

Online Appendix to “Managing Two-Dose COVID-19 Vaccine Rollouts with Limited Supply”

OA1. Technical Proofs

Proof of Lemma 1

No backlogs exist any time at or before t if and only if

$$(1 - \beta)y(u) \geq v_1^r(u - L) \text{ for all } u \leq t.$$

The above statement can be rewritten as

$$(1 - \beta)y(nL + \tau) \geq v_1^r((n - 1)L + \tau) \text{ for } n = 1, 2, \dots \text{ and all } \tau \in [0, L]. \quad (\text{OA1})$$

We prove (OA1) by induction. First, the case in which $n = 1$ and $n = 2$ obviously holds for all $\tau \in [0, L]$ given the first two conditions of the lemma. Next, suppose the statement holds for $n = 1, \dots, k$. Then, for $n = k + 1$, it holds that

$$\begin{aligned} v_1^r(kL + \tau) &= y(kL + \tau) - v_2^r(kL + \tau) \\ &= y(kL + \tau) - v_1^r((k - 1)L + \tau) \quad (\text{because } B(t) = 0 \text{ for all } t \leq kL + \tau) \\ &= y(kL + \tau) - y((k - 1)L + \tau) + v_1^r((k - 2)L + \tau) \\ &\leq y(kL + \tau) - \beta y((k - 1)L + \tau) \quad (\text{by induction assumption}). \end{aligned}$$

Because $(1 - \beta)y((k + 1)L + \tau) \geq y(kL + \tau) - \beta y((k - 1)L + \tau)$, the statement also holds for $n = k + 1$. Thus, it holds for any positive integer n by induction, which completes the proof. *Q.E.D.*

Proof of Lemma 2

The rates of vaccination are given by

$$v_1^h(t) = \frac{1}{2} + \frac{\alpha}{2}t$$

$$v_2^h(t) = \begin{cases} 0 & \text{for } t \in [0, L) \\ \frac{1}{2} + \frac{\alpha}{2}(t - L) & \text{otherwise.} \end{cases}$$

The result follows from integrating the vaccination rates with respect to t . *Q.E.D.*

Proof of Proposition 1

First, in $t \in [0, L)$, all available supply is released for first-dose appointments:

$$\begin{aligned} v_1^r(t) &= 1 + \alpha t \\ V_1^r(t) &= t + \frac{\alpha}{2}t^2 \end{aligned}$$

$$P_1^r(t) = \frac{1}{2}t^2 + \frac{\alpha}{6}t^3$$

$$v_2^r(t) = V_2^r(t) = P_2^r(t) = 0.$$

For $t \in [L, 2L)$, the demand for the second dose is $v_1^r(t-L) = 1 + \alpha(t-L)$ and the total supply available is $1 + \alpha t$. For no backlogs to exist, all second-dose demand will be fulfilled, that is, $v_2^r(t) = 1 + \alpha(t-L)$, and the supply available for first doses will be $y(t) - v_2^r(t) = \alpha L$. To ensure this amount exceeds the fraction β , we require $\beta \leq \frac{\alpha L}{1 + \alpha t}$ for all $t \in [L, 2L)$, that is, $\beta \leq \frac{\alpha L}{1 + 2\alpha L} = \frac{1}{2} - \frac{1}{2(1 + 2\alpha L)}$.

For $t \in [2L, 3L)$, the demand for the second dose is $v_1^r(t-L) = \alpha L$ and the supply is $y(t) = 1 + \alpha t$. Fulfilling all second-dose demand, the capacity available for first doses is given by $1 + \alpha(t-L)$. For this amount to exceed a β fraction of available supply, we require $\beta \leq \frac{1 + \alpha(t-L)}{1 + \alpha t}$ for all $t \in [2L, 3L)$, that is, $\beta \leq \frac{1 + \alpha L}{1 + 2\alpha L}$, which is implied by the previous necessary condition, $\beta \leq \frac{\alpha L}{1 + 2\alpha L} = \frac{1}{2} - \frac{1}{2(1 + 2\alpha L)}$.

Continuing this argument, the vaccination rate follows an oscillating pattern:

$$v_1^r(t) = \begin{cases} 1 + \alpha(t - nL) & \text{for } t \in [2nL, (2n+1)L) \\ \alpha(n+1)L & \text{for } t \in [(2n+1)L, (2n+2)L) \end{cases}. \quad (\text{OA2})$$

$$v_2^r(t) = \begin{cases} \alpha nL & \text{for } t \in [2nL, (2n+1)L) \\ 1 + \alpha(t - nL) & \text{for } t \in [(2n+1)L, (2n+2)L) \end{cases}. \quad (\text{OA3})$$

The upper bound on β increases over time. Thus, $\beta \leq \frac{1}{2} - \frac{1}{2(1 + 2\alpha L)}$ is sufficient. *Q.E.D.*

