1. Immune Correlates Analysis Using Vaccinees from Test Negative Designs Supplementary Materials

1.1 *Derivation of $P(X|W)$ and accuracy of regression calibration*

In this section we derive $P(D=1|w, Z=1)$ and compare it to the regression calibration approach. We are interested in the effect of the immune response of interest X on disease Y, but only see W. We assume that the probability of being a case (e.g. Ebola virus disease) for a vaccinee is given by

$$P(Y=1|X, Z=1) = \frac{\exp(\beta_0 + \beta_1 X)}{1 + \exp(\beta_0 + \beta_1 X)}$$

and the probability of being a control (e.g. non-Ebola virus disease) is given by

$$P(Y=0|X, Z=1) = \left(1 - \frac{\exp(\beta_0 + \beta_1 X)}{1 + \exp(\beta_0 + \beta_1 X)}\right)\pi.$$

The probabilities of arriving at the clinic are $P(A=1|Y=1, Z=1) = \theta_1, P(A=1|Y=0, Z=1) = \theta_0$, for cases and controls, respectively. Define $D=1$ if $Y=1 \cap A=1$ and $D=0$ if $Y=0 \cap A=1$. We have

$$P(D=1|X, Z=1) = P(Y=1 \cap A=1|X, Z=1) = \frac{\exp(\beta_0 + \beta_1 X)}{1 + \exp(\beta_0 + \beta_1 X)}\theta_1,$$

while the chance of a control arriving is

$$P(D=0|X, Z=1) = P(Y=0 \cap A=1|X, Z=1) = \left(1 - \frac{\exp(\beta_0 + \beta_1 X)}{1 + \exp(\beta_0 + \beta_1 X)}\right)\pi\theta_0.$$

We only see vaccinees who arrive, their disease status ($D=0$ or 1) and we measure W. The probability that a vaccinee with covariate W who arrives is a case is given by

$$\begin{aligned}
&P(D=1|W, D=1\cup D=0, Z=1)\\
&=\int P(D=1, X=x|W, D=1\cup D=0, Z=1)dx\\
&=\int \frac{P(D=1, X=x, W=w, Z=1)}{P(W, D=1\cup D=0, Z=1)}dx\\
&=\frac{\int \frac{\theta_1\exp(\beta_0+\beta_1 x)}{1+\exp(\beta_0+\beta_1 x)}\phi(x;\mu(w),\tau^2)\phi(w;\mu,\sigma^2)dx}{\int \frac{\pi\theta_0+\theta_1\exp(\beta_0+\beta_1 x)}{1+\exp(\beta_0+\beta_1 x)}\phi(x;\mu(w),\tau^2)\phi(w;\mu,\sigma^2)dx}\\
&=\frac{\int \frac{\theta_1\exp(\beta_0+\beta_1 x)}{1+\exp(\beta_0+\beta_1 x)}\phi(x;\mu(w),\tau^2)dx}{\int \frac{\pi\theta_0+\theta_1\exp(\beta_0+\beta_1 x)}{1+\exp(\beta_0+\beta_1 x)}\phi(x;\mu(w),\tau^2)dx}\\
&=\frac{\int \frac{\exp(\beta_0+\beta_1 x)}{1+\exp(\beta_0+\beta_1 x)}\phi(x;\mu(w),\tau^2)dx}{\int \frac{\pi\theta_0/\theta_1+\exp(\beta_0+\beta_1 x)}{1+\exp(\beta_0+\beta_1 x)}\phi(x;\mu(w),\tau^2)dx}\\
&=\frac{\int \frac{\exp(\beta_0+\beta_1 x)}{1+\exp(\beta_0+\beta_1 x)}\phi(x;\mu(w),\tau^2)dx}{\int \frac{\pi^*+\exp(\beta_0+\beta_1 x)}{1+\exp(\beta_0+\beta_1 x)}\phi(x;\mu(w),\tau^2)dx}\\
&=\frac{E_{X|w}(\frac{\exp(\beta_0-\log(\pi^*)+\beta_1 X)}{1+\exp(\beta_0+\beta_1 X)})}{E_{X|w}(\frac{1}{1+\exp(\beta_0+\beta_1 X)})+E_{X|w}(\frac{\exp(\beta_0-\log(\pi^*)+\beta_1 X)}{1+\exp(\beta_0+\beta_1 X)})}\\
&=p(w;\beta_0,\beta_1,\pi^*),
\end{aligned}$$

where $\phi(x;\mu(x),\tau^2)$ is the conditional distribution of $X|w$, $\phi(w;\mu,\sigma^2)$ the marginal distribution of W, $\pi^*=\pi\theta_0/\theta_1$, and the expectation is with respect to the conditional distribution of $X|w$.

