The influence of astrocytic leaflet motility on ionic signalling and homeostasis at active synapses – Supplementary Material

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Table 1. Transmembrane currents used in the synaptic cradle model.

Current	Description	Equation(s)	Source
$\boldsymbol{I_{vgnc,pre}}$	Voltage-gated Na^+ channel	$g_{vgnc,pre}m^3h(V_{pre}-E_{Na})$	1
$\boldsymbol{I_{vgkc,pre}}$	Voltage-gated K^+ channel	$g_{vgkc,pre}n^4(V_{pre}-E_K)$	1
$\boldsymbol{I_{l,pre}}$	Background leak channel	$g_{l,pre}(V_{pre}-E_l)$	1
$\boldsymbol{I_{vgcc,pre}}$	Voltage-gated Ca^{2+} channel	$g_{vgcc,pre}s(V_{pre}-E_{Ca})$	2
$\boldsymbol{\frac{dw}{dt}}$	Voltage-gated (in)activation	$\alpha_w(1-w)-\beta_w w$	1
$\boldsymbol{\alpha_m}$	Na^+ activation forward rate	$0.1\left(\frac{25-V_{pre}}{exp\left(\frac{25-V_{pre}}{10}\right)-1}\right)$	Adapted from [1]
$\boldsymbol{\beta_m}$	Na^+ activation backward rate	$4\left(\frac{-V_{pre}}{18}\right)$	Adapted from [1]
$\boldsymbol{\alpha_h}$	Na^+ inactivation forward rate	$0.07\,exp\left(\frac{-V_{pre}}{20}\right)$	Adapted from [1]
$\boldsymbol{\beta_h}$	Na^+ inactivation backward rate	$\left(exp\left(\frac{30-V_{pre}}{10}\right)+1\right)^{-1}$	Adapted from [1]
$\boldsymbol{\alpha_n}$	K^+ activation forward rate	$0.1\left(\frac{10-V_{pre}}{exp\left(\frac{10-V_{pre}}{10}\right)-1}\right)$	Adapted from [1]
$\boldsymbol{\beta_n}$	K^+ activation backward rate	$0.125\left(\frac{-V_{pre}}{80}\right)$	Adapted from [1]
$\boldsymbol{\alpha_s}$	Ca^{2+} activation forward rate	$\frac{1.6}{1+exp\left(-0.072(V_{pre}-5)\right)}$	Adapted from [2]
$\boldsymbol{\beta_s}$	Ca^{2+} activation backward rate	$\frac{0.02(V_{pre}-1.31)}{exp\left(\frac{V_{pre}-1.31}{5.36}\right)-1}$	Adapted from [2]
$\boldsymbol{I_{pmca,pre}}$	Plasma-membrane Ca^{2+}-ATPase (PMCA)	$\bar{I}_{pmca}\left(\frac{[Ca^{2+}]_{pre}}{K_{Ca,pre}+[Ca^{2+}]_{pre}}\right)$	3
$\boldsymbol{I_{nka,y}}$	Na^+-K^+-ATPase (NKA)	$\bar{I}_{nka}\left(\frac{[Na^+]_y^{1.5}}{K_{Na,y}^{1.5}+[Na^+]_y^{1.5}}\times\frac{[K^+]_y}{K_{K,y}+[K^+]_y}\right)$	4
$\boldsymbol{I_{ampa,post}}$	AMPA receptor	$g_{ampa}r(V_{post}-E_{ampa})$	5
$\boldsymbol{I_{nmda,post}}$	NMDA receptor	$g_{ampa}B(V_{post})r(V_{post}-E_{ampa})$	5