Proof of Proposition 2

The stock-release policy increases vaccination rates by:

$$\begin{aligned} \Delta v_1^r(t) &= \begin{cases} 1 + \alpha(t - nL) - \frac{1}{2}(1 + \alpha t) & \text{for } t \in [2nL, (2n+1)L) \\ \alpha(n+1)L - \frac{1}{2}(1 + \alpha t) & \text{for } t \in [(2n+1)L, (2n+2)L) \end{cases} \\ &= \begin{cases} \frac{1}{2} + \frac{\alpha}{2}(t - 2nL) > \frac{1}{2} & \text{for } t \in [2nL, (2n+1)L) \\ -\frac{1}{2} + \frac{\alpha}{2}[(2n+2)L - t] > -\frac{1}{2} & \text{for } t \in [(2n+1)L, (2n+2)L) \end{cases}. \end{aligned}$$

In the interval $[2nL, t)$ where $t \in [2nL, (2n+1)L)$, the release policy increases the cumulative vaccinations by:

$$\begin{aligned} \int_{2nL}^t \Delta v_1^r(\tau) d\tau &= \frac{1}{2}(t - 2nL) + \frac{\alpha}{2} \int_{2nL}^t (\tau - 2nL) d\tau \\ &= \frac{1}{2}(t - 2nL) + \frac{\alpha}{4}(t - 2nL)^2. \end{aligned}$$

Similarly, in the interval $[(2n+1)L, t)$, where $t \in [(2n+1)L, (2n+2)L)$, the release policy increases the cumulative vaccinations by

$$\begin{aligned} \int_{(2n+1)L}^t \Delta v_1^r(\tau) d\tau &= -\frac{1}{2}[t - (2n+1)L] + \frac{\alpha}{2} \int_{(2n+1)L}^t [(2n+2)L - \tau] d\tau \\ &= -\frac{1}{2}[t - (2n+1)L] + \frac{\alpha}{4} \{L^2 - [2(n+1)L - t]^2\}. \end{aligned}$$

The above implies that over $[2nL, (2n+2)L)$, the gain amounts to $\frac{\alpha}{2}L^2$. Thus, over the time period $t \in [0, t)$, the release policy increases the cumulative vaccinations by:

$$\Delta V_1^r(t) = \begin{cases} \frac{\alpha}{2}nL^2 + \frac{1}{2}(t-2nL) + \frac{\alpha}{4}(t-2nL)^2 & \text{for } t \in [2nL, (2n+1)L) \\ \frac{\alpha}{2}nL^2 + \frac{1}{2}L + \frac{\alpha}{2}L^2 - \frac{1}{2}[t-(2n+1)L] \\ -\frac{\alpha}{4}[2(n+1)L-t]^2 & \text{for } t \in [(2n+1)L, (2n+2)L). \end{cases}$$

Going through similar steps, in the interval $[2nL, t)$, where $t \in [2nL, (2n+1)L)$, the release policy increases the cumulative protection by:

$$\begin{aligned} \int_{2nL}^t \Delta V_1^r(\tau) d\tau &= \frac{\alpha}{2}nL^2(t-2nL) + \int_{2nL}^t \left[(\tau-2nL) + \frac{\alpha}{4}(\tau-2nL)^2 \right] d\tau \\ &= \frac{\alpha}{2}nL^2(t-2nL) + \frac{1}{4}(t-2nL)^2 + \frac{\alpha}{12}(t-2nL)^3 \\ &= \frac{1}{4}L^2 + \frac{\alpha}{2}L^3 \left(n + \frac{1}{6} \right) \quad (\text{when } t = (2n+1)L). \end{aligned}$$

In the interval $[(2n+1)L, t)$, where $t \in [(2n+1)L, (2n+2)L)$, the release policy increases the cumulative protection by:

$$\begin{aligned} \int_{(2n+1)L}^t \Delta V_1^r(\tau) d\tau &= \left(\frac{\alpha}{2}nL^2 + \frac{1}{2}L + \frac{\alpha}{2}L^2 \right) [t-(2n+1)L] - \int_{(2n+1)L}^t \left\{ [\tau-(2n+1)L] - \frac{\alpha}{4}[(2n+2)L-\tau]^2 \right\} d\tau \\ &= \left(\frac{\alpha}{2}nL^2 + \frac{1}{2}L + \frac{\alpha}{2}L^2 \right) [t-(2n+1)L] - \frac{1}{4}[t-(2n+1)]^2 - \frac{\alpha}{12} \{ L^3 - [(2n+2)L-t]^3 \} \\ &= \frac{1}{4}L^2 + \frac{\alpha}{2}L^3 \left(n + \frac{5}{6} \right) \quad (\text{when } t = (2n+2)L). \end{aligned}$$

