AK Software

Radiomics Parameters Description

GE Life Sciences

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Contents Guide

1. Histogram Parameters

Histogram parameters are concerned with properties of individual pixels. They describe the distribution of voxel intensities within the CT image through commonly used and basic metrics. Let X denote the three dimensional image matrix with N voxels and P the first order histogram divided by N_l discrete intensity levels. The following first order statistics were extracted:

1.1 Energy:

The energy feature measures the uniformity of the intensity level distribution. If the value is high, then the distribution is to a small number of intensity levels. Energy can be defined as:

$$\mathrm{energy} = \sum_{\mathrm{i}}^{\mathrm{N}} \mathrm{X(i)}^2$$

1.2 Entropy:

The entropy measures the randomness of the distribution of the coefficients values over the intensity levels. If the value of entropy is high, then the distribution is among more intensity levels in the image. This measurement is the inverse of energy. A simple image has low entropy while a complex image has high entropy. Entropy can be defined as:

$$entropy = -\sum_{i=1}^{N_l} P(i)\, log_2\, P(i)$$

1.3 MaxIntensity:

The maximum intensity value of X.

1.4 MinIntensity:

The minimum intensity value of X.

1.5 MeanValue:

The mean measures the average value of the intensity values.

$$mean = \frac{1}{N}\sum_{i}^{N} X(i)$$

1.6 Mean absolute deviation:

The mean of the absolute deviations of all voxel intensities around the mean intensity value.

1.7 MedianIntensity:

The median intensity value of X.

1.8 Range:

The range of intensity values of X.

1.9 Root mean square (RMS):

$$RMS = \sqrt{\frac{\sum_i^N X(i)^2}{N}}$$

1.10 Standard deviation: stdDeviation

Is a measure that is used to quantify the amount of variation or dispersion of a set of data values.

$$standard\ deviation = \left(\frac{1}{N-1}\sum_{i=1}^{N}(X(i) - \bar{X})^2\right)^{1/2}$$

where $\bar{X}$ is the mean of X.

1.11 Uniformity:

$$uniformity = \sum_{i=1}^{N_l} P(i)^2$$

1.12 Variance:

Is the average of the squared differences from the Mean.

$$variance = \frac{1}{N-1}\sum_{i=1}^{N}(X(i) - \bar{X})^2$$

where $\bar{X}$ is the mean of X.

1.13 Volume Count

Describe the size of the ROI.

1.14 Voxel Value Sum

Represents the Sum calculations for voxels in the ROI.

1.15 RelativeDeviation

Let $\bar{X}$ denote the mean of a set of quantities X_i , then the relative deviation is defined by:

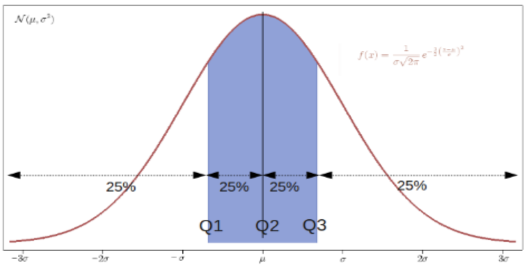
$$\frac{\Delta \mathrm{X_i}}{\bar{\mathrm{X}}} = \frac{|\mathrm{X_i} - \bar{\mathrm{X}}|}{\bar{\mathrm{X}}}$$

1.16 Frequency Size

1.17 Quantiles

Quantile normalization is a global adjustment method that assumes the statistical distribution of each sample is the same. The normalization is achieved by forcing the observed distributions to be the same and the average distribution, obtained by taking the average of each quantile across samples. They are cut points dividing the range of a probability distribution into contiguous intervals with equal probabilities, or dividing the observations in a sample in the same way.
For a finite population of *N* equally probable values indexed 1, …, *N* from lowest to highest, the *k*-th *q*-quantile of this population can equivalently be computed via the value of:

$$I_p = N\ k/q$$



The area below the red curve is the same in the intervals (-∞, *Q1*), (*Q1*, *Q2*), (*Q2*, *Q3*), and (*Q3*, +∞).

In AK software, we have 5 Quantiles:
Quantile0.025, Quantile0.25, Quantile0.5, Quantile0.75, Quantile0.975.

