Use of recently vaccinated individuals to detect bias in test-negative case-control studies of COVID-19 vaccine effectiveness

Supplementary Material

Hitchings et al

Supplementary Methods

Derivation of bias-indicator

The odds ratio comparing vaccination status between cases and controls is given by

$$OR_U^R(t) = \frac{C_{R+}(t)C_{U-}(t)}{C_{R-}(t)C_{U+}(t)}$$

$$= \frac{\pi_-\lambda_-[\mu_U\alpha_{U-}(1-v)t + \mu_P\alpha_{P-}vT_V]P}{\pi_+[\mu_U(1-e^{-\alpha_{U+}\lambda_+t})(1-v) + \mu_P(1-e^{-\alpha_{P+}\lambda_+T_V})v]P}\frac{\pi_+\mu_R(1-e^{-\alpha_{R+}\lambda_+T_P})vP}{\pi_-\mu_R\alpha_{R-}\lambda_-T_PvP}$$

$$\approx \frac{\alpha_{R+}}{\alpha_{R-}}\frac{[\mu_U\alpha_{U-}(1-v)t + \mu_P\alpha_{P-}vT_V]}{[\mu_U\alpha_{U+}(1-v)t + \mu_P\alpha_{P+}vT_V]},$$

as $1 - e^{-x} \approx x$ when x is small.

Derivation of bias-corrected vaccine effectiveness estimate

The odds ratio comparing vaccination status between cases and controls is given by

$$1 - OR_R^F(t) = 1 - \frac{C_{F+}(t)C_{R-}(t)}{C_{F-}(t)C_{R+}(t)}$$

$$= 1 - \left(\frac{\pi_+\mu_V[(1-\varphi)(1-e^{-\alpha_{V+}\lambda_+(t-T_P-T_V)}) + \varphi(1-e^{-\alpha_{V+}\theta\lambda_+(t-T_P-T_V)})]vP}{\pi_+\mu_V(1-e^{-\alpha_{V+}\lambda_+T_P})vP}\right)\frac{\pi_-\mu_V\alpha_{V-}\lambda_-T_PvP}{\pi_-\mu_V\alpha_{V-}\lambda_-(t-T_P-T_V)vP}$$

$$\approx 1 - \left(\frac{[(1-\varphi)\alpha_{V+}\lambda_+(t-T_P-T_V) + \varphi\alpha_{V+}\theta\lambda_+(t-T_P-T_V)]}{(\alpha_{V+}\lambda_+T_P)}\right)\frac{T_P}{(t-T_P-T_V)}$$

$$= \varphi(1-\theta).$$

Again, $1 - e^{-x} \approx x$ when x is small.

Derivation of bias-corrected vaccine effectiveness estimate with varying T_V

If vaccination occurred on a set of days T_{Vj}, and a proportion p_j of vaccinated individuals is vaccinated on each vaccination day, then

$$C_{F+}(t) = \sum_j C_{F+}(t|T_V = T_{Vj})p_j$$

$$C_{F-}(t) = \sum_j C_{F-}(t|T_V = T_{Vj})p_j$$

Expressions for $C_{R+}(t)$ and $C_{R-}(t)$ are independent of T_V, and therefore remain as in the main text. Thus,

$$1 - OR_R^F(t) = 1 - \frac{C_{F+}(t)C_{R-}(t)}{C_{F-}(t)C_{R+}(t)}$$

$$= 1 - \left(\frac{\sum_j [(1-\varphi)\left(1 - e^{-\alpha_{V+}\lambda_+(t-T_P-T_{Vj})}\right) + \varphi\left(1 - e^{-\alpha_{V+}\theta\lambda_+(t-T_P-T_{Vj})}\right)]p_j}{\left(1 - e^{-\alpha_{V+}\lambda_+T_P}\right)}\right)\frac{T_P}{\sum_j (t - T_P - T_{Vj})p_j}$$

$$\approx 1 - \left(\frac{\alpha_{V+}\lambda_+[(1-\varphi) + \varphi\theta]\sum_j (t - T_P - T_{Vj})p_j}{\alpha_{V+}\lambda_+ T_P}\right)\frac{T_P}{\sum_j (t - T_P - T_{Vj})p_j}$$

$$= \varphi(1-\theta).$$