

Supporting Information. Tang, J., and W.J. Riley. 2021. Finding Liebig's law of the minimum. *Ecological Applications*.

Appendix S1. Derivation of equation (7) based on Figure 1b.

The dynamic equations that follow the schema in Figure 1b are

$$\frac{d[EA]}{dt} = k_A^+[E][A] - k_B^+[B][EA], \quad (S1)$$

$$\frac{d[EB]}{dt} = k_B^+[E][B] - k_A^+[A][EB], \quad (S2)$$

$$\frac{d[EAB]}{dt} = k_A^+[EB][A] + k_B^+[EA][B] - k_2^+[EAB], \quad (S3)$$

$$[E]_T = [E] + [EA] + [EB] + [EAB], \quad (S4)$$

where subscript T means total concentration.

By aid of the steady-state approximation, equations (S1)-(S3) can be rewritten as

$$k_B^+[B][EA] = k_A^+[E][A], \quad (S5)$$

$$k_A^+[A][EB] = k_B^+[E][B], \quad (S6)$$

$$k_2^+[EAB] = k_A^+[EB][A] + k_B^+[EA][B]. \quad (S7)$$

By solving $[EA]$ from (S5), $[EB]$ from (S6), and then entering the result into (S7) to solve for $[EAB]$, we obtain

$$[EA] = \frac{k_A^+[A]}{k_B^+[B]} [E], \quad (S8)$$

$$[EB] = \frac{k_B^+[B]}{k_A^+[A]} [E], \quad (S9)$$

$$[EAB] = \frac{(k_A^+[A] + k_B^+[B])}{k_2^+} [E]. \quad (S10)$$

When equations (S8)-(S10) are entered into (S4), we have

$$[E] = \frac{[E]_T}{1 + \frac{k_A^+[A]}{k_B^+[B]} + \frac{k_B^+[B]}{k_A^+[A]} + \frac{(k_A^+[A] + k_B^+[B])}{k_2^+}}, \quad (S11)$$

which when combined with (S10) leads to

$$[EAB] = \frac{[E]_T}{1 + \frac{k_2^+}{k_B^+[B]} + \frac{k_2^+}{k_A^+[A]} + \frac{k_2^+}{(k_A^+[A] + k_B^+[B])}}, \quad (S12)$$

from which equation (7) in the main text can be obtained by calculating $k_2^+[EAB]$.