

## A Truncated normal distribution

In order to describe the effect of a cutoff we consider a truncated normal distribution. In general, such a distribution, in 3 dimensions, with mean  $\bar{\mu}$  and covariance matrix  $\Sigma$  is defined by the following probability density function (pdf):

$$f(\bar{x}, \bar{\mu}, \Sigma, S) = \mathcal{N} \frac{1}{\sqrt{(2\pi)^3 |\Sigma|}} \exp(-(\bar{x} - \bar{\mu})^T \Sigma^{-1} (\bar{x} - \bar{\mu})) \quad (\text{A.1})$$

for all  $x$  within a bounded set  $S$  and  $f = 0$  for all other  $x$ .  $\mathcal{N}$  is chosen such that  $f$  is normalized to 1. In the remainder of this work we consider only the case where  $\bar{\mu} = 0$ ,  $\Sigma = \text{diag}(\sigma^2)$  and  $S$  is defined by all  $\bar{x}$  for which  $|\bar{x}| < c\sigma$ .

While in this case  $\sigma$  is the standard deviation of the underlying normal distribution, it is not the standard deviation of the truncated normal distribution. We can determine this standard deviation of  $f$  along direction  $x_i$  by

$$\sigma_{c,i}^2 = \int x_i^2 f(\bar{x}, \sigma, c) d\bar{x} \quad (\text{A.2})$$

and since our problem is spherically symmetric,  $\sigma_{c,i} = \sigma_c$ , independent of the choice  $i$ . In spherical coordinates  $f$  becomes

$$f(r, \sigma, c) = \mathcal{N} \frac{1}{(2\pi)^{3/2} \sigma^3} \exp(-\frac{r^2}{2\sigma^2}) \quad (\text{A.3})$$

which can be integrated by parts to find

$$\mathcal{N} = (t_1 - t_2)^{-1} \quad (\text{A.4})$$

with

$$\begin{aligned} t_1 &= \text{erf}\left(\frac{c}{\sqrt{2}}\right) \\ t_2 &= c\sqrt{\frac{2}{\pi}} \exp(-\frac{1}{2}c^2). \end{aligned} \quad (\text{A.5})$$

In spherical coordinates we can write (A.2) as

$$\begin{aligned} \sigma_c^2 &= \frac{1}{3}(\sigma_{c,1}^2 + \sigma_{c,2}^2 + \sigma_{c,3}^2) \\ &= \frac{4\pi}{3} \int r^4 f(r, \sigma, c) dr \end{aligned} \quad (\text{A.6})$$

Combining (A.3), (A.4) and (A.6) results after several partial integrations in

$$\sigma_c^2 = \sigma^2 \left(1 - \frac{c^2}{3} \frac{t_2}{t_1 - t_2}\right). \quad (\text{A.7})$$

## B Simple analytic model

In this section, we provide analytic expressions for the margin. Our analysis is to a large extent analogous to the approach by Van Herk et al [16], only using the truncated normal distribution and an approximation for the finite number of fractions.

### B.1 Dose distribution

We consider a spherical target, with a spherical dose distribution described by a step function convolved with a normal distribution of width  $\sigma_p$  and zero mean. We place the origin in the center of the target, such that

$$w = s + m + \lambda, \quad (\text{B.1})$$

where  $w$  is the distance to the 50% isodose,  $s$  = target radius;  $m$  = margin and  $\lambda$  = distance between the prescription isodose line  $q$  and the 50% isodose line. The dose at position  $x$  is given by

$$d(x) = 1/2 \left( \operatorname{erf} \frac{w-x}{\sqrt{2}\sigma_p} + \operatorname{erf} \frac{w+x}{\sqrt{2}\sigma_p} \right). \quad (\text{B.2})$$

Under the assumption that  $s \gg \sigma_p$ ,  $\lambda$  is determined by  $d(m + s) = q$ :

$$\frac{1}{2} \left( \operatorname{erf} \frac{\lambda}{\sqrt{2}\sigma_p} + 1 \right) = q \quad (\text{B.3})$$

which e.g. for  $q = 0.95$  results in  $\lambda \approx 1.64\sigma_p$ . Now given a shift  $r$ , the dose becomes  $d(x - r)$  and thus at the edge of the CTV (again assuming  $s \gg \sigma_p$ )

$$d(s - r) = 1/2 \left( \operatorname{erf} \left( \frac{m + \lambda + r}{\sqrt{2}\sigma_p} \right) + 1 \right) \quad (\text{B.4})$$

### B.2 Margins for finite number of fractions

In this analysis, we assume the systematic error to be negligible. However, after a finite number of fractions  $N$ , the blurring due to the random error does not average out to zero, resulting in an effective systematic error. We assume the daily random variation is sampled from a normal distribution of mean zero and width  $\sigma$ . We approximate the finite number of fractions by

$$\begin{aligned} \sigma_N^2 &= \frac{N-1}{N} \sigma^2 \\ \Sigma_N^2 &= \frac{1}{N} \sigma^2, \end{aligned} \quad (\text{B.5})$$

where  $\sigma_N$  and  $\Sigma_N$  are the effective standard deviations of the random and systematic error respectively. A motivation and analysis for this approximation can be found in references [18-20].

