Web Appendix A - Proof of Theorem 1

We first recall Theorem 1 and then provide a proof that closely follows the one from Mao et al (2019).

Theorem 1. *The estimator $\hat{\tau}^{A-OW}_{t,t'}$ is consistent for $\tau^*_{t,t'}$ when the treatment model is correctly specified, whether the outcome model is correctly specified or not. When the outcome model is correctly specified, but the treatment model is misspecified, $\hat{\tau}^{A-OW}_{t,t'}$ is consistent for*

$$\tilde{\tau}^*_{t,t'} = \frac{\int_{\boldsymbol{X}\in\mathcal{X}} \{\mathbb{E}[Y(t)|\boldsymbol{X}] - \mathbb{E}[Y(t')|\boldsymbol{X}]\} f(\boldsymbol{X})\tilde{h}(\boldsymbol{X})\mu(d\boldsymbol{X})}{\int_{\boldsymbol{X}\in\mathcal{X}} f(\boldsymbol{X})\tilde{h}(\boldsymbol{X})\mu(d\boldsymbol{X})},$$

where $\tilde{h}(\boldsymbol{X})$ is the estimand of $\left[\sum_{l=1}^k 1/\hat{P}(T=l|\boldsymbol{X}_i)\right]^{-1}$ under the misspecified $\hat{P}(T=l|\boldsymbol{X}_i)$.

Proof. First, assume that the treatment model $\hat{P}(T=t|\boldsymbol{X}_i)$ is correctly specified. Then, the term $\hat{\tau}^{OW}_{t,t'} \xrightarrow{P} \tau^*_{t,t'}$ in $\hat{\tau}^{A-OW}_{t,t'}$. It remains to show that the other terms in $\hat{\tau}^{A-OW}_{t,t'}$ converge in probability to 0. These other terms can be written as

$$\left\{\frac{\sum_{i=1}^n h(\boldsymbol{X}_i)\hat{\mathbb{E}}(Y|T=t,\boldsymbol{X}_i)}{\sum_{i=1}^n h(\boldsymbol{X}_i)} - \frac{\sum_{i=1}^n I(T_i=t)\hat{\mathbb{E}}(Y|T=t,\boldsymbol{X}_i)w_i}{\sum_{i=1}^n I(T_i=t)w_i}\right\}$$
$$-\left\{\frac{\sum_{i=1}^n h(\boldsymbol{X}_i)\hat{\mathbb{E}}(Y|T=t',\boldsymbol{X}_i)}{\sum_{i=1}^n h(\boldsymbol{X}_i)} - \frac{\sum_{i=1}^n I(T_i=t')\hat{\mathbb{E}}(Y|T=t',\boldsymbol{X}_i)w_i}{\sum_{i=1}^n I(T_i=t')w_i}\right\}.$$

We have

$$\frac{\sum_{i=1}^n h(\boldsymbol{X}_i)\hat{\mathbb{E}}(Y|T=t,\boldsymbol{X}_i)}{\sum_{i=1}^n h(\boldsymbol{X}_i)} \xrightarrow{P} \frac{\mathbb{E}\left[\tilde{\mathbb{E}}(Y|T=t,\boldsymbol{X}_i)\right]}{\mathbb{E}[h(\boldsymbol{X}_i)]}$$
$$\frac{\sum_{i=1}^n h(\boldsymbol{X}_i)\hat{\mathbb{E}}(Y|T=t',\boldsymbol{X}_i)}{\sum_{i=1}^n h(\boldsymbol{X}_i)} \xrightarrow{P} \frac{\mathbb{E}\left[\tilde{\mathbb{E}}(Y|T=t',\boldsymbol{X}_i)\right]}{\mathbb{E}[h(\boldsymbol{X}_i)]},$$

where $\tilde{\mathbb{E}}(Y|T,\boldsymbol{X}_i)$ is the estimand of $\hat{\mathbb{E}}(Y|T,\boldsymbol{X}_i)$. When the outcome model is correctly specified, $\tilde{\mathbb{E}}(Y|T,\boldsymbol{X}_i) = \mathbb{E}(Y|T,\boldsymbol{X}_i)$.

The overlap weights benefit from a balancing property, which entails that the distribution of the covariates $\boldsymbol{X}_i$ is, on average, the same in all groups and is the same as the distribution in the population weighted according to $h(\boldsymbol{X}_i)$ (Li et al. 2019). As such,

$$\mathbb{E}[I(T=t)w_i\eta(\boldsymbol{X}_i)] = \mathbb{E}[I(T=t')w_i\eta(\boldsymbol{X}_i)] = \mathbb{E}[h(\boldsymbol{X}_i)\eta(\boldsymbol{X}_i)]$$

for any function $\eta(\boldsymbol{X}_i)$. As a result

$$\frac{\sum_{i=1}^n I(T_i=t)\hat{\mathbb{E}}(Y|T=t,\boldsymbol{X}_i)w_i}{\sum_{i=1}^n I(T_i=t)w_i} \xrightarrow{P} \frac{\mathbb{E}\left[\tilde{\mathbb{E}}(Y|T=t,\boldsymbol{X}_i)\right]}{\mathbb{E}[h(\boldsymbol{X}_i)]}$$
$$\frac{\sum_{i=1}^n I(T_i=t')\hat{\mathbb{E}}(Y|T=t',\boldsymbol{X}_i)w_i}{\sum_{i=1}^n I(T_i=t')w_i} \xrightarrow{P} \frac{\mathbb{E}\left[\tilde{\mathbb{E}}(Y|T=t',\boldsymbol{X}_i)\right]}{\mathbb{E}[h(\boldsymbol{X}_i)]}.$$

Hence,

$$\left\{\frac{\sum_{i=1}^n h(\boldsymbol{X}_i)\hat{\mathbb{E}}(Y|T=t,\boldsymbol{X}_i)}{\sum_{i=1}^n h(\boldsymbol{X}_i)} - \frac{\sum_{i=1}^n I(T_i=t)\hat{\mathbb{E}}(Y|T=t,\boldsymbol{X}_i)w_i}{\sum_{i=1}^n I(T_i=t)w_i}\right\}$$
$$-\left\{\frac{\sum_{i=1}^n h(\boldsymbol{X}_i)\hat{\mathbb{E}}(Y|T=t',\boldsymbol{X}_i)}{\sum_{i=1}^n h(\boldsymbol{X}_i)} - \frac{\sum_{i=1}^n I(T_i=t')\hat{\mathbb{E}}(Y|T=t',\boldsymbol{X}_i)w_i}{\sum_{i=1}^n I(T_i=t')w_i}\right\} \xrightarrow{P} 0.$$

This completes the proof the $\hat{\tau}^{A-OW}_{t,t'} \xrightarrow{P} \tau^*_{t,t'}$ when the treatment model is correctly specified, whether the outcome model is correctly specified or not.