1.18 Percentiles

A **percentile** (or a **centile**) is a measure used in statistics indicating the value below which a given percentage of observations in a group of observations fall.
The percentile, p%, of a distribution is defined as that value of the brightness *a* such that:
$P(a) = p\%$.
or equivalently: $\int_{-\infty}^{a} P(\alpha) = p\%$
The P-th percentile $0 < P \leq 100$ of a list of N ordered values (sorted from least to greatest) is the smallest value in the list such that P percent of the data is less than or equal to that value. This is obtained by first calculating the ordinal rank and then taking the value from the ordered list that corresponds to that rank. The ordinal rank *n* is calculated using this formula

$$n = \frac{P}{100} * N$$

AK Software have 19 Percentiles.
Percentile5, Percentile10, Percentile15, Percentile20, Percentile25, Percentile30, Percentile35, Percentile40, Percentile45, Percentile50, Percentile55, Percentile60, Percentile65, Percentile70, Percentile75, Percentile80, Percentile85, Percentile90, Percentile95

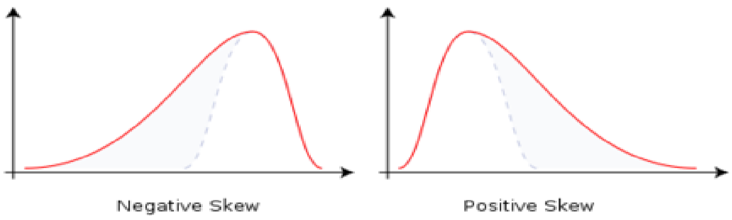
1.19 Skewness

Represents the degree of asymmetric distribution in the image histogram, this means that in some distribution of data, the right and the left of the distribution are perfect mirror images of one another, the mean, median and mode are all measures of the center of a set of data. The Skewness of the data can be determined by how these quantities are related to one another. [6]
High values of Skewness means that the distribution is asymmetric otherwise the image is more symmetric; negative skew is when the numerical distribution is relatively long also called negative Skewness distribution, the opposite is referred as positive Skewness distribution (positive skew). Its possible to use The positive and negative Skewness to draw comparisons between the uniform distribution curve.

Formula:

$$skewness = \frac{\frac{1}{N}\sum_{i=1}^{N}(X(i)-\bar{X})^3}{\left(\sqrt{\frac{1}{N}\sum_{i=1}^{N}(X(i)-\bar{X})^2}\right)^3}$$

where $\bar{X}$ is the mean of X.



1.20 Kurtosis

Kurtosis is a measure of whether the data are heavy-tailed or light-tailed relative to a normal distribution. That is, data sets with high kurtosis tend to have heavy tails, or outliers. Data sets with low kurtosis tend to have light tails, or lack of outliers. A uniform distribution would be the extreme case. [3]
When Kurtosis have small values, it shows more concentration, in contrast when the Kurtosis is bigger is more dispersed. Usually the size of positive and negative kurtosis is compared with the normal distribution curve. Positive Kurtosis indicates that the normal distribution curve is more smooth, on the other hand, the negative Kurtosis indicates that the normal distribution is more precipitous.

Kurtosis is unfortunately harder to picture than Skewness, but the illustrations below, should help. All of these three distributions have **mean of 0, standard deviation of 1, and Skewness of 0**, and all are plotted on the same horizontal and vertical scale**. Look at the progression from left to right, as kurtosis increases.** [3]

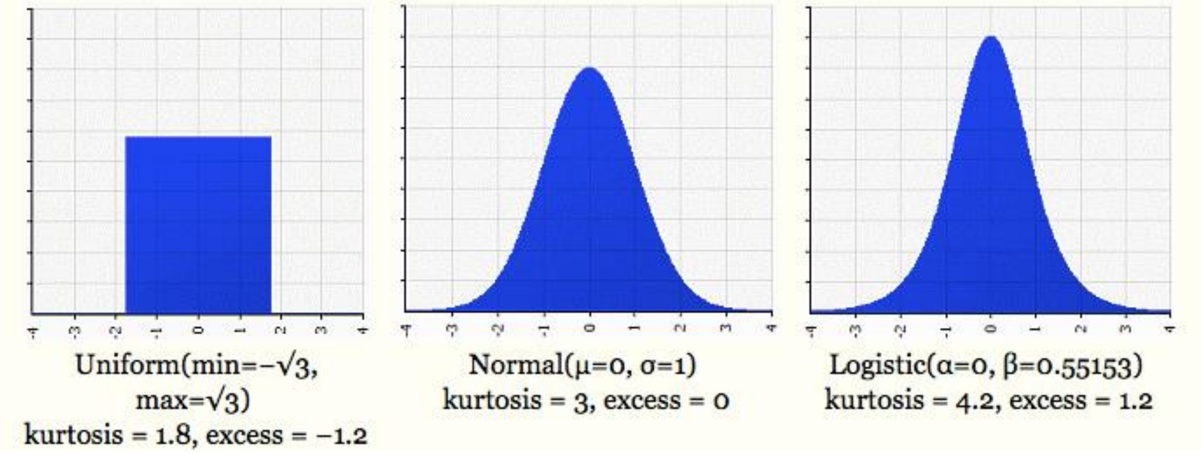


Image illustration of Kurtosis [4]

Formula:

$$kurtosis = \frac{\frac{1}{N}\sum_{i=1}^{N}(X(i)-\bar{X})^4}{\left(\sqrt{\frac{1}{N}\sum_{i=1}^{N}(X(i)-\bar{X})^2}\right)^2}$$

where $\bar{X}$ is the mean of X.