Thus, the gain in cumulative protection over the period $[0, t)$ is given by

$$\Delta P_1^r(t) = \begin{cases} \frac{n}{2}L^2 + \frac{\alpha}{2}L^3 (2\sum_{i=0}^n i) \\ + \frac{\alpha}{2}nL^2(t-2nL) + \frac{1}{4}(t-2nL)^2 + \frac{\alpha}{12}(t-2nL)^3 & \text{for } t \in [2nL, (2n+1)L) \\ \left(\frac{n}{2} + \frac{1}{4} \right) L^2 + \frac{\alpha}{2}L^3 (2\sum_{i=0}^n i + n + \frac{1}{6}) \\ + \left(\frac{\alpha}{2}nL^2 + \frac{1}{2}L + \frac{\alpha}{2}L^2 \right) [t-(2n+1)L] \\ - \frac{1}{4}[t-(2n+1)]^2 - \frac{\alpha}{12} [L^3 - [2(n+1)L-t]^3] & \text{for } t \in [(2n+1)L, (2n+2)L). \end{cases}$$

Over a full cycle $[0, 2nL)$, the release policy improves the cumulative protection by

$$\begin{aligned} \Delta P_1^r(2nL) &= \frac{n}{2}L^2 + \frac{\alpha}{2}L^3 \left(2\sum_{i=0}^n i \right) \\ &= \frac{n}{2}L^2 + \frac{\alpha}{2}L^3 n(n+1), \text{ and} \\ \Delta P_2^r(2nL) &= \Delta P_1^r((2n-1)L) \\ &= \left(\frac{n-1}{2} + \frac{1}{4} \right) L^2 + \frac{\alpha}{2}L^3 \left(2\sum_{i=0}^{n-1} i + n - 1 + \frac{1}{6} \right) \\ &= \left(\frac{n-1}{2} + \frac{1}{4} \right) L^2 + \frac{\alpha}{2}L^3 \left(n^2 - \frac{5}{6} \right). \end{aligned}$$

Q.E.D.

Proof of Proposition 3

Following Proposition 2, the cumulative first- and second-dose inoculations under the stock-release policy are given by:

$$V_1^r(t) = \begin{cases} \frac{t}{2} + \frac{\alpha t^2}{4} + \frac{\alpha}{2}nL^2 + \frac{1}{2}(t - 2nL) + \frac{\alpha}{4}(t - 2nL)^2 & \text{for } t \in [2nL, (2n+1)L) \\ \frac{t}{2} + \frac{\alpha t^2}{4} + \frac{\alpha}{2}nL^2 + \frac{1}{2}L + \frac{\alpha}{2}L^2 - \frac{1}{2}[t - (2n+1)L] & \text{for } t \in [(2n+1)L, (2n+2)L) \\ -\frac{\alpha}{4}[2(n+1)L - t]^2 & \text{for } t \in [(2n+1)L, (2n+2)L) \end{cases}$$

$$V_2^r(t) = V_1^r(t - L).$$

At $t = 2mL$, we have

$$\begin{aligned} V_1^r(2mL) &= mL + \alpha \left(m^2 + \frac{m}{2} \right) L^2 \\ V_2^r(2mL) &= \frac{(2m-1)L}{2} + \frac{\alpha}{4}(2m-1)^2 L^2 + \frac{\alpha}{2}(m-1)L^2 + \frac{1}{2}L + \frac{\alpha}{4}L^2 \\ &= mL + \frac{3\alpha}{4}L^2(2m^2 - 2m + 1). \end{aligned}$$

For the stretching policy, the same derivation holds with the increased lead time:

$$\begin{aligned} V_1^s(2mL) &= L^s + \alpha \left(1 + \frac{1}{2} \right) (L^s)^2 \\ &= mL + \frac{3\alpha m^2 L^2}{2} \\ &= V_1^r(2mL) + \alpha \cdot \frac{m^2 - m}{2} \cdot L^2 \\ V_2^s(2mL) &= L^s + \frac{3\alpha}{4}(L^s)^2 \\ &= mL + \frac{3\alpha}{4}L^2 m^2 \\ &= V_2^r(2mL) - \frac{3\alpha}{4}L^2(m^2 - 2m + 1). \end{aligned}$$

Similarly, following Proposition 2, the cumulative protection under the stock-release policy is given by:

$$\begin{aligned} P_1^r(2mL) &= m^2 L^2 + \frac{2\alpha}{3} m^3 L^3 + \frac{m}{2} L^2 + \frac{\alpha}{2} L^3 m(m+1) \\ &= \frac{2m^2 + m}{2} L^2 + \alpha L^3 \left[\frac{2}{3} m^3 + \frac{1}{2} m(m+1) \right] \\ P_2^r(2mL) &= \frac{1}{4}(2m-1)^2 L^2 + \frac{\alpha}{12}(2m-1)^3 L^3 + \left(\frac{m-1}{2} + \frac{1}{4} \right) L^2 + \frac{\alpha}{2} L^3 \left(m^2 - \frac{5}{6} \right) \\ &= \frac{3}{4}(2m^2 - 2m + 1)L^2 + \frac{\alpha L^3}{12}(8m^3 - 6m^2 + 6m - 6). \end{aligned}$$