Now, assume that the treatment model is incorrectly specified, but the outcome model is correctly specified. The augmented overlap weight estimator can be written as

$$\begin{aligned}\hat{\tau}_{t,t'}^{A-OW} =& \frac{\sum_{i=1}^{n} h(\boldsymbol{X}_i)\left[\hat{\mathbb{E}}(Y|T=t,\boldsymbol{X}_i)-\hat{\mathbb{E}}(Y|T=t',\boldsymbol{X}_i)\right]}{\sum_{i=1}^{n} h(\boldsymbol{X}_i)} \\ &+ \frac{\sum_{i=1}^{n} I(T_i=t)\left[Y_i-\hat{\mathbb{E}}(Y|T=t,\boldsymbol{X}_i)\right]w_i}{\sum_{i=1}^{n} I(T_i=t)w_i} \\ &- \frac{\sum_{i=1}^{n} I(T_i=t')\left[Y_i-\hat{\mathbb{E}}(Y|T=t',\boldsymbol{X}_i)\right]w_i}{\sum_{i=1}^{n} I(T_i=t')w_i}.\end{aligned}$$

Recall that $\tilde{h}(\boldsymbol{X}_i)$ denote the estimand of $h(\boldsymbol{X}_i)$ under the misspecified outcome model. Using this notation, we can note that

$$\begin{aligned}&\frac{\sum_{i=1}^{n} h(\boldsymbol{X}_i)\left[\hat{\mathbb{E}}(Y|T=t,\boldsymbol{X}_i)-\hat{\mathbb{E}}(Y|T=t',\boldsymbol{X}_i)\right]}{\sum_{i=1}^{n} h(\boldsymbol{X}_i)} \\ &\xrightarrow{P} \frac{\mathbb{E}_{\boldsymbol{X}}\left\{\tilde{h}(\boldsymbol{X})\left[\mathbb{E}(Y|T=t,\boldsymbol{X})-\mathbb{E}(Y|T=t',\boldsymbol{X})\right]\right\}}{\mathbb{E}_{\boldsymbol{X}}[\tilde{h}(\boldsymbol{X}_i)]} = \tilde{\tau}_{t,t'}^{*}\end{aligned}$$

invoking the common exchangeability, consistency and positivity assumptions. Thus, it remains to show that the second and third term in $\hat{\tau}_{t,t'}^{A-OW}$ as written above converge in probability to 0. We have

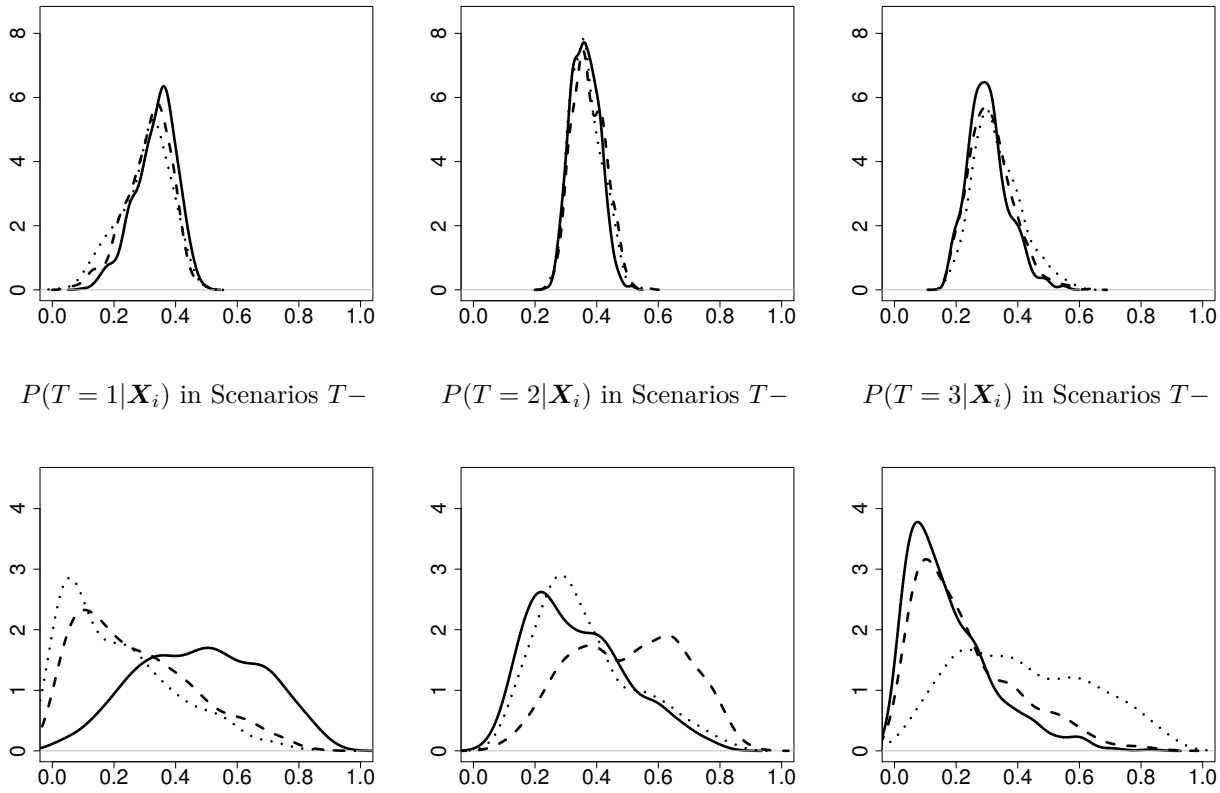
$$\begin{aligned}&\mathbb{E}\left\{I(T_i=t)\left[Y_i-\hat{\mathbb{E}}(Y|T=t,\boldsymbol{X}_i)\right]w_i\right\} \\ &= \mathbb{E}\left\{I(T_i=t)\left[Y_i^t-\hat{\mathbb{E}}(Y|T=t,\boldsymbol{X}_i)\right]w_i\right\} \\ &= \mathbb{E}_{\boldsymbol{X}}\left(\mathbb{E}\left\{I(T_i=t)\left[Y_i^t-\hat{\mathbb{E}}(Y|T=t,\boldsymbol{X}_i)\right]w_i|\boldsymbol{X}_i\right\}\right) \\ &= \mathbb{E}_{\boldsymbol{X}}\left(\mathbb{E}\left\{I(T_i=t)w_i|\boldsymbol{X}_i\right\}\right)\mathbb{E}_{\boldsymbol{X}_i}\left(\left[Y_i^t-\hat{\mathbb{E}}(Y|T=t,\boldsymbol{X}_i)\right]|\boldsymbol{X}_i\right),\end{aligned}$$

where the first equality is obtained by consistency, the second by the law of total probability, and the third is obtained by exchangeability, $Y \coprod w_i|X_i$ (w_i is a surjective function of X_i), $\hat{\mathbb{E}}(Y|T=t,\boldsymbol{X}_i) \coprod T$ and $\hat{\mathbb{E}}(Y|T=t,\boldsymbol{X}_i) \coprod w_i$ (because $\hat{\mathbb{E}}(Y|T=t,\boldsymbol{X}_i)$ is already conditional on T and $\boldsymbol{X}_i$). It now suffices to note that

$$\mathbb{E}_{\boldsymbol{X}_i}(Y_i^t|\boldsymbol{X}_i) = \mathbb{E}_{\boldsymbol{X}_i}\left[\hat{\mathbb{E}}(Y|T=t,\boldsymbol{X}_i)|\boldsymbol{X}_i\right] = \mathbb{E}[Y^t]$$

to show that the second term converges in probability to 0. The proof for the third term is conducted in the same fashion. This completes the proof that when the outcome model is correctly specified, but the treatment model is incorrectly specified, $\hat{\tau}_{t,t'}^{A-OW} \xrightarrow{P} \tilde{\tau}_{t,t'}^{*}$, thus completing the proof of Theorem 1. □