Current	Description	Equation(s)	Source
$\frac{dr}{dt}$	AMPA/NMDA (in)activation	$\alpha_r[Glu]_{ecs}(1-r) - \beta_r r$	5
$B(V_{post})$	NMDA magnesium block	$\left(1 + exp(-0.062V_{post})\frac{[Mg]_o}{3.57}\right)^{-1}$	6
$I_{eaat,psc}$	Excitatory amino-acid transporter (EAAT)	$\bar{I}_{eaat}\left(\frac{[Glu]_{ecs}}{K_{eaat} + [Glu]_{ecs}}\right)$	-
$I_{ncx,psc}$	Na^+/Ca^{2+} exchanger (NCX)	$\bar{I}_{ncx}\left(\left(\frac{[Na^+]_{psc}}{[Na^+]_{ecs}}\right)^3 exp\left(\frac{\gamma F V_{psc}}{RT}\right) - \left(\frac{[Ca^{2+}]_{psc}}{[Ca^{2+}]_{ecs}}\right) exp\left(\frac{(\gamma-1)FV_{psc}}{RT}\right)\right)$	4,7
$I_{kir,psc}$	K^+ inward rectifier channel ($K_{ir}4.1$)	$g_{kir}\sqrt{[K^+]_{ecs}}(V_{psc} - E_{kir})$	8
$I_{pf,x}$	Leaflet diffusion	$K_{pf,x}\frac{-E_x}{l_{lf}} exp\left(-\frac{Q\left(\varphi - \sqrt{\frac{Q(-E_x)}{l_{lf}\pi\epsilon}}\right)}{k_B T}\right)$	Adapted from 9
$I_{l,x,y}$	Passive channels	$g_{l,x,y}(V_y - E_x)$	3
$I_{ecs,x}$	ECS diffusion	$g_{ecs}\lambda(-E_{x,ecs})$	10
E_x	Equilibrium potential	$\frac{RT}{z_x F} ln\frac{[x]_o}{[x]_i}$	3
Note: $w \in \{m,h,n,s\}, x \in \{Na,K,Ca\}$ and $y \in \{pre,post,psc\}$			

Table 2. Parameters used in the synaptic cradle model.

Parameter	Description	Value	Unit(s)	Source
Constants				
dt	Time step	10	µs	-
F	Faraday's constant	96485	C/mol	-
R	Idela gas constant	8.314	J/mol.K	-
k_B	Boltzmann's constant	1.38×10^{-23}	J/K	-
T	Absolute temperature	310	K	-
Q	Elementary charge	1.6002×10^{-19}	C	-
z_K	Valency of K^+	1	-	-
z_{Na}	Valency of Na^+	1	-	-
z_{Ca}	Valency of Ca^{2+}	2	-	-
Neuronal parameters				

Parameter	Description	Value	Unit(s)	Source
V_{pre}	Presynaptic membrane potential	-70	mV	-
V_{post}	Postsynaptic membrane potential	-70	mV	-
C_m	Membrane capacitance	0.01	F/m^2	1
$g_{vgnc,pre}$	Maximal Na^+ conductance	120	mS/cm^2	1
E_{Na}	Na^+ equilibrium potential	45	mV	Adapted from [1]
$g_{vgkc,pre}$	Maximal K^+ conductance	36	mS/cm^2	1
E_K	K^+ equilibrium potential	-82	mV	Adapted from [1]
$g_{vgcc,pre}$	Maximal Ca^{2+} conductance	0.1	mS/cm^2	Adapted from [2]
E_{Ca}	Ca^{2+} equilibrium potential	$\frac{RT}{z_{Ca}F} ln \frac{[Ca^{2+}]_{ecs}}{[Ca^{2+}]_{pre}}$	V	-
$g_{l,pre}$	Maximal leak conductance	0.3	mS/cm^2	1
E_l	Leak equilibrium potential	-59	mV	Adapted from [1]
V_{max}	Maximal PMCA velocity	0.2×10^{-6}	mol/m^2s	3
$\bar{I}_{pmca}$	Maximal PMCA current	$V_{max}F$	A/m^2	3
$K_{Ca,pre}$	PMCA Ca^{2+} affinity	0.2	µM	3
P_{max}	Maximal NKA velocity	1.12×10^{-2}	mol/m^2s	4
$\bar{I}_{nka}$	Maximal NKA current	$P_{max}F$	A/m^2	4
$K_{Na,pre/post}$	NKA Na^+ affinity	10	mM	14
$K_{K,pre/post}$	NKA K^+ affinity	0.6	mM	14
g_{ampa}	Maximal AMPA conductance	0.18	S/m^2	5
E_{ampa}	AMPA equilibrium potential	0	mV	5
α_{ampa}	AMPA activation rate	1.1×10^6	$M^{-1}s^{-1}$	5
β_{ampa}	AMPA inactivation rate	190	s^{-1}	5
g_{nmda}	Maximal NMDA conductance	0.26	S/m^2	6
E_{nmda}	NMDA equilibrium potential	0	mV	6
α_{nmda}	NMDA activation rate	7.4×10^4	$M^{-1}s^{-1}$	6
β_{nmda}	NMDA inactivation rate	6.6	s^{-1}	6
$g_{l,Na,pre}$	Na^+ leak conductance	1.4184	S/m^2	Calculated
$g_{l,K,pre}$	K^+ leak conductance	7.6113	S/m^2	Calculated
$g_{l,Ca,pre}$	Ca^{2+} leak conductance	0.0186	S/m^2	Calculated
$g_{l,Na,post}$	Na^+ leak conductance	1.4184	S/m^2	Calculated
$g_{l,K,post}$	K^+ leak conductance	7.6113	S/m^2	Calculated
$g_{l,Ca,post}$	Ca^{2+} leak conductance	0.0186	S/m^2	Calculated
Astrocytic parameters				