2. Texture Parameters

Texture is one of the important characteristics used in identifying objects or regions of interest in an image, texture represents the appearance of the surface and how its elements are distributed. It is considered an important concept in machine vision, in a sense it assists in predicting the feeling of the surface (e.g. smoothness, coarseness …etc.) from image. Various texture analysis approaches tend to represent views of the examined textures form different perspectives, and due to multi-dimensionality of perceived texture, there is not an individual method that can be sufficient for all textures. Therefore, AK software is mainly concerned with texture classification accuracy improvement using textures features statistical based methods.

2.1 Energy

This feature Returns the sum of squared elements in the GLCM. Range = [0 1]
Energy is 1 for a constant image. Is high when image has very good homogeneity or when pixels are very similar The Property Energy is also known as uniformity, uniformity of energy, and angular second moment. [2]

Formula:

$$\sum_{i,j} g(i,j)^2$$

*g is a GLCM
Where i,j are the spatial coordinates of g (i,j).

2.2 Entropy

This is a measure of randomness, having its highest value when the elements of **g** are all equal. In the case of a checkerboard, the entropy would be low.

Formula

$$-\sum_{i,j} g(i,j)\log_2(i,j)$$

2.3 Correlation

Correlation measures the linear dependency of grey levels of neighboring pixels, in other words, it measures the similarity of the grey levels in neighboring pixels, tells how correlated a pixel is to its neighbor over the whole image. [2]
Range = [-1 1]. Correlation is 1 or -1 for a perfectly positively or negatively correlated image.

Formula

$$-\sum_{i,j}\frac{(i-\mu)(j-\mu)g(i,j)}{\sigma^2}$$

2.4 Inertia

It reflects the clarity of the image and texture groove depth. The contrast is proportional to the texture groove, high values of the groove produces more clarity, in contrast small values of the groove will result in small contrast and fuzzy image. [2]

Formula

$$\sum_{i,j}((i-j)^2 g(i,j))$$

2.5 Cluster Shade

Cluster analysis or **clustering** is the task of grouping a set of objects in such a way that objects in the same group (**cluster**) are more similar (in some sense or another) to each other than to those in other groups (clusters). It is a common technique for statistical data analysis.

Cluster Shade in clustered shading, we group similar view samples according to their position and, optionally, normal into clusters.
In AK Software we have the 36 parameters related to Cluster analysis, first we describe the 18 related to Cluster Shade.
(**ClusterShade_AllDirection_offset1, ClusterShade_AllDirection_offset1_SD,**
ClusterShade_angle0_offset1, ClusterShade_angle45_offset1,
ClusterShade_angle90_offset1, ClusterShade_angle135_offset1,
ClusterShade_AllDirection_offset4, ClusterShade_AllDirection_offset4_SD,
ClusterShade_angle0_offset4, ClusterShade_angle45_offset4,
ClusterShade_angle90_offset4, ClusterShade_angle135_offset4,
ClusterShade_AllDirection_offset7, ClusterShade_AllDirection_offset7_SD,
ClusterShade_angle0_offset7, ClusterShade_angle45_offset7,
ClusterShade_angle90_offset7, ClusterShade_angle135_offset7).

Formula

$$\sum_{i,j}((i-\mu)+(j-\mu))^3 g(i,j)$$

2.6 Cluster Prominence

Cluster Prominence is a measure of asymmetry of a given distribution, high values of this feature indicate that the symmetry of the image is low, in medical imaging low values of cluster prominence represent a smaller peak for the image grey level value and usually the grey level difference between the forms is small.
(**ClusterProminence_AllDirection_offset1, ClusterProminence_AllDirection_offset1_SD,**
ClusterProminence_angle0_offset1,
ClusterProminence_angle45_offset1,
ClusterProminence_angle90_offset1,
ClusterProminence_angle135_offset1,
ClusterProminence_AllDirection_offset4,
ClusterProminence_AllDirection_offset4_SD,
ClusterProminence_angle0_offset4,
ClusterProminence_angle45_offset4,
ClusterProminence_angle90_offset4,
ClusterProminence_angle135_offset4,
ClusterProminence_AllDirection_offset7,
ClusterProminence_AllDirection_offset7_SD,
ClusterProminence_angle0_offset7,
ClusterProminence_angle45_offset7,
ClusterProminence_angle90_offset7,
ClusterProminence_angle135_offset7)

