ON THE CONTROL OF PSYCHOLOGICAL NETWORKS - SUPPLEMENTAL MATERIALS

1. Network Model Estimation

Network models were estimated using the `mlVAR` R package (Epskamp, Deserno, & Bringmann, 2019). This approach fits a series of multi-level models to subjects' *integrated* symptom timeseries, each model predicting change in a symptom by all other changes in symptoms, with a random subject-level slope specified for each predictor. The fixed effect β coefficients from these regressions are then used as the edge weights in the *group-level networks*. To obtain subject specific networks, the subject specific random effects were added to the group-level fixed effects. Additionally, a single dichotomous treatment predictor was specified for each regression. This was 0 when subjects were not undergoing treatment, and 1 when subjects were undergoing treatment. No random slope of the treatment effect was specified.

2. Integrated VAR Models

The approach to estimating the network models outlined above corresponds to the use of a multi-level *integrated* vector autoregressive model (VARI), specified with an integration of 1 and a lag of 1. The choice to use a VARI model instead of a level VAR model was motivated by the need for the dynamical system to be non-stationary, and more specifically *unit-root*. A integrated VAR model is assumed to be stationary with respect to the relation of changes in symptoms, but forces a unit-root process on the level of the symptom values themselves.

The most important property of a unit-root process, and the one that effectively controlling a system relies on, is that a unit-root process is not mean-reverting. Put another way, when a unit-root process experiences a disturbance, either in the form of a control input or an unmodelled shock to the system, the changes induced by the disturbance are permanent. Contrast this to the behavior of a stationary process, in which any effect of a disturbance will eventually be erased, and the expected value of the process will revert back to the *a priori* expected value.

This unit-root vs. stationary process issue has major implications for the use of control theory methods. In a unit-root process, control inputs can, if the system is controllable, drive the system to a desired input in a finite amount of time for a finite cost. However, in a stationary system, controlling the system would require an infinite amount of time (and correspondingly, infinite cost), as the control inputs would have to be applied permanently for the effect of the control to remain indefinitely.

A VARI(1,1) model is of the following form:

$$\Delta x_{t+1} = \mathbf{A}\Delta x_t + \varepsilon_{t+1},\ \varepsilon_{t+1} \sim MVN(\mathbf{0}, \Sigma) \tag{1}$$

where $\Delta x_t = x_t - x_{t-1}$ is the vector of symptom measurements at time t, A is the matrix of lag-1 relations ε is white (multivariate normal, uncorrelated in time) noise, and Σ is the covariance matrix of ε, representing contemporaneous relations between symptoms. By substituting back in the expression for $\Delta \mathbf{x}_t$, the VARI(1,1) model can be transformed into a level VAR of lag-2 (VAR(2))

$$x_{t+1} = (\mathbf{I} + \mathbf{A})x_t - \mathbf{A}x_{t-1} + \varepsilon_{t+1},\ \varepsilon_{t+1} \sim MVN(\mathbf{0}, \Sigma) \tag{2}$$

where $\mathbf{I}$ is the identity matrix. Finally, this VAR(2) model is transformed into the corresponding VAR(1) model:

$$\begin{bmatrix} x_{t+1} \\ x_t \end{bmatrix} = \begin{bmatrix} (\mathbf{I} + \mathbf{A}) & -\mathbf{A} \\ \mathbf{I} & 0 \end{bmatrix} \begin{bmatrix} x_t \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{t+1} \\ 0 \end{bmatrix} \tag{3}$$

This reduced form expression of our original VARI(1,1) model as a VAR(1) model allows us to use the discrete linear time invariant system framework described in the main text. However, several of the control matrices must be correctly specified. In the case of our complicated grief example with 10 symptoms, the combined dynamics matrix

$$\mathbf{A}^* = \begin{bmatrix} (\mathbf{I} + \mathbf{A}) & -\mathbf{A} \\ \mathbf{I} & 0 \end{bmatrix} \tag{4}$$

is a 20×20 matrix, rather than a 10×10 matrix. In order to specify the control inputs as only impacting x_t, and for the cost function to only weight x_{t+1} the following control matrices were used:

$$\mathbf{S}^* = \mathbf{Q}^* = \begin{bmatrix} \mathbf{S} & \mathbf{0}_{10\times10} \\ \mathbf{0}_{10\times10} & \mathbf{0}_{10\times10} \end{bmatrix} \tag{5}$$

and

$$\mathbf{B}^* = \begin{bmatrix} \mathbf{B} \\ 0_{10\times q} \end{bmatrix} \tag{6}$$

where q is the number of interventions modelled.