Parameter	Description	Value	Unit(s)	Source
V_{psc}	Membrane potential	-80.7	mV	10
V_{eaat}	Maximal EAAT velocity	3 x 10^{-6}	mol/m^2s	11
eff_{eaat}	Average EAAT efficiency	0.5	-	12
$\bar{I}_{eaat}$	Maximal EAAT current	$V_{eaat} eff_{eaat} F$	A/m^2	-
K_{eaat}	EAAT Glu affinity	20	µM	13
$K_{Na,psc}$	NKA Na^+ affinity	10	mM	14
$K_{K,psc}$	NKA K^+ affinity	3.6	mM	14
$\bar{I}_{ncx}$	Maximal NCX current	1	A/m^2	7
γ	NCX energy partition	0.5	-	7
g_{kir}	Maximal K_{ir}4.1 conductance	144	S/m^2	10
E_{kir}	K_{ir}4.1 equilibrium potential	$\frac{RT}{z_K F} ln \frac{[K^+]_{ecs}}{[K^+]_{psc}}$	V	-
K_{pf}	Poole-Frenkel channel constant	0.018	S/m	9
φ	Well activation energy	10	J	9
ϵ	Dynamic permittivity	$\epsilon_0 \epsilon_r$	-	9
ϵ_0	Vacuum permittivity	8.85 x 10^{-12}	F/m	9
ϵ_r	Relative permittivity of brain tissue	0.82	F/m	9
$g_{l,Na,psc}$	Na^+ leak conductance	2.3771	S/m^2	Calculated
$g_{l,K,psc}$	K^+ leak conductance	33.0159	S/m^2	Calculated
$g_{l,Ca,psc}$	Ca^{2+} leak conductance	1.6 x 10^{-11}	S/m^2	Calculated
Extracellular parameters				
g_{ecs}	Maximal diffusion conductance	1	S/m^2	10
λ	Conductance scaling factor	10	-	-
$E_{x,ecs}$	ECS equilibrium potential	$\frac{RT}{z_x F} ln \frac{[x]_{gecs}}{[x]_{ecs}}$	V	10
Concentrations				
$[K^+]_{psc}$	Initial K^+ in PsC	100	mM	14
$[K^+]_{ecs}$	Initial K^+ in ECS	4	mM	14
$[K^+]_{pre}$	K^+ in Pre	100	mM	14
$[K^+]_{post}$	K^+ in Post	100	mM	14
$[K^+]_{gecs}$	K^+ in GECS	4	mM	14
$[Na^+]_{psc}$	Initial Na^+ in PsC	15	mM	14
$[Na^+]_{ecs}$	Initial Na^+ in ECS	135	mM	14
$[Na^+]_{pre}$	Na^+ in Pre	15	mM	14

Parameter	Description	Value	Unit(s)	Source
$[Na^+]_{post}$	Na^+ in Post	15	mM	14
$[Na^+]_{gecs}$	Na^+ in GECS	135	mM	14
$[Ca^{2+}]_{psc}$	Initial Ca^{2+} in PsC	100	nM	14
$[Ca^{2+}]_{ecs}$	Initial Ca^{2+} in ECS	1.5	mM	14
$[Ca^{2+}]_{pre}$	Initial Ca^{2+} in Pre	50	nM	14
$[Ca^{2+}]_{post}$	Ca^{2+} in Post	50	nM	14
$[Ca^{2+}]_{gecs}$	Ca^{2+} in GECS	1.5	mM	14
Morphology parameters				
Vol_{psc}	PsC volume	0.031416	fL	Calculated
Vol_{pre}	Pre volume	0.014314	fL	Calculated
Vol_{post}	Post volume	0.014314	fL	Calculated
$Vol_{ecs,a}$	ECS volume (A configuration)	0.001145	fL	Calculated
$Vol_{ecs,b}$	ECS volume (B configuration)	0.00786	fL	Calculated
$Vol_{ecs,c}$	ECS volume (C configuration)	0.055022	fL	Calculated
SA_{psc}	PsC surface area	0.23562	μm^2	Calculated
SA_{pre}	Pre surface area	0.21206	μm^2	Calculated
SA_{post}	Post surface area	0.21206	μm^2	Calculated
SA_{ecs}	ECS diffusion surface area	0.015	μm^2	Calculated
CSA_{lf}	Leaflet cross-sectional area	0.007854	μm^2	Calculated
l_{lf}	Leaflet length	2	µm	-

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