Web Appendix B - Additional simulation results



$P(T = 1|\boldsymbol{X}_i)$ in Scenarios $T-$ $P(T = 2|\boldsymbol{X}_i)$ in Scenarios $T-$ $P(T = 3|\boldsymbol{X}_i)$ in Scenarios $T-$

$P(T = 1|\boldsymbol{X}_i)$ in Scenarios $T+$ $P(T = 2|\boldsymbol{X}_i)$ in Scenarios $T+$ $P(T = 3|\boldsymbol{X}_i)$ in Scenarios $T+$

Figure 1: Distribution of the treatment probabilities according to treatment group and Scenarios with a weak ($T-$) or a strong ($T+$) association between covariates and treatment.
Legend: —: T = 1, − − −: $T = 2$ and $\cdots$: $T = 3$

Table 1: Estimate of the treatment effect in the Scenario with a weak association between covariates and treatment, a weak association between covariates and outcome ($T - Y-$), and a sample size of 500.

Implementation	Approach	Bias		Std		RMSE		Coverage IC	
		$\hat{\tau}_{21}$	$\hat{\tau}_{31}$	$\hat{\tau}_{21}$	$\hat{\tau}_{31}$	$\hat{\tau}_{21}$	$\hat{\tau}_{31}$	$\hat{\tau}_{21}$	$\hat{\tau}_{31}$
Cor.param.	Crude	0.239	0.339	0.145	0.157	0.279	0.374	0.672	0.441
	stan	0.001	0.000	0.111	0.116	0.111	0.116	0.961	0.956
	IPW	0.004	0.002	0.144	0.148	0.144	0.148	0.991	0.985
	match	0.024	0.024	0.139	0.140	0.141	0.142	0.962	0.971
	BCM	0.001	-0.004	0.136	0.139	0.136	0.139	0.965	0.971
	TMLE	0.001	-0.001	0.115	0.119	0.115	0.119	0.959	0.959
	OW	0.001	-0.002	0.123	0.128	0.123	0.128	0.987	0.983
	A-OW	0.001	-0.000	0.113	0.118	0.113	0.118	0.946	0.939
Inc.param.	stan	0.046	0.054	0.131	0.138	0.139	0.148	0.927	0.936
	IPW	0.091	0.082	0.142	0.147	0.169	0.168	0.932	0.940
	match	0.041	0.038	0.136	0.144	0.142	0.149	0.949	0.955
	BCM	0.031	0.020	0.135	0.143	0.138	0.144	0.955	0.956
	TMLE	0.076	0.073	0.135	0.145	0.155	0.162	0.934	0.941
	OW	0.082	0.073	0.135	0.139	0.158	0.158	0.941	0.947
	A-OW	0.068	0.065	0.131	0.138	0.148	0.152	0.912	0.922
M.Learning	stan	0.004	0.002	0.114	0.119	0.114	0.119	0.943	0.944
	IPW	0.133	0.128	0.150	0.175	0.200	0.217	0.878	0.850
	match*	0.066	0.059	0.158	0.181	0.171	0.190	0.922	0.902
	BCM*	-0.004	-0.004	0.136	0.143	0.136	0.143	0.963	0.958
	TMLE	0.006	0.004	0.116	0.121	0.116	0.121	0.960	0.955
	OW	0.125	0.121	0.146	0.172	0.192	0.211	0.881	0.854
	A-OW	0.005	0.004	0.115	0.120	0.115	0.120	0.948	0.941

Legend: Cor.param=correct parametric models, Inc.param=incorrect parametric models, M.Learning=machine learning, Crude=Unadjusted, stan=standardization, IPW=inverse probability weighting, match=matching, BCM=bias-corrected matching, TMLE=targeted maximum likelihood, OW=overlap weights, A-OW=augmented overlap weights. *: 137 replications were discarded due to errors when running the code for *match* and BCM.

Table 2: Estimate of the treatment effect in the Scenario with a weak association between covariates and treatment, a weak association between covariates and outcome ($T - Y-$), and a sample size of 2000.

Implementation	Approach	Bias		Std		RMSE		Coverage IC	
		$\hat{\tau}_{21}$	$\hat{\tau}_{31}$	$\hat{\tau}_{21}$	$\hat{\tau}_{31}$	$\hat{\tau}_{21}$	$\hat{\tau}_{31}$	$\hat{\tau}_{21}$	$\hat{\tau}_{31}$
Cor.param.	Crude	0.243	0.345	0.075	0.080	0.254	0.355	0.107	0.006
	stan	0.001	0.003	0.058	0.058	0.058	0.058	0.949	0.963
	IPW	0.001	0.001	0.073	0.070	0.073	0.070	0.989	0.995
	match	0.012	0.011	0.070	0.068	0.071	0.069	0.955	0.960
	BCM	0.002	0.001	0.070	0.068	0.070	0.068	0.953	0.963
	TMLE	0.001	0.002	0.059	0.059	0.059	0.059	0.961	0.958
	OW	0.001	0.001	0.065	0.064	0.065	0.064	0.982	0.988
	A-OW	0.001	0.002	0.059	0.058	0.059	0.059	0.941	0.943
Inc.param.	stan	0.047	0.058	0.070	0.069	0.084	0.089	0.887	0.867
	IPW	0.093	0.088	0.075	0.073	0.119	0.114	0.810	0.839
	match	0.024	0.024	0.069	0.069	0.073	0.073	0.941	0.955
	BCM	0.019	0.016	0.068	0.068	0.071	0.070	0.955	0.959
	TMLE	0.077	0.077	0.072	0.072	0.105	0.105	0.826	0.855
	OW	0.084	0.079	0.072	0.070	0.111	0.105	0.826	0.860
	A-OW	0.069	0.070	0.070	0.069	0.098	0.098	0.814	0.830
M.Learning	stan	0.001	0.004	0.060	0.059	0.060	0.059	0.931	0.941
	IPW	0.086	0.072	0.075	0.072	0.114	0.102	0.836	0.892
	match	0.025	0.014	0.072	0.069	0.077	0.071	0.950	0.964
	BCM	-0.004	-0.003	0.069	0.068	0.069	0.068	0.963	0.966
	TMLE	0.002	0.004	0.061	0.060	0.061	0.060	0.950	0.964
	OW	0.078	0.064	0.072	0.069	0.106	0.094	0.853	0.906
	A-OW	0.001	0.004	0.060	0.059	0.060	0.060	0.934	0.947

Legend: Cor.param=correct parametric models, Inc.param=incorrect parametric models, M.Learning=machine learning, Crude=Unadjusted, stan=standardization, IPW=inverse probability weighting, match=matching, BCM=bias-corrected matching, TMLE=targeted maximum likelihood, OW=overlap weights, A-OW=augmented overlap weights

Table 3: Estimate of the treatment effect in the Scenario with a strong association between covariates and treatment, a weak association between covariates and outcome ($T + Y-$), and a sample size of 500.