Formula

$$\sum_{i,j}((i-\mu)+(j-\mu))^4 g(i,j)$$

3. Form Factor Parameters

These group of features includes descriptors of the three-dimensional size and shape of the tumor region. Let in the following definitions V denote the volume and A the surface area of the volume of interest. We determined the following shape and size based features:

3.1 Sphericity:

$$sphericity = \frac{\pi^{\frac{1}{3}}(6V)^{\frac{2}{3}}}{A}$$

3.2 Surface area:

The surface area is calculated by triangulation (i.e. dividing the surface into connected triangles) and is defined as:

$$A = \sum_{i=1}^{N} \frac{1}{2} |a_i b_i \times a_i c_i|$$

3.3 Compactness 1:

$$compactness\ 1 = \frac{V}{\sqrt{\pi} A^{\frac{2}{3}}}$$

3.4 Compactness 2:

$$compactness\ 2 = 36\pi \frac{V^2}{A^3}$$

3.5 Maximum 3D diameter:

The maximum three-dimensional tumor diameter is measured as the largest pairwise Euclidean distance, between voxels on the surface of the tumor volume.

3.6 Spherical disproportion:

$$spherical\ disproportion = \frac{A}{4\pi R^2}$$

Where R is the radius of a sphere with the same volume as the tumor.

Where N is the total number of triangles covering the surface and a, b and c are edge vectors of the triangles.

3.7 Surfacetovolumeratio:

$$surface\ to\ volume\ ratio = \frac{A}{V}$$

3.8 Volume:

The volume (V) of the tumor is determined by counting the number of pixels in the tumor region and multiplying this value by the voxel size.

3.9 VolumeCC and VolumeMM.

The maximum 3D diameter, surface area and volume provide information on the size of the lesion. Measures of compactness, spherical disproportion, sphericity and the surface to volume ratio describe how spherical, rounded, or elongated the shape of the tumor is.

4. GLCM Parameters

The Grey level co-occurrence matrix (GLCM) $\mathbf{P}\,(\mathbf{i},\mathbf{j}|\boldsymbol{\theta},\mathbf{d})$ represents the joint probability of certain sets of pixels having certain grey-level values. It calculates how many times a pixel with grey-level **i** occurs jointly with another pixel having a grey value **j**. By varying the displacement vector **d** between each pair of pixels.

The advantage of the co-occurrence matrix calculations is that the co-occurring pairs of pixels can be spatially related in various orientations with reference to distance and angular spatial relationships, as on considering the relationship between two pixels at a time. As a result, the combination of grey levels and their positions are exhibited apparently. Therefore, it is defined as "A two dimensional histogram of gray levels for pair of pixels, which are separated by a fixed spatial relationship". However, the matrix is sensitive to rotation. With the change of different offsets define pixel relationships by varying directions.

The rotation angle of an offset: $0°, 45°, 90°, 135°$ and displacement vectors (distance to the neighbor pixel: 1, 2, 3 ...), different co-occurrence distributions from the same image of reference. GLCM of an image is computed using displacement vector d defined by its radius, (distance or count to the next adjacent neighbor preferably is equal to one) and rotational angles.

Example:

Image

(a)

10	11	0
7	0	10
10	0	8
10	0	10

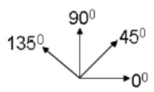
(b)

$\longrightarrow i$

Gray values	0	7	8	10	11
0	(0,0)	(0,7)	(0,8)	(0,10)	(0,11)
7	(7,0)	(7,7)	(7,8)	(7,10)	(7,11)
8	(8,0)	(8,7)	(8,8)	(8,10)	(8,11)
10	(10,0)	(10,7)	(10,8)	(10,10)	(10,11)
11	(0,0)	(0,7)	(0,8)	(0,10)	(0,11)

$\downarrow i$

(c)



(d)

0	1	1	4	1
1	0	0	0	0
1	0	0	0	0
4	0	0	0	1
1	0	0	1	0

(e)

2	0	1	3	0
0	0	0	0	1
1	0	0	0	0
3	0	0	0	0
0	1	0	0	0

(f)

Fig. 1 (a) test image; (b) test image intensity values in matrix form; (c) generalized fromof GLCM of test image; (d) roation offsets defines the pixel spatial relationships;(e) and (f) GLCMs of the image at an angle of 0^0 and 45^0.