To evaluate the accuracy of regression calibration, we assume that X, W are bivariate normal with common mean 3.00 and common standard deviation 0.50 which allows us to evaluate $E_{X|w}()$. Figure 1 graphs the true curve, $p(w;\beta_0,\beta_1,\pi^*)$, vs the regression calibration curve, $\exp(\beta_0^{**}+\beta_1 E(X|w))/[1+\exp\{\beta_0^{**}+\beta_1 E(X|w)\}]$, where β_0^{**} best approximates $p(w;\beta_0,\beta_1,\pi^*)$ for $\rho=0.90$ and 0.20. Clearly, regression calibration provides an excellent approximation for both values of ρ.

We note that if $\rho=1$ so that $W=X$ then

$$p(w;\beta_0,\beta_1,\pi^*)=\frac{\exp\{\beta_0-\log(\pi^*)+\beta_1 x\}}{1+\exp\{\beta_0-\log(\pi^*)+\beta_1 x\}}$$

and β_0 cannot be determined. Thus the model is unidentifiable for $\rho=1$. Since it is hard to see how an unidentifiable model can be made identifiable by introducing noise, i.e. replacing X with $W|X$, the model seems unidentifiable in general. Admittedly, this is not really a proof of unidentifiability.

We also note that if $\theta_1 = \theta_0 = \pi = 1.00$ and $\pi^* = 1$ we obtain $p(w; \beta_0, \beta_1, 1)$ which becomes

$$P(D = 1|W, D = 1 \cup D = 0, Z = 1) = E_{X|w}\left(\frac{\exp(\beta_0 + \beta_1 X)}{1 + \exp(\beta_0 + \beta_1 X)}\right).$$

This corresponds to an (identifiable) prospective model for $P(Y = 1|W, Z = 1)$ with no missing data.

1.2 *Graphical Representation of the six scenarios*

To help understand the scenarios, Figure 2 provides the hazards for the case disease (solid curves) at $X(t) = 2.5$ and $X(t) = 4.0$ and for the control disease (dashed curves) as a function of time since study start. Scenarios 3-6 have non-constant and dissimilar hazards for the case and control diseases. Also graphed, as tics at $D = 0$ and 1, are the arrival times of the symptomatic vaccinees for the last simulated dataset. For scenario six, $\lambda_1(t)/\lambda_0(t)$ at $t/T = .90$ divided by the same ratio at $t/T = .10$ is 1/9. Thus during follow-up there is an extreme decrease in the number of cases relative to controls as in an imploding outbreak. The analogous ratios for scenarios 3, 4, and 5 are 3, 1/3, and 9, respectively.

1.3 *Additional simulations including the maximum likelihood estimator*

In this subsection, we provide a more extensive set of simulations to evaluate different methods of estimation. We use the setup of section 3 but evaluate sample sizes 1/10th, 1, and 10 times that of section 3. We evaluate the three approaches described in section 3 as well as two maximum likelihood estimators which we explain below.