Implementation	Approach	Bias		Std		RMSE		Coverage IC	
		$\hat{\tau}_{21}$	$\hat{\tau}_{31}$	$\hat{\tau}_{21}$	$\hat{\tau}_{31}$	$\hat{\tau}_{21}$	$\hat{\tau}_{31}$	$\hat{\tau}_{21}$	$\hat{\tau}_{31}$
Cor.param.	Crude	0.461	0.791	0.143	0.171	0.482	0.810	0.106	0.007
	stan	-0.002	-0.005	0.117	0.133	0.117	0.133	0.952	0.959
	IPW	0.052	0.055	0.360	0.384	0.363	0.388	0.874	0.889
	match	0.125	0.129	0.170	0.199	0.211	0.237	0.885	0.887
	BCM	0.018	0.006	0.166	0.195	0.167	0.195	0.948	0.939
	TMLE	-0.000	0.003	0.145	0.181	0.145	0.181	0.949	0.935
	OW	0.004	0.001	0.153	0.168	0.153	0.168	0.952	0.961
	A-OW	-0.001	0.001	0.138	0.155	0.138	0.155	0.944	0.946
Inc.param.	stan	-0.132	-0.046	0.145	0.160	0.196	0.167	0.841	0.929
	IPW	0.209	0.187	0.229	0.268	0.310	0.327	0.721	0.816
	match	0.153	0.156	0.169	0.198	0.228	0.252	0.850	0.873
	BCM	0.097	0.055	0.164	0.195	0.190	0.202	0.921	0.929
	TMLE	0.152	0.110	0.189	0.229	0.243	0.254	0.872	0.914
	OW	0.112	0.097	0.161	0.173	0.196	0.198	0.878	0.898
	A-OW	0.059	0.059	0.148	0.168	0.159	0.177	0.919	0.930
M.Learning	stan	-0.011	-0.003	0.128	0.149	0.129	0.149	0.938	0.941
	IPW	0.176	0.163	0.237	0.269	0.295	0.314	0.768	0.845
	match*	0.145	0.149	0.170	0.195	0.223	0.246	0.863	0.880
	BCM*	0.006	0.014	0.162	0.191	0.162	0.191	0.940	0.948
	TMLE	0.011	0.010	0.149	0.186	0.149	0.186	0.940	0.928
	OW	0.085	0.074	0.158	0.170	0.180	0.186	0.911	0.916
	A-OW**	-0.005	-0.003	0.141	0.156	0.141	0.156	0.933	0.938

Legend: Cor.param=correct parametric models, Inc.param=incorrect parametric models, M.Learning=machine learning, Crude=Unadjusted, stan=standardization, IPW=inverse probability weighting, match=matching, BCM=bias-corrected matching, TMLE=targeted maximum likelihood, OW=overlap weights, A-OW=augmented overlap weights. *: 2 replications were discarded due to errors when running the code for *match* and *BCM*, **: in 9 replications, the confidence intervals could not be computed.

Table 4: Estimate of the treatment effect in the Scenario with a strong association between covariates and treatment, a weak association between covariates and outcome ($T + Y-$), and a sample size of 2000.

Implementation	Approach	Bias		Std		RMSE		Coverage IC	
		$\hat{\tau}_{21}$	$\hat{\tau}_{31}$	$\hat{\tau}_{21}$	$\hat{\tau}_{31}$	$\hat{\tau}_{21}$	$\hat{\tau}_{31}$	$\hat{\tau}_{21}$	$\hat{\tau}_{31}$
Cor.param.	Crude	0.461	0.790	0.067	0.084	0.465	0.795	0.000	0.000
	stan	-0.000	0.000	0.059	0.068	0.059	0.068	0.957	0.958
	IPW	0.012	0.014	0.328	0.332	0.328	0.332	0.854	0.889
	match	0.075	0.072	0.088	0.101	0.116	0.124	0.861	0.896
	BCM	0.015	0.010	0.086	0.099	0.087	0.099	0.956	0.952
	TMLE	0.000	0.002	0.073	0.085	0.073	0.085	0.951	0.953
	OW	-0.000	0.000	0.072	0.081	0.072	0.081	0.975	0.970
	A-OW	-0.001	0.000	0.067	0.074	0.067	0.074	0.946	0.955
Inc.param.	stan	-0.129	-0.041	0.073	0.083	0.148	0.093	0.530	0.897
	IPW	0.202	0.183	0.128	0.158	0.239	0.242	0.450	0.646
	match	0.103	0.097	0.087	0.100	0.135	0.139	0.785	0.842
	BCM	0.071	0.043	0.084	0.098	0.110	0.107	0.873	0.939
	TMLE	0.151	0.112	0.099	0.117	0.181	0.162	0.615	0.830
	OW	0.109	0.096	0.079	0.085	0.134	0.128	0.708	0.815
	A-OW	0.057	0.059	0.072	0.083	0.092	0.102	0.865	0.884
M.Learning	stan	0.001	0.017	0.064	0.076	0.064	0.077	0.940	0.931
	IPW	0.101	0.096	0.198	0.204	0.222	0.225	0.728	0.808
	match	0.089	0.084	0.089	0.101	0.126	0.131	0.813	0.862
	BCM	0.002	0.007	0.086	0.098	0.086	0.099	0.953	0.956
	TMLE	0.007	0.007	0.076	0.090	0.076	0.090	0.942	0.947
	OW	0.043	0.039	0.080	0.084	0.091	0.093	0.908	0.937
	A-OW*	-0.005	-0.003	0.068	0.076	0.068	0.076	0.952	0.949

Legend: Cor.param=correct parametric models, Inc.param=incorrect parametric models, M.Learning=machine learning, Crude=Unadjusted, stan=standardization, IPW=inverse probability weighting, match=matching, BCM=bias-corrected matching, TMLE=targeted maximum likelihood, OW=overlap weights, A-OW=augmented overlap weights. *: in 1 replications, the confidence intervals could not be computed.

Table 5: Estimate of the treatment effect in the Scenario with a weak association between covariates and treatment, a strong association between covariates and outcome ($T - Y+$), and a sample size of 500.