Similarly, under the dose-stretching policy, the cumulative protection can be written as:

$$P_1^s(2mL) = \frac{3}{2}(L^s)^2 + \frac{5}{3}\alpha(L^s)^3$$

$$\begin{aligned}
&= P_1^r(2mL) + \frac{m^2 - m}{2}L^2 + \alpha L^3[2m^3 - m(m+1)] \\
P_2^s(2mL) &= \frac{3}{4}m^2L^2 + \frac{\alpha L^3}{12}(2m^3) \\
&= P_2^r(2mL) - \frac{3}{4}(m^2 - 2m + 1)L^2 - \frac{\alpha L^3}{2}(m^3 - m^2 + m - 1) \\
&= P_2^r(2mL) - \frac{3}{4}(m-1)^2L^2 - \frac{\alpha L^3}{2}(m-1)(m^2 + 1),
\end{aligned}$$

which completes the proof.

Q.E.D.

Proof of Proposition 4

Following Proposition 3, dose stretching increases efficacy-weighted protection by:

$$\begin{aligned}
\Delta P^s(2mL) &= \lambda(1 - \theta_1) [\Delta P_1^s(2mL) + \zeta \Delta P_2^s(2mL)] \\
&= \frac{L^2}{4}\lambda(1 - \theta_1) [2m(m-1) - 3(m-1)^2\zeta] + \frac{\alpha L^3}{2}\lambda(1 - \theta_1) [4m^3 - 2m(m+1) - (m-1)(m^2 + 1)\zeta] \\
&= \frac{L^2}{4}\lambda(1 - \theta_1) [2m(m-1) - 3(m-1)^2\zeta] + \frac{\alpha L^3}{2}\lambda(1 - \theta_1) [2(m-1)(2m^2 + m) - (m-1)(m^2 + 1)\zeta].
\end{aligned}$$

If $m = 1$ or $\zeta \leq \frac{2m}{3(m-1)}$, the term $[2m(m-1) - 3(m-1)^2\zeta] \geq 0$. Also, for any $m \geq 1$, $2m^2 + m > m^2 + 1$. Thus the term $[2(m-1)(2m^2 + m) - (m-1)(m^2 + 1)\zeta] > 0$ if $\zeta \leq 2$. Because $\frac{2m}{3(m-1)} < 2$ for all $m > 1$, $\Delta P^s(2mL) > 0$ as long as $\zeta \leq \frac{2m}{3(m-1)}$.

Q.E.D.

Proof of Proposition 5

Because $v^d(t) = 1 + \alpha t$, we have $V^d(t) = t + \frac{\alpha}{2}t^2$ and $P^d(t) = \frac{t^2}{2} + \frac{\alpha}{6}t^3$. These expressions directly lead to the comparisons with the hold-back policy.

Comparing against the stock-release policy gives

$$\begin{aligned}
V_1^d(2nL) - V_1^r(2nL) &= 2nL + \frac{\alpha}{2}(2nL)^2 - \frac{2nL}{2} - \frac{\alpha}{4}(2nL)^2 - \frac{\alpha}{2}nL^2 \\
&= nL + \frac{\alpha}{2}(2n^2 - n) \\
P_1^d(2nL) - P_1^r(2nL) &= \frac{1}{4}(2nL)^2 + \frac{\alpha}{12}(2nL)^3 - \frac{n}{2}L^2 - \frac{\alpha}{2}L^3n(n+1) \\
&= \frac{n}{2}L^2 + \frac{\alpha}{2}L^3 \left[\frac{4}{3}n^3 - n(n+1) \right],
\end{aligned}$$

which completes the proof.

Q.E.D.

Proof of Proposition 6

Recall from the proof of Proposition 4 that

$$\begin{aligned}
P_1^r(2nL) &= \frac{2n^2 + n}{2}L^2 + \alpha L^3 \left[\frac{2}{3}n^3 + \frac{1}{2}n(n+1) \right] \\
P_2^r(2nL) &= \frac{3}{4}(2n^2 - 2n + 1)L^2 + \frac{\alpha L^3}{12}(8n^3 - 6n^2 + 6n - 6).
\end{aligned}$$

