Supplementary Materials for "Transformation model based regression with dependently truncated and independently censored data"

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Table S1 presents the values of parameters ϕ, a(or θ), r under each simulation scenario in Section 4 of the main paper. Table S2 and Table S6 present the average bias for the transformation parameter estimates for the simulation setups in Section 4.1 and in Section 4.2 of the main paper, respectively. Table S3 presents empirical bias of $\hat{\beta}$ under simulation setup in Section 4.1 with 10,000 replications. Table S4 lists the power analysis for the hypothesis tests $H_0 : \beta = 0.8$ versus $H_1 : \beta > 0.8$ based on the estimates of β under the simulation setup in Section 4.1. Table S5 presents additional summary statistics for the simulation setup in Section 4.1, including the value of minimum $||\hat{\alpha}||$ in equation (4) for Cox1 and Cox2 approaches, and the p-value of the test for conditional independence between X and T given covariate Z. The test for conditional independence follows the global likelihood ratio test described in Section 4.3 of the main paper.

Table S1: Values of the parameters ϕ, a(or θ), r in simulation setups in Section 4[†]

censoring	$\tau = 0.2$			$\tau = 0.35$		
rate	ϕ	a (or θ)	r	ϕ	a(or θ)	r
	Simulation setups in Section 4.1					
0%	1.60	-0.14	–	1.60	-0.33	–
15%	1.47	-0.13	0.07	1.35	-0.28	0.08
30%	1.28	-0.11	0.14	1.18	-0.24	0.17
50%	1.13	-0.09	0.24	1.01	-0.19	0.27
	Simulation setups in Section 4.2					
0%	2.20	-0.20	–	1.50	-0.50	–
15%	3.00	-0.25	4.00	1.50	-0.40	4.50
	Simulation setups in Section 4.3					
0%	1.00	2.00	–	0.80	6.00	–
15%	1.00	1.64	0.65	1.00	3.00	0.50

[†]a, the parameter in the linear transformation model; θ, the Clayton copula parameter.

Table S2: Absolute bias of $\hat{a}$, the estimator for transformation parameter a, under simulation setup in Section 4.1[†]

τ	cen	bias$(\hat{a})$			bias$(\hat{a})$		
		$\hat{a}_{CK}$	$\hat{a}_{Cox1}$	$\hat{a}_{Cox2}$	$\hat{a}_{CK}$	$\hat{a}_{Cox1}$	$\hat{a}_{Cox2}$
		$n = 200$			$n = 500$		
0.2	0%	<0.001	<0.001	0.007	<0.001	<0.001	0.003
	15%	0.001	0.002	0.010	0.001	<0.001	0.004
	30%	<0.001	<0.001	0.009	<0.001	<0.001	0.002
	50%	0.002	0.003	0.009	<0.001	0.002	0.005
0.35	0%	<0.001	<0.001	0.005	<0.001	<0.001	0.002
	15%	<0.001	<0.001	0.007	0.001	<0.001	0.004
	30%	0.001	0.001	0.008	<0.001	<0.001	0.003
	50%	0.002	0.003	0.010	<0.001	<0.001	0.004

[†]the subscripts for $\hat{a}$ represent different proposed methods; CK, unadjusted transformation via conditional Kendall's tau; Cox1, adjusted transformation via Cox model with continuous $T'(a)$; Cox2, adjusted transformation via Cox model with the first and second tertiles of $T'(a)$.

Table S3: Bias of $\hat{\beta}$ under simulation setup in Section 4.1 with 10,000 replications[†]

| τ | cen | bias($\hat{\beta}$) | | | | |
		Cox QI	JC	CK	Cox1	Cox2
				$n = 200$		
0.2	0%	-0.0822	-0.1157	0.0038	0.0022	0.0033
	15%	-0.0731	-0.1046	0.0035	0.0014	0.0035
	30%	-0.0564	-0.0823	0.0013	-0.0013	0.0013
	50%	-0.0420	-0.0609	0.0043	-0.0076	0.0048
0.35	0%	-0.2804	-0.4273	0.0038	0.0022	0.0020
	15%	-0.2241	-0.3665	0.0034	0.0014	0.0028
	30%	-0.1836	-0.3158	-0.0001	-0.0027	-0.0007
	50%	-0.1338	-0.2423	-0.0076	-0.0106	-0.0083

[†]Cox QI, Cox model assuming independent truncation; JC, Jones & Crowley's method; CK, unadjusted transformation via conditional Kendall's tau; Cox1, adjusted transformation via Cox model with continuous $T'(a)$; Cox2, adjusted transformation via Cox model with first and second tertiles of $T'(a)$.