Implementation	Approach	Bias		Std		RMSE		Coverage IC	
		$\hat{\tau}_{21}$	$\hat{\tau}_{31}$	$\hat{\tau}_{21}$	$\hat{\tau}_{31}$	$\hat{\tau}_{21}$	$\hat{\tau}_{31}$	$\hat{\tau}_{21}$	$\hat{\tau}_{31}$
Cor.param.	Crude	0.560	0.803	0.241	0.269	0.609	0.847	0.443	0.166
	stan	0.001	0.000	0.111	0.116	0.111	0.116	0.983	0.984
	IPW	0.007	0.004	0.220	0.225	0.220	0.225	0.998	0.998
	match	0.052	0.057	0.162	0.164	0.170	0.173	0.984	0.985
	BCM	0.001	-0.006	0.149	0.153	0.149	0.153	0.990	0.989
	TMLE	0.001	-0.001	0.115	0.119	0.115	0.119	0.959	0.959
	OW	0.000	-0.003	0.153	0.159	0.153	0.159	0.999	0.997
	A-OW	0.001	-0.000	0.113	0.118	0.113	0.118	0.946	0.939
Inc.param.	stan	0.046	0.054	0.131	0.138	0.139	0.148	0.927	0.936
	IPW	0.091	0.082	0.142	0.147	0.169	0.168	0.932	0.940
	match	0.041	0.038	0.136	0.144	0.142	0.149	0.949	0.955
	BCM	0.031	0.020	0.135	0.143	0.138	0.144	0.955	0.956
	TMLE	0.076	0.073	0.135	0.145	0.155	0.162	0.934	0.941
	OW	0.082	0.073	0.135	0.139	0.158	0.158	0.941	0.947
	A-OW	0.068	0.065	0.131	0.138	0.148	0.152	0.912	0.922
M.Learning	stan	0.004	0.002	0.114	0.119	0.114	0.119	0.943	0.944
	IPW	0.133	0.128	0.150	0.175	0.200	0.217	0.878	0.850
	match*	0.077	0.076	0.160	0.186	0.178	0.201	0.913	0.895
	BCM*	-0.001	0.003	0.139	0.150	0.139	0.150	0.962	0.957
	TMLE	0.006	0.004	0.116	0.121	0.116	0.121	0.960	0.955
	OW	0.125	0.121	0.146	0.172	0.192	0.211	0.881	0.854
	A-OW	0.005	0.004	0.115	0.120	0.115	0.120	0.948	0.941

Legend: Cor.param=correct parametric models, Inc.param=incorrect parametric models, M.Learning=machine learning, Crude=Unadjusted, stan=standardization, IPW=inverse probability weighting, match=matching, BCM=bias-corrected matching, TMLE=targeted maximum likelihood, OW=overlap weights, A-OW=augmented overlap weights.*: 2 replications were discarded due to errors when running the code for *match* and *BCM*.

Table 6: Estimate of the treatment effect in the Scenario with a weak association between covariates and treatment, a strong association between covariates and outcome ($T - Y+$), and a sample size of 2000.

Implementation	Approach	Bias		Std		RMSE		Coverage IC	
		$\hat{\tau}_{21}$	$\hat{\tau}_{31}$	$\hat{\tau}_{21}$	$\hat{\tau}_{31}$	$\hat{\tau}_{21}$	$\hat{\tau}_{31}$	$\hat{\tau}_{21}$	$\hat{\tau}_{31}$
Cor.param.	Crude	0.572	0.815	0.122	0.138	0.584	0.827	0.002	0.000
	stan	0.001	0.003	0.058	0.058	0.058	0.058	0.982	0.986
	IPW	0.002	0.001	0.106	0.102	0.106	0.102	0.998	1.000
	match	0.024	0.023	0.077	0.075	0.081	0.078	0.983	0.986
	BCM	0.003	-0.001	0.075	0.073	0.076	0.073	0.986	0.989
	TMLE	0.001	0.002	0.059	0.059	0.059	0.059	0.961	0.958
	OW	0.001	-0.001	0.080	0.079	0.080	0.079	1.000	1.000
	A-OW	0.001	0.002	0.059	0.058	0.059	0.059	0.941	0.943
Inc.param.	stan	0.047	0.058	0.070	0.069	0.084	0.089	0.887	0.867
	IPW	0.093	0.088	0.075	0.073	0.119	0.114	0.810	0.839
	match	0.024	0.024	0.069	0.069	0.073	0.073	0.941	0.955
	BCM	0.019	0.016	0.068	0.068	0.071	0.070	0.955	0.959
	TMLE	0.077	0.077	0.072	0.072	0.105	0.105	0.826	0.855
	OW	0.084	0.079	0.072	0.070	0.111	0.105	0.826	0.860
	A-OW	0.069	0.070	0.070	0.069	0.098	0.098	0.814	0.830
M.Learning	stan	0.001	0.004	0.060	0.059	0.060	0.059	0.931	0.941
	IPW	0.086	0.072	0.075	0.072	0.114	0.102	0.836	0.892
	match	0.025	0.014	0.072	0.069	0.077	0.071	0.950	0.964
	BCM	-0.004	-0.003	0.069	0.068	0.069	0.068	0.963	0.966
	TMLE	0.002	0.004	0.061	0.060	0.061	0.060	0.950	0.964
	OW	0.078	0.064	0.072	0.069	0.106	0.094	0.853	0.906
	A-OW	0.001	0.004	0.060	0.059	0.060	0.060	0.934	0.947

Legend: Cor.param=correct parametric models, Inc.param=incorrect parametric models, M.Learning=machine learning, Crude=Unadjusted, stan=standardization, IPW=inverse probability weighting, match=matching, BCM=bias-corrected matching, TMLE=targeted maximum likelihood, OW=overlap weights, A-OW=augmented overlap weights

Table 7: Estimate of the treatment effect in the Scenario with a strong association between covariates and treatment, a strong association between covariates and outcome ($T + Y+$), and a sample size of 500.

Implementation	Approach	Bias		Std		RMSE		Coverage IC	
		$\hat{\tau}_{21}$	$\hat{\tau}_{31}$	$\hat{\tau}_{21}$	$\hat{\tau}_{31}$	$\hat{\tau}_{21}$	$\hat{\tau}_{31}$	$\hat{\tau}_{21}$	$\hat{\tau}_{31}$
Cor.param.	Crude	1.174	1.947	0.226	0.292	1.195	1.969	0.001	0.000
	stan	-0.002	-0.005	0.117	0.133	0.117	0.133	0.979	0.982
	IPW	0.117	0.128	0.693	0.713	0.703	0.725	0.841	0.868
	match	0.274	0.294	0.204	0.232	0.341	0.375	0.801	0.820
	BCM	0.036	0.007	0.182	0.210	0.185	0.210	0.966	0.970
	TMLE	-0.000	0.003	0.145	0.181	0.145	0.181	0.949	0.932
	OW	0.009	0.003	0.191	0.206	0.191	0.206	0.980	0.985
	A-OW	-0.001	0.001	0.138	0.155	0.138	0.155	0.944	0.946
Inc.param.	stan	-0.132	-0.046	0.145	0.160	0.196	0.167	0.841	0.929
	IPW	0.209	0.187	0.229	0.268	0.310	0.327	0.721	0.816
	match	0.153	0.156	0.169	0.198	0.228	0.252	0.850	0.873
	BCM	0.097	0.055	0.164	0.195	0.190	0.202	0.921	0.929
	TMLE	0.152	0.110	0.189	0.229	0.243	0.254	0.872	0.914
	OW	0.112	0.097	0.161	0.173	0.196	0.198	0.878	0.898
	A-OW	0.059	0.059	0.148	0.168	0.159	0.177	0.919	0.930
M.Learning	stan	-0.011	-0.003	0.128	0.149	0.129	0.149	0.938	0.941
	IPW	0.176	0.163	0.237	0.269	0.295	0.314	0.768	0.845
	match*	0.145	0.149	0.170	0.195	0.223	0.246	0.863	0.880
	BCM*	0.006	0.014	0.162	0.191	0.162	0.191	0.940	0.948
	TMLE	0.011	0.010	0.149	0.186	0.149	0.186	0.940	0.928
	OW	0.085	0.074	0.158	0.170	0.180	0.186	0.911	0.916
	A-OW**	-0.005	-0.003	0.141	0.156	0.141	0.156	0.933	0.938

Legend: Cor.param=correct parametric models, Inc.param=incorrect parametric models, M.Learning=machine learning, Crude=Unadjusted, stan=standardization, IPW=inverse probability weighting, match=matching, BCM=bias-corrected matching, TMLE=targeted maximum likelihood, OW=overlap weights, A-OW=augmented overlap weights.*: 3 replications were discarded due to errors when running the code for *match* and *BCM*, **: in 9 replications, the confidence intervals could not be computed.