For the single-dose regimen,

$$P_1^d(2nL) = 2n^2L^2 + \frac{4\alpha}{3}n^3L^3.$$

Comparing the efficacy-weighted protection levels,

$$\begin{aligned} \Delta P^d(2nL) &= \lambda(1 - \theta_1)\zeta^d P_1^d(2nL) - \lambda(1 - \theta_1)[P_1^r(2nL) + \zeta P_2^r(2nL)] \\ &= \lambda(1 - \theta_1) \left\{ \zeta^d \left(2n^2L^2 + \frac{4\alpha}{3}n^3L^3 \right) - \frac{2n^2 + n}{2}L^2 - \alpha L^3 \left[\frac{2}{3}n^3 + \frac{1}{2}n(n+1) \right] \right. \\ &\quad \left. - \frac{3}{4}\zeta(2n^2 - 2n + 1)L^2 - \zeta \frac{\alpha L^3}{12}(8n^3 - 6n^2 + 6n - 6) \right\} \\ &= \lambda(1 - \theta_1)L^2 \left[2n^2\zeta^d - \frac{2n^2 + n}{2} - \frac{3\zeta}{4}(2n^2 - 2n + 1) \right] \\ &\quad + \lambda(1 - \theta_1)\frac{\alpha L^3}{12} [16n^3\zeta^d - 8n^3 - 6n^2 - 6n - \zeta(8n^3 - 6n^2 + 6n - 6)] \\ &= \lambda(1 - \theta_1)L^2 \left[2n^2(\zeta^d - 1) + \frac{n-1}{2} + \left(\frac{1}{2} - \frac{3\zeta}{4} \right) (2n^2 - 2n + 1) \right] \\ &\quad + \lambda(1 - \theta_1)\frac{\alpha L^3}{12} [(16\zeta^d - 16)n^3 - 12n + 6 + (1 - \zeta)(8n^3 - 6n^2 + 6n - 6)]. \end{aligned}$$

In the above, the L^2 term is non-negative if $\zeta^d \geq 1$ and $\zeta \leq \frac{2}{3}$. The L^3 term is non-negative if $\zeta \leq 1$ and $\zeta^d \geq \frac{11}{8}$, which ensures $(16\zeta^d - 16)n^3 \geq 6n^3$. Thus, a sufficient condition for $\Delta P^d(2nL) \geq 0$ is that $\zeta^d \geq \frac{11}{8}$ and $\zeta \leq \frac{2}{3}$.

Q.E.D.

OA2. SEIR Epidemic Model

In this section, we first provide technical details of our SEIR model in [Section OA2.1](#). Next, we provide the rationale for the parameter values used in our baseline case in [Section OA2.2](#).

OA2.1. SEIR Model Description

We consider a variant of the standard SEIR model to incorporate the effect of vaccination on the evolution of the pandemic.

- We consider the population to comprise of a high-risk group (age 65 and above) and a low-risk group (64 or below), indexed by $i \in \{H, L\}$. The initial population size of group i is denoted by N_i .¹

- At any time t , following the vaccine rollout policy, the high- and low-risk groups can further be divided into three subgroups each: those who have received k doses of the vaccine, where $k = 0, 1, 2$. We refer to individuals in risk group i who have received k vaccine doses as type (i, k) .

¹ We thank the Associate Editor for proposing to use the age as the basis for determining the cutoff between two risk groups. We note that under the cutoff of 65, the death rate of the elderly is more than 1,000% higher than the younger group. In addition, the data regarding vaccine efficacy rate, hospitalization, transmission rate, etc. are available for these two groups.

- The structure of the compartmental model follows that of Keskinocak et al. (2020). At any time, a each type (i, k) patient is in one of the following states: susceptible (S), exposed (E), infectious-presymptomatic (IP), infectious-asymptomatic (IA), infectious-symptomatic (IS), hospitalized (H), recovered (R), or deceased (D). See Figure 1 in Section 7.1 for a graphical illustration.
- We assume that vaccination reduces susceptibility, infectiousness, probability of symptomatic disease, probability of hospitalization and probability of death.

Infection. Similar to Bubar et al. (2020) and consistent with the notion of contact discussed by Diekmann and Heesterbeek (2000), a susceptible person of type (i, k) faces a rate of exposure expressed by

$$\lambda_{ik}^E = u_i \theta_k \sum_{j \in \{H, L\}} c_{ij} \frac{\sum_{l=0}^2 \phi_l (IS_{jl} + \delta^P IP_{jl} + \delta^A IA_{jl})}{N_j - \Omega_j},$$

where u_i is the probability of virus transmission given contact with an infectious individual and c_{ij} is the expected number of contacts between a group i and a group j individual (per unit time). The adjustment factors δ^P and δ^A reflects the lower infectiousness of presymptomatic and asymptomatic patients. The parameters θ_k and ϕ_k reflect susceptibility (of an uninfected individual) and infectiousness (of an infected individual) after receiving k doses of the vaccine ($k \in \{0, 1, 2\}$). Naturally, $0 \leq \theta_2 \leq \theta_1 \leq \theta_0 = 1$ and $0 \leq \phi_2 \leq \phi_1 \leq \phi_0 = 1$, i.e., vaccination reduces susceptibility and infectiousness. The variables IP_{jl} , IS_{jl} , and IA_{jl} denote the type (j, l) individuals who are in the infectious presymptomatic, symptomatic and asymptomatic states, respectively, and Ω_j denotes the cumulative number of deceased individuals in risk group j . We assume that hospitalized individuals are isolated and do not transmit the virus to others. The term $\frac{\sum_{l=0}^2 \phi_l (IS_{jl} + \delta^P IP_{jl} + \delta^A IA_{jl})}{N_j - \Omega_j}$ can be interpreted as the probability that a randomly-encountered group- j person is infectious.