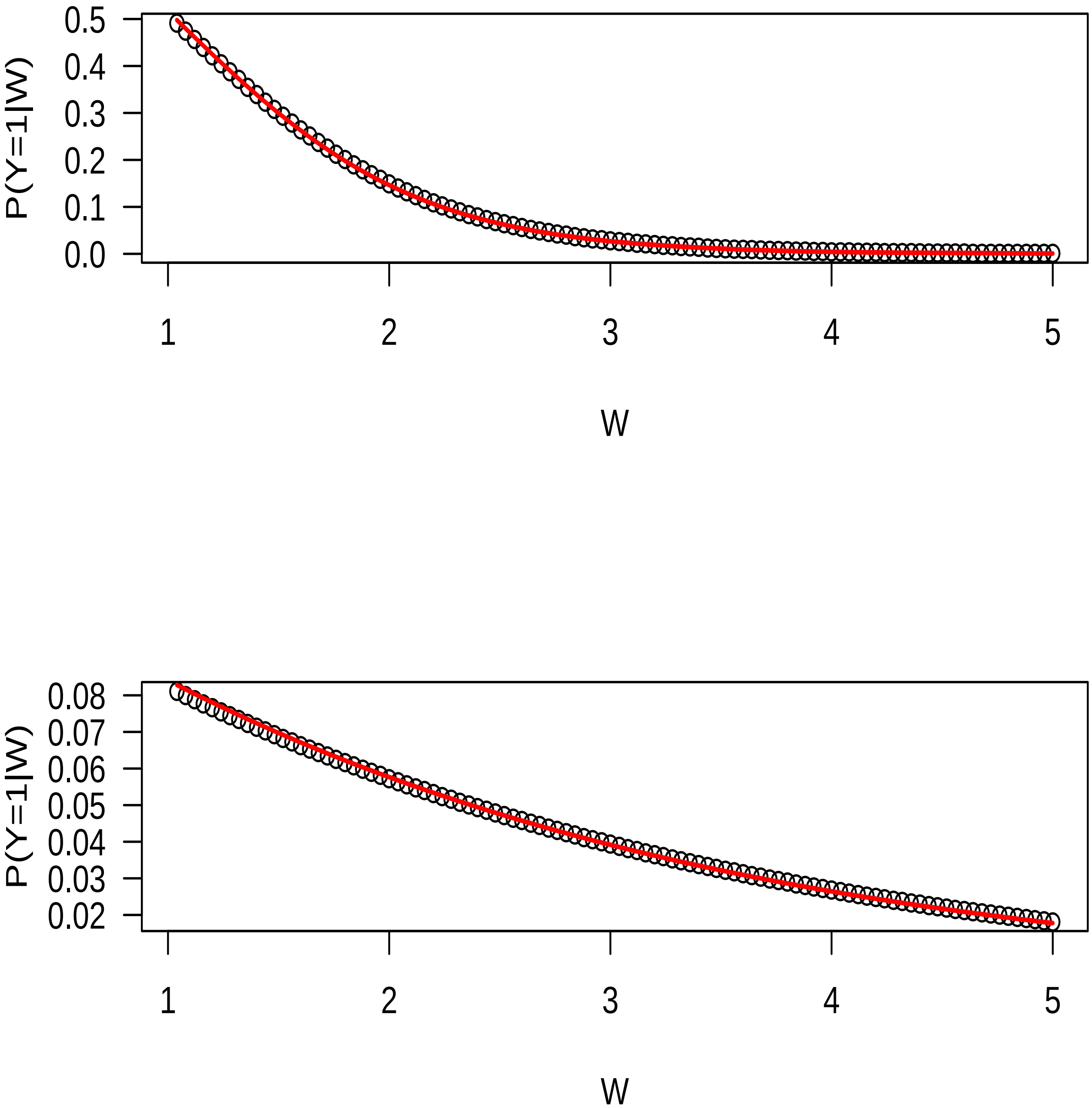

Fig. 1. The true risk curve $P(D=1|w) = p(w;\beta_0,\beta_1,\pi^*)$ (black dots) and regression calibration curve, logit $p(D=1|w)) = \beta_0^{**} + \beta_1 E(X|w)$ (red line) curves. Here $(\beta_0,\beta_1,\pi^*) = (-2.11,-2.03,0.10)$ and β_0^{**} is the best fitting (by eye) curve to $P(D=1|w)$. We evaluate $\rho = 0.90$ (top panel) and $\rho = 0.20$ (bottom panel). Here W,X are bivariate normal with common mean 3 and common standard deviation .5.

