Supplementary Information S1 - Time-dependant ROC AUC and Brier score indicators

The predictive performances of the model can be assessed in terms of discrimination (i.e. ability to separate patients according to their prognosis) with time-dependent Area Under the Curve (AUC), and calibration (i.e. accuracy of predictions) with time-dependent Brier score. We note $\pi_i(l+t|l) = 1 - s_i(l+t|l)$, the probability for patient i to experience event 1 between l and $l+t$. At a landmark time l and an horizon time t, we define a case as a patient who experienced the event of interest (event 1) between l and $l+t$, and a control as a patient who did not experience any event or experienced the competing event (the event 2) between l and $l+t$.

In the following, we present definitions and estimates of AUC and Brier score according to Blanche et al. works (2013) [1]. Estimates are provided without accounting for censoring. If data contains censored observations, refined methods to handle censoring based on inverse probability of censoring weighting are preferable, also described in Blanche et al. (2013)[1].

The AUC for a landmark time l and a prediction horizon t is defined as:

$$AUC(l,t) = \mathbb{P}(\pi_i(l+t|l) > \pi_j(l+t|l) \mid D_i(l,t) = 1, D_j(l,t) = 0, T_i > l, T_j > l) \quad (1)$$

where $D_i(l,t) = \mathbb{1}_{(l<T_i \leq l+t, K_i=1)}$; that is to say, $D_i(l,t) = 1$ if the subject experiences the event of interest between time l and $l+t$, and $D_i(l,t) = 1$ if the subject experiences the competing event between time l and $l+t$ or is event-free at time $l+t$. AUC is estimated by:

$$\widehat{AUC}(l,t) = \frac{\sum_{i=1}^{N_l}\sum_{j=1}^{N_l} \mathbb{1}_{\hat{\pi}_i(l+t|l) > \hat{\pi}_j(l+t|l)} D_i(l,t)\big(1 - D_j(l,t)\big)}{\sum_{i=1}^{N_l}\sum_{j=1}^{N_l} D_i(l,t)\big(1 - D_j(l,t)\big)} \quad (2)$$

where N_l denotes the number of at-risk patients at time l.

The Brier score for a landmark time l and a prediction horizon t is defined as:

$$BS(l,t) = \mathbb{E}\Big[\big(D(l,t) - \pi(l+t|l)\big)^2 \mid T > l\Big] \quad (3)$$

and estimated by:

$$\widehat{BS}(l,t) = \frac{1}{N_l}\sum_{i=1}^{N_l}\big(D_i(l,t) - \hat{\pi}_i(l+t|l)\big)^2 \quad (4)$$

A method for the derivation of asymptotic confidence intervals is proposed in the literature [1].

References

[1] Blanche P, Dartigues JF, Jacqmin-Gadda H. Estimating and comparing time-dependent areas under receiver operating characteristic curves for censored event times with competing risks. *Stat. Med.* 2013; 32: 5381-5397