To determine the required margin due to random errors  $m_{rand}$ , we assume the dose distribution after  $N$  fractions can be approximated by the dose distribution (B.4), convolved with a normal distribution of width  $\sigma_N$ . Effectively this means replacing  $\sigma_p$  with  $\sqrt{\sigma_p^2 + \sigma_N^2}$ . As is explained in the manuscript,

this approach is only approximate, since with a finite number of fractions, the convolution is not a perfect Gaussian. The margin required is then found by solving for  $d = q$  and using the definition of  $\lambda$  from (B.3)

$$m_{rand} = \sqrt{2} \operatorname{erf}^{-1}(2 * q - 1) (\sqrt{\sigma_N^2 + \sigma_p^2} - \sigma_p) \quad (\text{B.6})$$

which for  $q = 0.95$  and  $\sigma_N = \sigma$  is the non-linear van Herk margin recipe.

The margin for the systematic error  $m_{sys}$  should be chosen such that a certain fraction of all patients has a displacement less than  $m_{sys}$ . The probability that  $r < m_{sys}$  is

$$p(r < m_{sys}) = \mathcal{N} \left( \operatorname{erf} \left( \frac{m_{sys}}{\sqrt{2} \Sigma_N} \right) - \sqrt{\frac{2}{\pi}} \frac{m_{sys}}{\Sigma_N} \exp \left( -\frac{1}{2} \frac{m_{sys}^2}{\Sigma_N^2} \right) \right), \quad (\text{B.7})$$

with  $\mathcal{N}$  as in (A.4). Solving (B.7) for  $m_{sys}$  provides the margin required for the systematic component. For  $p = 0.9$  and  $c \rightarrow \infty$  (such that  $\mathcal{N} = 1$ ), this results in  $m_{sys} \approx 2.5 \Sigma_N$ . The final margin required for  $N$  fractions is

$$m = m_{rand} + m_{sys} \quad (\text{B.8})$$

### B.3 Margins in the presence of a cut-off

We approximate the margin required due to random errors in the presence of a cut-off by simply taking  $\sigma_N$  from (B.5) and replace  $\sigma$  with  $\sigma_c$  from (A.7), resulting in  $\sigma_{c,N}$ . The margin is then given by (B.6) with the appropriate substitution. To incorporate the cut-off in the systematic error, we only need to correct for the change in population due to the truncation of the gaussian. This is in effect taken into account by the normalization  $\mathcal{N}$  in (B.7). Notice that we do not need to substitute  $\Sigma_N$  with  $\Sigma_{N,c}$ , since within the truncation, the distribution is properly described by  $\sigma$  and not  $\sigma_c$ .

We denote the margin in the presence of a cut-off with  $m_c$  and will next study  $m_c/m$ .

#### B.3.1 dependency on $\sigma$

It follows from (A.7) and (B.5) that  $\sigma_{c,N}$  and  $\Sigma_N$  are both linear in  $\sigma$ . Therefore, in the regime where  $\sigma$  is not too large and  $m_{rand}$  is approximately linear in  $\sigma_{c,N}$ , it follows that  $m$  is approximately linear in  $\sigma$  (which is equivalent to the regime where the van Herk recipe is linear). From this it immediately follows that  $m_c/m$  is approximately independent of  $\sigma$ .

#### B.3.2 dependency on $N$

to determine the dependency of  $m_c/m$  on  $N$ , we consider the following statements to make the dependency on  $c$  and  $N$  explicit (again in the linear regime)

$$\begin{aligned} m(N) &= \alpha \Sigma_N + \gamma \sigma_N \\ m(N = \infty) &= \gamma \sigma \\ m_c(N) &= \alpha_c \Sigma_N + \gamma \sigma_{c,N} \\ m_c(N = \infty) &= \gamma \sigma_c \end{aligned} \quad (\text{B.9})$$

and then study the ratio of  $m_c/m$  for finite  $N$  with the same for infinite  $N$ .