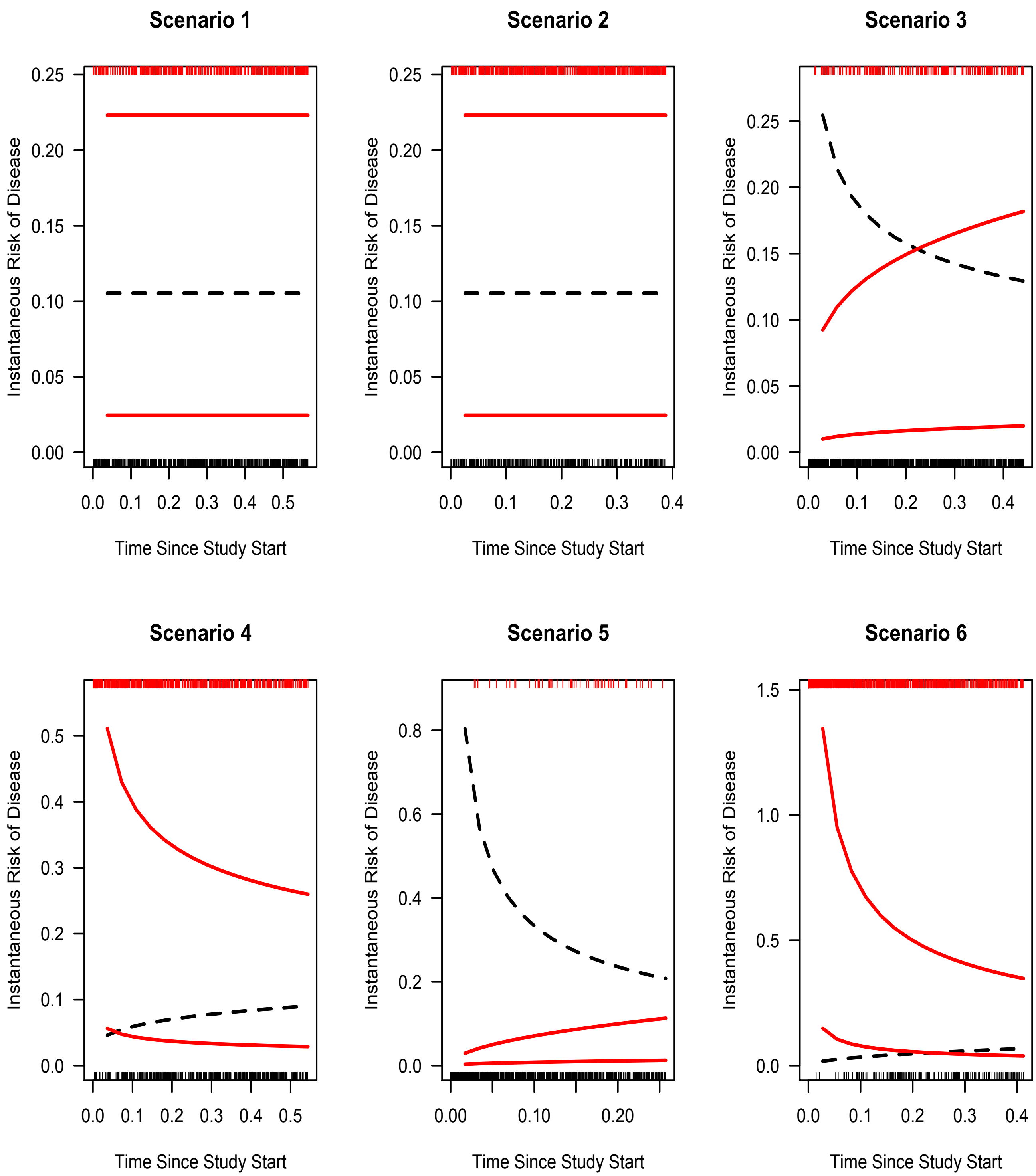

Fig. 2. Hazards of case disease at $X(t) = 4.0$ and $X(t) = 2.5$ (solid curves) and control disease (dashed curve) for the six scenarios. The first two scenarios have $(\alpha_0, \alpha_1) = (1, 1)$ while the last four have α_0, α_1=(1.25,0.75), (0.75,1.25), (1.50,0.50) and (0.50,1.50). The arrival times are given by tics, for cases (D=1) and controls (D=0), for the last simulated dataset of each scenario. Red denotes the case disease while black denotes the control disease.

In the previous subsection we showed that

$$P(D = 1|w, Z = 1) = p(w; \beta_0, \beta_1, \pi^*)$$

The proper likelihood based on data $(D_i, W_i), i = 1, \ldots, n$ is therefore

$$L(\beta_0, \beta_1, \pi^*) = \prod_i p(W_i; \beta_0, \beta_1, \pi^*)^{D_i}\{1 - p(W_i; \beta_0, \beta_1, \pi^*)\}^{1-D_i}.$$

We consider two mles, the first maximizes $L(\beta_0, \beta_1, \pi^*)$ while the second sets π^* to its true value of 0.01 and maximizes $L(\beta_0, \beta_1, 0.01)$. We do this as a reference approach because when $\rho = 1$, $p(w; \beta_0, \beta_1, \pi^*)$ is only identifiable for β_1.

As for regression calibration, we discard the known Xs in the controls for whom X is known. If one uses actual the Xs for the controls, the mle of β_1 is substantially biased towards zero. This makes sense as we observed substantial bias towards zero of the estimate of β_1 when using X in the controls for regression calibration and as shown in the previous section, the derived $P(Y|w)$ is virtually identical to the regression calibration $P(Y)$ for the parameters used in our simulations.

Interestingly, when we set $\pi = 1$ and $\theta_1 = \theta_0 = 1$ and use $\pi^* = 1$ as known, we have defined a standard prospective study with no missing data. In this case, at least in limited simulations, the asymmetric use of $P(Y = 1|X, Z = 1) = \exp(\beta_0 + \beta_1 X)/\{1 + \exp(\beta_0 + \beta_1 X)\}$ in controls, and $P(Y = 1|W, Z = 1) = p(w; \beta_0, \beta_1, 1)$ in cases does not produce a biased estimate and has similar performance to regression calibration.