Disease Severity. Let T^E and T^P be the average time that an infected patient spends in the exposed and infected-presymptomatic states, respectively.

At the end of the presymptomatic state, a group i patient develops symptoms with probability η_i^S . In this case, the patient spends an average of T_i^S in the infected-symptomatic state, at the end of which the patient can either become hospitalized (with probability η_i^H) or recovered.

To reflect how the fatality rate depends on the load of the healthcare system (e.g., ICU utilization), we consider a two-step function of death rates. In particular, if the total number of hospitalizations remains lower than a threshold K (effective capacity of the healthcare system), a hospitalized group i patient transitions to the deceased state at the base probability of η_i^D ; if the number of hospitalized individuals exceeds K , the probability becomes $\kappa \eta_i^D$, where $\kappa > 1$.

Similar to Keskinocak et al. (2020), we assume that all fatalities go through the hospitalized (H) state, i.e., a patient cannot transition directly from infected-symptomatic (IS) to deceased

(D). Thus, one can interpret the H state in our model as cases of severe disease that *require* hospitalization, although not all patients in this state may be admitted to hospital in reality due to factors such as access or capacity. Nevertheless, all such cases constitute the potential load on the healthcare system and are thus counted toward the capacity threshold for higher death rates.

On the other hand, if a patient becomes asymptomatic (milder disease) following the presymptomatic stage, the patient recovers after an average time of T^A .

To reflect the vaccine's effect on disease severity, we consider that group i patients who have received the k -th dose will see their probabilities of symptomatic disease, hospitalization and death (unvacinated) drop to $\tau_k \eta_i^S$, $\gamma_k \eta_i^H$ and $\rho_k \eta_i^D$, respectively. Similarly, once the capacity threshold for hospitalization is reached, the death probability rises to $\rho_k \kappa \eta_i^D$.

Vaccination Rollout. The external vaccine supply process is specified as follows. The supply rate $y(t) = 0$ for $t \in [0, 50)$; thereafter, we have $y(t) = y(0) + \alpha t$ for $t \geq 50$, where $y(0) = 0.001$ (supply at day 50 equals 0.1% of the population) and $\alpha = 0.02y(0)$ (2% increase in supply rate per day).

At time t , following the vaccination rates determined from the chosen rollout policy (as discussed in Sections 4 to 6), $v_1(t)$ and $v_2(t)$ doses are made available for first and second doses. We assume that these are proportionally allocated to the H and L groups; that is, the rate at which type (i, k) patients are vaccinated is denoted by

$$v_{ik}(t) = \begin{cases} \pi \cdot v_k(t) & \text{if } i = H \\ (1 - \pi) \cdot v_k(t) & \text{if } i = L \end{cases}.$$

With a finite population and the probability of mortality, it is possible that the allocated vaccination rates exceed the number of remaining eligible individuals (particularly in group H). In such case, the excess supplies are reallocated to group L .

Consistent with practice, we assume that all individuals without symptoms (i.e., the S , E , IP , IA , and R compartments) have uniform probability of receiving the vaccine allocated to the risk group i . As an approximation, we do not consider a minimum time gap between recovery and eligibility to receive the vaccine.

Differential Equation Formulation. Based on the assumptions discussed above, the SEIR model can be formulated as follows (suppressing (t) for brevity):

$$S'_{ik} = -S_{ik} \lambda_{ik}^E + \frac{v_{i,k-1} S_{i,k-1}}{S_{i,k-1} + E_{i,k-1} + IP_{i,k-1} + IA_{i,k-1} + R_{i,k-1}} - \frac{v_{ik} S_{ik}}{S_{ik} + E_{ik} + IP_{i,k} + IA_{i,k} + R_{ik}} \quad (\text{OA4})$$

$$E'_{ik} = S_{ik} \lambda_{ik}^E - E_{ik} \frac{1}{T^E} + \frac{v_{i,k-1} E_{i,k-1}}{S_{i,k-1} + E_{i,k-1} + IP_{i,k-1} + IA_{i,k-1} + R_{i,k-1}} - \frac{v_{ik} E_{ik}}{S_{i,k} + E_{i,k} + IP_{i,k} + IA_{i,k} + R_{i,k}} \quad (\text{OA5})$$

$$IP'_{ik} = E_{ik} \frac{1}{T^E} - IP_{ik} \frac{1}{T^P} + \frac{v_{i,k-1} IP_{i,k-1}}{S_{i,k-1} + E_{i,k-1} + IP_{i,k-1} + IA_{i,k-1} + R_{i,k-1}} - \frac{v_{ik} IP_{ik}}{S_{i,k} + E_{i,k} + IP_{i,k} + IA_{i,k} + R_{i,k}} \quad (\text{OA6})$$