$$\begin{aligned}
\frac{m_c(N)}{m_c(N=\infty)} \frac{m(N=\infty)}{m(N)} &= \frac{\alpha_c \Sigma_N + \gamma \sigma_{c,N}}{\gamma \sigma_c} \frac{\gamma \sigma}{\alpha \Sigma_N + \gamma \sigma_N} \\
&= \frac{\frac{1}{\sqrt{N}} \frac{\alpha_c}{\gamma} \frac{\sigma}{\sigma_c} + \frac{\sigma_N}{\sigma}}{\frac{1}{\sqrt{N}} \frac{\alpha}{\gamma} + \frac{\sigma_N}{\sigma}} \\
&= 1 + \frac{1}{\sqrt{N}} \frac{\frac{\alpha_c}{\gamma} \frac{\sigma}{\sigma_c} - \frac{\alpha}{\gamma}}{\frac{1}{\sqrt{N}} \frac{\alpha}{\gamma} + \frac{\sigma_N}{\sigma}} \tag{B.10} \\
&= 1 + \frac{\frac{\alpha_c}{\gamma} \frac{\sigma}{\sigma_c} - \frac{\alpha}{\gamma}}{\frac{\alpha}{\gamma} + \sqrt{N-1}} \\
&= 1 + \frac{\frac{\alpha_c}{\alpha} \frac{\sigma}{\sigma_c} - 1}{1 + \frac{\gamma}{\alpha} \sqrt{N-1}},
\end{aligned}$$

where we used (B.5) and, following from (A.7), that

$$\frac{\sigma_{N,c}}{\sigma_c} = \frac{\sigma_N}{\sigma}. \tag{B.11}$$

(B.10) implies that  $m_c(N)/m(N)$  approaches its value at  $N = \infty$  with a correction that decreases as  $1/\sqrt{N}$ . In other words: for  $N$  large enough,  $m_c/m$  approaches

$$\begin{aligned}
\frac{m_c(N=\infty)}{m(N=\infty)} &= \frac{\sigma_c}{\sigma} \\
&= 1 - \frac{c^2}{3} \frac{t_2}{t_1 - t_2}, \tag{B.12}
\end{aligned}$$

which is independent of  $N$  and where we used (A.7).

## C Supplementary Tables

Table C1. Overview of the most important symbols used.

$c$	Truncation parameter
$\sigma$	Standard deviation random error
$\Sigma$	Standard deviation systematic error
$\sigma_p$	Penumbra of dose distribution
$N$	Number of fractions
$\sigma_N$	Effective standard deviation of random errors, corrected for finite number of fractions (Eq. 1)
$\Sigma_N$	Effective standard deviation of systematic errors, corrected for finite number of fractions (Eq. 2)
$\sigma_c$	Standard deviation of truncated Gaussian with standard deviation $\sigma$ and truncation $c$ . (Eq. A7)
$\sigma_{N,c}$	Effective standard deviation of random errors, $\sigma_N$ , with $\sigma$ in Eq. 2 replaced with $\sigma_c$ .
$M$	Margin in the absence of truncation
$m_c$	Margin in the presence of truncation (Eq. 3)
$m_0$	Margin with truncation $c=0$ ; In other words all errors are corrected for except for any residual, uncorrected, errors. (Eq. 4)
$\alpha_c$	Systematic factor of the margin (Eq. 3)
$\beta$	Random factor of the margin (Eq. 3)
$\varepsilon$	Correction parameter for model 2 where in the analytic approximation the effective standard deviation is fitted as $\sigma \rightarrow \varepsilon\sigma$
$\omega$	Parameter fitted using the linear approximation $m_c / m = \omega c$
$W$	Distance to the 50% isodose (Eq. B1)
$Q$	Prescription isodose
$\lambda$	Distance between the 50% isodose and the prescription isodose (Eq. B3)

Table C2: Margins required (in mm) for different values of the standard deviation  $\sigma$ , number of fractions  $N$  and the truncation parameter  $c$  for model 1 (discrete displacement), based on the Monte Carlo simulations. The values in this table correspond to the dots in figure 1.