Table S4: Power analysis for hypothesis test $H_0 : \beta = 0.8$ v.s. $H_1 :$ $\beta > 0.8$ under simulation setup in Section 4.1[†]

τ	cen	Power (%)				
		Cox QI	JC	CK	Cox1	Cox2
		$n = 200$				
0.2	0%	20.4	12.0	51.6	49.2	44.5
	15%	23.3	13.7	49.2	46.1	44.3
	30%	22.5	16.5	38.2	36.3	36.8
	50%	26.9	18.5	37.5	35.1	34.3
0.35	0%	0	0	51.6	49.2	39.4
	15%	1.0	0	48.8	45.7	42.1
	30%	1.8	0	38.2	36.3	36.8
	50%	7.5	0.6	34.8	32.7	31.1
		$n = 500$				
0.2	0%	43.3	20.9	90.6	87.9	88.5
	15%	47.8	24.0	87.5	86.0	86.2
	30%	54.2	37.7	80.8	79.2	79.3
	50%	61.6	46.0	75.9	73.5	75.5
0.35	0%	0	0	90.6	87.9	87.7
	15%	1.0	0	87.9	86.0	85.5
	30%	3.0	0	79.7	78.5	79.0
	50%	13.0	0.1	76.3	74.5	75.7

[†]See footnotes of Table S3.

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Table S5: Summary of statistics under simulation setup in Section 4.1[†]

τ	cen	value of minimum $||\hat{\alpha}||$ in eqn. (4)		p-value of test assessing $X \perp T \| Z$	
		Cox1	Cox2	%< 0.05	median
			$n = 200$		
0.2	0%	<0.001	0.098	84.1	3.68×10^{-3}
	15%	<0.001	0.103	78.8	6.44×10^{-3}
	30%	<0.001	0.116	71.7	1.08×10^{-2}
	50%	<0.001	0.131	62.8	2.11×10^{-2}
0.35	0%	<0.001	0.099	100	4.24×10^{-8}
	15%	<0.001	0.108	100	7.46×10^{-8}
	30%	<0.001	0.121	100	2.95×10^{-7}
	50%	<0.001	0.139	100	2.20×10^{-6}
			$n = 500$		
0.2	0%	<0.001	0.063	99.3	1.74×10^{-6}
	15%	<0.001	0.068	99.5	7.31×10^{-6}
	30%	<0.001	0.078	98.7	2.97×10^{-5}
	50%	<0.001	0.092	95.4	2.17×10^{-4}
0.35	0%	<0.001	0.064	100	2.19×10^{-19}
	15%	<0.001	0.073	100	1.52×10^{-18}
	30%	<0.001	0.082	100	3.60×10^{-17}
	50%	<0.001	0.095	100	1.66×10^{-14}

[†]Cox1, adjusted transformation via Cox model with continuous $T'(a)$; Cox2, adjusted transformation via Cox model with the first and second tertiles of $T'(a)$.

Table S6: Bias for $\hat{a}$ under simulation setup in Section 4.2[†]

		bias($\hat{a}$)				bias($\hat{a}$)		
τ	cen	$\hat{a}_{\mathrm{CK}}$	$\hat{a}_{\mathrm{Cox1}}$	$\hat{a}_{\mathrm{Cox2}}$		$\hat{a}_{\mathrm{CK}}$	$\hat{a}_{\mathrm{Cox1}}$	$\hat{a}_{\mathrm{Cox2}}$
			$n = 200$				$n = 500$	
0.2	0%	-0.308	-0.004	-0.017		-0.306	-0.002	-0.011
	15%	-0.285	-0.017	-0.045		-0.280	-0.010	-0.021
0.35	0%	-0.170	-0.003	-0.011		-0.172	-0.001	-0.007
	15%	-0.210	-0.015	-0.024		-0.204	-0.005	-0.009

[†]CK, unadjusted transformation via conditional Kendall's tau; Cox1, adjusted transformation via Cox model with continuous $T'(a)$; Cox2, adjusted transformation via Cox model with the first and second tertiles of $T'(a)$.