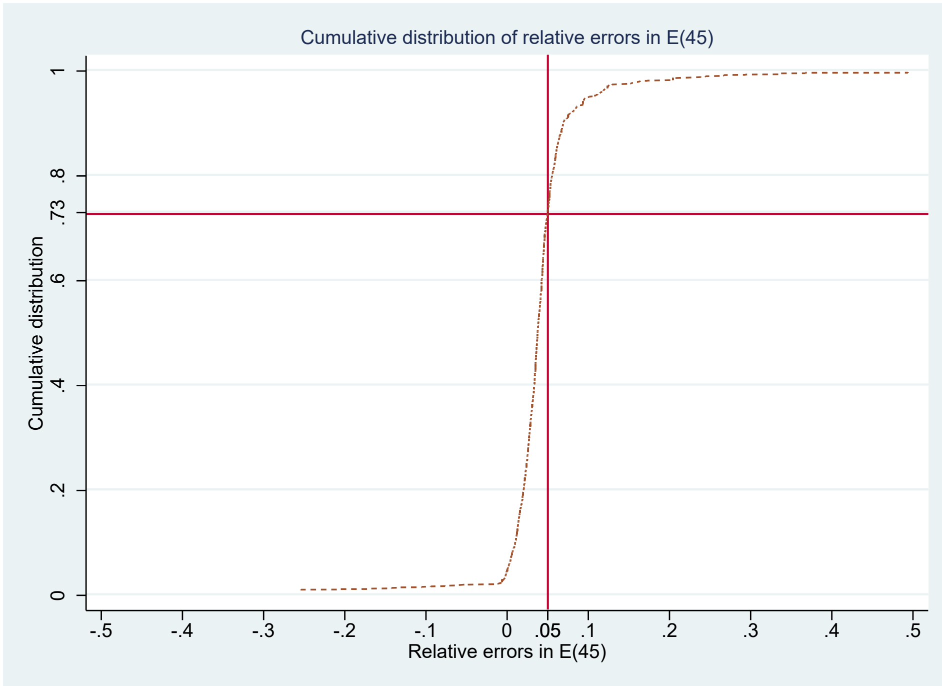
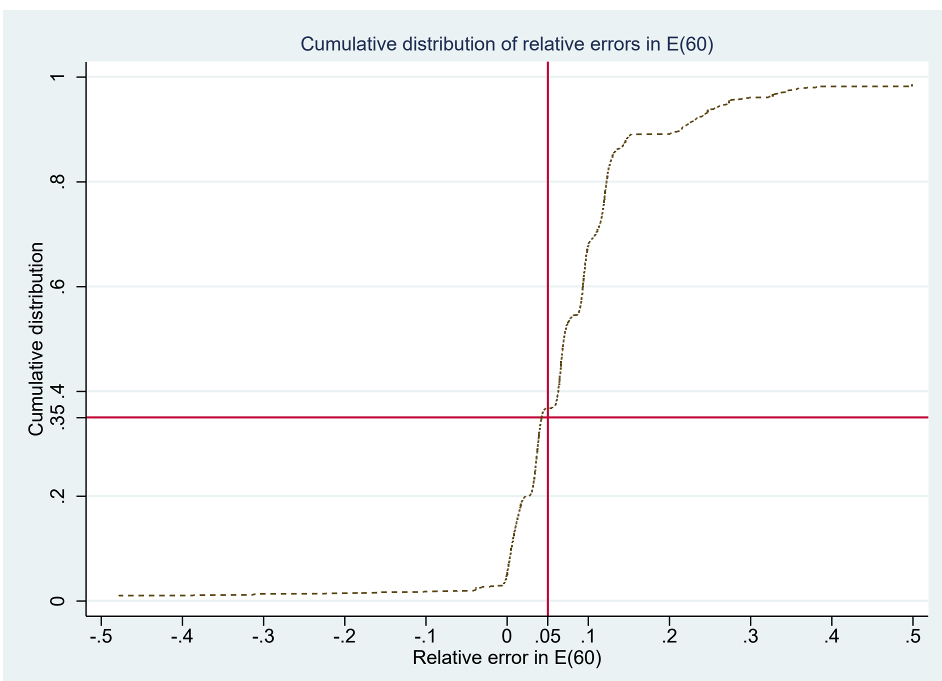


Figure E.2: Cumulative distribution of errors in estimates of E(45) and E(60) (sensitivity to misidentification of standard schedule of age misreporting).





Source: Costa Rica Special study of 2000 population census.