4.1 Energy of GLCM

This feature Returns the sum of squared elements in the GLCM. Range = [0 1]
Energy is 1 for a constant image. Is high when image has very good homogeneity or when pixels are very similar The Property Energy is also known as uniformity, uniformity of energy, and angular second moment.

In AK Software we have 18 parameters related to the GLCM Energy : **GLCMEnergy_AllDirection_offset1**, **GLCMEnergy_AllDirection_offset1_SD**, **GLCMEnergy_angle0_offset1**, **GLCMEnergy_angle45_offset1**, **GLCMEnergy_angle90_offset1**, **GLCMEnergy_angle135_offset1**, **GLCMEnergy_AllDirection_offset4**, **GLCMEnergy_angle0_offset4**, **GLCMEnergy_angle45_offset4**, **GLCMEnergy_angle90_offset4**,

GLCMEnergy_angle135_offset4, GLCMEnergy_AllDirection_offset4_SD, GLCMEnergy_AllDirection_offset7, GLCMEnergy_angle0_offset7, GLCMEnergy_angle45_offset7, GLCMEnergy_angle90_offset7, GLCMEnergy_angle135_offset7, GLCMEnergy_AllDirection_offset7_SD

(Formula)

$$\sum_{i,j} g(i,j)^2$$

*g is a GLCM
Where i,j are the spatial coordinates of g (i,j).

4.2 Entropy of GLCM

Entropy is a measure of randomness of intensity image.
Entropy shows the amount of information of the image that is needed for the image compression. Entropy measures the loss of information or message in a transmitted signal and also measures the image information.
In AK Software we have the 18 parameters related to the GLCM Entropy (**GLCMEntropy_AllDirection_offset1, GLCMEntropy_AllDirection_offset1_SD, GLCMEntropy_angle0_offset1, GLCMEntropy_angle45_offset1, GLCMEntropy_angle90_offset1, GLCMEntropy_angle135_offset1, GLCMEntropy_AllDirection_offset4,GLCMEntropy_AllDirection_offset4_SD, GLCMEntropy_angle0_offset4, GLCMEntropy_angle45_offset4, GLCMEntropy_angle90_offset4, GLCMEntropy_angle135_offset4, GLCMEntropy_AllDirection_offset7, GLCMEntropy_AllDirection_offset7_SD, GLCMEntropy_angle0_offset7, GLCMEntropy_angle45_offset7, GLCMEntropy_angle90_offset7, GLCMEntropy_angle135_offset7)**

Formula

$$-\sum_{i,j} g(i,j)\log_2(i,j)$$

4.3 Inertia of GLCM

It reflects the clarity of the image and texture groove depth. The contrast is proportional to the texture groove, high values of the groove produces more clarity, in contrast small values of the groove will result in small contrast and fuzzy image.

In AK Software we have the 18 parameters related to the Inertia (**Inertia_AllDirection_offset1, Inertia _AllDirection_offset1_SD, Inertia _angle0_offset1, Inertia _angle45_offset1, Inertia _angle90_offset1, Inertia _angle135_offset1, Inertia _AllDirection_offset4, Inertia _AllDirection_offset4_SD, Inertia _angle0_offset4, Inertia _angle45_offset4, Inertia _angle90_offset4, Inertia _angle135_offset4, Inertia _AllDirection_offset7, Inertia _AllDirection_offset7_SD, Inertia _angle0_offset7, Inertia _angle45_offset7, Inertia _angle90_offset7, Inertia _angle135_offset7)**

Formula

$$\sum_{\boldsymbol{i,j}} ((\boldsymbol{i}-\boldsymbol{j})^2 \boldsymbol{g}(\boldsymbol{i},\boldsymbol{j}))$$

4.4 Correlation

Image-based
Correlation measures the similarity of the grey levels in neighboring pixels, tells how correlated a pixel is to its neighbor over the whole image.
Range = [-1 1]. Correlation is 1 or -1 for a perfectly positively or negatively correlated image.

In AK Software we have the 18 parameters related to the Correlation
(**Correlation_AllDirection_offset1, Correlation _AllDirection_offset1_SD,**
Correlation _angle0_offset1, Correlation _angle45_offset1,
Correlation _angle90_offset1, Correlation _angle135_offset1,
Correlation _AllDirection_offset4, Correlation _AllDirection_offset4_SD,
Correlation _angle0_offset4, Correlation _angle45_offset4,
Correlation _angle90_offset4, Correlation _angle135_offset4,
Correlation _AllDirection_offset7, Correlation _AllDirection_offset7_SD,
Correlation _angle0_offset7, Correlation _angle45_offset7,
Correlation _angle90_offset7, Correlation _angle135_offset7)

Formula

$$-\sum_{i,j}\frac{(i-\mu)(j-\mu)g(i,j)}{\sigma^2}$$

4.5 Inverse Difference Moment

Inverse Difference Moment (IDM) is the local homogeneity. It is high when local gray level is uniform and inverse GLCM is high. IDM weight value is the inverse of the Contrast weight.