$N$	$c = 1$				$c = 2$				$c = 3$				no cut-off			
	$\sigma = 1 \text{ mm}$	$\sigma = 2.5 \text{ mm}$	$\sigma = 5 \text{ mm}$	$\sigma = 7.5 \text{ mm}$	$\sigma = 1 \text{ mm}$	$\sigma = 2.5 \text{ mm}$	$\sigma = 5 \text{ mm}$	$\sigma = 7.5 \text{ mm}$	$\sigma = 1 \text{ mm}$	$\sigma = 2.5 \text{ mm}$	$\sigma = 5 \text{ mm}$	$\sigma = 7.5 \text{ mm}$	$\sigma = 1 \text{ mm}$	$\sigma = 2.5 \text{ mm}$	$\sigma = 5 \text{ mm}$	$\sigma = 7.5 \text{ mm}$
1	1.0	2.4	4.8	7.2	1.9	4.7	9.2	13.8	2.5	5.9	12.0	18.0	2.6	6.3	12.6	18.7
2	0.8	1.9	3.9	6.3	1.4	3.8	8.5	13.2	1.9	5.5	12.0	18.3	2.0	5.8	12.9	19.8
3	0.7	1.7	3.7	5.8	1.2	3.4	7.9	12.7	1.6	5.0	11.6	18.3	1.8	5.5	12.9	20.3
4	0.6	1.6	3.4	5.5	1.1	3.2	7.4	12.2	1.5	4.7	11.3	18.0	1.6	5.2	12.8	20.3
5	0.6	1.5	3.2	5.3	1.0	3.0	7.1	11.8	1.4	4.4	11.0	17.8	1.5	5.0	12.7	20.3
6	0.5	1.4	3.1	5.1	0.9	2.9	6.9	11.4	1.3	4.1	10.7	17.6	1.4	4.8	12.5	20.4
7	0.5	1.3	3.0	5.0	0.9	2.8	6.8	11.2	1.2	4.0	10.4	17.4	1.3	4.6	12.4	20.3
8	0.5	1.3	2.9	4.9	0.9	2.7	6.6	11.0	1.1	3.8	10.1	17.1	1.2	4.5	12.2	20.1
9	0.5	1.2	2.8	4.8	0.8	2.6	6.5	10.9	1.1	3.7	9.8	16.8	1.2	4.4	12.0	19.9
10	0.5	1.2	2.8	4.7	0.8	2.5	6.4	10.7	1.0	3.6	9.6	16.5	1.1	4.2	11.8	19.8
11	0.4	1.1	2.7	4.6	0.8	2.5	6.3	10.6	1.0	3.6	9.4	16.2	1.1	4.1	11.5	19.7
12	0.4	1.1	2.7	4.6	0.8	2.4	6.2	10.5	1.0	3.5	9.2	15.9	1.0	4.0	11.3	19.5
13	0.4	1.0	2.6	4.5	0.7	2.4	6.1	10.4	0.9	3.4	9.1	15.7	1.0	3.9	11.0	19.3
14	0.4	1.0	2.6	4.5	0.7	2.3	6.0	10.3	0.9	3.3	9.0	15.5	1.0	3.8	10.9	19.1
15	0.4	1.0	2.6	4.4	0.7	2.3	5.9	10.2	0.9	3.3	8.9	15.3	1.0	3.8	10.7	18.9
16	0.4	1.0	2.5	4.4	0.7	2.3	5.9	10.1	0.9	3.2	8.8	15.1	1.0	3.7	10.5	18.6
17	0.4	1.0	2.5	4.3	0.7	2.2	5.8	10.0	0.9	3.2	8.7	15.0	0.9	3.6	10.3	18.3
18	0.4	1.0	2.5	4.3	0.7	2.2	5.8	9.9	0.9	3.1	8.6	14.9	0.9	3.6	10.1	18.0
19	0.4	0.9	2.4	4.2	0.7	2.2	5.7	9.9	0.9	3.1	8.5	14.8	0.9	3.5	9.9	17.6
20	0.4	0.9	2.4	4.2	0.7	2.1	5.7	9.8	0.8	3.0	8.5	14.7	0.9	3.5	9.8	17.3

Table C3: Margins required (in mm) for different values of the standard deviation  $\sigma$ , number of fractions  $N$  and the truncation parameter  $c$  for model 2 (continuous movement), based on the Monte Carlo simulations. The values in this table correspond to the dots in figure 2.