As in section 3, we generate X, W as bivariate normal with mean 3.00, standard deviation 0.50 and set $\rho = 0.40$, 0.70, and 1.00. We generate $Y = 0, 1, 2$ for both vaccinated subjects (Z=1) and unvaccinated subjects ($Z = 0$) as

$$P(Y = 1|Z, X) = \frac{\exp\{\beta_0 + \beta_1 X + \beta_2(1 - Z)\}}{1 + \exp\{\beta_0 + \beta_1 X + \beta_2(1 - Z)\}}$$

and generate $Y = 0$ as

$$P(Y = 0|Z, X) = \left(1 - \frac{\exp\{\beta_0 + \beta_1 X + \beta_2(1 - Z)\}}{1 + \exp\{\beta_0 + \beta_1 X + \beta_2(1 - Z)\}}\right)\pi.$$

For the unvaccinated, we set $X = 0$. We set $(\beta_0, \beta_1, \beta_2)$ as (-5.51, 0.00, 0.92), (-4.05,-0.60, -0.54), (-1.12, -1.81, -3.47), and set $\pi = .01$. We set the arrival probabilities $P(A = 1|Z = 1, Y = 1) = P(A = 1|Z = 1, Y = 0) = P(A = 1|Z = 0, Y = 1) = P(A = 1|Z = 0, Y = 0) = 0.50$. We vary N as 15000, 150000, and 1500000.

In addition to the summaries we report in section 3, we additionally report the vaccine efficacy as a function of the quartiles of X. Under the assumptions of the test negative design, and the assumption that the counterfactual X that would've been realized, had the unvaccinated been vaccinated, has no effect on risk in the unvaccinated, the VE is given by

$$\mathrm{VE}_X(q) = 1 - \exp\{\beta_1 X(q) - \beta_2\},$$

where $X(q)$ is the x corresponding to the qth percentile of the distribution of X. We provide

$$\mathrm{VE}_V(\frac{o}{8})$$

for $V = X, \widehat{X}(W)$, and W, and for $o = 1$ and 7.

Table 1. Use of W as IR with n=15000, 150000, 1500000.

scen	β_1	ρ	imp	$\overline{\widehat{\beta}_1}$	$S^2(\widehat{\beta}_1)$	% Reject	$\overline{\text{VE}(\frac{1}{8})}$	$\overline{\text{VE}(\frac{7}{8})}$	$\overline{n}_V$	$\overline{n}_{VD}$	$\overline{n}_{VU}$
1	0.000	1.000	0.000	-0.002	0.217	0.048	0.584	0.588	105	30	149
2	-0.600	1.000	0.000	-0.612	0.291	0.177	0.578	0.781	97	22	148
3	-1.810	1.000	0.000	-1.906	0.524	0.753	0.562	0.941	90	16	149
4	0.000	0.700	0.000	0.003	0.213	0.052	0.581	0.585	104	30	149
5	-0.600	0.700	0.000	-0.442	0.288	0.123	0.611	0.759	97	22	149
6	-1.810	0.700	0.000	-1.345	0.454	0.483	0.617	0.907	90	16	149
7	0.000	0.400	0.000	0.021	0.211	0.049	0.587	0.579	105	30	148
8	-0.600	0.400	0.000	-0.268	0.273	0.084	0.639	0.732	97	22	149
9	-1.810	0.400	0.000	-0.765	0.413	0.207	0.681	0.858	90	15	149
1	0.000	1.000	0.000	0.001	0.020	0.061	0.598	0.597	1049	302	1493
2	-0.600	1.000	0.000	-0.602	0.023	0.976	0.595	0.797	972	225	1493
3	-1.810	1.000	0.000	-1.821	0.040	1.000	0.596	0.949	909	161	1492
4	0.000	0.700	0.000	-0.000	0.020	0.052	0.598	0.598	1049	302	1494
5	-0.600	0.700	0.000	-0.421	0.024	0.772	0.628	0.770	971	224	1492
6	-1.810	0.700	0.000	-1.286	0.035	1.000	0.637	0.916	908	160	1492
7	0.000	0.400	0.000	0.006	0.019	0.050	0.598	0.595	1049	302	1493
8	-0.600	0.400	0.000	-0.243	0.024	0.338	0.662	0.744	972	224	1491
9	-1.810	0.400	0.000	-0.720	0.030	0.986	0.697	0.867	910	160	1494
1	0.000	1.000	0.000	0.001	0.002	0.045	0.600	0.599	10491	3021	14926
2	-0.600	1.000	0.000	-0.599	0.002	1.000	0.597	0.798	9732	2254	14926
3	-1.810	1.000	0.000	-1.814	0.004	1.000	0.600	0.950	9094	1606	14920
4	0.000	0.700	0.000	-0.000	0.002	0.055	0.599	0.599	10492	3021	14926
5	-0.600	0.700	0.000	-0.418	0.002	1.000	0.629	0.771	9731	2252	14926
6	-1.810	0.700	0.000	-1.263	0.003	1.000	0.640	0.916	9092	1607	14923
7	0.000	0.400	0.000	-0.000	0.002	0.040	0.599	0.599	10490	3022	14919
8	-0.600	0.400	0.000	-0.239	0.002	0.999	0.661	0.742	9731	2249	14918
9	-1.810	0.400	0.000	-0.723	0.003	1.000	0.698	0.869	9091	1606	14929