Table 8: Estimate of the treatment effect in the Scenario with a strong association between covariates and treatment, a strong association between covariates and outcome ($T + Y+$), and a sample size of 2000.

Implementation	Approach	Bias		Std		RMSE		Coverage IC	
		$\hat{\tau}_{21}$	$\hat{\tau}_{31}$	$\hat{\tau}_{21}$	$\hat{\tau}_{31}$	$\hat{\tau}_{21}$	$\hat{\tau}_{31}$	$\hat{\tau}_{21}$	$\hat{\tau}_{31}$
Cor.param.	Crude	1.172	1.939	0.105	0.141	1.177	1.944	0.000	0.000
	stan	-0.000	0.000	0.059	0.068	0.059	0.068	0.984	0.984
	IPW	0.024	0.027	0.708	0.709	0.708	0.710	0.835	0.862
	match	0.159	0.162	0.103	0.116	0.189	0.199	0.727	0.770
	BCM	0.029	0.016	0.094	0.107	0.098	0.108	0.978	0.978
	TMLE	0.001	0.002	0.073	0.085	0.073	0.085	0.947	0.953
	OW	0.001	0.000	0.090	0.101	0.090	0.101	0.988	0.990
	A-OW	-0.001	0.000	0.067	0.074	0.067	0.074	0.946	0.955
Inc.param.	stan	-0.129	-0.041	0.073	0.083	0.148	0.093	0.530	0.897
	IPW	0.202	0.183	0.128	0.158	0.239	0.242	0.450	0.646
	match	0.103	0.097	0.087	0.100	0.135	0.139	0.785	0.842
	BCM	0.071	0.043	0.084	0.098	0.110	0.107	0.873	0.939
	TMLE	0.151	0.112	0.099	0.117	0.181	0.162	0.615	0.830
	OW	0.109	0.096	0.079	0.085	0.134	0.128	0.708	0.815
	A-OW	0.057	0.059	0.072	0.083	0.092	0.102	0.865	0.884
M.Learning	stan	0.001	0.017	0.064	0.076	0.064	0.077	0.940	0.931
	IPW	0.101	0.096	0.198	0.204	0.222	0.225	0.728	0.808
	match	0.089	0.084	0.089	0.101	0.126	0.131	0.813	0.862
	BCM	0.002	0.007	0.086	0.098	0.086	0.099	0.953	0.956
	TMLE	0.007	0.007	0.076	0.090	0.076	0.090	0.942	0.947
	OW	0.043	0.039	0.080	0.084	0.091	0.093	0.908	0.937
	A-OW*	-0.005	-0.003	0.068	0.076	0.068	0.076	0.952	0.949

Legend: Cor.param=correct parametric models, Inc.param=incorrect parametric models, M.Learning=machine learning, Crude=Unadjusted, stan=standardization, IPW=inverse probability weighting, match=matching, BCM=bias-corrected matching, TMLE=targeted maximum likelihood, OW=overlap weights, A-OW=augmented overlap weights. *: in 1 replications, the confidence intervals could not be computed.

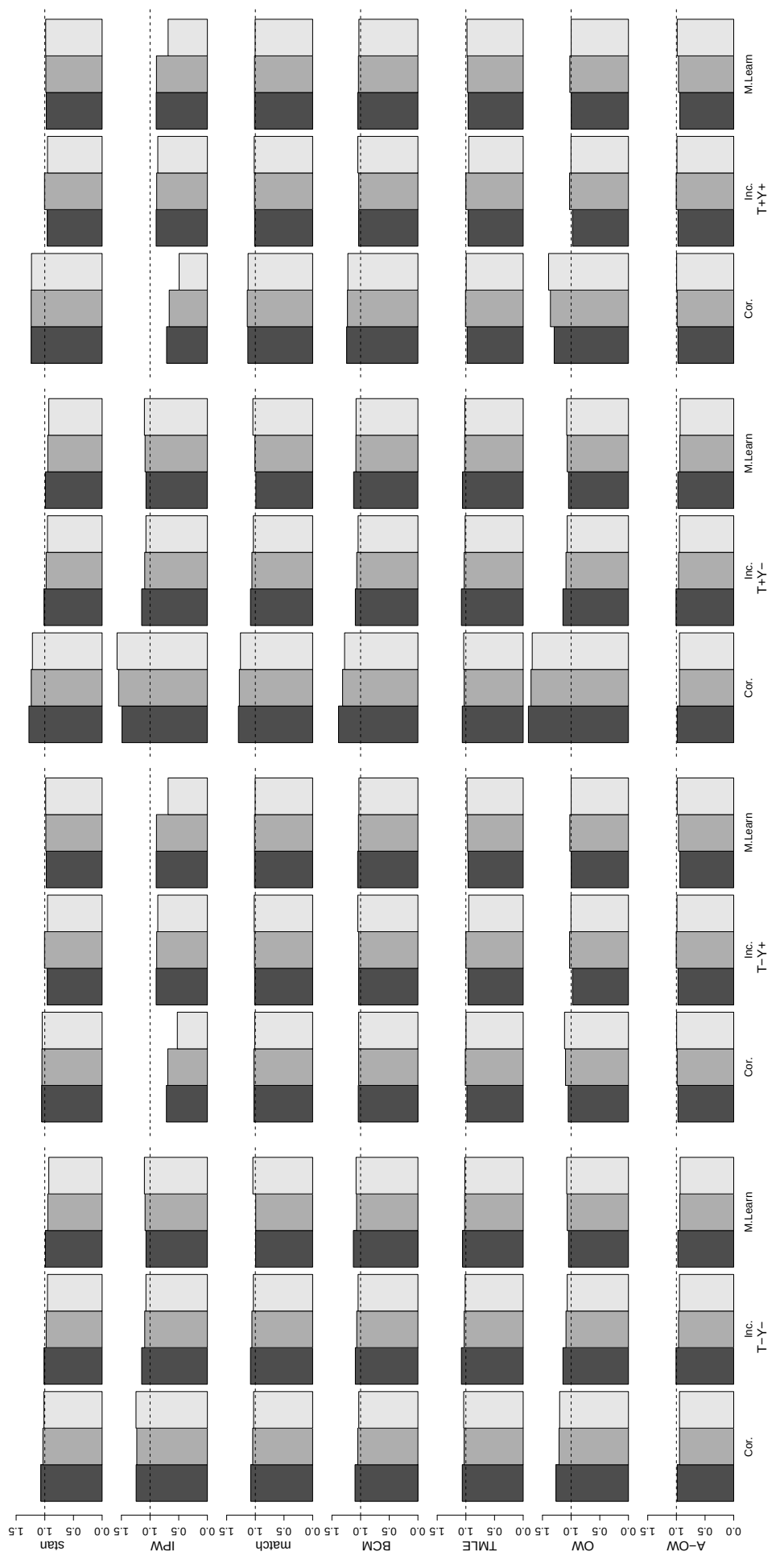


Figure 2: Ratio of the mean estimated standard error to the standard deviation of the estimates of τ_{21} for the correct parametric implementation according to approach (rows), sample size (dark gray = 500, gray = 1000, light gray = 2000), the strength of the association between covariates and treatment (columns; weak=$T-$ or strong=$T+$) and between the covariates and outcome (weak=$Y-$ or strong=$Y+$), and implementation (Cor = Correct parametric, Inc = Incorrect parametric, ML = machine learning)