$N$	$c = 1$				$c = 2$				$c = 3$				no cut-off			
	$\sigma = 1 \text{ mm}$	$\sigma = 2.5 \text{ mm}$	$\sigma = 5 \text{ mm}$	$\sigma = 7.5 \text{ mm}$	$\sigma = 1 \text{ mm}$	$\sigma = 2.5 \text{ mm}$	$\sigma = 5 \text{ mm}$	$\sigma = 7.5 \text{ mm}$	$\sigma = 1 \text{ mm}$	$\sigma = 2.5 \text{ mm}$	$\sigma = 5 \text{ mm}$	$\sigma = 7.5 \text{ mm}$	$\sigma = 1 \text{ mm}$	$\sigma = 2.5 \text{ mm}$	$\sigma = 5 \text{ mm}$	$\sigma = 7.5 \text{ mm}$
1	0.6	1.4	3.0	4.8	1.0	2.8	6.5	10.5	1.4	3.8	8.8	14.2	1.4	4.1	9.3	15.0
2	0.5	1.0	2.3	3.8	0.8	2.1	5.4	9.3	1.0	3.2	8.2	13.8	1.1	3.5	9.1	15.1
3	0.4	0.9	2.0	3.5	0.7	1.9	4.7	8.4	0.8	2.7	7.6	13.1	0.9	3.0	8.5	14.5
4	0.3	0.8	1.9	3.2	0.6	1.7	4.4	7.8	0.8	2.4	6.9	12.3	0.8	2.8	7.9	13.9
5	0.3	0.8	1.8	3.0	0.6	1.6	4.2	7.4	0.7	2.2	6.4	11.5	0.8	2.6	7.5	13.3
6	0.3	0.7	1.7	2.9	0.5	1.5	3.9	7.2	0.7	2.1	5.9	10.8	0.7	2.4	7.0	12.7
7	0.3	0.7	1.6	2.8	0.5	1.4	3.8	7.0	0.6	2.0	5.7	10.3	0.7	2.3	6.6	12.0
8	0.3	0.7	1.5	2.7	0.5	1.4	3.7	6.8	0.6	1.9	5.5	10.0	0.6	2.1	6.3	11.5
9	0.3	0.6	1.5	2.6	0.5	1.3	3.6	6.6	0.6	1.8	5.3	9.8	0.6	2.0	6.1	11.1
10	0.3	0.6	1.4	2.5	0.4	1.3	3.5	6.4	0.6	1.8	5.2	9.6	0.6	2.0	5.9	10.8
11	0.2	0.6	1.4	2.5	0.4	1.2	3.4	6.3	0.5	1.7	5.0	9.4	0.6	1.9	5.7	10.5
12	0.2	0.6	1.4	2.4	0.4	1.2	3.3	6.2	0.5	1.7	4.9	9.2	0.6	1.9	5.6	10.3
13	0.2	0.6	1.3	2.4	0.4	1.2	3.2	6.1	0.5	1.7	4.8	9.0	0.5	1.8	5.4	10.0
14	0.2	0.6	1.3	2.3	0.4	1.1	3.2	5.9	0.5	1.6	4.7	8.9	0.5	1.8	5.3	9.8
15	0.2	0.5	1.3	2.3	0.4	1.1	3.1	5.9	0.5	1.6	4.7	8.7	0.5	1.7	5.2	9.7
16	0.2	0.5	1.2	2.2	0.4	1.1	3.1	5.8	0.5	1.5	4.6	8.6	0.5	1.7	5.1	9.5
17	0.2	0.5	1.2	2.2	0.4	1.1	3.0	5.7	0.5	1.5	4.5	8.5	0.5	1.7	5.0	9.4
18	0.2	0.5	1.2	2.2	0.4	1.0	3.0	5.7	0.5	1.5	4.4	8.4	0.5	1.6	4.9	9.2
19	0.2	0.5	1.2	2.1	0.4	1.0	2.9	5.6	0.5	1.5	4.4	8.2	0.5	1.6	4.9	9.1
20	0.2	0.5	1.2	2.1	0.4	1.0	2.9	5.6	0.4	1.5	4.3	8.1	0.5	1.6	4.8	9.0

Table C4: Difference in the margin  $m_c$  (in mm) between the Monte Carlo calculation and a calculation based on the linear fit  $m_c/m = 0.3 c$ , for different values of the standard deviation  $\sigma$  and number of fractions  $N$ . The difference is calculated over the range of the truncation parameter  $c = \{0...3\}$  and the reported values are the median (lower quartile ; upper quartile).

<b>Model 1</b>	<b><math>\sigma</math> (mm)</b>	<b>N = 1</b>	<b>N = 3</b>	<b>N = 5</b>	<b>N = 20</b>
	1	0.00 (0.00 ; 0.00)	0.00 (0.00 ; 0.00)	0.00 (0.00 ; 0.00)	0.00 (0.00 ; 0.00)
	2,5	0.01 (0.01 ; 0.02)	0.00 (0.00 ; 0.00)	-0.00 (-0.00 ; 0.00)	-0.00 (-0.00 ; 0.00)
	5	0.04 (0.02 ; 0.05)	-0.00 (-0.00 ; 0.00)	-0.02 (-0.02 ; -0.01)	-0.02 (-0.02 ; -0.01)
	7,5	0.07 (0.05 ; 0.10)	0.01 (-0.00 ; 0.02)	-0.02 (-0.02 ; -0.01)	-0.04 (-0.04 ; -0.03)
<b>Model 2</b>					
	1	0.00 (0.00 ; 0.00)	0.00 (0.00 ; 0.00)	0.00 (0.00 ; 0.00)	0.00 (0.00 ; 0.00)
	2,5	0.00 (0.00 ; 0.01)	0.00 (0.00 ; 0.00)	0.00 (0.00 ; 0.00)	0.00 (0.00 ; 0.00)
	5	0.02 (0.00 ; 0.02)	-0.01 (-0.01 ; -0.01)	-0.01 (-0.02 ; -0.01)	-0.00 (-0.01 ; 0.00)
	7,5	0.03 (0.01 ; 0.04)	-0.01 (-0.03 ; -0.00)	-0.02 (-0.03 ; -0.02)	-0.00 (-0.02 ; 0.01)