Formula

$$\sum\sum\frac{\mathbf{1}}{\mathbf{1}+(\mathbf{i}-\mathbf{j})^{\mathbf{2}}}\mathbf{g}(\mathbf{i},\mathbf{j})$$

4.6 Haralick features

$P_{ij} = P(i,j)$ = matrix of relative frequencies with which two neighboring resolution cells separated by distance d occur on the image,
$p(i,j) = P(i,j)/R$ = (i,j)th entry in a normalized gray-tone spatial dependence matrix,
N_g = number of distinct gray levels in the quantized image (the *EBImage haralick.nbins* parameter in the function *computerFeatures.haralick* defaults to 32 gray levels),
R = normalizing constant = number of neighboring resolution cell pairs used in computing a particular gray-tone spatial-dependence matrix.

$R = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} P(i,j)$ = sum of all elements of co-occurrence frequency matrix

$p(i,j) = \frac{P(i,j)}{R}$ = co-occurence probability matrix

$p_x(i) = \sum_{j=1}^{N_g} p(i,j)$ = i-th entry in the marginal-probability matrix obtained by summing the rows of $p(i,j)$.

$p_y(j) = \sum_{i=1}^{N_g} p(i,j)$ = j-th entry in the marginal-probability matrix obtained by summing the columns of $p(i,j)$.

$$p_{x+y}(k) = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} \delta_{i+j,k} p(i,j), \qquad k = 2, 3, \ldots, 2N_g$$

$$p_{x-y}(k) = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} \delta_{|i-j|,k} p(i,j), \qquad k = 0, 1, \ldots, N_g - 1$$

4.6.1 Haralick Correlation

Measures the degree of similarity of the gray level of the image in the row or column direction. Represents the local grey level correlation, the greater its value, the greater the correlation;

(**HaralickCorrelation_AllDirection_offset1**, **HaralickCorrelation_AllDirection_offset1_SD**,
HaralickCorrelation_angle0_offset1,
HaralickCorrelation_angle45_offset1,
HaralickCorrelation_angle90_offset1,
HaralickCorrelation_angle135_offset1,
HaralickCorrelation_AllDirection_offset4,
HaralickCorrelation_AllDirection_offset4_SD,
HaralickCorrelation_angle0_offset4,
HaralickCorrelation_angle45_offset4,
HaralickCorrelation_angle90_offset4,
HaralickCorrelation_angle135_offset4,
HaralickCorrelation_AllDirection_offset7,
HaralickCorrelation_AllDirection_offset7_SD,
HaralickCorrelation_angle0_offset7,
HaralickCorrelation_angle45_offset7,
HaralickCorrelation_angle90_offset7,
HaralickCorrelation_angle135_offset7)

Formula

$$-\sum_{i,j} \frac{(i,j)g(i,j) - \mu_t^2}{\sigma_t^2}$$

* where μ_t and σ_t are the mean and standard deviation of the row (or column, due to symmetry) sums.

4.6.2 Angular Second Moment

$$f_1 = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} \left(\frac{P(i,j)}{R}\right)^2 = \sum_i \sum_j p(i,j)^2$$

4.6.3 Contrast

The contrast feature, is a difference moment of the P matrix and is a measure of the contrast or the amount of local variations present in the image.

$$f_2 = \sum_{k=0}^{N_g-1} k^2 \left\{ \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} \delta_{|i-j|,k} p(i,j) \right\} = \sum_{k=0}^{N_g-1} k^2 p_{x-y}(k)$$

4.6.4 Haralick Entropy

$$f_9 = -\sum_{i=1}^{N_g} \sum_{j=1}^{N_g} p(i,j) \log(p(i,j))$$

4.6.5 HaraVariance

$$f_4 = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} (i-\mu)^2 p(i,j)$$

4.6.6 sumAverage

$$f_6 = \sum_{i=2}^{2N_g} i p_{x+y}(i)$$

4.6.7 sumVariance

$$f_7 = \sum_{i=2}^{2N_g} (i - f_8)^2 p_{x+y}(i)$$

4.6.8 sumEntropy

$$f_8 = -\sum_{i=2}^{2N_g} p_{x+y}(i) \log(p_{x+y}(i))$$

4.6.9 differenceVariance

$$f_{10} = \text{variance of } p_{x-y}$$

4.6.10 differenceEntropy

$$f_{11} = -\sum_{i=0}^{N_g-1} p_{x-y}(i) \log(p_{x-y}(i))$$

4.6.11 inverseDifferenceMoment

$$f_5 = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} \frac{1}{1 + (i - j)^2} p(i,j)$$