Table 2. Use of $\widehat{X}(W)$ as IR with n=15000, 150000, 1500000.

scen	β_1	ρ	imp	$\overline{\widehat{\beta}_1}$	$S^2(\widehat{\beta}_1)$	% Reject	$\overline{\mathrm{VE}(\frac{1}{8})}$	$\overline{\mathrm{VE}(\frac{7}{8})}$	$\overline{n}_V$	$\overline{n}_{VD}$	$\overline{n}_{VU}$
1	0.000	1.000	1.000	-0.002	0.217	0.048	0.584	0.588	105	30	149
2	-0.600	1.000	1.000	-0.612	0.291	0.177	0.578	0.781	97	22	148
3	-1.810	1.000	1.000	-1.906	0.524	0.753	0.562	0.941	90	16	149
4	0.000	0.700	1.000	0.007	0.463	0.052	0.581	0.585	104	30	149
5	-0.600	0.700	1.000	-0.641	0.639	0.117	0.611	0.759	97	22	149
6	-1.810	0.700	1.000	-1.960	1.047	0.450	0.617	0.907	90	16	149
7	0.000	0.400	1.000	0.044	1.903	0.040	0.587	0.579	105	30	148
8	-0.600	0.400	1.000	-0.717	2.310	0.079	0.639	0.732	97	22	149
9	-1.810	0.400	1.000	-1.374	548.190	0.001	0.681	0.858	90	15	149
1	0.000	1.000	1.000	0.001	0.020	0.061	0.598	0.597	1049	302	1493
2	-0.600	1.000	1.000	-0.602	0.023	0.976	0.595	0.797	972	225	1493
3	-1.810	1.000	1.000	-1.821	0.040	1.000	0.596	0.949	909	161	1492
4	0.000	0.700	1.000	-0.000	0.042	0.050	0.598	0.598	1049	302	1494
5	-0.600	0.700	1.000	-0.600	0.049	0.769	0.628	0.770	971	224	1492
6	-1.810	0.700	1.000	-1.843	0.077	1.000	0.637	0.916	908	160	1492
7	0.000	0.400	1.000	0.016	0.125	0.049	0.598	0.595	1049	302	1493
8	-0.600	0.400	1.000	-0.609	0.155	0.338	0.662	0.744	972	224	1491
9	-1.810	0.400	1.000	-1.821	0.217	0.979	0.697	0.867	910	160	1494
1	0.000	1.000	1.000	0.001	0.002	0.045	0.600	0.599	10491	3021	14926
2	-0.600	1.000	1.000	-0.599	0.002	1.000	0.597	0.798	9732	2254	14926
3	-1.810	1.000	1.000	-1.814	0.004	1.000	0.600	0.950	9094	1606	14920
4	0.000	0.700	1.000	-0.001	0.004	0.056	0.599	0.599	10492	3021	14926
5	-0.600	0.700	1.000	-0.598	0.005	1.000	0.629	0.771	9731	2252	14926
6	-1.810	0.700	1.000	-1.807	0.007	1.000	0.640	0.916	9092	1607	14923
7	0.000	0.400	1.000	-0.001	0.011	0.042	0.599	0.599	10490	3022	14919
8	-0.600	0.400	1.000	-0.597	0.014	0.999	0.661	0.742	9731	2249	14918
9	-1.810	0.400	1.000	-1.814	0.025	1.000	0.698	0.869	9091	1606	14929

Table 3. Use of $\widehat{X}(W)$ as IR with n=150000 and $\theta_0 = \theta_{U0} = 0.25$ rather than 0.50 (first third of table), $\theta_{U0} = 0.25$ instead of 0.50 (second third of table), and π for unvaccinated equal 0.025 rather than 0.01 (last third of table).