$$IA'_{ik} = IP_{ik} \frac{1 - \tau_k \eta_i^S}{T^P} - IA_{ik} \frac{1}{T^A} + \frac{v_{i,k-1} IA_{i,k-1}}{S_{i,k-1} + E_{i,k-1} + IP_{i,k-1} + IA_{i,k-1} + R_{i,k-1}} - \frac{v_{ik} IA_{ik}}{S_{i,k} + E_{i,k} + IP_{i,k} + IA_{i,k} + R_{i,k}} \quad (\text{OA7})$$

$$IS'_{ik} = IP_{ik} \frac{\tau_k \eta_i^S}{T^P} - IS_{ik} \frac{1}{T^S} \quad (\text{OA8})$$

$$H'_{ik} = IS_{ik} \frac{\gamma_k \eta_i^H}{T^S} - H_{ik} \frac{1}{T^H} \quad (\text{OA9})$$

$$D'_{ik} = \begin{cases} H_{ik} \frac{\rho_k \eta_i^D}{T^H} & \text{if } \sum_{i,k} H_{ik} \leq K \\ H_{ik} \frac{\rho_k \kappa \eta_i^D}{T^H} & \text{if } \sum_{i,k} H_{ik} > K \end{cases} \quad (\text{OA10})$$

$$R_{ik} = \begin{cases} IA_{ik} \frac{1}{T^A} + IS_{ik} \frac{1 - \gamma_k \eta_i^H}{T^S} + H_{ik} \frac{1 - \rho_k \eta_i^D}{T^H} + \frac{v_{i,k-1} R_{i,k-1}}{S_{i,k-1} + E_{i,k-1} + IP_{i,k-1} + IA_{i,k-1} + R_{i,k-1}} - \frac{v_{ik} R_{ik}}{S_{i,k} + E_{i,k} + IP_{i,k} + IA_{i,k} + R_{i,k}} & \text{if } \sum_{i,k} H_{ik} \leq K \\ IA_{ik} \frac{1}{T^A} + IS_{ik} \frac{1 - \gamma_k \eta_i^H}{T^S} + H_{ik} \frac{1 - \rho_k \kappa \eta_i^D}{T^H} + \frac{v_{i,k-1} R_{i,k-1}}{S_{i,k-1} + E_{i,k-1} + IP_{i,k-1} + IA_{i,k-1} + R_{i,k-1}} - \frac{v_{ik} R_{ik}}{S_{i,k} + E_{i,k} + IP_{i,k} + IA_{i,k} + R_{i,k}} & \text{if } \sum_{i,k} H_{ik} > K. \end{cases} \quad (\text{OA11})$$

OA2.2. Parameter Calibration for the SEIR model

In Table OA1, we provide the rationale for the choice of parameter values used in the simulation of our baseline case (Section 7.2).

References

- Bubar KM, Reinholt K, Kissler SM, Lipsitch M, Cobey S, Grad YH, Larremore DB (2021) Model-informed COVID-19 vaccine prioritization strategies by age and serostatus. *Sci.* 371(6532) 916–921.
- Centers for Disease Control and Prevention (CDC) (2021) COVID-19 pandemic planning scenarios. Accessed May 10, 2021, <https://www.cdc.gov/coronavirus/2019-ncov/hcp/planning-scenarios.html>.
- Diekmann, O., J. A. P. Heesterbeek. 2000. *Mathematical Epidemiology of Infectious Diseases* (John Wiley & Sons, West Sussex, England).
- Haas EJ, Angulo FJ, McLaughlin JM, Anis E, Singer SR, Khan F, Brooks N, et al. (2021) Impact and effectiveness of mRNA BNT162b2 vaccine against SARS-CoV-2 infections and COVID-19 cases, hospitalisations, and deaths following a nationwide vaccination campaign in Israel: an observational study using national surveillance data. *Lancet* 397(10287):1819–1829.
- Harris RJ, Hall JA, Zaidi A, Andrews NJ, Dunbar JK, Dabrera G (2021) Impact of vaccination on household transmission of SARS-COV-2 in England. Accessed May 16, 2021, <https://khub.net/documents/135939561/390853656/Impact+of+vaccination+on+household+transmission+of+SARS-COV-2+in+England.pdf/35bf4bb1-6ade-d3eb-a39e-9c9b25a8122a?t=1619551571214>.