5. RLM Parameters

The grey level run-length matrix (RLM) $\mathbf{P_r(i, j \mid \theta)}$ is defined as the numbers of runs with pixels of gray level *i* and run length *j* for a given direction θ. RLMs is generated for each sample image segment having directions (0°,45°,90° &135°), then the following ten statistical features were derived: short run emphasis, long run emphasis, grey level non-uniformity, run length non-uniformity, Low Grey Level Run Emphasis, High Grey Level Run Emphasis, Short Run Low Grey Level Emphasis, Short Run High Grey Level Emphasis, Long Run Low Grey Level Emphasis and Long Run High Grey Level Emphasis.

5.1 Short Run Emphasis (18 Parameters)

ShortRunEmphasis_AllDirection_offset1,
ShortRunEmphasis_AllDirection_offset1_SD,
ShortRunEmphasis_angle0_offset1,
ShortRunEmphasis_angle45_offset1,
ShortRunEmphasis_angle90_offset1,
ShortRunEmphasis_angle135_offset1,
ShortRunEmphasis_AllDirection_offset4,
ShortRunEmphasis_AllDirection_offset4_SD,
ShortRunEmphasis_angle0_offset4,
ShortRunEmphasis_angle45_offset4,
ShortRunEmphasis_angle90_offset4,
ShortRunEmphasis_angle135_offset4,
ShortRunEmphasis_AllDirection_offset7,
ShortRunEmphasis_AllDirection_offset7_SD,
ShortRunEmphasis_angle0_offset7,
ShortRunEmphasis_angle45_offset7,
ShortRunEmphasis_angle90_offset7,
ShortRunEmphasis_angle135_offset7

Formula:

$$SRE(\theta) = \frac{1}{n_r} \sum_{i=1}^{M} \sum_{j=1}^{N} \frac{p(i, j, \theta)}{j^2}$$

5.2 Long Run Emphasis (18Parameters)

$$LRE(\theta) = \frac{1}{n_r} \sum_{i=1}^{M} \sum_{j=1}^{N} p(i, j, \theta)\, j^2$$

5.3 Grey Level Non-uniformity(18Parameters)

$$GLN(\theta) = \frac{1}{n_r} \sum_{i=1}^{M} \left(\sum_{j=1}^{N} p(i, j, \theta)\right)^2$$

5.4 Run Length Non-uniformity(18Parameters)

$$RLN(\theta) = \frac{1}{n_r} \sum_{j=i}^{N} \left(\sum_{i=1}^{M} p(i, j, \theta)\right)^2$$

5.5 Low Grey Level Run Emphasis(18Parameters)

$$LGRE(\theta) = \frac{1}{n_r} \sum_{j=i}^{N} \sum_{i=1}^{M} \frac{p(i, j, \theta)}{i^2}$$

5.6 High Grey Level Run Emphasis(18Parameters)

$$HGRE(\theta)=\frac{1}{n_r}\sum_{j=i}^{N}\sum_{i=1}^{M}p(i,j,\theta)i^2$$

5.7 Short Run Low Grey Level Emphasis(18Parameters)

$$SRLGE(\theta)=\frac{1}{n_r}\sum_{j=i}^{N}\sum_{i=1}^{M}\frac{p(i,j,\theta)}{i^2 j^2}$$

5.8 Short Run High Grey Level Emphasis(18Parameters)

$$SRHGE(\theta)=\frac{1}{n_r}\sum_{j=i}^{N}\sum_{i=1}^{M}\frac{p(i,j,\theta)i^2}{j^2}$$

5.9 Long Run Low Grey Level Emphasis(18Parameters)

$$LRLGE(\theta)=\frac{1}{n_r}\sum_{j=i}^{N}\sum_{i=1}^{M}\frac{p(i,j,\theta)j^2}{i^2}$$

5.10 Long Run High Grey Level Emphasis(18Parameters)

$$LRHGE(\theta)=\frac{1}{n_r}\sum_{j=i}^{N}\sum_{i=1}^{M}p(i,j,\theta)i^2 j^2$$

where n_r is the total number of runs and n_p is the number of pixels in the image.