## D Supplementary figures

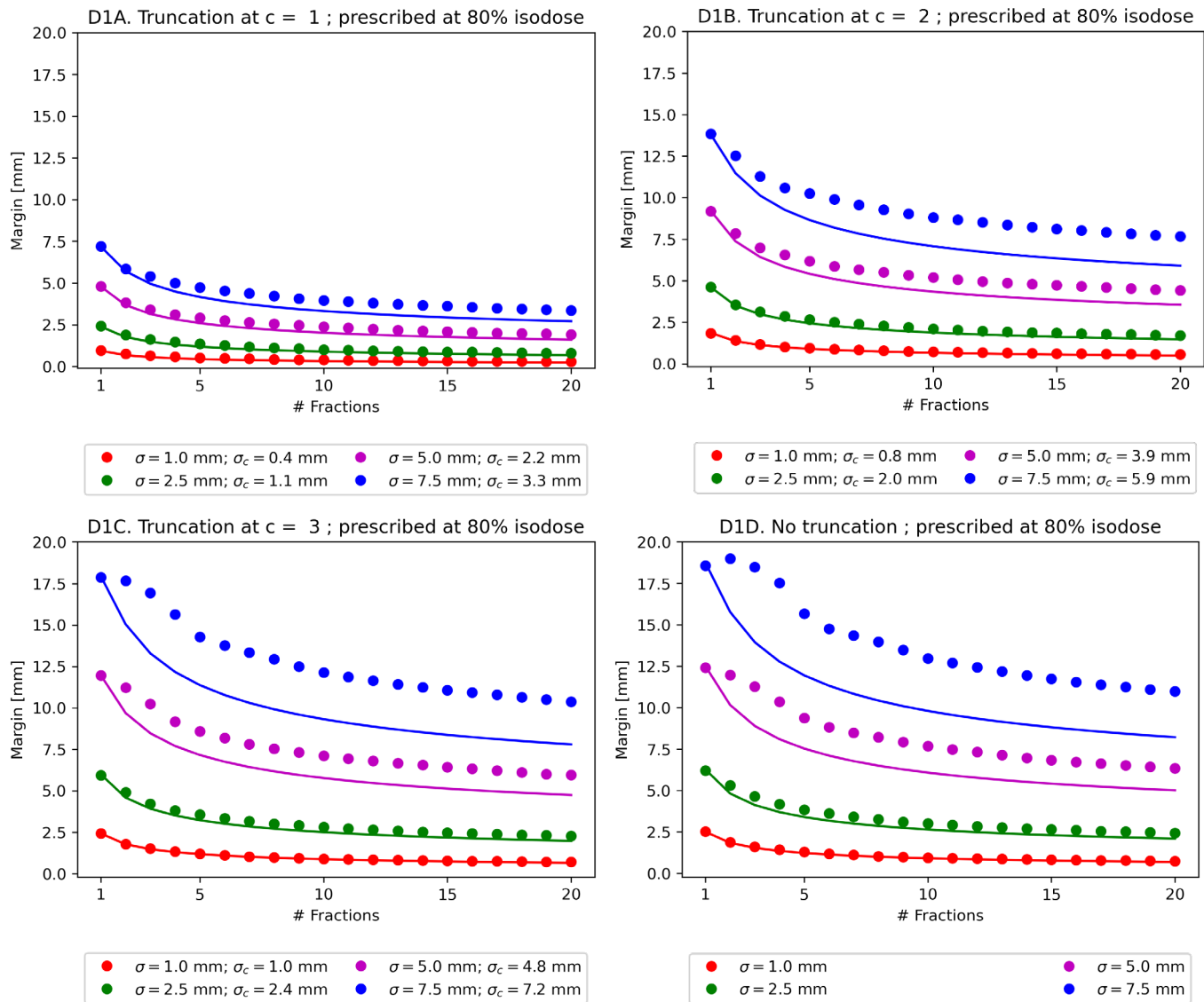


Figure D1. Margin required for different values of standard deviation  $\sigma$ , truncation parameter  $c$  and number of fractions  $N$  for model 1 (discrete displacement), using a prescription dose of 80%. Dots show the Monte Carlo simulation result and the solid line shows the analytic approximation given by the van Herk margin formula, corrected for finite fractions and truncation (equation 3).  $\sigma_c$  is the standard deviation of the truncated distribution (given by equation A7).

