APPENDIX

| Conditioning of Standard MUSE Model

The effect of additional phase on aliased voxels for an even-odd line decomposition in the MUSE model is:

$$\begin{bmatrix} Y_{\mathrm{er}_1} \\ Y_{\mathrm{or}_1} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} P_{\mathrm{r}_1} & 0 \\ 0 & P_{\mathrm{r}_2} \end{bmatrix} \begin{bmatrix} M_{\mathrm{r}_1} \\ M_{\mathrm{r}_2} \end{bmatrix} = \begin{bmatrix} P_{\mathrm{r}_1} & P_{\mathrm{r}_2} \\ P_{\mathrm{r}_1} & -P_{\mathrm{r}_2} \end{bmatrix} \begin{bmatrix} M_{\mathrm{r}_1} \\ M_{\mathrm{r}_2} \end{bmatrix} \tag{A1}$$

The system in Equation A1 can be factored into two orthogonal matrices and has non-zero determinant $\forall P_{\mathrm{r}_1}, P_{\mathrm{r}_2}$. This implies the even-odd line reconstruction of MUSE cannot be singular.

Incorporating coil sensitivities into Equation A1 gives:

$$\begin{bmatrix} Y_{\mathrm{er}_1} \\ Y_{\mathrm{or}_1} \end{bmatrix} = A_{\mathrm{MUSE\ ss}} \begin{bmatrix} M_{\mathrm{r}_1} \\ M_{\mathrm{r}_2} \end{bmatrix} = \begin{bmatrix} C_{\mathrm{r}_1} P_{\mathrm{r}_1} & C_{\mathrm{r}_2} P_{\mathrm{r}_2} \\ C_{\mathrm{r}_1} P_{\mathrm{r}_1} & -C_{\mathrm{r}_2} P_{\mathrm{r}_2} \end{bmatrix} \begin{bmatrix} M_{\mathrm{r}_1} \\ M_{\mathrm{r}_2} \end{bmatrix} \tag{A2}$$

The information matrix of $A_{\mathrm{MUSE\ ss}}$ is:

$$(A_{\mathrm{MUSE\ ss}})^H A_{\mathrm{MUSE\ ss}} = \begin{bmatrix} 2C_{\mathrm{r}_1}^H C_{\mathrm{r}_1} & 0 \\ 0 & 2C_{\mathrm{r}_2}^H C_{\mathrm{r}_2} \end{bmatrix} \tag{A3}$$

From Equation A3, the conditioning of the even-odd line combination using a MUSE system is invariant to P.

| Calculation of Single-Shot non-CPMG Information Matrix

Intermediate steps for calculation of $(A_{\mathrm{non-CPMG\ ss}})^H A_{\mathrm{non-CPMG\ ss}}$ are as follows:

$$(A_{\mathrm{non-CPMG\ ss}})^H A_{\mathrm{non-CPMG\ ss}} = \begin{bmatrix} C_{\mathrm{r}_1}^H P_{\mathrm{r}_1}^* & C_{\mathrm{r}_1}^H P_{\mathrm{r}_1} \\ C_{\mathrm{r}_2}^H P_{\mathrm{r}_2}^* & -C_{\mathrm{r}_2}^H P_{\mathrm{r}_2} \end{bmatrix} \begin{bmatrix} C_{\mathrm{r}_1} P_{\mathrm{r}_1} & C_{\mathrm{r}_2} P_{\mathrm{r}_2} \\ C_{\mathrm{r}_1} P_{\mathrm{r}_1}^* & -C_{\mathrm{r}_2} P_{\mathrm{r}_2}^* \end{bmatrix} \tag{A4}$$

$$(A_{\mathrm{non-CPMG\ ss}})^H A_{\mathrm{non-CPMG\ ss}} = \begin{bmatrix} 2C_{\mathrm{r}_1}^H C_{\mathrm{r}_1} & C_{\mathrm{r}_1}^H C_{\mathrm{r}_2} (P_{\mathrm{r}_1}^* P_{\mathrm{r}_2} - P_{\mathrm{r}_1} P_{\mathrm{r}_2}^*) \\ C_{\mathrm{r}_2}^H C_{\mathrm{r}_1} (P_{\mathrm{r}_1} P_{\mathrm{r}_2}^* - P_{\mathrm{r}_1}^* P_{\mathrm{r}_2}) & 2C_{\mathrm{r}_2}^H C_{\mathrm{r}_2} \end{bmatrix} \tag{A5}$$

Equation 10 can be obtained from Equation A5 by applying Euler's identity and angle-sum formulas.

| Calculation of SENSE Information Matrix

A conventional $R = 2$ SENSE system of equations is:

$$\begin{bmatrix} Y_{1 \ldots N_c} \end{bmatrix} = A_{SENSE} \begin{bmatrix} M_{\mathrm{r}_1} \\ M_{\mathrm{r}_2} \end{bmatrix} = \begin{bmatrix} C_{\mathrm{r}_1} & C_{\mathrm{r}_2} \end{bmatrix} \begin{bmatrix} M_{\mathrm{r}_1} \\ M_{\mathrm{r}_2} \end{bmatrix} \tag{A6}$$

To obtain the singular values of $A_{SENSE} \in \mathbb{C}^{N_c \times 2}$, we must calculate $(A_{SENSE})^H A_{SENSE}$:

$$(A_{SENSE})^H A_{SENSE} = \begin{bmatrix} C_{\mathrm{r}_1}^H C_{\mathrm{r}_1} & C_{\mathrm{r}_1}^H C_{\mathrm{r}_2} \\ C_{\mathrm{r}_2}^H C_{\mathrm{r}_1} & C_{\mathrm{r}_2}^H C_{\mathrm{r}_2} \end{bmatrix} \tag{A7}$$

The characteristic polynomial of $(A_{SENSE})^H A_{SENSE}$ is:

$$\det((A_{SENSE})^H A_{SENSE} - \lambda I) = (C_{\mathrm{r}_1}^H C_{\mathrm{r}_1} - \lambda)(C_{\mathrm{r}_2}^H C_{\mathrm{r}_2} - \lambda) - (C_{\mathrm{r}_1}^H C_{\mathrm{r}_2})(C_{\mathrm{r}_2}^H C_{\mathrm{r}_1}) \tag{A8}$$

Equation A8 is equal to the characteristic polynomial of the information matrix in Equation 12.

| Eigenvalues of a 2×2 Information Matrix

The information matrix has an $A^H A$ decomposition and is therefore positive semi-definite with Hermitian symmetry. For a 2×2 information matrix:

$$A^H A = \begin{bmatrix} a & b \\ b^* & c \end{bmatrix} \geq 0, \tag{A9}$$

the eigenvalues are non-negative, given by:

$$\lambda_1 = \frac{(a + c) + \sqrt{(a + c)^2 - 4ac + 4|b|^2}}{2} \tag{A10}$$

$$\lambda_2 = \frac{(a + c) - \sqrt{(a + c)^2 - 4ac + 4|b|^2}}{2} \tag{A11}$$

The condition number increases with the magnitude of the off-diagonal term $|b|$ if a and c are constant. Relating to Equation A7, a and c constant assumes that the sum-of-squares over the coil dimension at voxels r_1 and r_2 is constant but the coupling $|b|$ between coils varies.