scen	β_1	ρ	imp	$\overline{\widehat{\beta}_1}$	$S^2(\widehat{\beta}_1)$	% Reject	$\overline{\text{VE}(\frac{1}{8})}$	$\overline{\text{VE}(\frac{7}{8})}$	$\overline{n}_V$	$\overline{n}_{VD}$	$\overline{n}_{VU}$
1	0.000	1.000	1.000	0.000	0.025	0.050	0.597	0.597	676	302	1121
2	-0.600	1.000	1.000	-0.605	0.030	0.950	0.592	0.796	599	225	1121
3	-1.810	1.000	1.000	-1.822	0.051	1.000	0.593	0.949	534	161	1120
4	0.000	0.700	1.000	-0.003	0.055	0.047	0.596	0.598	676	302	1122
5	-0.600	0.700	1.000	-0.608	0.061	0.664	0.626	0.770	597	224	1121
6	-1.810	0.700	1.000	-1.851	0.098	1.000	0.635	0.916	534	160	1121
7	0.000	0.400	1.000	0.016	0.155	0.046	0.597	0.594	675	302	1121
8	-0.600	0.400	1.000	-0.619	0.201	0.259	0.660	0.743	597	224	1120
9	-1.810	0.400	1.000	-1.828	0.276	0.945	0.696	0.866	535	160	1121
1	0.000	1.000	1.000	0.000	0.025	0.050	0.195	0.194	676	302	1493
2	-0.600	1.000	1.000	-0.605	0.030	0.950	0.185	0.592	599	225	1493
3	-1.810	1.000	1.000	-1.822	0.051	1.000	0.184	0.897	534	161	1492
4	0.000	0.700	1.000	-0.003	0.055	0.047	0.194	0.197	676	302	1494
5	-0.600	0.700	1.000	-0.608	0.061	0.664	0.250	0.538	597	224	1492
6	-1.810	0.700	1.000	-1.851	0.098	1.000	0.270	0.832	534	160	1492
7	0.000	0.400	1.000	0.016	0.155	0.046	0.193	0.188	675	302	1493
8	-0.600	0.400	1.000	-0.619	0.201	0.259	0.321	0.485	597	224	1491
9	-1.810	0.400	1.000	-1.828	0.276	0.945	0.391	0.732	535	160	1494
1	0.000	1.000	1.000	0.001	0.020	0.061	-0.005	-0.006	1049	302	2606
2	-0.600	1.000	1.000	-0.602	0.023	0.976	-0.013	0.492	972	225	2607
3	-1.810	1.000	1.000	-1.821	0.040	1.000	-0.011	0.873	909	161	2605
4	0.000	0.700	1.000	-0.000	0.042	0.050	-0.005	-0.005	1049	302	2608
5	-0.600	0.700	1.000	-0.600	0.049	0.769	0.070	0.425	971	224	2604
6	-1.810	0.700	1.000	-1.843	0.077	1.000	0.093	0.790	908	160	2605
7	0.000	0.400	1.000	0.016	0.125	0.049	-0.005	-0.011	1049	302	2606
8	-0.600	0.400	1.000	-0.609	0.155	0.338	0.153	0.358	972	224	2605
9	-1.810	0.400	1.000	-1.821	0.217	0.979	0.243	0.667	910	160	2609

Table 4. Maximum likelihood over β_0, β_1 with π set at its true value

scen	β_1	ρ	imp	$\overline{\widehat{\beta}_1}$	$S^2(\widehat{\beta}_1)$	% Reject	$\overline{\text{VE}(\frac{1}{8})}$	$\overline{\text{VE}(\frac{7}{8})}$	$\overline{n}_V$	$\overline{n}_{VD}$	$\overline{n}_{VU}$
1	0.000	1.000	2.000	-0.002	0.217	0.049	0.996	0.996	105	30	149
2	-0.600	1.000	2.000	-0.612	0.291	0.176	0.996	0.998	97	22	148
3	-1.810	1.000	2.000	-1.907	0.525	0.753	0.996	0.999	90	16	149
4	0.000	0.700	2.000	0.005	0.467	0.052	0.996	0.996	104	30	149
5	-0.600	0.700	2.000	-0.645	0.645	0.118	0.996	0.998	97	22	149
6	-1.810	0.700	2.000	-2.752	620.490	0.001	0.996	0.999	90	16	149
7	0.000	0.400	2.000	-2.619	7115.016	0.001	0.996	0.996	105	30	148
8	-0.600	0.400	2.000	-5.963	9856.639	0.003	0.996	0.998	97	22	149
9	-1.810	0.400	2.000	-18.942	33284.757	0.009	0.996	0.999	90	15	149
1	0.000	1.000	2.000	0.001	0.020	0.061	0.996	0.996	1049	302	1493
2	-0.600	1.000	2.000	-0.602	0.023	0.976	0.996	0.998	972	225	1493
3	-1.810	1.000	2.000	-1.822	0.041	1.000	0.996	0.999	909	161	1492
4	0.000	0.700	2.000	-0.000	0.042	0.050	0.996	0.996	1049	302	1494
5	-0.600	0.700	2.000	-0.601	0.049	0.769	0.996	0.998	971	224	1492
6	-1.810	0.700	2.000	-1.849	0.078	1.000	0.996	0.999	908	160	1492
7	0.000	0.400	2.000	0.006	0.142	0.043	0.996	0.996	1049	302	1493
8	-0.600	0.400	2.000	-0.612	0.159	0.329	0.996	0.998	972	224	1491
9	-1.810	0.400	2.000	-1.830	0.220	0.980	0.996	0.999	910	160	1494
1	0.000	1.000	2.000	0.001	0.002	0.048	0.996	0.996	10491	3021	14926
2	-0.600	1.000	2.000	-0.599	0.002	1.000	0.996	0.998	9732	2254	14926
3	-1.810	1.000	2.000	-1.814	0.004	1.000	0.996	1.000	9094	1606	14920
4	0.000	0.700	2.000	-0.010	0.021	0.005	0.996	0.996	10492	3021	14926
5	-0.600	0.700	2.000	-0.598	0.005	1.000	0.996	0.998	9731	2252	14926
6	-1.810	0.700	2.000	-1.811	0.007	1.000	0.996	0.999	9092	1607	14923
7	0.000	0.400	2.000	-0.001	0.011	0.043	0.996	0.996	10490	3022	14919
8	-0.600	0.400	2.000	-0.603	0.022	0.995	0.996	0.998	9731	2249	14918
9	-1.810	0.400	2.000	-1.820	0.026	1.000	0.996	0.999	9091	1606	14929