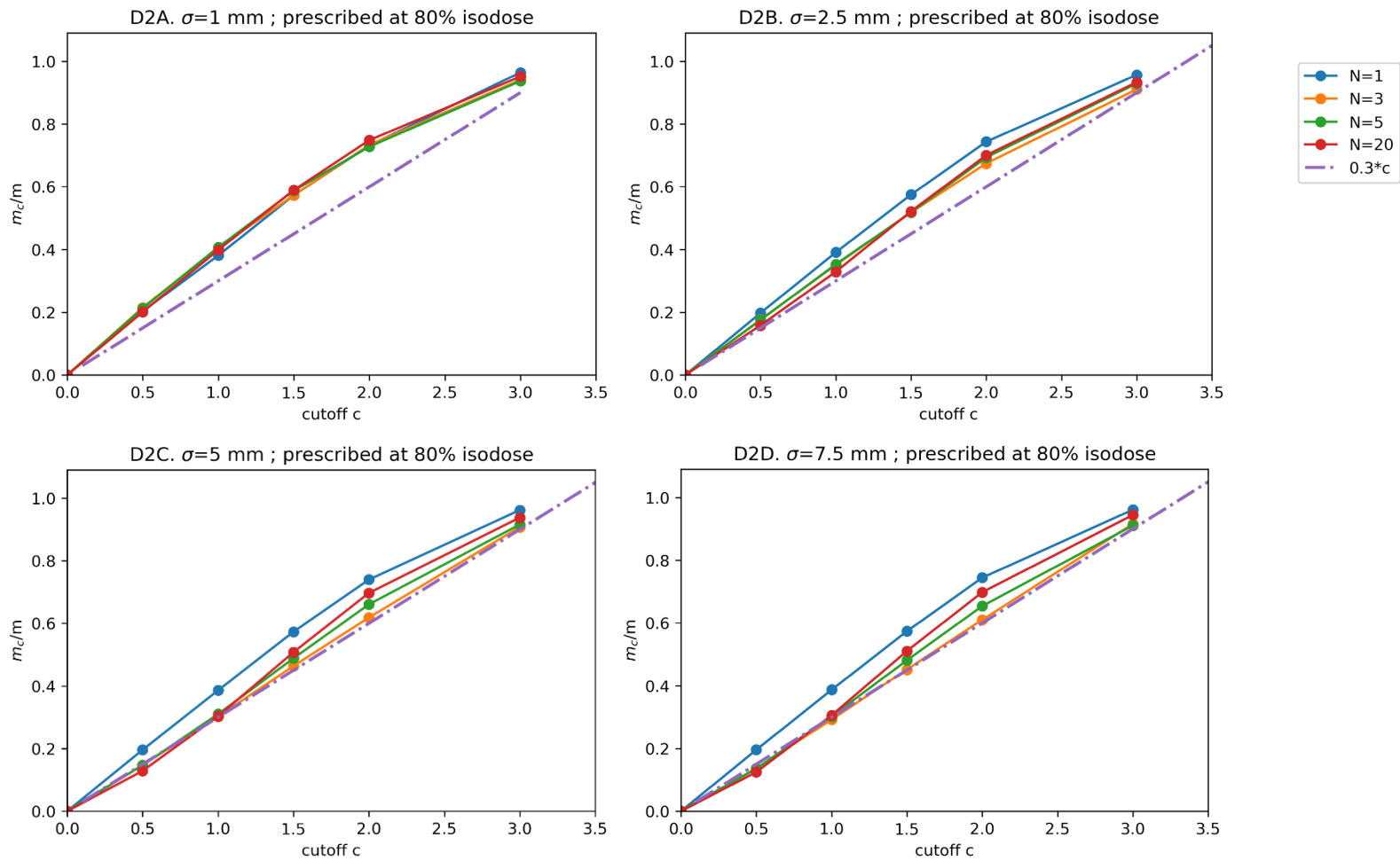


Figure D2. Margin relative to the original (no truncation) margin ( $m_c/m$ ) for different values of the standard deviation  $\sigma$  and number of fractions  $N$  as a function of the truncation parameter  $c$  for model 1 (discrete displacement), using a prescription dose of 80%. Dots are the Monte Carlo result and the solid lines are a linear interpolation. The dash-dotted line is the linear relation with  $m_c/m = 0.3c$ .