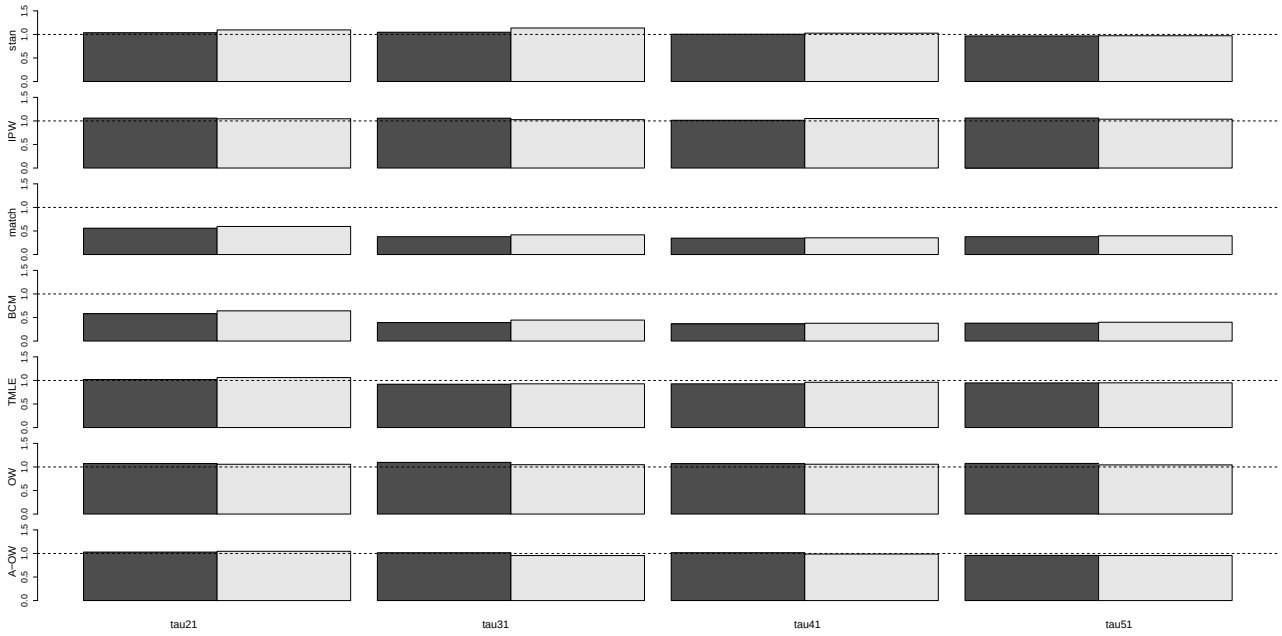
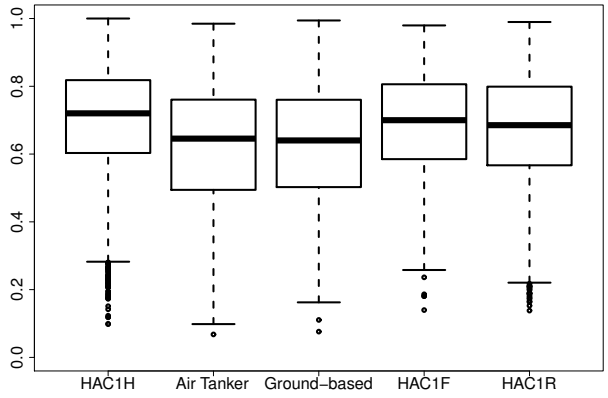
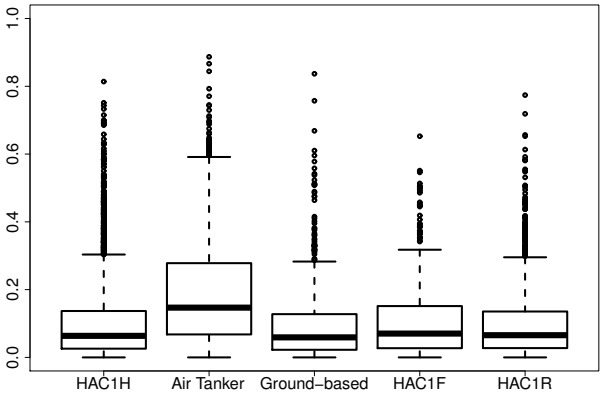


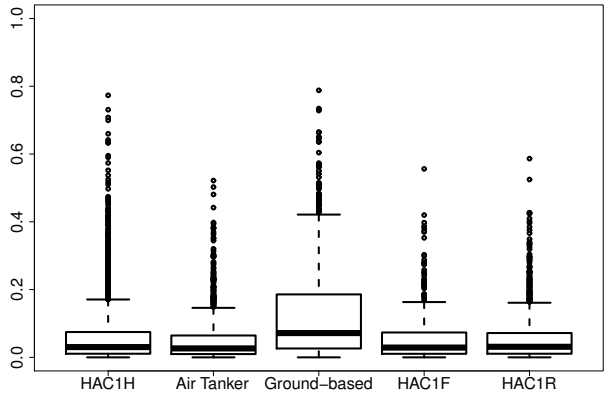
Figure 3: Ratio of the mean estimated standard error to the standard deviation of the estimates of the plasmode simulations, according to approach (rows), model parameter (columns) and implementation (dark gray = parametric, light gray = machine learning)



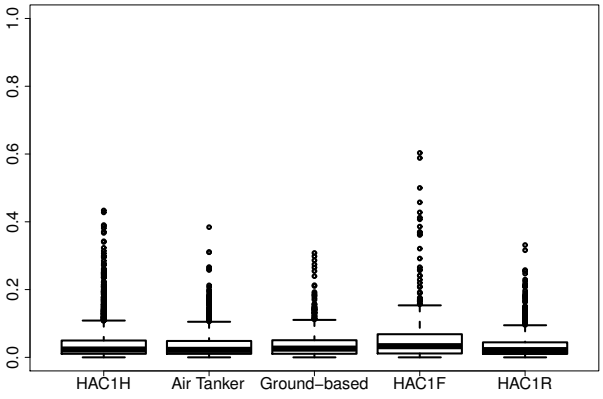
$P(T = \text{HAC1H}|\boldsymbol{X}_i)$



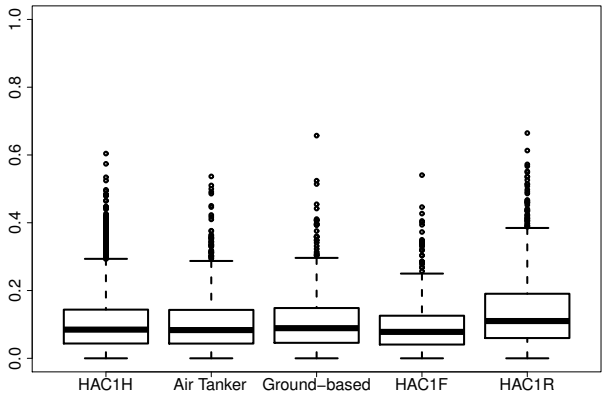
$P(T = \text{Air Tanker}|\boldsymbol{X}_i)$



