Appendix 1 Features of the dSIR and drSIR filters including use of them for T_1 mapping

The signals S_s and S_i for two long TR IR T_1-filters with short and intermediate TIs, TI_s and TI_i respectively are given by:

$$S_S = 1 - 2e(-TI_S/T_1) \quad [19]$$

and

$$S_i = 1 - 2e(-TI_i/T_1) \quad [20]$$

Performing the subtraction: magnitude of the IR signal $|S_s|$ in Eq. [19] minus magnitude of the IR signal $|S_i|$ in Eq. [20] gives the signal of the SIR filter S_{SIR} which is equal to $-S_s - S_i$ i.e.:

$$S_{SIR} = 2e(-TI_S/T_1) + 2e(-TI_i/T_1) - 2 \quad [21]$$

Addition of the magnitudes of the two IR signals $|S_s|$ and $|S_i|$ in Eqs. [19,20] S_{AIR} is equal to $-S_s + S_i$ i.e.:

$$S_{AIR} = 2e(-TI_S/T_1) - 2e(-TI_i/T_1) \quad [22]$$

Division of the signal of the subtraction filter S_{SIR} in Eq. [21] by the signal of the addition filter S_{AIR} in Eq. [22] gives the signal of the S_{dSIR} filter:

$$S_{dSIR} = \frac{e(-TI_S/T_1) + e(-TI_i/T_1) - 1}{e(-TI_S/T_1) - e(-TI_i/T_1)} \quad [23]$$

While this expression is accurate, it does not provide easy insight into the properties of the S_{dSIR} filter. To do this a linear regression of the form y = mx + c between the end-points of the mD produced by fitting a straight line between the first and last points of the mD (ie first point x = TI_s / ln 2 and y = 1, and last point x = TI_i / ln 2 and y = –1) can be used as an approximation for the S_{dSIR} filter so:

$$S_{dSIR} \approx \frac{\ln 4}{\Delta TI} T_1 - \frac{\Sigma TI}{\Delta TI} \quad [24]$$

Where $\Delta TI = TI_s - TI_i$ and $\Sigma TI = TI_s + TI_i$

The same applies to the drSIR filter except that it has a negative slope and a positive offset. Its signal equation is:

$$S_{drSIR} \approx -\frac{\ln 4}{\Delta TI} T_1 + \frac{\Sigma TI}{\Delta TI} \quad [25]$$

The expressions in Eq. [24,25] capture four key features of the dSIR filter, firstly, they show linear change of signal with T_1 in the mD, secondly, they have slopes equal to ln 4/ΔTI and -ln 4/ΔTI respectively, thirdly they show high sensitivity to small changes in T_1 when ΔTI is small, and fourthly the equations can be used to map T_1 since for S_{dSIR} and S_{drSIR}:

$$T_1 \approx \frac{\Delta TI}{\ln 4} S_{dSIR} - \frac{\Sigma TI}{\ln 4} \quad [26]$$

$$T_1 \approx -\frac{\Delta TI}{\ln 4} S_{drSIR} + \frac{\Sigma TI}{\ln 4} \quad [27]$$

The S_{dSIR} and S_{drSIR} maps show high contrast and high spatial resolution as for the two source images since they are linear voxel rescalings of these images (e.g., *Figure 37*) with the two caveats (i) it only applies to T_1s in the mD, and (ii) the reasoning applies to long TR IR images. If the TR is not long enough, correction of the T1 values is likely to be needed.

For absolute contrast, Cab from Eqs. [24,25] and using a linear X axis:

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$$C_{ab} = \Delta S_{dSIR} \approx \frac{\ln 4}{\Delta TI} \Delta T_1 \quad [28]$$

and

$$C_{ab} = \Delta S_{drSIR} \approx -\frac{\ln 4}{\Delta TI} \Delta T_1 \quad [29]$$

Thus the absolute contrast for the dSIR and drSIR filters is proportional to the reciprocal of ΔTI as well as the difference/change in T_1.

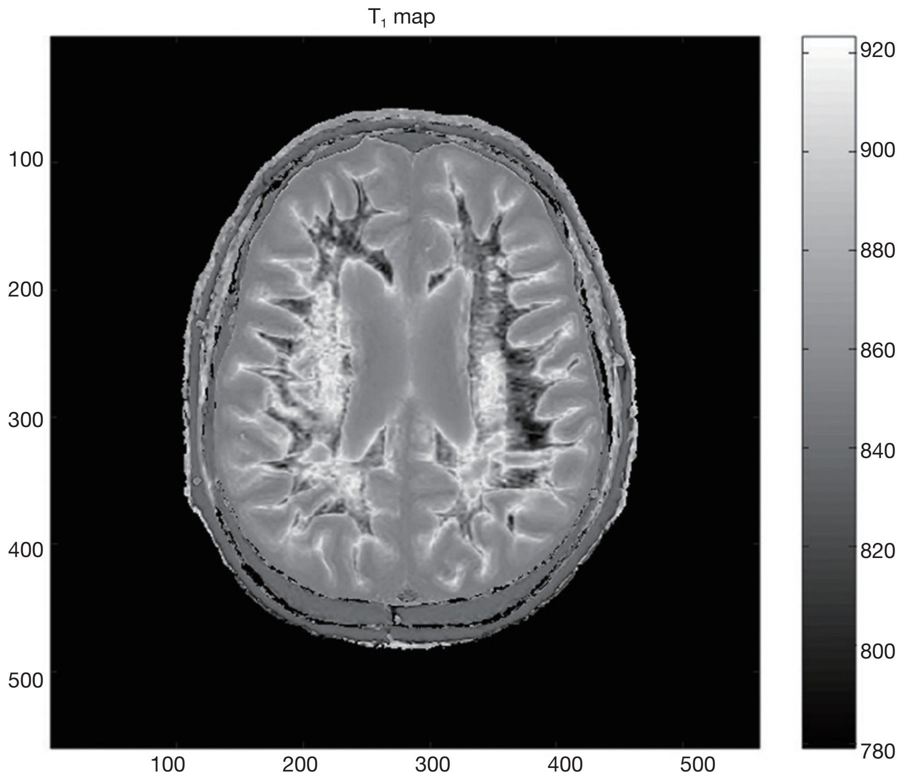

Figure S1 Rescaled dSIR image and T_1 map in a patient with small vessel disease showing T_1 values within the mD which is in white matter (TI_s=540 ms, TI_i=640 ms, ΔTI=19%, TR=6,000 ms at 3T, contrast amplification compared to TIs equal to 15 times). The gray-scale shows T_1 values over a range from 780 ms (i.e., 540/ln 2 ms) to 924 ms (i.e., 640/ln 2 ms) with the dark low signal representing shorter normal T_1 values of about 780 ms and higher signal representing abnormal T_1 values up to a maximum of about 924 ms. Lesions with T_1 values greater than the maximum in the mD "overshoot" (i.e., greater than about 924 ms) and appear mid-gray in their centers (where their T_1 values are unreliable). The T_1 maps of lesions that overshoot are surrounded by high signal boundaries. The T_1 maps are only valid in the mD and are obtained using long TR IR images, as in this case. If TR is short, the T_1 values may be too low and need to be corrected. dSIR, divided subtracted inversion recovery.

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