Table 5. Maximum likelihood over β_0, β_1, π

scen	β_1	ρ	imp	$\overline{\widehat{\beta}_1}$	$S^2(\widehat{\beta}_1)$	% Reject	$\overline{\text{VE}(\frac{1}{8})}$	$\overline{\text{VE}(\frac{7}{8})}$	$\overline{n}_V$	$\overline{n}_{VD}$	$\overline{n}_{VU}$
1	0.000	1.000	2.000	-0.002	0.217	0.048	0.722	0.708	105	30	149
2	-0.600	1.000	2.000	-0.612	0.291	0.177	0.746	0.863	97	22	148
3	-1.810	1.000	2.000	-1.906	0.524	0.753	0.835	0.973	90	16	149
4	0.000	0.700	2.000	0.008	0.499	0.053	0.723	0.699	104	30	149
5	-0.600	0.700	2.000	-0.672	0.754	0.112	0.736	0.864	97	22	149
6	-1.810	0.700	2.000	-3.744	1135.734	0.002	0.710	0.965	90	16	149
7	0.000	0.400	2.000	-2.054	4281.964	0	-13620*	-12108*	105	30	148
8	-0.600	0.400	2.000	-7.535	5305.414	0	-12707*	-13162*	97	22	149
9	-1.810	0.400	2.000	-43.459	32840.794	0.055	-124692*	0.942	90	15	149
1	0.000	1.000	2.000	0.001	0.020	0.061	0.723	0.721	1049	302	1493
2	-0.600	1.000	2.000	-0.602	0.023	0.976	0.762	0.881	972	225	1493
3	-1.810	1.000	2.000	-1.821	0.040	1.000	0.872	0.983	909	161	1492
4	0.000	0.700	2.000	-0.000	0.042	0.050	0.724	0.722	1049	302	1494
5	-0.600	0.700	2.000	-0.606	0.051	0.760	0.761	0.881	971	224	1492
6	-1.810	0.700	2.000	-1.908	0.094	1.000	0.851	0.983	908	160	1492
7	0.000	0.400	2.000	0.016	0.128	0.049	0.722	0.709	1049	302	1493
8	-0.600	0.400	2.000	-0.624	0.173	0.305	0.761	0.885	972	224	1491
9	-1.810	0.400	2.000	-1.943	0.321	0.950	0.832	0.979	910	160	1494
1	0.000	1.000	2.000	0.001	0.002	0.044	0.731	0.731	10491	3021	14926
2	-0.600	1.000	2.000	-0.599	0.002	1.000	0.761	0.880	9732	2254	14926
3	-1.810	1.000	2.000	-1.814	0.004	1.000	0.830	0.979	9094	1606	14920
4	0.000	0.700	2.000	-0.001	0.004	0.055	0.725	0.726	10492	3021	14926
5	-0.600	0.700	2.000	-0.601	0.005	1.000	0.759	0.880	9731	2252	14926
6	-1.810	0.700	2.000	-1.869	0.009	1.000	0.842	0.982	9092	1607	14923
7	0.000	0.400	2.000	-0.001	0.011	0.042	0.703	0.703	10490	3022	14919
8	-0.600	0.400	2.000	-0.604	0.015	0.999	0.757	0.879	9731	2249	14918
9	-1.810	0.400	2.000	-1.878	0.032	1.000	0.908	0.990	9091	1606	14929

* value was truncated