$P(T = \text{Ground-based action}|\boldsymbol{X}_i)$



$P(T = \text{HAC1F}|\boldsymbol{X}_i)$



$P(T = \text{HAC1R}|\boldsymbol{X}_i)$

Figure 4: Distribution of the "true" treatment probabilities according to treatment group in the complete sample.

Web Appendix C - Descriptive Statistics of the Wildfire Data

Table 9: Characteristics of lightning-caused wildfires in Alberta, Canada, between 1996 and 2004 in provincial or public lands, according to initial intervention for suppressing the fire.

	HAC1H	Air tanker	Ground-based	HAC1F	HAC1R	SMD
ISI -2 days, mean(sd)	4.90 (3.83)	4.81 (3.55)	5.28 (8.85)	5.17 (3.98)	5.81 (11.74)	0.067
ISI -1 day, mean(sd)	5.17 (3.89)	5.46 (3.64)	5.21 (4.05)	5.71 (4.11)	5.48 (3.83)	0.068
ISI, mean(sd)	4.94 (3.76)	5.79 (3.88)	5.05 (4.50)	5.21 (3.92)	5.26 (3.80)	0.097
ISI +1 day, mean(sd)	3.50 (3.81)	3.83 (3.75)	3.45 (3.68)	3.73 (4.18)	3.71 (3.47)	0.052
ISI +2 days, mean(sd)	3.28 (3.62)	3.52 (3.57)	3.31 (3.69)	3.70 (3.91)	3.47 (3.63)	0.057
FWI -2 days, mean(sd)	11.50 (8.99)	11.52 (8.87)	12.15 (10.88)	12.30 (8.89)	12.67 (11.87)	0.063
FWI -1 day, mean(sd)	12.31 (9.24)	13.18 (8.77)	12.57 (9.54)	13.68 (9.28)	13.33 (9.46)	0.076
FWI, mean(sd)	12.10 (9.11)	14.15 (8.89)	12.43 (10.40)	13.01 (8.90)	13.09 (9.06)	0.103
FWI -1 day, mean(sd)	8.70 (9.06)	9.76 (9.23)	8.88 (9.52)	9.34 (9.57)	9.62 (9.05)	0.062
FWI -2 days, mean(sd)	8.17 (8.99)	9.00 (8.99)	8.21 (9.25)	9.36 (9.78)	8.98 (9.24)	0.068
Year						0.349
2003	248 (8.2)	61 (6.6)	38 (8.1)	57 (10.6)	52 (10.5)	
2004	277 (9.2)	151 (16.4)	47 (10.1)	82 (15.2)	78 (15.8)	
2005	177 (5.9)	89 (9.7)	24 (5.1)	31 (5.7)	27 (5.5)	
2006	335 (11.1)	95 (10.3)	78 (16.7)	67 (12.4)	56 (11.3)	
2007	242 (8.0)	79 (8.6)	31 (6.6)	22 (4.1)	40 (8.1)	
2008	363 (12.0)	129 (14.0)	46 (9.9)	74 (13.7)	65 (13.1)	
2009	272 (9.0)	57 (6.2)	45 (9.6)	63 (11.7)	48 (9.7)	
2010	354 (11.7)	88 (9.6)	76 (16.3)	56 (10.4)	62 (12.5)	
2011	109 (3.6)	13 (1.4)	14 (3.0)	13 (2.4)	14 (2.8)	
2012	217 (7.2)	36 (3.9)	37 (7.9)	26 (4.8)	17 (3.4)	
2013	149 (4.9)	31 (3.4)	9 (1.9)	14 (2.6)	11 (2.2)	
2014	274 (9.1)	91 (9.9)	22 (4.7)	35 (6.5)	25 (5.1)	
Fire discovery						0.252
Air patrol	738 (24.5)	146 (15.9)	77 (16.5)	136 (25.2)	103 (20.8)	
Lookout	1482 (49.1)	553 (60.1)	188 (40.3)	248 (45.9)	228 (46.1)	
Unplanned	797 (26.4)	221 (24.0)	202 (43.3)	156 (28.9)	164 (33.1)	
Ecological region						0.356
Clear Hills Upland	831 (27.5)	169 (18.4)	171 (36.6)	107 (19.8)	174 (35.2)	
Mid-Boreal Uplands	1764 (58.5)	680 (73.9)	194 (41.5)	348 (64.4)	251 (50.7)	
Other	422 (14.0)	71 (7.7)	102 (21.8)	85 (15.7)	70 (14.1)	
Fuel type						0.234
Boreal Spruce	2077 (68.8)	729 (79.2)	263 (56.3)	378 (70.0)	334 (67.5)	
Boreal Mixedwood – Green	543 (18.0)	84 (9.1)	106 (22.7)	82 (15.2)	77 (15.6)	
Other	397 (13.2)	107 (11.6)	98 (21.0)	80 (14.8)	84 (17.0)	
PM Period	2646 (87.7)	834 (90.7)	398 (85.2)	486 (90.0)	439 (88.7)	0.082
Month						0.180
May or June	1061 (35.2)	400 (43.5)	161 (34.5)	209 (38.7)	188 (38.0)	
July	1373 (45.5)	427 (46.4)	201 (43.0)	254 (47.0)	243 (49.1)	
August to October	583 (19.3)	93 (10.1)	105 (22.5)	77 (14.3)	64 (12.9)	
Response time, mean(sd)	3939.45 (9187.46)	2697.44 (6535.54)	7199.87 (11900.25)	3747.22 (8888.37)	3316.27 (7686.02)	0.202
Active fires, mean(sd)	59.31 (61.56)	81.84 (89.55)	79.95 (88.70)	71.66 (66.45)	64.63 (75.76)	0.158
ln(initial size), mean(sd)	-2.84 (1.96)	-1.72 (2.32)	-2.36 (2.65)	-2.75 (1.97)	-3.22 (1.90)	0.320

*N*umbers are n (%), unless otherwise indicated. Abbreviations: SMD = standardized mean difference, ISI = Initial Spread Index, FWI = Fire Weather Index, HAC1H = heli-attack crew with helicopter but no rappel capability, HAC1R = heli-attack crew with helicopter and rappel capability, HAC1F = fire-attack crew with or without a helicopter and no rappel capability.

Note: SMD were calculated using the `tableone` R package. References on formulas for computing SMD in the multivariate case are provided within the documentation.