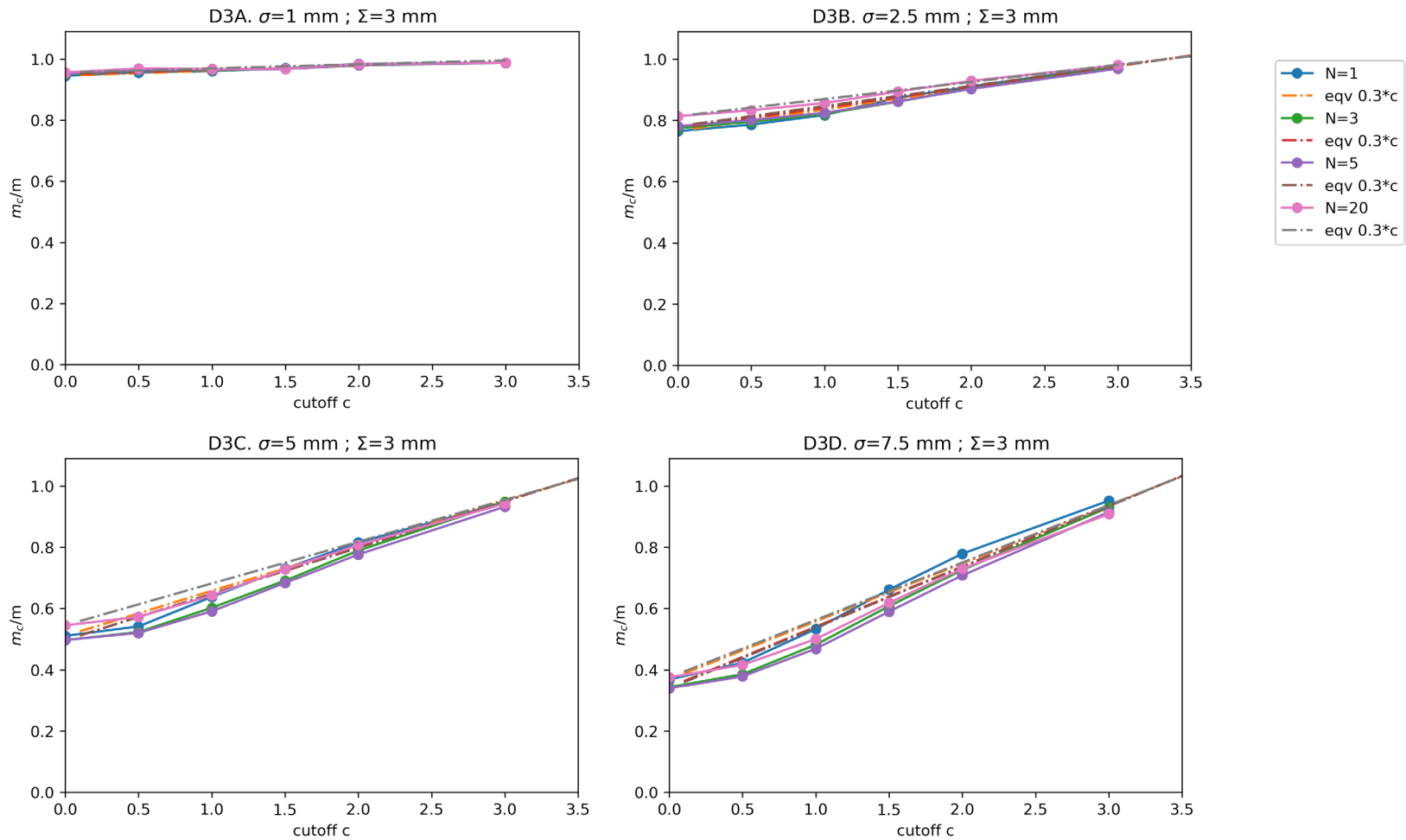


Figure D3. Margin relative to the original (no truncation) margin ( $m_c/m$ ) for different values of the standard deviation  $\sigma$  and number of fractions  $N$  as a function of the truncation parameter  $c$  for model 1 (discrete displacement) in the presence of a residual systematic error of  $\Sigma = 3$  mm. Dots are the Monte Carlo result and the solid lines are a linear interpolation. The dash-dotted line is the equivalent of the linear relation  $m_c/m = 0.3c$  in the presence of a residual error (equation 4).