6. GLZSM Parameters

The gray level Size Zone Matrix (SZM) is the starting point of Thibault matrices.[8]
For a texture image f with N gray levels, it is denoted GSf(s, g) and provides a statistical representation by the estimation of a bivariate conditional probability density function of the image distribution values. It is calculated according to the pioneering Run Length Matrix principle: the value of the matrix GSf(s, g) is equal to the number of zones of size s and of gray level g. The resulting matrix has a fixed number of lines equal to N, the number of gray levels, and a dynamic number of columns, determined by the size of the largest zone as well as the size quantization.
The more homogeneous the texture, the wider and flatter the matrix. SZM does not required computation in several directions, contrary to RLM and co-occurrences matrix (COM). However, it has been empirically proved that the degree of gray level quantization still has an important impact on the texture classification performance. For a general application it is usually required to test several gray level quantization in order to find the optimal one with respect to a training dataset. Empirically, 32 provides often the best result.
More precisely, this matrix is particularly efficient to characterize the texture homogeneity, non periodicity or speckle like texture; it had provided betters characterizations than granulometry (or COM, RLM, etc.) for the classification of cell nuclei, dermis, road quality (bitumen condition) and some textures in PET images.
Two examples of matrix filling for textures 4x4 with four gray levels.

1	2	3	4
1	3	4	4
3	2	2	2
4	1	4	1

Level	Size zone, s		
g	1	2	3
1	2	1	0
2	1	0	1
3	0	0	1
4	2	0	1

Texture

1	1	3	4
1	3	4	4
3	2	4	4
3	2	1	1

$\Longrightarrow$

Gray level	Size zone (j)				
i	1	2	3	4	5
1	0	1	1	0	0
2	0	1	0	0	0
3	0	0	0	1	0
4	0	0	0	0	1

Let P define the GLSZM of a quantized volume V (x,y,z) with isotropic voxel size. P(i,j) represents the number of 3D zones of gray-levels i and of size j in V , Ng represents the pre-defined number of quantized gray-levels set in V , and Lzrepresents the size of the largest zone (of any gray-level) in V . One GLSZM of size Ng× Lz is computed per volume V by adding up all possible largest zone-sizes, with zones constructed from 26-connected neighbours of the same gray-level in 3D space (one voxel can be part of only one zone). The entry (i,j) of the normalized GLSZM is then defined as:

$$\mathrm{p(i,j)} = \frac{P(i,j)}{\sum_{i=1}^{Ng}\sum_{j=1}^{Lz} P(i,j)}$$

The following quantities are also defined:

$$\mu_i = \sum_{i=1}^{Ng} i \sum_{j=1}^{Lz} p(i,j)$$

$$\mu_j = \sum_{j=1}^{Lz} j \sum_{i=1}^{Ng} p(i,j)$$

6.1 Small Zone Emphasis[9,12]

$$\mathrm{SZE} = \sum_i \sum_j \frac{p(i,j)}{j^2}$$

6.2 Large Zone Emphasis[9,12]

$$\mathrm{LZE} = \sum_i \sum_j j^2 p(i,j)$$

6.3 Gray-Level Nonuniformity[9,12]

$$\mathrm{GLN} = \sum_i \left(\sum_j p(i,j)\right)^2$$

6.4 Zone-Size Nonuniformity[9,12]

$$\mathrm{ZSN} = \sum_j \left(\sum_i p(i,j)\right)^2$$

6.5 Zone Percentage[9,12]

$$\mathrm{ZP} = \sum_i \sum_j \frac{\sum_i \sum_j p(i,j)}{\sum_j j \sum_i p(i,j)}$$

6.6 Low Gray-Level Zone Emphasis[10,12]

$$\mathrm{LGZE} = \sum_i \sum_j \frac{p(i,j)}{i^2}$$

6.7 High Gray-Level Zone Emphasis[10,12]

$$\mathrm{HGZE} = \sum_i \sum_j i^2 p(i,j)$$

6.8 Small Zone Low Gray-Level Emphasis[11,12]

$$\text{SZLGE} = \sum_i \sum_j \frac{p(i,j)}{i^2 j^2}$$

6.9 Small Zone High Gray-Level Emphasis[11,12]

$$\text{SZLGE} = \sum_i \sum_j \frac{j^2 p(i,j)}{i^2}$$

6.10 Large Zone Low Gray-Level Emphasis[11,12]

$$\text{LZLGE} = \sum_i \sum_j \frac{j^2 p(i,j)}{i^2}$$

6.11 Large Zone High Gray-Level Emphasis[11,12]

$$\text{LZHGE} = \sum_i \sum_j i^2 j^2 p(i,j)$$

6.12 Gray-Level Variance[12]

$$\text{GLV} = \frac{1}{Ng \times Lz} \sum_i \sum_j (ip(i,j) - \mu_i)^2$$

6.13 Zone-Size Variance[12]

$$\text{ZSV} = \frac{1}{Ng \times Lz} \sum_i \sum_j (jp(i,j) - \mu_j)^2$$

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