3. Absolute Out-Strength Ranking of Symptoms

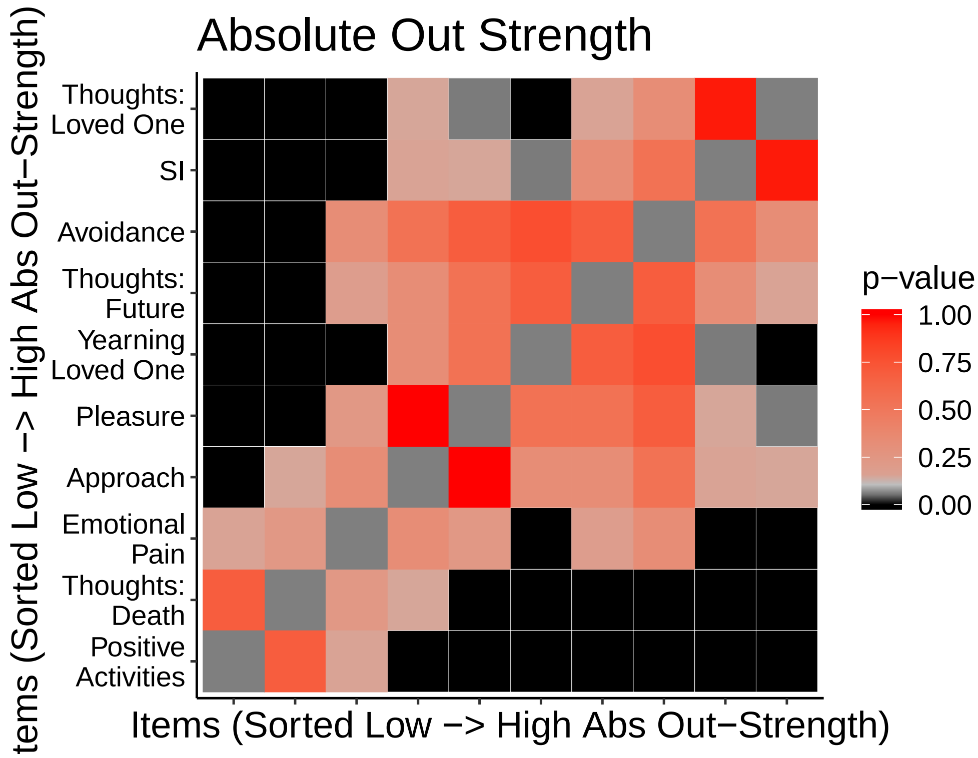

Figure 1.
Pairwise Rank Tests for Absolute Out-Strength. Symptoms are sorted in ascending order. Tile colors represent the p-value of the associated Wilcoxian signed-rank sum test (after FDR correction). Black: $p < .05$, Grey: $.05 \leq p \geq .1$, Red: $p > .1$.

The analysis of the ranking of absolute outstrength indicates that while the overall ordering of absolute out-strength and average controllability are comparable, absolute outstrength shows substantially more variability in ranking for the highest ranked symptoms. For example, while both Thoughts: Loved Ones and SI are ranked highest, they are not distinguishable from many other symptoms. Whereas in average, and to a lesser extent, modal controllability, these symptoms are more distinguishable in terms of ranking.

References

Epskamp, S., Deserno, M. K., & Bringmann, L. F. (2019). *mlVAR: Multi-level vector autoregression.* Retrieved from `https://CRAN.R-project.org/package=mlVAR` (R package version 0.4.3)