Table OA1 Parameter Values Used in the Baseline Case (Section 7.2)

Parameter	Value	Rationale
$[N_L, N_H]$	[87, 13]	In 2019, 13% of the U.S. population (over 20) was age 65 or above (U.S. Census Bureau 2021). We normalize $N_L + N_H = 100$.
$E_{ik}(0)$	0.01 for $i = H, L$ and $k = 0$; 0 otherwise	$E_{ik}(t)$ is the size of infectious group for type (i, k) . We assume the pandemic begins with 0.01% (1 in 10,000) of the population becoming exposed in each of the two groups.
$[u_L, u_H]$	[0.84, 0.84]	u_i is the base probability of virus transmission given contact. We take the mid-point of values calibrated for above-20 age groups in Bubar et al. (2021).
$\begin{bmatrix} c_{LL} & c_{HL} \\ c_{LH} & c_{HH} \end{bmatrix}$	$\begin{bmatrix} 0.841 & 0.071 \\ 0.258 & 0.318 \end{bmatrix}$	The number of group- j individuals contacted by an group- i individual per day. We first regroup the U.S. contact matrix from Prem et al. (2017) into the 20-64 ($i = L$) and 65+ ($i = H$) bins. Then, the matrix is rescaled such that the reproductive coefficient is 1.5.
$[\delta^P, \delta^A]$	[0.48, 0.48]	Transmissibility adjustment factors in pre/asymptomatic states (Keskinocak et al. 2020).
T^E	4.6 days	The average duration of exposed state (Keskinocak et al. 2020).
T^P	0.5 days	The average duration of presymptomatic state (Keskinocak et al. 2020).
T^S	2.9 days	The average duration of symptomatic state (Keskinocak et al. 2020).
T^H	10.4 days	The average duration of hospitalization (Keskinocak et al. 2020).
T^A	1.93 days	The average duration of asymptomatic state, computed using the symptomatic-asymptomatic duration ratio of 1.5 (Keskinocak et al. 2020).
$[\eta_L^S, \eta_H^S]$	[0.431, 0.653]	The probability of symptomatic given infection (from Israel's nationwide rollout, Haas et al. 2021).
$[\eta_L^H, \eta_H^H]$	[0.104, 0.593]	The probability of hospitalization given symptomatic (from Israel's nationwide rollout, Haas et al. 2021).
$[\eta_L^D, \eta_H^D]$	[0.043, 0.303]	The probability of death given hospitalization (from Israel's nationwide rollout, Haas et al. 2021).
K	0.108	On average, about 25% of hospitalized patients require ICU admission (CDC 2021). The U.S. ICU capacity is 0.027 beds per 100 population (Kaiser Family Foundation 2021), which translates into $0.027/0.25 = 0.108$ effective hospitalizations.
κ	1.92	When ICU capacity is reached, fatality rates increase by 92% (Wilde et al. 2021).
$[\theta_1, \theta_2]$	$[0.423, 0.047]$ for Pfizer, $[0.34, 0.34]$ for Johnson & Johnson	One minus the first- and second-dose efficacy rates of the Pfizer vaccine (from Israel's nationwide rollout, Haas et al. 2021)
$[\phi_1, \phi_2]$	[0.57, 0.57]	The Pfizer vaccine reduces transmissibility by 43% (from study by Public Health England, Harris et al. 2021).
$[\tau_1, \tau_2]$	[0.929, 0.884]	Reduction factors for the conditional probability of symptomatic disease given infection (from Israel's nationwide rollout, Haas et al. 2021) with vaccination.
$[\gamma_1, \gamma_2]$	[0.901 2.477]	Reduction factors for the conditional probability of hospitalization given symptomatic disease (from Israel's nationwide rollout, Haas et al. 2021) with vaccination. The second dose increases the <i>conditional</i> probability, despite reducing the overall probability of hospitalization.
$[\rho_1, \rho_2]$	[1.077, 1.799]	Reduction factors for the conditional probability of death given hospitalization (from Israel's nationwide rollout, Haas et al. 2021) with vaccination. Vaccine increases the <i>conditional</i> probabilities, despite reducing the overall probabilities of death.
π	1 (except in Section 7.4, where its value is 0.5)	The fraction of vaccine supply allocated to H group is 100%, following age-based prioritization (e.g., in the U.S. and U.K.). We consider the effect of prioritization by considering allocating only half of supplies to H group in Section 7.4.

- Kaiser Family Foundation (2021) State health facts – ICU beds. Accessed May 10, 2021, <https://www.kff.org/other/state-indicator/icu-beds/>.
- Keskinocak P, Oruc BE, Baxter A, Asplund J, Serban N (2020) The impact of social distancing on COVID19 spread: State of Georgia case study. *PLoS ONE* 15(10):e0239798.
- Prem K, Cook AR, Jit M (2017) Projecting social contact matrices in 152 countries using contact surveys and demographic data. *PLoS Comput. Biol.* 13(9):e1005697.
- U.S. Census Bureau (2021) U.S. and world population clock. Accessed May 10, 2021, <https://www.census.gov/popclock/>.
- Wilde H, Mellan T, Hawryluk I, Dennis JM, Denaxas S, Pagel C, Duncan A, et al. (2021) The association between mechanical ventilator availability and mortality risk in intensive care patients with COVID-19: A national retrospective cohort study. *medRxiv* 2021.01.11.21249461.